Pair correlations of internal quantum numbers and entangled two-particle states

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Main topics

- Nonfactorizable two-particle quantum states
- Two-particle spin density matrix and correlation tensor
- Spin correlations at the generation of pairs of identical particles with close momenta
- Spin correlations and angular correlations in the decays of two unstable particles
- Spin correlations at the generation of $\Lambda\bar{\Lambda}$ pairs in multiple processes
- Angular correlations in the decays of $\Lambda\bar{\Lambda}$ pairs and the “mixed phase”
- “Incoherence” inequalities for correlation tensor components and their violations in two-particle quantum systems
- Quantum character of spin correlations; processes

  $\pi^+ + {}^4He \to p + {}^3He$; $e^+e^- \to \mu^+\mu^-, \tau^+\tau^-$; $\gamma\gamma \to e^+e^-, \mu^+\mu^-, \tau^+\tau^-$
- Correlations of polarizations in the system of two photons
- Correlations of pairs of neutral *K* mesons in inclusive multiparticle processes with strangeness conservation
- Correlations of pairs of neutral heavy mesons ($D^0\bar{D}^0, B^0\bar{B}^0, B_s^0\bar{B}_s^0$)

1. Nonfactorizable two-particle quantum states

- Nonfactorizable (entangled) states of two particles cannot be presented, in principle, as a mere direct product of two one-particle states :

$$|\Phi\rangle^{(1,2)} = \sum_i \sum_k C_{ik} \, |i\rangle^{(1)} \otimes |k\rangle^{(2)} \neq \sum_i b_i \, |i\rangle^{(1)} \otimes \sum_k d_k \, |k\rangle^{(2)}$$

superposition of pairs of two-particle states,

$$\sum_i \sum_k |\, c_{ik} \,|^2 = 1.$$

- When a two-particle state is nonfactorizable, then, due to quantum correlations, the character of measurements performed for particle 1 determines the readout of the detector analyzing the state of particle 2 – without any direct force action .

- Selection of the different states $|L\rangle^{(1)}$ and $|M\rangle^{(1)}$ for particle 1 only leads to two different states of particle 2:

$$|\,\Psi\rangle_L^{(2)} = \sum_i \sum_k c_{ik} \langle L \mid i\rangle \mid k\rangle^{(2)}, \qquad |\,\Psi\rangle_M^{(2)} = \sum_i \sum_k c_{ik} \langle M \mid i\rangle \mid k\rangle^{(2)}.$$

In the case of a pure two-particle nonfactorizable state, the respective one-particle states are "mixed" -- they should be described by one-particle density matrices, but not by vectors of state (wave functions) $\Rightarrow$ quantum-mechanical Einstein – Podolsky – Rosen effect, which is often considered as a "paradox" .

- For the "mixed" two-particle nonfactorizable state ,

  the two-particle density matrix cannot be presented as a sum of direct products of one-particle density matrices with non-negative coefficients :

$$\hat{\rho}^{(1,2)} \neq \sum b_{ik} \hat{\rho}^{(1)} \otimes \hat{\rho}^{(2)}, b_{ik} \geq 0$$

2. Two-particle spin density matrix and correlation tensor

- Spin density matrix of two particles with spin ½ :

$$\hat{\rho}^{(1,2)} = \frac{1}{4}[\hat{I}^{(1)} \otimes \hat{I}^{(2)} + (\hat{\vec{\sigma}}^{(1)}\vec{P}_1) \otimes \hat{I}^{(2)} + \hat{I}^{(1)} \otimes (\hat{\vec{\sigma}}^{(2)}\vec{P}_2) + \sum_{i=1}^{3}\sum_{k=1}^{3} T_{ik}\hat{\sigma}_i^{(1)} \otimes \hat{\sigma}_k^{(2)}]$$

$$tr_{(1,2)}\hat{\rho}^{(1,2)} = 1$$

$\hat{I}$ - two-row unit matrix ; $\hat{\vec{\sigma}} = \{\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3\}$ - vector Pauli operator;

$\vec{P}_1 = \langle\hat{\vec{\sigma}}^{(1)}\rangle$ and $\vec{P}_2 = \langle\hat{\vec{\sigma}}^{(2)}\rangle$ - polarization vectors for particles 1 and 2;

$T_{ik} = \langle\hat{\sigma}_i^{(1)} \otimes \hat{\sigma}_k^{(2)}\rangle$ - correlation tensor .

- One-particle density matrices :

$$\hat{\rho}^{(1)} = tr_{(2)}\hat{\rho}^{(1,2)} = \tfrac{1}{2}(\hat{I} + \hat{\vec{\sigma}}^{(1)}\vec{P}_1),$$

$$\hat{\rho}^{(2)} = tr_{(1)}\hat{\rho}^{(1,2)} = \tfrac{1}{2}(\hat{I} + \hat{\vec{\sigma}}^{(2)}\vec{P}_2)$$

In the absence of correlations, factorization takes place :

$$T_{ik} = P_{1i}P_{2k}, \qquad \hat{\rho}^{(1,2)} = \hat{\rho}^{(1)} \otimes \hat{\rho}^{(2)}.$$

In the general case :

$C_{ik} = T_{ik} - P_{1i}P_{2k} \neq 0$

- Singlet state (total spin $S = 0$) – nonfactorizable spin state :

$$\boxed{|\,\Psi\rangle_{S=0} = \frac{1}{\sqrt{2}}(|\,+1/2\rangle_z^{(1)} \otimes |\,-1/2\rangle_z^{(2)} - |\,-1/2\rangle_z^{(1)} \otimes |\,+1/2\rangle_z^{(2)}).}$$

Spin projections are tightly correlated : they are opposite at the choice of any quantization axis z .

Two-particle density matrix : $$\hat{\rho}^{(s)} = \frac{1}{4}[\hat{I}^{(1)} \otimes \hat{I}^{(2)} - \hat{\vec{\sigma}}^{(1)} \otimes \hat{\vec{\sigma}}^{(2)}]$$

Polarization vectors : $\vec{P}_1 = \vec{P}_2 = 0$

Correlation tensor : $T_{ik} = -\delta_{ik}$

“Trace” of the correlation tensor : $$T = \sum_{i=1}^{3} T_{ii} = \left\langle \hat{\vec{\sigma}}^{(1)} \otimes \hat{\vec{\sigma}}^{(2)} \right\rangle = -3$$

- Triplet states :

$$|\psi_{t,+1}\rangle = |+1/2\rangle_z^{(1)} \otimes |+1/2\rangle_z^{(2)}; \quad |\psi_{t,-1}\rangle = |-1/2\rangle_z^{(1)} \otimes |-1/2\rangle_z^{(2)};$$

$$|\psi_{t,0}\rangle = \frac{1}{\sqrt{2}}(|+1/2\rangle_z^{(1)} \otimes |-1/2\rangle_z^{(2)} + |-1/2\rangle_z^{(1)} \otimes |+1/2\rangle_z^{(2)}).$$

- Among these states, the nonfactorizable one is the state $|\psi_{t,0}\rangle$ with the zero projection onto the axis z. For any triplet state $T=1$.

- In case of an arbitrary state of two particles with spin ½ :
$T = W_t - 3W_s$ (W_s and W_t – relative weights of the singlet and triplet states, respectively) .

- If particles 1 and 2 have different relativistic momenta, their polarization vectors and correlation tensor components with "left" and "right" indexes are specified in the rest frames of particles 1 and 2 – in the coordinate axes of the c.m. frame of two particles .

Registration of spin correlations

Two analyzers , selecting the states of particles 1 and 2 with the polarization vectors $\vec{\varsigma}^{(1)}$ and $\vec{\varsigma}^{(2)}$ $\Longrightarrow$ probability of registration of the two-particle state by one-particle detectors :

$$W = \frac{1}{4}[1 + \vec{P}_1\vec{\zeta}^{(1)} + \vec{P}_2\vec{\zeta}^{(2)} + \sum_{i=1}^{3}\sum_{k=1}^{3} T_{ik}\zeta_i^{(1)}\zeta_k^{(2)}].$$

If the polarization vector for particle 1 is measured only, then the components of polarization vector for particle 2 :

$$\tilde{P}_{2k} = (P_{2k} + \sum_{i=1}^{3} T_{ik}\zeta_i^{(1)})/(1 + \vec{\zeta}^{(1)}\vec{P}_1) \quad .$$

If the one-particle states are unpolarized ($\vec{P}_1 = \vec{P}_2 = 0$) , $\Longrightarrow$ spin effects are entirely determined by the correlation tensor

$$\boxed{\widetilde{P}_{2k} = \sum_{i=1}^{3} T_{ik}\varsigma_i^{(1)}}$$

If, in doing so, correlations are absent ($T_{ik} = 0$), then $\vec{\widetilde{P}}_2 = 0$ at any choice of vector $\vec{\varsigma}^{(1)}$.

3. Spin correlations at the generation of pairs of identical particles with close momenta

- Effect of Bose or Fermi statistics leads not only to the momentum-energy correlations at low relative momenta (correlation femtoscopy), but to the spin correlations as well .
- Consequence of symmetrization or antisymmetrization of the total wave function of two identical particles with nonzero spin : **$(-1)^{S+L} = 1$**

  S – total spin, L – orbital momentum in the c.m. frame .
- At the 4-momentum difference $q \to 0$, states with the nonzero orbital momenta "die out" -- only states with $L = 0$ and even total spin S survive.

  Spin ½ -- identical particles are generated in the singlet state only

  (protons, Λ particles) .

- At the momentum difference $q \neq 0$, there are also triplet states produced together with the singlet state.

- Further we will use the conventional model of one-particle sources emitting unpolarized particles, which is the most adequate one for relativistic heavy ion collisions . In the framework of this model , the triplet states with spin projections +1, 0 and -1 are generated with equal probabilities .

- Relative weights of the singlet and triplet states

$$W_s(q) = \frac{1}{4}\left[1 + \langle\cos(qx)\rangle\right] + 2B_{int}(q)\right]\frac{1}{R(q)}, \qquad W_t(q) = \frac{3}{4}\left[1 - \langle\cos(qx)\rangle\right]\frac{1}{R(q)},$$

q – difference of 4-momenta, *x* –difference of 4-coordinates of two sources,

$$\langle\cos(qx)\rangle = \int W(x)\cos(qx)\,d^4x$$

W(x) – distribution of the 4-coordinate difference for two sources,

$$R(q) = 1 - \frac{1}{2}\langle\cos(qx)\rangle + \frac{1}{2}B_{int}(q)$$ - function describing the momentum-energy correlations of the interacting identical particles,

$\langle\cos(qx)\rangle$ - quantum statistics contribution for the non-interacting particles,

$B_{int}(q)$ – contribution of the *s*-wave final-state interaction .

- Two-particle spin density matrix

$$\hat{\rho}^{(1,2)} = \frac{1}{R(q)}\left[\hat{I}^1 \otimes \hat{I}^2 + B_{int}(q) - \frac{1}{2}(1 + \boldsymbol{\sigma}^1 \otimes \boldsymbol{\sigma}^2)(\langle\cos(qx)\rangle + 2B_{int}(q))\right]$$

- Correlation tensor

$$\boxed{T_{ik} = -\frac{\langle\cos(qx)\rangle + B_{int}(q)}{2 - \langle\cos(qx)\rangle + B_{int}(q)}\delta_{ik}}$$

$|\vec{q}| \to 0,$ $T_{ik} = -\delta_{ik}$ (singlet state)

At sufficiently large $|\vec{q}|$ $\langle\cos(qx)\rangle \to 0$, $R(q) \to 1$, $B_{int}(q) \to 0$, $T_{ik} \to 0$ (correlations are absent).

- At the emission of unpolarized identical particles with arbitrary spin j (assuming that the interaction does not depend on total spin)

$$\hat{\rho}^{(1,2)} = \frac{1}{(2j+1)^2}\left[\hat{I}^{(1)} \otimes \hat{I}^{(2)} + B_{int}(q) + (-1)^{2j}\hat{P}_{exch}(\langle\cos(qx)\rangle + B_{int}(q))\right]\frac{1}{R(q)}$$

$\hat{P}_{exch}$- operator of permutation of spin projections :

$$(\hat{P}_{exch})_{m_1 m_1' ; m_2 m_2'} = \delta_{m_1 m_2'} \delta_{m_2 m_1'}$$

$$R(q) = 1 + B_{int}(q) + \frac{(-1)^{2j}}{2j+1}(\langle\cos(qx)\rangle + B_{int}(q))$$

For $j = 1/2$ $\Longrightarrow$ $$\hat{P}_{exch} = \frac{1}{2}(\hat{I}^{(1)} \otimes \hat{I}^{(2)} + \boldsymbol{\sigma}^{(1)} \otimes \boldsymbol{\sigma}^{(2)})$$

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- Spin correlations of identical particles at low relative momenta, as well as the momentum-energy correlations, depend on the space-time parameters of the generation region. The advantage of spin correlations is the fact that, in this case, the problem of non-correlated background, upon which the effects of quantum statistics and final-state interaction manifest themselves, is eliminated . Usually this background is constructed by the way of mixing different events, which leads to some uncertainties .

4. Spin correlations and angular correlations in the decays of two unstable particles

- Angular correlations in the decays of pairs of unstable particles, generated in the same act of collision, are conditioned by the spin correlations in nonfactorizable quantum states.

- The normalized angular distribution for the flight direction $\vec{n}$ of one of the particles produced in the two-body decay of an unstable particle with spin *j* , or for the direction of normal to the three-body decay plane, or for the direction of some vector characterizing the multiparticle decay, has the following structure (in the rest frame of decaying particle):

$$dW(\vec{n}) = \frac{2j+1}{4\pi}\left(\sum_m \sum_{m'} \sum_\Lambda D^{(j)}_{\Lambda m}(\vec{n}) D^{(j)}_{\Lambda m'}(\vec{n}) R_\Lambda \rho_{mm'}\right) d\Omega_{\vec{n}} .$$

$D^j_{\Lambda m}(\vec{n}) = d^j_{\Lambda m}(\theta) e^{im\varphi}$ - elements of the matrix of finite rotations ;

R_Λ - non-negative parameters -- probabilities of the event that projections of spin of the unstable particle onto the vector $\vec{n}$ take the values Λ $(\sum_\Lambda R_\Lambda = 1)$; in the case of two-body decay Λ is the difference of helicities ; $\rho_{mm'}$ – elements of the spin density matrix .

- The normalized double angular distribution at the decay of two unstable particles with spins j_1 and j_2:

$$d^2W(\vec{n}_1,\vec{n}_2) = \frac{(2j_1+1)(2j_2+1)}{16\pi^2}\sum_{m_1}\sum_{m_1'}\sum_{m_2}\sum_{m_2'}\sum_{\Lambda_1}\sum_{\Lambda_2} D^{(j_1)}_{\Lambda_1 m_1}(\vec{n}_1) D^{*(j_1)}_{\Lambda_1 m_1'}(\vec{n}_1)\times$$
$$\times D^{(j_2)}_{\Lambda_2 m_2}(\vec{n}_2) D^{*(j_2)}_{\Lambda_2 m_2'}(\vec{n}_2) R_{\Lambda_1}\widetilde{R}_{\Lambda_2}\,\rho^{(1,2)}_{m_1 m_1';m_2 m_2'}\,d\Omega_{\vec{n}_1}d\Omega_{\vec{n}_2},$$

$\rho_{m_1 m_1';m_2 m_2'}$ - elements of the two-particle spin density matrix .

The vectors $\vec{n}_1$ and $\vec{n}_2$ are defined in the rest frames for the first and second unstable particle, within the common coordinate axes of the c.m. frame for the particle pair .

- In the presence of spin correlations

$$\rho^{(1,2)}_{m_1 m_1';m_2 m_2'} \neq \rho^{(1)}_{m_1 m_1'}\rho^{(2)}_{m_2 m_2'}.$$

$$d^2W(\vec{n}_1,\vec{n}_2) \neq dW(\vec{n}_1)dW(\vec{n}_2).$$

- Integration of the double angular distribution over all angles, except the angle β between the vectors $\vec{n}_1$ and $\vec{n}_2$, leads to the general formula for the angular correlation between the flight directions for the products of decay :

$$\boxed{dN(\beta)=\frac{1}{2}\sum_L(2L+1)T^{(1)}_{L0}T^{(2)}_{L0}K_L P_L(\cos\beta)\sin(\beta)d\beta.}$$

$P_L(\cos\beta)$ – Legendre polynomials

$$T_{L0}^{(1)} = \sum_{\Lambda_1} R_{\Lambda_1} C_{j_1 \Lambda_1 \, L0}^{j_1 \Lambda_1}, \qquad T_{L0}^{(2)} = \sum_{\Lambda_2} \widetilde{R}_{\Lambda_2} C_{j_2 \Lambda_2 \, L0}^{j_2 \Lambda_2}$$

- Multipole parameters of decay Clebsh-Jordan coeffiicients

$$K_L = \sum_S a_{SL} \rho_S$$

ρ_S – relative weight of the states with total spin S ($\sum_S \rho_S = 1$)

$$a_{SL} = \sqrt{(2j_1+1)(2j_2+1)}(-1)^{S-j_1-j_2} W(j_1 j_2 j_1 j_2; SL).$$

Racah coefficients

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If $\vec{n}$ is a polar vector, then, under space parity conservation, $R_\Lambda = R_{-\Lambda}$,
$T_{L0} = 0$ at odd L .

Weak decays of two particles with spin $j_1 = j_2 = ½$

$$K_0 = 1 \quad , \quad K_1 = -\rho_0 + \frac{\rho_1}{3} \equiv -\rho_S + \frac{\rho_t}{3} \quad , \quad K_{L>1} = 0$$

Parity is not conserved: $R_{1/2} \neq R_{-1/2}$ $\quad T_{10}^{(1)} = \frac{\alpha_1}{\sqrt{3}}, \quad T_{10}^{(2)} = \frac{\alpha_2}{\sqrt{3}}$

α_1 and α_2 – P – odd asymmetry coefficients

Angular distributions : $dW_1 = \frac{1}{4\pi}(1 + \alpha_1 \vec{P}_1 \vec{n}_1)\, d\Omega_{\vec{n}_1}$, $dW_2 = \frac{1}{4\pi}(1 + \alpha_2 \vec{P}_2 \vec{n}_2)\, d\Omega_{\vec{n}_2}$

$\vec{P}_1, \vec{P}_2$ – polarization vectors: decay is the spin analyzer for the unstable particle .

Double angular distribution :

$$d^2W(\vec{n}_1, \vec{n}_2) = \frac{1}{16\pi^2}\left[1 + \alpha_1 \vec{P}_1 \vec{n}_1 + \alpha_2 \vec{P}_2 \vec{n}_2 + \alpha_1 \alpha_2 \sum_{i=1}^{3} \sum_{k=1}^{3} T_{ik} n_{1i} n_{2k}\right] d\Omega_{\vec{n}_1} d\Omega_{\vec{n}_2}$$

correlation tensor components

The polarization parameters and components of the correlation tensor can be determined from the angular distribution of products of two decays by the method of moments -- as a result of averaging combinations of trigonometric functions of flight angles over the double angular distribution :

$$\boxed{P_{1i} = \frac{3}{\alpha_1} \langle n_{1i} \rangle, \quad P_{2k} = \frac{3}{\alpha_2} \langle n_{2k} \rangle, \quad T_{ik} = \frac{9}{\alpha_1 \alpha_2} \langle n_{1i} n_{2k} \rangle}$$

Here

$$\langle .... \rangle \equiv \int (....) \left(\frac{d^2 W (\vec{n}_1, \vec{n}_2)}{d\Omega_{\vec{n}_1} d\Omega_{\vec{n}_2}} \right) d\Omega_{\vec{n}_1} d\Omega_{\vec{n}_2} \qquad ;$$

$n_{1x} = \sin\theta_1 \cos\varphi_1$; $n_{1y} = \sin\theta_1 \sin\varphi_1$; $n_{1z} = \cos\theta_1$;
$n_{2x} = \sin\theta_2 \cos\varphi_2$; $n_{2y} = \sin\theta_2 \cos\varphi_2$; $n_{2z} = \cos\theta_2$,

where θ_1 and φ_1 , θ_2 and φ_2 are the polar and azimuthal angles of emission of decay products in the rest frames of the first and second particle, respectively – with respect to the unified system of coordinate axes of c.m. frame of pair;

$d\Omega_{\vec{n}_1} = \sin\theta_1 \, d\theta_1 \, d\varphi_1$ and $d\Omega_{\vec{n}_2} = \sin\theta_2 \, d\theta_2 \, d\varphi_2$

are the elements of solid angles of decay product emission .

Angular correlations between the directions $\vec{n}_1$ and $\vec{n}_2$ at the decays of two unstable particles :

$$\boxed{dN(\beta) = \frac{1}{2}\left(1 + \frac{\alpha_1\alpha_2}{3} T \cos\beta\right) \sin\beta \, d\beta}$$

irrespective of the polarization vectors $\vec{P}_1$ and $\vec{P}_2$, which may be equal to zero $(\cos\beta = \vec{n}_1\vec{n}_2)$.

$T = \sum_{i=1}^{3} T_{ii} = \rho_t - 3\rho_s$ - " trace" of the correlation tensor .

P – odd asymmetry coefficients :

$\Lambda \to p + \pi^-$; *α* = 0.642; $\overline{\Lambda} \to \overline{p} + \pi^+$; *α* = – 0.642 (*CP* invariance)

Angular correlations between the directions of proton flight at the decays of two Λ – particles into the channel $\Lambda \to p + \pi^-$:

$$dN(\beta) = \frac{1}{2}\left(1 + \frac{\alpha^2}{3} T \cos\beta\right) \sin\beta \, d\beta, \qquad T = (\rho_t - 3\rho_s)$$

At the decays $\Lambda \to p + \pi^-$, $\overline{\Lambda} \to \overline{p} + \pi^+$, angular correlations between the flight directions for the proton and antiproton :

$$dN(\beta) = \frac{1}{2}\left(1 - \frac{\alpha^2}{3} T \cos\beta\right) \sin\beta \, d\beta$$

P – odd asymmetry coefficient for electron flight at the decay $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$, averaged over the electron energy spectrum: α = -⅓ . At the decay $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$: α = +⅓ .

Angular correlations between the directions of electron and positron emission in the decay of muon pair ($\mu^+ \mu^-$) :

$$dN(\beta) = \frac{1}{2}\,(1 - \frac{1}{27}\,T\cos\beta)\,\sin\beta\,d\beta$$

In case of muon pair production in the process $e^+ e^- \rightarrow \mu^+ \mu^-$: T = 1

Angular correlations in generation of pairs of unpolarized identical particles with spin *j* and close momenta (within the model of one-particle sources)

$$dN(\beta) = \frac{1}{2}\left[1 + 2\,K\left(A(\beta) - \frac{1}{2j+1}\right)\right]\sin\beta\,d\beta$$

$$K = (-1)^{2j}\,\frac{\langle\cos(qx)\rangle + B_{int}(q)}{2\,R(q)}$$

$$R(q) = 1 + B_{int}(q) + \frac{(-1)^{2j}}{2j+1}\,[\langle\cos(qx)\rangle + B_{int}(q)].$$

$\langle\cos(qx)\rangle$ – contribution of the effect of quantum statistics for non-interacting particles ;

$B_{int}(q)$ – contribution of the *S*-wave final-state interaction

$$A(\beta) = \sum_{\Lambda_1} \sum_{\Lambda_2} R_{\Lambda_1} \widetilde{R}_{\Lambda_2} \left(d^{(j)}_{\Lambda_1 \Lambda_2}(\beta) \right)^2$$

Taking into account the normalization of *d*-functions and the parameters R_{Λ_1} and R_{Λ_2} :

$$\int_0^{\pi} A(\beta)\, \sin\beta\, d\beta = \frac{2}{2j+1},$$

angular correlation is normalized by unity .

In particular, for the above-considered case of decays of two Λ particles with close momenta into the channel $\Lambda \to p + \pi^-$ we have :

j = ½ ,

$$R_{1/2} = \widetilde{R}_{1/2} = \frac{1+\alpha}{2}, \qquad R_{-1/2} = \widetilde{R}_{-1/2} = \frac{1-\alpha}{2}$$

$$A(\beta) = \frac{1}{2}\left[1 + \alpha^2 \cos\beta\right]$$

$$dN(\beta) = \frac{1}{2}(1 + K\alpha^2 \cos\beta)\sin\beta\, d\beta,$$

K = ⅓*T*

“trace” of the correlation tensor

5. Spin correlations at the generation of $\Lambda\bar{\Lambda}$ pairs in multiple processes

- Spin and angular correlations at the decays of two Λ particles, being identical particles, with taking into account Fermi statistics and final-state interaction , were considered previously .
- Let us consider now spin correlations in the decays of $\Lambda\bar{\Lambda}$ pairs . In the framework of the model of independent one-particle sources, spin correlations in the $\Lambda\bar{\Lambda}$ system arise only on account of the difference between the interaction in the final triplet state ($S = 1$) and the interaction in the final singlet state. At small relative momenta, the *s* -wave interaction plays the dominant role as before, but, contrary to the case of identical particles ($\Lambda\Lambda$) , in the case of non-identical particles ($\Lambda\bar{\Lambda}$) the total spin may take both the values $S = 1$ and $S = 0$ at the orbital momentum $L = 0$. In doing so, the interference effect, connected with quantum statistics, is absent .

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If the sources emit unpolarized particles, then, in the case under consideration, the correlation function describing momentum-energy correlations has the following structure

(in the c.m. frame of the $\boxed{\Lambda\bar{\Lambda}}$ pair) :

$$R\,(\mathbf{k},\mathbf{v}) = 1 + \frac{3}{4}\,B_t^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v}) + \frac{1}{4}\,B_s^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v}).$$

Here $B_t^{(\Lambda\bar{\Lambda})}$ and $B_s^{(\Lambda\bar{\Lambda})}$ -- contributions of interaction of Λ and $\boxed{\bar{\Lambda}}$ in the final triplet (singlet) state, which are expressed through the amplitudes of scattering of non-identical particles Λ and $\boxed{\bar{\Lambda}}$, and depend on space-time dimensions of the generation region of $\boxed{\Lambda\bar{\Lambda}}$ -pair ,

k – momentum of $\boxed{\bar{\Lambda}}$ in the c.m. frame of the pair, **v** – velocity of the pair

The spin density matrix of the $\boxed{\Lambda\bar{\Lambda}}$ pair is given by the formula :

$$\hat{\rho}^{(\Lambda\bar{\Lambda})} = \hat{I}^{(1)} \otimes \hat{I}^{(2)} + \frac{B_t^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v}) - B_s^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v})}{4\,R\,(\mathbf{k},\mathbf{v})}\;\hat{\boldsymbol{\sigma}}^{(1)} \otimes \hat{\boldsymbol{\sigma}}^{(2)} \quad ,$$

and the components of the correlation tensor are as follows:

$$\boxed{T_{ik} = \frac{B_t^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v}) - B_s^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v})}{4 + 3B_t^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v}) + B_s^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v})}\,\delta_{ik}}$$

- At sufficiently large values of k , one should expect that :

$$B_s^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v}) = 0\,, \quad B_t^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v}) = 0 \qquad .$$

In this case the angular correlations in the decays $\Lambda \to p + \pi^-$ and $\boxed{\bar{\Lambda} \to p + \pi^+}$, connected with the final-state interaction, are absent :

$$T_{ik} = 0, \qquad T = 0\ .$$

6. Angular correlations in the decays $\Lambda \to p + \pi^-$ and $\boxed{\bar{\Lambda} \to \bar{p} + \pi^+}$ and the "mixed phase"

- Thus, at sufficiently large relative momenta (for $k >> m_\pi$) one should expect that the angular correlations in the decays $\Lambda \to p + \pi^-$ and $\boxed{\bar{\Lambda} \to \bar{p} + \pi^+}$, in the framework of the model of one-particle sources are absent. In this case two-particle (and multiparticle) sources may be, in principle, the cause of the spin correlations. Such a situation may arise, if at the considered energy the dynamical trajectory of the system passes through the so-called "mixed phase"; then the two-particle sources, consisting of the free quark and antiquark , start playing a noticeable role . For example, the process $\boxed{s\bar{s} \to \Lambda\bar{\Lambda}}$ may be discussed .

  The CP parity of the fermion-antifermion pair is $\boxed{CP = (-1)^{S+1}}$.

- In the case of one-gluon exchange, $CP = 1$, and then $S = 1$, i.e. the $\boxed{\Lambda\bar{\Lambda}}$ pair is generated in the triplet state; in doing so, the "trace" of the correlation tensor $T = 1$.

- Even if the frames of one-gluon exchange are overstepped, the quarks s and $\bar{s}$, being ultrarelativistic, interact in the triplet state ($S = 1$) . In so doing, the primary CP parity $CP = 1$, and, due to the CP parity conservation, the $\boxed{\Lambda\bar{\Lambda}}$ pair is also produced in the triplet state. Let us denote the contribution of two-quark sources by x . Then at large relative momenta $T = x > 0$.

- Apart from the two-quark sources, there are also two-gluon sources being able to play a comparable role. Analogously with the annihilation process $\boxed{\gamma\gamma \to e^+e^-}$, in this case the “trace” of the correlation tensor is described by the formula (the process $\boxed{gg \to \Lambda\bar{\Lambda}}$ is implied) :

$$T = 1 - \frac{4(1-\beta^2)}{1 + 2\beta^2 \sin^2\theta - \beta^4 - \beta^4 \sin^4\theta} \quad ,$$

where β is the velocity of Λ (and $\boxed{\overline{\Lambda}}$) in the c.m. frame of the $\boxed{\Lambda\overline{\Lambda}}$ pair, θ is the angle between the momenta of one of the gluons and Λ in the c.m. frame . At small β ($\beta << 1$) the $\boxed{\Lambda\overline{\Lambda}}$ pair is produced in the singlet state (total spin $S = 0$, $T = -3$), whereas at $\beta \approx 1$ – in the triplet state ($S = 1$, $T = 1$) . Let us remark that at ultrarelativistic velocities β (i.e. at extremely large relative momenta of Λ and $\boxed{\overline{\Lambda}}$) both the two-quark and two-gluon mechanisms lead to the triplet state of the $\boxed{\Lambda\overline{\Lambda}}$ pair ($T = 1$) .

V.L.Lyuboshitz, V.V.Lyuboshitz (2010) // Yad. Fiz. **73** (5), 836 [Phys. At. Nucl. **73** (5), 805]

V.L.Lyuboshitz, V.V.Lyuboshitz (2011) – in NICA White Paper, pp. 164 – 167

V.L.Lyuboshitz, V.V.Lyuboshitz (2011) // Proc. of ISHEPP XX, JINR E1,2-2011-121, v.II, pp.54-60

In the general case, the appearance of angular correlations in the decays $\Lambda \rightarrow p + \pi^-$ and $\boxed{\overline{\Lambda} \rightarrow \overline{p} + \pi^+}$ with the nonzero values of the “trace” of the correlation tensor T at large relative momenta of the Λ and $\boxed{\overline{\Lambda}}$ particles may testify to the passage of the system through the “mixed phase” .

7. "Incoherence" inequalities for the correlation tensor and their violations in two-particle quantum systems

- Incoherent mixture of factorizable spin states of two particles with spin ½

$$\hat\rho^{(1,2)} = \sum_s \sum_t b_{st}\, \hat\rho_s^{(1)} \otimes \hat\rho_t^{(2)}, \quad b_{st} > 0, \quad \sum_s \sum_t b_{st} = 1$$

$$\hat\rho_s^{(1)} = \frac{1}{2}(\hat I^{(1)} + \vec P_s^{(1)} \hat{\vec\sigma}^{(1)}), \quad \hat\rho_t^{(2)} = \frac{1}{2}(\hat I^{(2)} + \vec P_t^{(2)} \hat{\vec\sigma}^{(2)})$$

V. L.Lyuboshitz (2000) // Proc. 34th Wint. Sc. St. Petersburg, 402-424.

R.Lednicky, V.L.Lyuboshitz (2001) // Phys. Lett. B 508, 146.

$\hat{\vec\sigma}$ -- vector Pauli operator ; $|\vec P_s^{(1)}| \le 1$, $|\vec P_t^{(2)}| \le 1$

Correlation tensor components : $T_{ik} = \sum_s \sum_t b_{st} P_{si}^{(1)} P_{tk}^{(2)}$ $\quad i, k = \{1,2,3\} \to \{x,y,z\}$

From the restrictions upon the parameters b_{st} , $\vec P_s^{(1)}$, $\vec P_t^{(2)}$,
the following inequalities for correlation tensor components arise :

$\lvert T \rvert = \lvert T_{xx} + T_{yy} + T_{zz} \rvert \le 1$	$\lvert T_{xx} + T_{yy} \rvert \le 1$	$\lvert T_{xx} + T_{zz} \rvert \le 1$	$\lvert T_{yy} + T_{zz} \rvert \le 1$

These inequalities are more simple than the well-known Bell inequality

$$Q = \left| \langle (\hat{\vec{\sigma}}^{(1)} \vec{n}) \otimes (\hat{\vec{\sigma}}^{(2)} \vec{m}) \rangle + \langle (\hat{\vec{\sigma}}^{(1)} \vec{n}) \otimes (\hat{\vec{\sigma}}^{(2)} \vec{m}') \rangle + \right.$$
$$\left. + \langle (\hat{\vec{\sigma}}^{(1)} \vec{n}' \otimes (\hat{\vec{\sigma}}^{(2)} \vec{m}) \rangle - \langle (\hat{\vec{\sigma}}^{(1)} \vec{n}') \otimes (\hat{\vec{\sigma}}^{(2)} \vec{m}') \rangle \right| \leq 2,$$

where $\vec{n}, \vec{m}, \vec{n}', \vec{m}'-$ arbitrary unit vectors ;

$$\langle (\vec{\sigma}^{(1)} \vec{n}) \otimes (\vec{\sigma}^{(2)} \vec{m}) \rangle = \sum_i^3 \sum_k^3 T_{ik} n_i m_k$$

$\Longrightarrow$ which is also valid for incoherent mixtures of factorizable states .

In the case of nonfactorizable coherent superpositions of two-particle states, the "incoherence" inequalities may be violated .

For the singlet state all the inequalities are violated :

$$\boxed{T_{xx} + T_{yy} = T_{xx} + T_{zz} = T_{yy} + T_{zz} = -2\,, \quad T = -3}$$

In the case of nonfactorizable triplet state with zero projection of total spin onto the axis z , one of the restrictions is not satisfied :

instead of the inequality $|T_{xx} + T_{yy}| < 1$, the equality $T_{xx} + T_{yy} = 2$ (>1) holds .

Quantum character of spin correlations

- Reaction $\pi^+ + {}^4He \to p + {}^3He$

V.V. Lyuboshitz, V. L. Lyuboshitz (2000) // Yad. Fiz. 63, 837; V. L. Lyuboshitz, V. V. Lyuboshitz (2005) // Proc. Int. Spin Symp. (Trieste, 2004), P.251

It follows from the conservation of space parity and angular momentum, taking into account the negative internal parity, that in the reaction $\pi^+ + {}^4He \to p + {}^3He$ the (p, ^{3}He) system is produced in the triplet state (total spin 1), which represents a nonfactorizable superposition of triplet states of two spin -½ particles with total spin projections onto the normal to the reaction plane, equaling +1 and -1.

Helicity amplitudes $R_1(E,\theta) = -R_{-1}(E,\theta)$, $R_0(E,\theta)$ (E – total energy, θ – emission angle in the c.m. frame)

$$|\ \psi\rangle = \frac{1}{(|R_0(E,\theta)|^2 + 2|R_1(E,\theta)|^2)^{1/2}} \times$$

$$\times \Big[R_1(E,\theta) \left(|+1/2, \vec{l}\rangle^{(p)} \otimes |+1/2, \vec{l}\rangle^{(He)} - |-1/2, \vec{l}\rangle^{(p)} \otimes |-1/2, \vec{l}\rangle^{(He)} \right) +$$

$$+ \frac{1}{\sqrt{2}} R_0(E,\theta) \left(|+1/2, \vec{l}\rangle^{(p)} \otimes |-1/2, \vec{l}\rangle^{(He)} + |-1/2, \vec{l}\rangle^{(p)} \otimes |+1/2, \vec{l}\rangle^{(He)} \right) \Big]$$

$\vec{l}$ - unit vector along the proton momentum in the c.m. frame .

- Or

$$|\psi\rangle = \frac{1}{(|R_0|^2 + 2|R_1|^2)^{\frac{1}{2}}}[(R_1 - \tfrac{i}{\sqrt{2}}R_0)|+\tfrac{1}{2},\vec{n}\rangle^{(p)} \otimes |+\tfrac{1}{2},\vec{n}\rangle^{(He)} + (R_1 + \tfrac{i}{\sqrt{2}}R_0)|-\tfrac{1}{2},\vec{n}\rangle^{(p)} \otimes |-\tfrac{1}{2},\vec{n}\rangle^{(He)}]$$

$\vec{n}$ - unit vector along the normal to the reaction plane .

Polarization vectors

$$\vec{P}^{(p)} = \vec{P}^{(He)} = -2\sqrt{2}\,\frac{\mathrm{Im}(R_1 R_0^*)}{|R_0|^2 + 2|R_1|^2}\,\vec{n};$$

In the system of coordinate axes $z \parallel \vec{l},\ y \parallel \vec{n},\ x \parallel [\vec{n}\vec{l}\,]$, the correlation tensor component have the form :

$$T_{xx} = \frac{|R_0|^2 - 2|R_1|^2}{|R_0|^2 + 2|R_1|^2}, \quad T_{yy} = 1, \quad T_{zz} = \frac{2|R_1|^2 - |R_0|^2}{|R_0|^2 + 2|R_1|^2} = -T_{xx}\,;$$

$$T_{xz} = T_{zx} = 2\sqrt{2}\,\frac{\mathrm{Re}(R_1 R_0^*)}{|R_0|^2 + 2|R_1|^2}, \quad T_{xy} = T_{yx} = T_{yz} = T_{zy} = 0\,.$$

Irrespective of the concrete values of helicity amplitudes, one of the incoherence inequalities for the correlation tensor is violated :

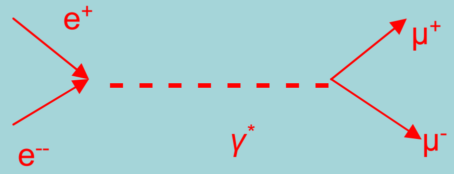
if $|R_0|^2 - 2|R_1|^2 > 0$, then $T_{xx} + T_{yy} > 1$;

if $|R_0|^2 - 2|R_1|^2 < 0$, then $T_{zz} + T_{yy} > 1$.

Processes $e^+e^- \to \mu^+\mu^-, \tau^+\tau^-$

V. L. Lyuboshitz, V. V. Lyuboshitz (2009) // Yad. Fiz., v.72, p.340.

In the first approximation over the constant $e^2/\hbar c$, the process of conversion of the electron-positron pair into the muon one (or $\tau^+\tau^-$) is described by the one-photon diagram :



The virtual photon with time-like momentum transfers angular momentum J=1 and negative parity. The internal parities of μ^+ and μ^- are opposite: the ($\mu^+\mu^-$) pair is generated in triplet states (total spin S=1), with total angular momentum J=1 and negative space parity .

Helicity amplitudes:

$$f_{\Lambda'\Lambda}(\theta,\phi) = R_{\Lambda'\Lambda}(E)\, d^{(1)}_{\Lambda'\Lambda}(\theta)\, \exp\left(i\Lambda\phi\right)$$

d- functions

θ and φ – polar and azimuthal angles of flight direction of μ^+ with respect to positron momentum in the c.m. frame of the reaction; Λ' – difference of helicities of μ^+ and μ^-, Λ - difference of helicities of e^+ and e^-; E – total energy in the c.m. frame .

Factorization: $$R_{\Lambda'\Lambda}(E) = r_{\Lambda'}^{(\mu)}(E)\, r_{\Lambda}^{(e)}(E)$$

Parity conservation: $$r_{+1}^{(\mu)}(E) = r_{-1}^{(\mu)}(E), \quad r_{+1}^{(e)}(E) = r_{-1}^{(e)}(E)$$

As follows from the structure of electromagnetic current for pairs $(e^+ e^-)$ and $(\mu^+ \mu^-)$:

$$r_0^{(\mu)}(E) = \frac{m_\mu}{E} r_1^{(\mu)}(E) = \sqrt{1-\beta_\mu^2}\, r_1^{(\mu)}(E), \quad r_0^{(e)}(E) = \frac{m_e}{E} r_1^{(e)}(E)$$

m_μ and m_e - muon and electron masses , β_μ - muon velocity .

Since always $E \geq m_\mu >> m_e$, the contribution of states of electron and positron with antiparallel spins (equal helicities) is negligibly small : $r_0^{(e)}(E) \approx 0$, $R_{\Lambda' 0}(E) \approx 0$.

At the annihilation of electron and positron being totally polarized in the direction parallel to positron momentum in the reaction c.m. frame, the $(\mu^+ \mu^-)$ system is generated in the triplet state :

$$|\Psi\rangle^{(+1)} = \frac{\sqrt{2}}{\sqrt{2-\beta_\mu^2 \sin^2\theta}} \left(\frac{1+\cos\theta}{2} |+1\rangle - \sqrt{1-\beta_\mu^2}\, \frac{\sin\theta}{\sqrt{2}} |0\rangle + \frac{1-\cos\theta}{2} |-1\rangle \right)$$

$$|+1\rangle = |+1/2\rangle^{(\mu^+)} \otimes |+1/2\rangle^{(\mu^-)}, \quad |-1\rangle = |-1/2\rangle^{(\mu^+)} \otimes |-1/2\rangle^{(\mu^-)},$$

$$|0\rangle = \frac{1}{\sqrt{2}} \left(|+1/2\rangle^{(\mu^+)} \otimes |-1/2\rangle^{(\mu^-)} + |-1/2\rangle^{(\mu^+)} \otimes |+1/2\rangle^{(\mu^-)} \right)$$

- states with projections of total spin of $(\mu^+ \mu^-)$ pair onto the μ^+ momentum direction in the c.m.s. +1, -1 and 0

- If the electron and positron are totally polarized in the direction being antiparallel to the positron momentum in the c.m. frame :

$$|\Psi\rangle^{(-1)} = \frac{\sqrt{2}}{\sqrt{2-\beta_\mu^2 \sin^2\theta}} \left(\frac{1-\cos\theta}{2} |+1\rangle + \sqrt{1-\beta_\mu^2}\, \frac{\sin\theta}{\sqrt{2}} |0\rangle + \frac{1+\cos\theta}{2} |-1\rangle \right)$$

- When the primary electron and positron are not polarized, the nonfactorizable states $|\psi\rangle^{(+1)}$ and $|\psi\rangle^{(-1)}$ are generated with equal probabilities .
- In the one-photon approximation, the generated muons are unpolarized but their spins are strongly correlated . Correlation tensor components at the choice of axis *z* along the relative momentum of muons in the c.m. frame and axis *y* – along the normal to the reaction plane :

$$T_{zz}^{(\mu^+\mu^-)} = \frac{2\cos^2\theta + \beta_\mu^2 \sin^2\theta}{2-\beta_\mu^2 \sin^2\theta},$$

$$T_{xx}^{(\mu^+\mu^-)} = \frac{(2-\beta_\mu^2)\sin^2\theta}{2-\beta_\mu^2 \sin^2\theta}, \quad T_{yy}^{(\mu^+\mu^-)} = -\frac{\beta_\mu^2 \sin^2\theta}{2-\beta_\mu^2 \sin^2\theta},$$

$$T_{xz}^{(\mu^+\mu^-)} = T_{zx}^{(\mu^+\mu^-)} = -\frac{(1-\beta_\mu^2)^{1/2}\sin 2\theta}{2-\beta_\mu^2 \sin^2\theta},$$

$$T_{xy}^{(\mu^+\mu^-)} = T_{yx}^{(\mu^+\mu^-)} = T_{yz}^{(\mu^+\mu^-)} = T_{yz}^{(\mu^+\mu^-)} = 0.$$

The “trace” of the correlation tensor

$$T^{(\mu+\mu^-)} = T_{xx}^{(\mu+\mu^-)} + T_{yy}^{(\mu^+\mu^-)} + T_{zz}^{(\mu^+\mu^-)} = 1,$$

just as it should hold for any triplet states .

- One of the “incoherence” inequalities is always violated in the process $e^+ e^{--} \to \mu^+ \mu^{--}$ at $\theta \neq 0$:

$$T_{xx}^{(\mu^+\mu^-)} + T_{zz}^{(\mu^+\mu^-)} = \frac{2}{2 - \beta_\mu^2 \sin^2\theta} > 1.$$

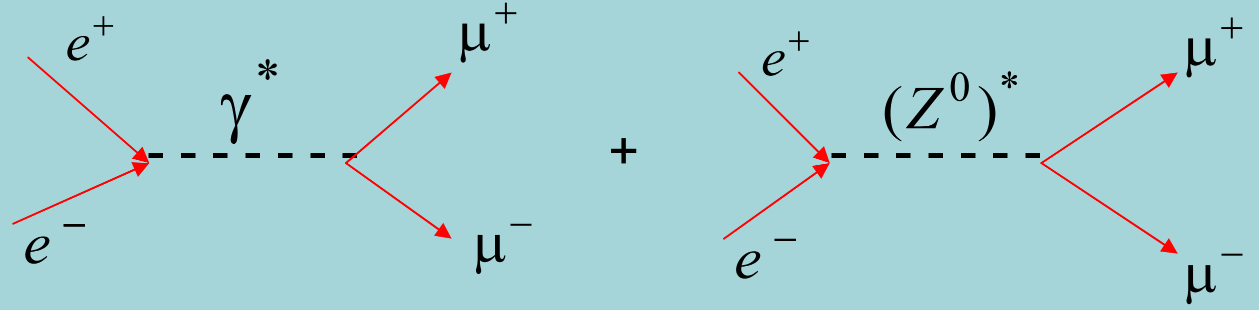
- Analogous consideration -- for the process $e^+e^- \to \tau^+\tau^-,$ ($m_\mu \to m_\tau, \quad \beta_\mu \to \beta_\tau$) .

- At very high energies ($\beta_\mu \to 1$, $\beta_\tau \to 1$) , the nonzero components of the correlation tensor are as follows :

$$T_{xx}^{(\mu^+\mu^-)} = -T_{yy}^{(\mu^+\mu^-)} = \frac{\sin^2\theta}{1+\cos^2\theta}, \quad T_{zz}^{(\mu^+\mu^-)} = 1. \qquad T_{xx}^{(\mu^+\mu^-)} + T_{zz}^{(\mu^+\mu^-)} = \frac{2}{1+\cos^2\theta} > 1$$

- At high energies the annihilation processes

$e^+e^- \to \mu^+\mu^-, \; e^+e^- \to \tau^+\tau^-$ are conditioned not only by electromagnetic interaction but also by the weak interaction of neutral currents through the virtual Z^0 boson :



- Interference of the amplitudes of the purely electromagnetic and weak interaction $\longrightarrow$ leads to the charge asymmetry in lepton emission and to the space parity violation.

$\mu^+\mu^-, \; \tau^+\tau^- \to$ generated in the triplet states : ${}^3S_1, \; {}^3D_1$ and ${}^3P_1$

due to weak interaction

(J = 1, positive CP parity)

It follows from the structure of "left" and "right" components of neutral currents that the nonzero helicity amplitudes of the processes $e^+e^- \to \mu^+\mu^-,\ e^+e^- \to \tau^+\tau^-$ in the c.m. frame take the form :

$$R_{11}(E) = \frac{e^2}{2E}\left[1 + x\left(\xi - \frac{1}{2}\right)^2\right] \quad ; \qquad R_{-1-1}(E) = \frac{e^2}{2E}\left[1 + x\,\xi^2\right] \ .$$

$$R_{-11}(E) = R_{1-1}(E) = \frac{e^2}{2E}\left[1 + x\,\xi\left(\xi - \frac{1}{2}\right)\right] \quad ;$$

($R_{0\lambda}$, $R_{\lambda'0}$ $\to$ turn practically to zero at high energies) .

Here: $\xi = \sin^2\theta_W$, where θ_W is the Weinberg angle (angle of gauge boson mixing) ; parameter x $\to$ determines the relative contribution of weak interaction :

$$x = \frac{1}{\sin^2\theta_W \cos^2\theta_W}\ \frac{s}{s - \left(M_{Z^0} - i\frac{\Gamma_{Z^0}}{2}\right)^2} \qquad (s = (2E)^2)$$

According to the standard model :

$$\frac{1}{\sin^2\theta_W \cos^2\theta_W} = \frac{1}{\xi(1-\xi)} \approx 6 = \frac{\sqrt{2}\,G_F\,M_{Z^0}^{\ 2}}{\pi\,\alpha}$$

$G_F = 1.166\cdot 10^{-5}\ GeV^{-2}$ $\to$ universal Fermi constant of weak interaction, $\alpha = 1/137$

Finally, performing the further analysis, we obtain, in particular, that :

1) Due to the weak interaction through the Z^0 boson, the final leptons with opposite charges , generated at the annihilation of the unpolarized electron and positron, acquire the equal longitudinal polarizations and opposite average helicities

(whereas, if the weak interaction contribution is neglected, the final leptons are correlated but unpolarized).

At the energies below and above the resonance energy, the average helicities of the final leptons have different signs :

$$E < \frac{M_{Z^0}}{2} \rightarrow x < 0, \ \overline{\lambda}_+ < 0, \ \overline{\lambda}_- > 0 \ ;$$

$$E > \frac{M_{Z^0}}{2} \rightarrow x > 0, \ \overline{\lambda}_+ > 0, \ \overline{\lambda}_- < 0 \ .$$

2) Structure of the correlation tensor of the final leptons is, on the whole, similar to that for the case of purely electromagnetic annihilation at high energies .

In doing so, $T_{zz} = 1$ as before ;

the expression for T_{xx} changes , but, as before, $T_{xx} = -T_{yy}$:

$$\boxed{T_{xx} = -T_{yy} = \sin^2\theta \frac{[\,1 + x\,(\,\xi^2 - \frac{1}{2}\,\xi + \frac{1}{8}\,)\,]\,[\,1 + x\,\xi\,(\xi - \frac{1}{2}\,)\,]}{a_+(E)(\,1 + \cos^2\theta) + 2\,a_-(E)\cos\theta}}$$

where

$$a_+(E) = 1 + \frac{1}{2}\,x\left(\frac{1}{2} - 2\xi\right)^2 + \frac{1}{4}\,x^2\left[\left(\frac{1}{2} - \xi\right)^2 + \xi^2\right]^2 \quad ,$$

$$a_-(E) = \frac{1}{8}\,x + \frac{1}{4}\,x^2\left(\frac{1}{4} - \xi\right)^2 \quad .$$

Again, one of the incoherence inequalities for the correlation tensor components is violated : $T_{xx} + T_{zz} > 1$.

Process of electron–positron pair production by two photons , $\gamma\,\gamma \rightarrow e^{+}e^{-}$ (and analogous processes $\gamma\,\gamma \rightarrow \mu^{+}\,\mu^{-}$, $\gamma\,\gamma \rightarrow \tau^{+}\,\tau^{-}$)

V. L. Lyuboshitz, V. V. Lyuboshitz (2010) // Proc. of DSPIN-09, JINR E1,2-2010-13, pp.91-94

- In the first nonvanishing approximation over the electromagnetic constant (Born approximation) : if the primary photons are unpolarized, then the electron and positron are unpolarized as well but their spins are correlated .

- Extending the analysis made previously in the paper

W.H.McMaster (1961) // Rev. Mod. Phys. 33, N 1, p.8 ,

we obtain the following expressions for the components of the correlation tensor of the electron–positron pair, generated in the interaction of unpolarized γ quanta :

$$T_{zz} = 1 - \frac{2(1-\beta^2)\ [\ \beta^2 (1+\sin^2\theta) + 1\ ]}{1 + 2\beta^2 \sin^2\theta - \beta^4 - \beta^4 \sin^4\theta}$$

$$T_{yy} = \frac{(1-\beta^2)\ [\ \beta^2 (1+\sin^2\theta) - 1\ ] - \beta^2 \sin^4\theta}{1 + 2\beta^2 \sin^2\theta - \beta^4 - \beta^4 \sin^4\theta}$$

$$T_{xx} = \frac{(1-\beta^2)\ [\ \beta^2 (1+\sin^2\theta) - 1\ ] + \beta^2 \sin^4\theta}{1 + 2\beta^2 \sin^2\theta - \beta^4 - \beta^4 \sin^4\theta}$$

- Here the axis z is aligned along the positron momentum in the c.m. frame, the axis x lies in the reaction plane, the axis y is directed along the normal to the reaction plane ;

  $\beta = \frac{v}{c}$, **v** – positron velocity in the c.m. frame ;

- $1 - \beta^2 = \frac{m_e c^2}{E_+}$, E_+ -- positron (or electron) energy in the c.m.

  frame ; θ – angle between the positron momentum and the momentum of one of the photons in the c.m. frame .

- The differential cross section of the process $\gamma\gamma \to e^+e^-$ in the c.m. frame has the form :

$$\frac{d\sigma}{d\Omega} = r_0{}^2 \, \frac{1-\beta^2}{4}\beta \left[\frac{1+2\beta^2\sin^2\theta - \beta^4 - \beta^4\sin^4\theta}{(1-\beta^2\cos^2\theta)^2} \right]$$

where $r_0 = \dfrac{e^2}{m_e c^2}$.

- The "trace" of the correlation tensor is determined by the formula

$$\boxed{T = T_{xx} + T_{yy} + T_{zz} = 1 - \frac{4(1-\beta^2)}{1+2\beta^2\sin^2\theta - \beta^4 - \beta^4\sin^4\theta}}$$

- In doing so, the relative fraction of the triplet states is as follows [3] :

$$\boxed{W_t = \frac{T+3}{4} = 1 - \frac{1-\beta^2}{1+2\beta^2\sin^2\theta - \beta^4 - \beta^4\sin^4\theta}}\quad ,$$

and the relative fraction of the singlet state (total spin $S = 0$) equals

$$\boxed{W_s = \frac{1-T}{4} = 1 - W_t = \frac{1-\beta^2}{1+2\beta^2\sin^2\theta - \beta^4 - \beta^4\sin^4\theta}}\quad .$$

- At $\beta \ll 1$ we have $W_t \approx 0$, $W_s \approx 1$.
- If $\beta \approx 1$, $2\sin^2\theta - \sin^4\theta > 1 - \beta^2$, then $W_t \approx 1$, $W_s \approx 0$

- Let us consider the cases $\theta = 0$ and $\theta = \pi$. Then, according to the formula for $\frac{d\sigma}{d\Omega}$, we obtain :

$$\frac{d\sigma}{d\Omega} = \frac{1}{4} r_0^{\,2} \, \beta \, (1 + \beta^2) \qquad .$$

- In the ultrarelativistic limit at $\theta = 0$ and $\theta = \pi$

$$\frac{d\sigma}{d\Omega} = \frac{r_0^{\,2}}{2} \quad .$$

- According to the above formulas for the correlation tensor components, at $\theta = 0$ and $\theta = \pi$ we have :

$$T_{zz} = 1 - \frac{2\,(1+\beta^2)\,(1-\beta^2)}{1-\beta^4} = -1 \quad ; \quad T_{xx} = T_{yy} = -\frac{1-\beta^2}{1+\beta^2}$$

- In doing so, the "trace" of the correlation tensor equals

$$T = 1 - \frac{4}{1+\beta^2} = -\frac{3-\beta^2}{1+\beta^2} \quad .$$

The relative fraction of the triplet states amounts to

$$W_t = \frac{T+3}{4} = \frac{\beta^2}{1+\beta^2} \quad ,$$

and the relative fraction of the singlet state equals

$$W_s = \frac{1-T}{4} = \frac{1}{1+\beta^2} \quad .$$

- At nonrelativistic velocities, just as one would expect,

$$W_t \approx 0 \, , \, W_s \approx 1 \quad ,$$

whereas at $\beta \to 1$ we have : $W_t = W_s = \frac{1}{2}$.

In the processes $\gamma\gamma \to e^+e^-$ ($\gamma\gamma \to \mu^+\mu^-$, $\gamma\gamma \to \tau^+\tau^-$) we also observe the violation of the "incoherence" inequalities for correlation tensor components.

Indeed, at $\theta = 0$ and $\theta = \pi$, in particular, we obtain :

$$| T_{zz} + T_{xx} | = | T_{zz} + T_{yy} | = \frac{2}{1 + \beta^2} > 1 ,$$

since $\beta < 1$.

Thus, spin correlations of the final leptons in the considered processes have the strongly pronounced quantum character .

8. Correlations of polarizations in the system of two photons

V. L. Lyuboshitz, V. V. Lyuboshitz (2005) // EChAYa, 36, No. 7Б, 123
(Proceedings of the Bogolyubov-2004 Conference)

Photon mass equals zero ; definition of spin in the rest frame is not applicable .

The internal state of a photon is described by a superposition of two mutually perpendicular vectors, which are perpendicular to the photon momentum in any frame .

Polarization states :

$$|1,+\rangle = \frac{\vec{e}_1 + \vec{e}_2}{\sqrt{2}}, \quad |1,-\rangle = \frac{\vec{e}_1 - \vec{e}_2}{\sqrt{2}}, \qquad |2,+\rangle = \frac{\vec{e}_1 + i\,\vec{e}_2}{\sqrt{2}}, \quad |2,-\rangle = \frac{\vec{e}_1 - i\,\vec{e}_2}{\sqrt{2}}, \qquad |3,+\rangle = \vec{e}_1, \quad |3,-\rangle = \vec{e}_2$$

The axis z is directed along the photon momentum, axis x – along the vector $\vec{e}_2$, axis y – along the vector $\vec{e}_1$.

Stokes parameters : $$\epsilon_1 = W_1^{(+)} - W_1^{(-)}, \quad \epsilon_2 = W_2^{(+)} - W_2^{(-)}, \quad \epsilon_3 = W_3^{(+)} - W_3^{(-)}$$

$W_i^{(+)}$ и $W_i^{(-)}$ – probabilities of registration of states $|i,+\rangle$ and $|i,-\rangle$
($W_i^{(+)} + W_i^{(-)} = 1$, $i = 1,2,3$) ; ε_2 - degree of linear polarization;

$$r = \sqrt{\varepsilon_1^2 + \varepsilon_3^2}$$ - degree of circular polarization (average helicity) .

- System of two photons with momenta $\vec{k}_1$, $\vec{k}_2$: two systems of coordinate axes --(x, y, z) with axis z parallel to momentum $\vec{k}_1$, and $(\tilde{x}, \tilde{y}, \tilde{z})$ with axis $\tilde{z}$ parallel to momentum $\vec{k}_2$; the axes x and $\tilde{x}$ are mutually parallel and, in doing so, perpendicular to the plane passing through the photon momenta $\vec{k}_1$ and $\vec{k}_2$.

- Polarization density matrix of the two-photon system is analogous on structure to the spin density matrix for two particles with spin ½ :

$$\hat{\rho}^{(1,2)} = \frac{1}{4}\left[\hat{I}^{(1)} \otimes \hat{I}^{(2)} + \sum_{i=1}^{3} \epsilon_i^{(1)} \hat{\sigma}_i^{(1)} \otimes \hat{I}^{(2)} + \sum_{k=1}^{3} \epsilon_k^{(2)} \hat{I}^{(1)} \otimes \hat{\sigma}_k^{(2)} + \sum_{i=1}^{3}\sum_{k=1}^{3} T_{ik} \hat{\sigma}_i^{(1)} \otimes \hat{\sigma}_k^{(2)}\right]$$

$\hat{\sigma}_i^{(1)}$, $\hat{\sigma}_k^{(2)}$ - Pauli matrices ; $\varepsilon_i^{(1)}$ - Stokes parameters for photon 1, $\varepsilon_k^{(2)}$ - Stokes parameters for photon 2 ; T_{ik} – correlation “tensor” in Stokes space, describing the correlation of polarizations for photons 1 and 2:

$$T_{ik} = W_{i,k}^{(+,+)} - W_{i,k}^{(-,+)} - W_{i,k}^{(+,-)} + W_{i,k}^{(-,-)}, \qquad |T_{ik}| \leq 1$$

$W_{i,k}^{(+,+)},\ W_{i,k}^{(+,-)},\ W_{i,k}^{(-,+)},\ W_{i,k}^{(-,-)}$ -- probabilities of joint registration of photons 1 and 2 in the states

$$|\,i,+\rangle\,,|\,k\,,+\rangle\,;\ |\,i,+\rangle\,,|\,k\,,-\rangle\,;\ |\,i,-\rangle\,,|\,k\,,+\rangle\,;\ |\,i,-\rangle\ |\,k\,,-\rangle$$

$$W_{i,k}^{(+,+)} + W_{i,k}^{(+,-)} + W_{i,k}^{(-,+)} + W_{i,k}^{(-,-)} = 1$$

In the case of unpolarized non-correlated photons $\varepsilon_i^{(1)} = 0,\ \varepsilon_k^{(2)} = 0,\ T_{ik} = 0$.
At the Lorentz transformation from the laboratory frame to the frame moving with velocity $\vec{v}$, the basis unit vectors of polarization for photons 1 and 2 turn around the vectors $[\vec{k}_1\vec{v}]$, $[\vec{k}_2\vec{v}]$ by the angles θ_1 and θ_2 being equal to the aberration angles; Stokes parameters and correlation tensor components are relativistic invariants .
At the transition to the c.m. frame of two photons moving with velocity $\vec{v} = \frac{\vec{k}_1 + \vec{k}_2}{|\vec{k}_1| + |\vec{k}_2|}$, the polarization unit vectors for photons 1 and 2 turn around the vector $[\vec{k}_1\vec{k}_2]$ in opposite directions . In the c.m. frame

$$\vec{k}_1 = -\vec{k}_2,\ \vec{e}_1^{(1)} = \vec{e}_1^{(2)},\ \vec{e}_2^{(1)} = -\vec{e}_2^{(2)}.$$

Examples :

- Decays of pseudoscalar mesons $\pi^0 \to 2\gamma$, $\eta \to 2\gamma$

In the meson rest frame : $A_{2\gamma} \sim ([\vec{e}^{(1)*}\vec{e}^{(2)*}]\vec{n})$

$\vec{n}$ - unit vector along the momentum of one of photons in this frame .

System of two γ-quanta is generated in the nonfactorizable state with mutually perpendicular polarizations :

$$|\Psi\rangle^{(1,2)} = \frac{1}{\sqrt{2}} \left(|\vec{e}_1^{\,(1)}\rangle \otimes |\vec{e}_2^{\,(2)}\rangle + |\vec{e}_2^{\,(1)}\rangle \otimes |\vec{e}_1^{\,(2)}\rangle \right)$$

$$\varepsilon_i^{(1)} = \varepsilon_k^{2} = 0, \quad T_{11} = 1, \quad T_{22} = 1, \quad T_{33} = -1, \quad T_{ik} = 0 \text{ at } i \neq k$$

Equality $T_{22} = 1$ means that the helicities of photons 1 and 2 are the same (spinless particle) .

Quantum character of correlations of polarizations $\Longrightarrow$ $T_{11} + T_{22} = 2 \quad (> 1)$ - the incoherence inequality is violated .

- Decay $K_s^0 \to 2\gamma$ (positive *CP* parity) ; $A_{2\gamma} \sim (\vec{e}^{\,(1)*} \vec{e}^{\,(2)*})$

System of two γ-quanta is produced in the nonfactorizable state

$$|\Psi\rangle^{(1,2)} = \frac{1}{\sqrt{2}} \left(|\vec{e}_1^{\,(1)}\rangle \otimes |\vec{e}_1^{\,(2)}\rangle - |\vec{e}_2^{\,(1)}\rangle \otimes |\vec{e}_2^{\,(2)}\rangle \right)$$

$$\varepsilon_{\mathrm{i}}^{(1)} = \varepsilon_k^{(2)} = 0, \; T_{11} = -1, \; T_{22} = 1, \; T_{33} = 1, \; T_{ik} = 0 \text{ at } i \neq k; \quad T_{22} + T_{33} > 1$$

quantum correlations

Taking into account the change of orientation of basis vectors, the polarization parameters remain invariable in any frame (e.g., in the laboratory frame, where the decaying particle is moving) .

9. Correlations of pairs of neutral *K* mesons

V. L. Lyuboshitz, V. V. Lyuboshitz (2007) // Pis'ma v EChAYa, 4, № 5 (141), 654

- Internal states of the neutral kaon with definite strangeness:

$$|K^0\rangle \quad (S=1), \qquad |\overline{K}^0\rangle \quad (S=-1)$$

- Internal states of the neutral kaon with definite *CP* parity (neglecting weak effects of *CP* nonconservation) :

$$|K_S^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle) \Longrightarrow$$

CP = +1, short-lived neutral kaon decaying into two π mesons ;

$$|K_L^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle) \Longrightarrow$$

CP = -1, long-lived neutral kaon decaying into three π mesons .

Analogy with spin ½: K^0 и $\overline{K}^0$ $\longrightarrow$ projections +½ и -½ onto axis *z* ;
K_S^0 и K_L^0 $\longrightarrow$ onto axis *x*.

In inclusive processes with strangeness conservation, pairs $K^0K^0 \quad (S=2), \quad \overline{K}^0\overline{K}^0 \quad (S=-2)$ are generated incoherently. The internal state of pair $K^0\overline{K}^0 \quad (S=0)$ is non-factorizable at given momenta $\vec{p}_1$, $\vec{p}_2$:

non-diagonal elements of the density matrix between the states

$|K^0\rangle^{(\mathbf{p}_1)} \otimes |\bar{K}^0\rangle^{(\mathbf{p}_2)}$ and $|\bar{K}^0\rangle^{(\mathbf{p}_1)} \otimes |K^0\rangle^{(\mathbf{p}_2)}$ are not equal to zero .

As follows from the Bose symmetry with respect to full permutation, *CP* parity of the system $K^0\overline{K}^0$ is always positive (*C* = (-1)*ᴸ* , *P* = (-1)*ᴸ* , *L* is the orbital momentum) .

Symmetric internal state of the pair $K^0\overline{K}^0$, corresponding to even orbital momenta :

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle^{(\mathbf{p}_1)} \otimes |\bar{K}^0\rangle^{(\mathbf{p}_2)} + |\bar{K}^0\rangle^{(\mathbf{p}_1)} \otimes |K^0\rangle^{(\mathbf{p}_2)}) =$$
$$= \frac{1}{\sqrt{2}}(|K^0_S\rangle^{(\mathbf{p}_1)} \otimes |K^0_S\rangle^{(\mathbf{p}_2)} - |K^0_L\rangle^{(\mathbf{p}_1)} \otimes |K^0_L\rangle^{(\mathbf{p}_2)})$$

Decomposition into the schemes $K^0_S K^0_S$ and $K^0_L K^0_L$ (analogue of the triplet state with zero projection of total spin onto the axis z) .

- Antisymmetric internal state, corresponding to odd orbital momenta :

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle^{(\mathbf{p}_1)} \otimes |\bar{K}^0\rangle^{(\mathbf{p}_2)} - |\bar{K}^0\rangle^{(\mathbf{p}_1)} \otimes |K^0\rangle^{(\mathbf{p}_2)}) =$$
$$= \frac{1}{\sqrt{2}}(|K^0_S\rangle^{(\mathbf{p}_1)} \otimes |K^0_L\rangle^{(\mathbf{p}_2)} - |K^0_L\rangle^{(\mathbf{p}_1)} \otimes |K^0_S\rangle^{(\mathbf{p}_2)})$$

Decomposition into the scheme $K^0_S K^0_L$ (analogue of the singlet state) .

At the selection of the pairs of neutral kaons over decays, the structure functions (double inclusive cross sections) are invariant with respect to the permutation of momenta $\vec{p}_1$ and $\vec{p}_2$ and replacement $K_S^0 \Leftrightarrow K_L^0$.

$$f_{SS}(\mathbf{p}_1, \mathbf{p}_2) = f_{LL}(\mathbf{p}_1, \mathbf{p}_2) = f_{SL}(\mathbf{p}_1, \mathbf{p}_2) + \mathrm{Re}\,\rho_{K^0\bar{K}^0 \to \bar{K}^0 K^0}(\mathbf{p}_1, \mathbf{p}_2)$$

$\rho_{K^0\bar{K}^0 \to \bar{K}^0 K^0} = \rho^*_{\bar{K}^0 K^0 \to K^0 \bar{K}^0}$ - non-diagonal element of the density matrix of two neutral kaons

- Pair momentum-energy correlations of neutral kaons with small relative momenta

In the framework of the conventional model of one-particle sources, correlation functions R_{SS} and R_{LL}, normalized by 1 at large momentum differences:

$$\begin{aligned} R_{SS}(\mathbf{k}) = R_{LL}(\mathbf{k}) = \lambda_{K^0K^0}\,[1 + F_{K^0}(2\mathbf{k}) + 2\,b_{\mathrm{int}}(\mathbf{k})] + \\ +\lambda_{\bar{K}^0\bar{K}^0}\left[1 + F_{\bar{K}^0}(2\mathbf{k}) + 2\,\tilde{b}_{\mathrm{int}}(\mathbf{k})\right] + \\ +\lambda_{K^0\bar{K}^0}\,[1 + F_{K^0\bar{K}^0}(2\mathbf{k}) + 2\,B_{\mathrm{int}}(\mathbf{k})] \end{aligned}$$

$\vec{k}$ - momentum of one of the kaons in the c.m. frame of the kaon pair ,

$\lambda_{K^0K^0}, \lambda_{\bar{K}^0\bar{K}^0}, \lambda_{K^0\bar{K}^0}$ are relative weights of the pairs $K^0\overline{K}^0$, K^0K^0, $\overline{K}^0\overline{K}^0$

($\lambda_{K^0K^0} + \lambda_{\bar{K}^0\bar{K}^0} + \lambda_{K^0\bar{K}^0} = 1$) .

«Form factors» $F_{K^0}(2\mathbf{k}), F_{\bar{K}^0}(2\mathbf{k}), F_{K^0\bar{K}^0}(2\mathbf{k})$ describe the contribution of Bose statistics without taking into account final-state interaction ;

$b_{\text{int}}(\vec{k})$, $\widetilde{b}_{\text{int}}(\vec{k})$ $\longrightarrow$ *S* – wave interaction of two K^0 mesons and two $\overline{K}^0$ mesons ; $B_{\text{int}}(\vec{k}) \longrightarrow$ *S* – wave interaction between the K^0 meson and $\overline{K}^0$ meson .

- If a pair of non-identical neutral kaons $K^0\overline{K}^0$ is generated, but the states $K_S^0K_S^0$ (or $K_L^0K_L^0$) are registered over decays, then the two-particle momentum-energy correlations at small relative momenta have the same character as in the case of ordinary identical bosons (pions) with zero spin .

- For pairs of non-identical states $K_S^0 K_L^0$:

$$R_{SL}(\mathbf{k}) = R_{LS}(\mathbf{k}) = \lambda_{K^0K^0}\left[1 + F_{K^0}(2\mathbf{k}) + 2\,b_{\rm int}(\mathbf{k})\right] +$$
$$+\lambda_{\bar{K}^0\bar{K}^0}\left[1 + F_{\bar{K}^0}(2\mathbf{k}) + 2\,\tilde{b}_{\rm int}(\mathbf{k})\right] +$$
$$+\lambda_{K^0\bar{K}^0}\left[1 - F_{K^0\bar{K}^0}(2\mathbf{k})\right]$$

- At the generation of pairs of non-identical neutral kaons $K^0\overline{K}^0$ and registration of the state $K_S^0 K_L^0$ over decays, pair correlations are analogous to the correlations of identical fermions with equal spin projections (since in this case the pair $K_S^0 K_L^0$ has odd orbital momentum) .

$$R_{SS}(\mathbf{k}) - R_{SL}(\mathbf{k}) = 2\,\lambda_{K^0\bar{K}^0}\left[F_{K^0\bar{K}^0}(2\mathbf{k}) + B_{\rm int}(\mathbf{k})\right]$$

The difference between the correlation functions for pairs of identical neutral kaons $K_S^0K_S^0$ and pairs of non-identical neutral kaons $K_S^0 K_L^0$ is conditioned exclusively by the generation of $K^0\overline{K}^0$ pairs .

- Form factors $F_{K^0}(2\vec{k})$, $F_{\overline{K}^0}(2\vec{k})$, $F_{K^0\overline{K}^0}(2\vec{k})$ and functions $b_{\text{int}}(\vec{k})$, $\widetilde{b}_{\text{int}}(\vec{k})$ and $B_{\text{int}}(\vec{k})$ contain the information on space-time parameters of the generation region of neutral kaons and tend to zero at large relative momenta $q = 2|\vec{k}|$:

$$F_{K^0}(2\mathbf{k}) = \int W_{K^0}(\mathbf{r}) \cos(2\mathbf{kr})\, d^3\mathbf{r}, \qquad F_{\bar{K}^0}(2\mathbf{k}) = \int W_{\bar{K}^0}(\mathbf{r}) \cos(2\mathbf{kr})\, d^3\mathbf{r},$$

$$F_{K^0\bar{K}^0}(2\mathbf{k}) = \int W_{K^0\bar{K}^0}(\mathbf{r}) \cos(2\mathbf{kr})\, d^3\mathbf{r}.$$

$$b_{\text{int}}(\mathbf{k}) = \int W_{K^0}(\mathbf{r})\, b(\mathbf{k},\mathbf{r}) d^3\mathbf{r}, \quad \tilde{b}_{\text{int}}(\mathbf{k}) = \int W_{\bar{K}^0}(\mathbf{r})\, \tilde{b}(\mathbf{k},\mathbf{r}) d^3\mathbf{r}.$$

(due to *CP* invariance $b(\mathbf{k},\mathbf{r}) = \tilde{b}(\mathbf{k},\mathbf{r})$) ;

$$B_{\text{int}}(\mathbf{k}) = \int W_{K^0\bar{K}^0}(\mathbf{r}) B(\mathbf{k},\mathbf{r}) d^3\mathbf{r},$$

$W_{K^0}(\mathbf{r})$, $W_{\bar{K}^0}(\mathbf{r})$, $W_{K^0\bar{K}^0}(\mathbf{r})$ are the distributions of distances between sources of emission of two K^0 mesons, two $\overline{K}^0$ mesons, a K^0 meson and a $\overline{K}^0$ meson, respectively -- in the c.m. frame of the kaon pair .

- Connection of the contribution of final-state interaction into the pair momentum-energy correlations of kaons at small relative momenta with the parameters of S -wave low-energy scattering

R. Lednicky, V.V. Lyuboshitz, V.L.Lyuboshitz (1998) // Yad. Fiz. 61, 2161

Approximate formula :

$$B(\mathbf{k},\mathbf{r}) = \left(|A_{K^0\bar{K}^0\to K^0\bar{K}^0}(k)|^2 + |A_{K^+K^-\to K^0\bar{K}^0}(k)|^2 \right) \frac{1}{r^2} + $$
$$+2\,\mathrm{Re}\left(A_{K^0\bar{K}^0\to K^0\bar{K}^0}(k)\, \frac{\exp(ikr)\cos \mathbf{kr}}{r} \right),$$

$k = |\vec{k}|$, $r = |\vec{r}|$, $A_{K^0\bar{K}^0\to K^0\bar{K}^0}(k)$, -- amplitude of S -wave elastic $K^0\overline{K}^0$ - scattering ; $A_{K^+K^-\to K^0\bar{K}^0}(k)$ -- amplitude of the reaction $K^+K^- \to K^0\overline{K}^0$ at the momentum of final K^0 meson equaling k in the c.m.s. of pair $K^0\overline{K}^0$

(cross section of the process $K^+K^- \to K^0\overline{K}^0$:

$$\sigma_{K^+K^-\to K^0\bar{K}^0}(k) = 4\pi\, |A_{K^+K^-\to K^0\bar{K}^0}(k)|^2\, \frac{k}{\tilde{k}},$$

$\tilde{k} = \sqrt{k^2 + (m_0^2 - m_+^2)}$ - momentum of the charged kaon in the c.m. frame .

10. Correlations of pairs of neutral heavy mesons

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- Formally, analogous relations are valid also for the neutral heavy mesons D^0 , B^0 and B_s^0 . In doing so, the role of strangeness conservation is played, respectively, by the conservation of charm and beauty in inclusive multiple processes with production of these mesons . In these cases the quasistationary states are also states with definite *CP* parity, neglecting the effects of *CP* nonconservation .

For example, $|B_S^0\rangle = \frac{|B^0\rangle + |\overline{B}^0\rangle}{\sqrt{2}}$, *CP* parity + 1 ;

$|B_L^0\rangle = \frac{|B^0\rangle - |\overline{B}^0\rangle}{\sqrt{2}}$, *CP* parity – 1;

- In accordance with the mechanism of mixing a particle with the respective antiparticle due to weak interaction through the exchange of two virtual *W* bosons, states with *CP* parity (– 1) have the greater mass and the larger lifetime than states with *CP* parity (+ 1) . The difference of masses is very insignificant in all the cases, ranging from 10^{-12} MeV for K^0 mesons up to 10^{-8} MeV for B_s^0 mesons .

- Concerning the lifetimes, in the case of K^0 mesons they differ by 600 times, but for D^0, B^0 and B_s^0 mesons the respective difference is very inconsiderable. In connection with this, it is practically impossible to distinguish the states of D^0, B^0 and B_s^0 mesons with definite *CP* parity by the difference in their lifetimes. These states, in principle, can be identified through the purely *CP*-even and purely *CP*-odd decay channels; however, in fact the branching ratio for such decays is very small. For example,

$$Br\,(D^0 \to \pi^+\pi^-) = 1.62 \cdot 10^{-3} \quad (CP = +1)\ ;$$

$$Br\,(D^0 \to K^+K^-) = 4.25 \cdot 10^{-3} \quad (CP = +1)\ ;$$

$$Br\,(B_s^0 \to J/\Psi\ \pi^0) < 1.2 \cdot 10^{-3} \quad (CP = +1)\,;$$

$$Br\,(B^0 \to J/\Psi\ K_S^0) = 9 \cdot 10^{-4} \quad (CP = -1);$$

Just as in the case of neutral *K* mesons, the correlation functions for the pairs of states of neutral *D*, *B* and B_s mesons with the same *CP* parity ($R_{SS} = R_{LL}$) and for the pairs of states with different *CP* parity (R_{SL}) do not coincide, and the difference between them is conditioned exclusively by the production of pairs $D^0\overline{D}^0$, $B^0\overline{B}^0$ and $B_s^0\overline{B}_s^0$, respectively. In particular, for B_s^0 mesons the following relation holds:

$$R_{SS}(k) - R_{SL}(k) = 2\,\lambda_{B_s^0\overline{B}_s^0}\,[\,F_{B_s^0\overline{B}_s^0}(2\mathbf{k}) + B_{\text{int}}(\mathbf{k})\,] \quad ;$$

here $\lambda_{B_s^0\overline{B}_s^0}$ is the relative fraction of generated pairs $B_s^0\overline{B}_s^0$,

$$F_{B_s^0\overline{B}_s^0}(2\mathbf{k}) = \int W_{B_s^0\overline{B}_s^0}(\,\mathbf{r}\,)\cos(2\mathbf{kr})\,d^3\mathbf{r} \quad ,$$

$$B_{\text{int}}(\mathbf{k}) = \int W_{B_s^0\overline{B}_s^0}(\,\mathbf{r}\,)\,B\,(\mathbf{k},\mathbf{r}\,)\,d^3\mathbf{r} \quad ,$$

$$B(\mathbf{k},\mathbf{r}) = |\,A_{B_S^0\overline{B}_S^0}(k)\,|^2\,\frac{1}{r^2} + 2\,\mathrm{Re}\left(A_{B_S^0\overline{B}_S^0}(k\,)\,\frac{\exp(ikr)\cos\mathbf{kr}}{r}\right) \quad ,$$

where $A_{B_S^0\overline{B}_S^0}(k\,) \equiv A_{B_S^0\overline{B}_S^0 \to B_s^0\overline{B}_s^0}(k\,)$ is the amplitude of *S*-wave $B_s^0\overline{B}_s^0$ – scattering, k = | **k** | , r = | **r** | . Let us remark that the B_s^0 and $\overline{B}_s^0$ mesons do not have charged partners (the isotopic spin equals zero) and, on account of that, in the given case the transition similar to $K^+K^- \to K^0\overline{K}^0$ is absent.

So, the study of pair correlations of internal quantum numbers is very important for understanding the dynamics of various physical processes – especially the processes of multiple production of particles .

Thank you !