Pion-Nucleus Microscopic Optical Potential at Intermediate Energies and In-Medium Effect on the Elementary πN Scattering Amplitude

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[2] *Presentation of two works (titles see below)*

- **ANALYSIS OF THE PION-NUCLEUS ELASTIC SCATTERING USING THE MICROSCOPIC OPTICAL POTENTIAL**

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- **A MODELING OF THE PION-NUCLEUS MICROSCOPIC OPTICAL POTENTIAL AT ENERGIES OF 33-RESONANCE AND IN-MEDIUM EFFECT ON THE PION-NUCLEON AMPLITUDE OF SCATTERING**

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Motivation

- There is a great number of papers on pion-nucleus scattering at different energies
- In theoretical study two approaches are usually employed: *First*, microscopic Kisslinger potential based on s, p, $d-$ phases of the πN amplitudes having six and more parameters obtained from analysis of πN scattering data. *Second*, the Glauber high-energy approximation (HEA) that uses analytic form of the πN amplitude inherent in high energy scattering.
- Here we utilize our HEA-based microscopic optical potential[5] for calculation of π-nucleus elastic scattering.
- The aim of our study is an explanation of experimental data in the region of (3 3)-resonance energies and estimation of the "in-medium" effect on the elementary pion-nucleon amplitude.

[5] V.Lukyanov *et al.* Phys.At.Nucl.**73**(2010)1443

How and what we deal with

- Our microscopic optical potential (OP) is constructed as an optical limit of a Glauber theory. It is defined by known nuclear density distributions and by the elementary πN-amplitude of scattering.
- The πN-amplitude depends on three parameters: total cross section σ, the ratio $\alpha = \Re e\, f(0)/\Im m\, f(0)$, and the slope parameter β. For free πN amplitudes they are obtained from πN scattering data, while for the "in-medium"πN amplitudes one should analyze the data on πA scattering.
- The established best-fit "in-medium" πN parameters are compared with the corresponding parameters of the "free" πN scattering amplitudes.

Basic equations

The cross sections are calculated by solving the Klein-Gordon equation in its form at conditions $E \gg U$

$$\left(\Delta + k^2\right) \psi(\vec{r}) = 2\bar{\mu} U(r) \psi(\vec{r}), \quad U(r) = U^H(r) + U_C(r)$$

Here k is relativistic momentum of pion in c.m. system,

$$k = \frac{M_A k^{\mathrm{lab}}}{\sqrt{(M_A + m_\pi)^2 + 2 M_A T^{\mathrm{lab}}}}, \quad k^{\mathrm{lab}} = \sqrt{T^{\mathrm{lab}} \left(T^{\mathrm{lab}} + 2 m_\pi\right)},$$

and $\bar{\mu} = \dfrac{E M_A}{E + M_A}$ – relativistic reduced mass, $E = \sqrt{k^2 + m_\pi^2}$ – total energy, m_π and M_A – the pion and nucleus masses.

Microscopic OP based on $f_{\pi p}$ and $f_{\pi p}$ amplitudes

HEA-based microscopic OP

$$U^H = -\frac{\hbar c \beta_c}{(2\pi)^2} \sum_{N=p,n} \sigma_{\pi N} \left(\alpha_{\pi N} + i\right) \cdot \int_0^\infty dq\, q^2 j_0(qr) \rho_N(q) f_{\pi N}(q),$$

where $\beta_c = k/E$; $f_{\pi N}(q) = \exp[\frac{-\beta_{\pi N} q^2}{2}]$ – formfactor of πN-amplitude; $\rho(q)$ – formfactor of a nuclear density distribution.

Nuclear density is taken as the symmetrized Fermi-function:

$$\rho_{SF}(r) = \rho_0 \frac{\sinh\left(R/a\right)}{\cosh\left(R/a\right) + \cosh\left(r/a\right)}, \; \rho_0 = \frac{A}{1.25\pi R^3} \left[1 + \left(\frac{\pi a}{R}\right)^2\right]^{-1}$$

Parameters R and a known from electron-nucleus scattering data.

Pion-nucleus 6-parameters fit; $T_{lab} = 291 MeV$

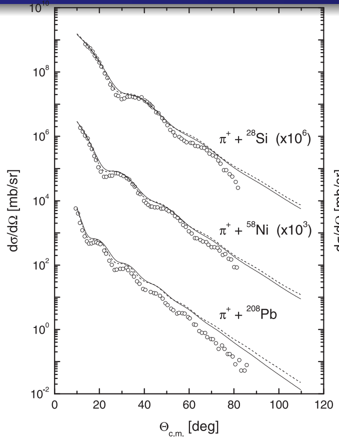


Fig.1. πA calculations with 6 parameters of free $f_{\pi p(n)}$ amplitudes taken from Nucl.Ph.**B38** (1972)221. Here $\chi^2(\pi^-)$=4.67 to 85.32; $\chi^2(\pi^+)$=24.05 to 173.33.

Microscopic OP based on $f_{\pi N}$ amplitude

For pion scattering on nuclei with Z=A-Z, the charge-independent principle $f_{\pi^\pm p} = f_{\pi^\mp n}$ makes available to use only 3 tabulated parameters at different energies

$$\sigma = \frac{1}{2}[\sigma_{\pi^+ p} + \sigma_{\pi^- p}], \quad \alpha = \frac{1}{2}[\alpha_{\pi^+ p} + \alpha_{\pi^- p}], \quad \beta = \frac{1}{2}[\beta_{\pi^+ p} + \beta_{\pi^- p}]$$

Thus the microscopic OP takes the simple shape

$$U^H = -\frac{\hbar c \beta_c}{(2\pi)^2}\sigma\,(\alpha + i)\cdot \int_0^\infty dq\, q^2 j_0(qr)\rho(q) f_{\pi N}(q), \quad f_{\pi N}(q) = e^{\frac{-\beta q^2}{2}}$$

Pion-nucleus 2-parameters fit; $T_{lab} = 291 MeV$

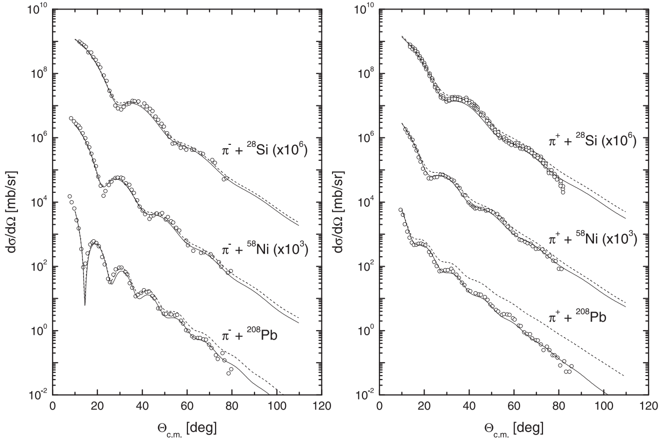


Fig.2. Solid: Fit of 2 from 3 parameters of $f_{\pi N}$ "in-medium" amplitude; Dashed: calculations with "free" $f_{\pi N}$.

2D χ^2-plots of fitted "in-medium"parameters

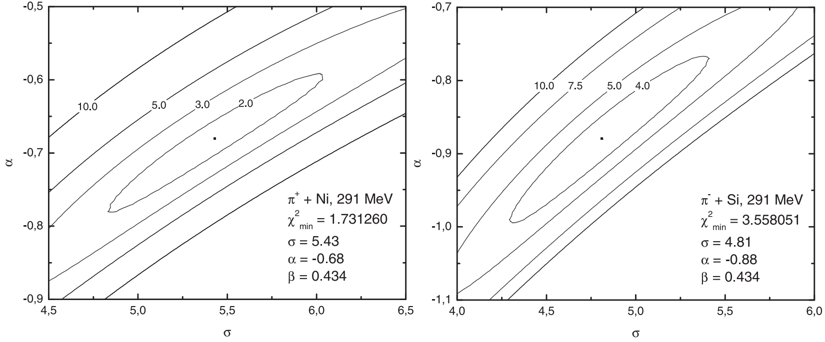


Fig.3. Fit of "in-medium"parameters σ and α of $f_{\pi N}$ amplitude at fixed β=0.434 and T$_{lab}$=291 MeV.

2D χ^2-plots of fitted "in-medium"parameters

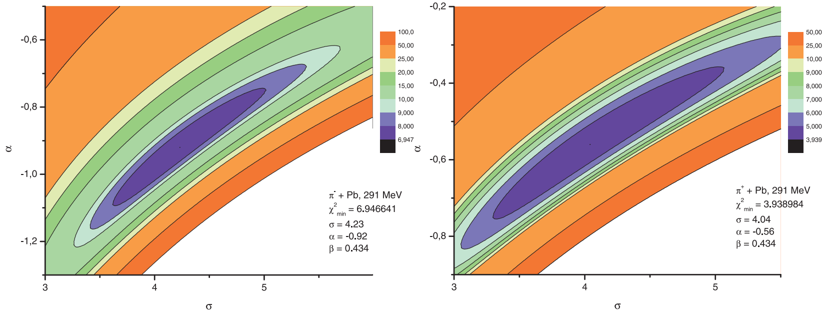


Fig.4. Fit of "in-medium"parameters σ and α of $f_{\pi N}$ amplitude at fixed β=0.434 and T_{lab}=291 MeV.

The σ and α best-fit πN parameters at T_{lab}=291 MeV

reaction	T_{lab}	σ	α	β	χ^2/k
$\pi^- + {}^{28}Si$	291	4.81	-0.88	0.434	3.558
$\pi^+ + {}^{28}Si$		5.55	-0.64	0.434	2.305
$\pi^- + {}^{58}Ni$		4.09	-1.02	0.434	4.255
$\pi^+ + {}^{58}Ni$		5.43	-0.68	0.434	1.731
$\pi^- + {}^{208}Pb$		4.23	-0.92	0.434	6.947
$\pi^+ + {}^{208}Pb$		4.04	-0.56	0.434	3.939
average		4.69	-0.783	0.434	3.789
free $\pi + N$		4.76	-0.95	0.434	

1. Fit at 291 MeV with fixed β yields two "in-medium" parameters σ and α close to "free" one.
2. One sees overall negative α.
3. One should analyze the data at lower energies and by fitting 3 "in-medium" parameters σ, α, and β.

Many parameter fitting technique

Three parameters of the πN scattering amplitude are obtained by fitting to the experimental πA differential cross sections:

- σ, total cross section πN,
- α, ratio of real to imaginary part of the forward πN amplitude,
- β, the slope parameter.

We minimize the function

$$\chi^2 = f(\sigma, \alpha, \beta) = \sum_i \frac{(y_i - \hat{y}_i(\sigma, \alpha, \beta))^2}{s_i^2},$$

where $y_i = \log \frac{d\sigma}{d\Omega}$ and $\hat{y}_i = \log \frac{d\sigma}{d\Omega}(\sigma, \alpha, \beta)$ are, respectively, experimental and theoretical differential cross sections, s_i – experimantal errors. The asynchronous differential evolution technique[16] is applied

[16] E.Zhabitskaya, M.Zhabitsky. Springer Lect.Notes Comp.Sci.**7125**(2012)328

Pion-nucleus elastic scattering; $T_{lab} = 291 MeV$

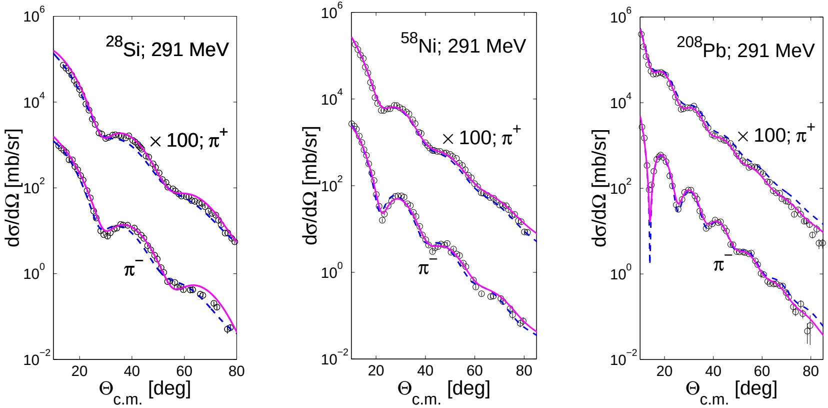


Fig.5. Solid purple: fit to the data[18] of 3 "in-medium" parameters for $f_{\pi N}$ amplitude. Dashed blue: fit ("by hands") of 6 parameters for both $f_{\pi p}$ and $f_{\pi n}$ amplitudes.

[18]Geesaman *et al.* Phys.Rev.**C23**,6(1981)2635

Pion-nucleus elastic scattering; $T_{lab} = 162 MeV$

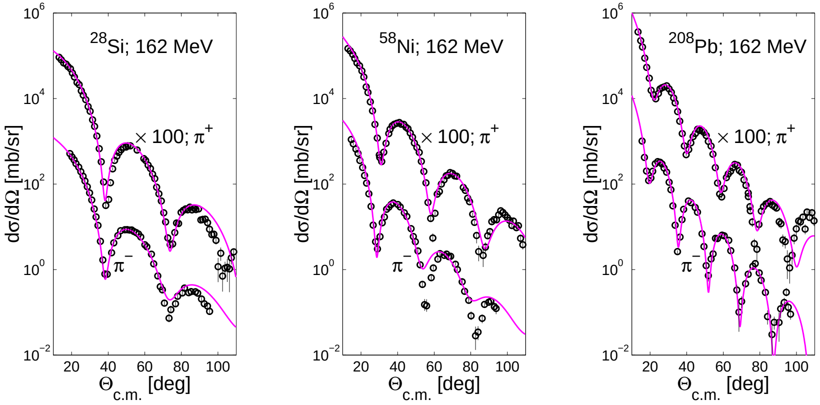


Fig.6. Fit to the data[20] at 162 MeV. Agreement is comparable with that obtained by using Kisslinger potential.

[20] Olmer *et al.* Phys.Rev.**C21**(1980)254

Pion-nucleus elastic scattering; $T_{lab} = 130$ and 180 MeV

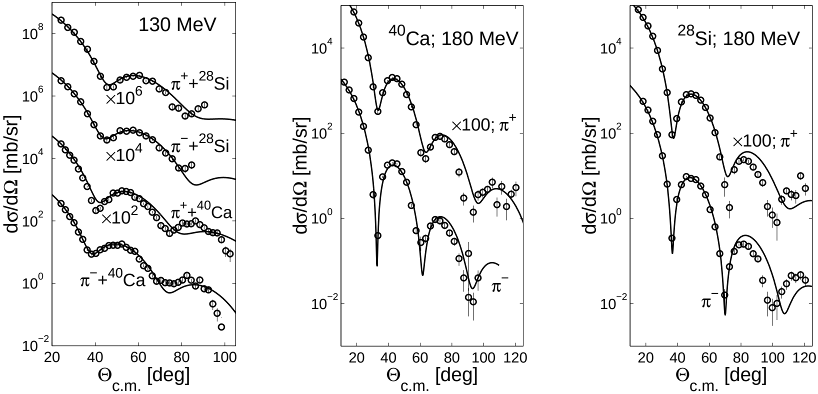
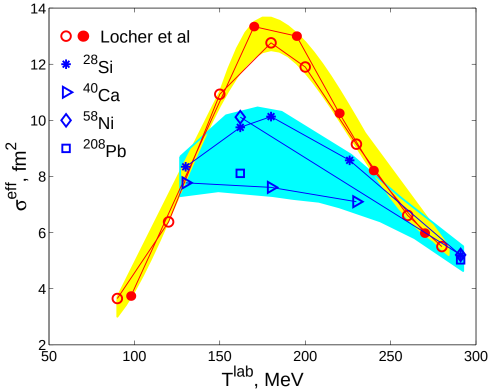


Fig.7. Fit to the data at 180 MeV[22] and 130 MeV[23].
Some dissimilarity is observed at large angles.

[22] Preedom *et al.* Nucl.Phys.**A326**(1979)385
[23] Gretillat *et al.* Nucl.Phys.**A364**(1981)270

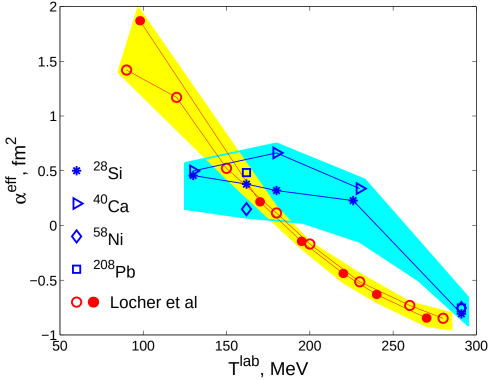
reaction	T_{lab}	χ^2/k	σ	α	β
π^-+^{28}Si	130	2.1	7.08±0.16	0.87±0.05	0.81±0.05
π^++^{28}Si		5.5	9.61±0.14	0.04±0.02	0.85±0.04
π^-+^{40}Ca		3.9	6.97±0.11	0.89±0.01	0.87±0.03
π^++^{40}Ca		13.3	8.58±0.08	0.11±0.01	0.76±0.02
π^-+^{28}Si	162	3.5	11.02±0.1	0.04±0.02	0.39±0.02
π^++^{28}Si		6.7	8.48±0.06	0.71±0.01	0.71±0.01
π^-+^{58}Ni		10.7	10.95±0.1	-0.146±0.01	1.08±0.02
π^++^{58}Ni		7.5	9.28±0.04	-0.444±0.01	0.77±0.01
π^-+^{208}Pb		3.7	9.62±0.09	0.36±0.01	1.02±0.01
π^++^{208}Pb		10.3	6.60±0.03	0.61±0.01	0.01±0.01
π^-+^{28}Si	180	10.5	10.03±0.06	0.33±0.01	0.266±0.01
π^++^{28}Si		12.1	10.24±0.07	0.31±0.01	0.323±0.01
π^-+^{40}Ca		3.3	9.44±0.11	0.25±0.02	0.29±0.01
π^++^{40}Ca		4.2	5.78±0.07	1.08±0.02	0.70±0.02
π^-+^{28}Si	291	3.7	4.17±0.08	1.08±0.02	0.04±0.01
π^++^{28}Si		3.5	3.71±0.07	1.63±0.01	0.32±0.01
π^-+^{58}Ni		3.8	4.78±0.08	-0.85±0.02	0.28±0.02
π^++^{58}Ni		2.6	5.63±0.15	-0.66±0.02	0.36±0.01
π^-+^{208}Pb		4.1	4.50±0.07	-1.06±0.02	0.666±0.02
π^++^{208}Pb		3.0	5.56±0.15	-0.45±0.02	0.588±0.02

In-medium effect on σ^{eff} of πN amplitude



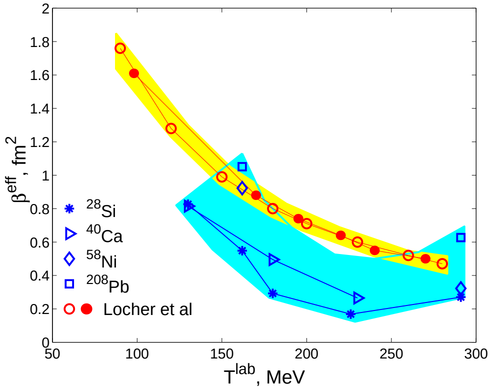
Yellow: parameters of "free" $\pi^{\pm}N$-amplitude from Nucl.Phys.B27(1971)593. Blue: the best fit $\sigma^{eff} = (\sigma_{\pi^+} + \sigma_{\pi^-})/2$.

In-medium effect on α^{eff} of πN amplitude



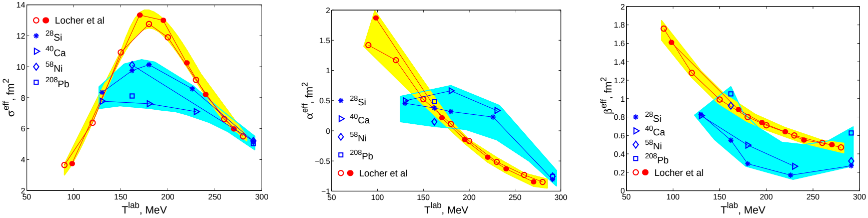
Yellow: parameters of "free" $\pi^{\pm}N$-amplitude from Nucl.Phys.B27(1971)593. Blue: the best fit $\alpha^{eff} = (\alpha_{\pi^+} + \alpha_{\pi^-})/2$.

In-medium effect on β^{eff} of πN amplitude



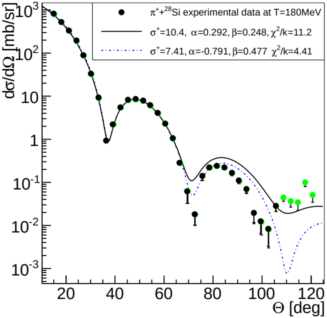
Yellow: parameters of "free" $\pi^{\pm}N$-amplitude from Nucl.Phys.B27(1971)593. Blue: the best fit $\beta^{eff} = (\beta_{\pi^+} + \beta_{\pi^-})/2$.

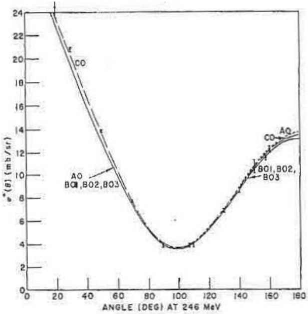
In-medium effect on πN scattering



- Bell-like forms of σ^{free} and $\sigma^{eff}(T^{lab})$ have maximum at the same T^{lab}.
- "Blue" domain σ^{eff} is located below the "yellow" σ^{free} region.
- "In-medium" α^{eff}: refraction increases at energy $T^{lab} > T^{lab}_{res}) \simeq 170$ MeV.
- "Blue" and "yellow" regions become closer at $T^{lab} > 250$ MeV.

Problem of large angles. The case $\pi^+ + ^{28}$Si at 180 MeV:

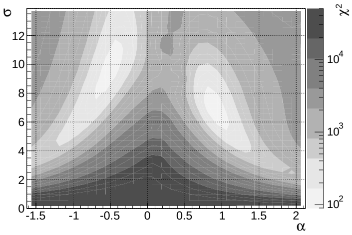


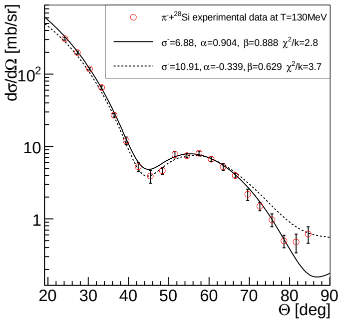


- Left panel: Agreement with experiment is improved as we remove a few experimental points (green) at large angles.
- Right panel[30]: Gaussian form $f_\pi(q) = \exp[\frac{-\beta q^2}{2}]$ is not realistic for large angles.

[30] reproduced from Roper *et al.* Phys.Rev.**B138**,48(1965)

Ambiguity problem. The case $\pi^- + {}^{28}$Si at 130 MeV:





- Left panel: two minima exist on the (α, σ)-plane $(\beta = 0.9)$.
- Right panel: Two sets of parameters provide almost the same agreement with experimental data. Additional information (total cross section) is needed to make a choice.

Summary

- We show that the HEA-based three-parametric microscopic OP provides a reasonable agreement with experimental data of pion-nucleus elastic scattering at intermediate energies between 130 and 290 MeV.
- Comparison of σ^{free} and σ^{eff} shows that, at (3 3)-resonance energies, the πN-interaction in nuclear matter is weaker than in the case of free πN collisions.
- Behavior of parameter α indicates that the refraction increases at energies more than $T^{lab}_{res} \simeq 170$ MeV.
- Total cross section data are desirable to resolve the ambiguity problem.

Thank you for your attention!