TOP QUARK AS RESONANCE: RENORMALIZATION AND SPIN EFFECT

A.E. Kaloshin[1], V.P. Lomov[2]

[1]Irkutsk State University, Irkutsk

[2]Institute for System Dynamics and Control Theory of SB RAS, Irkutsk
Irkutsk State Technical University, Irkutsk

XXI INTERNATIONAL BALDIN SEMINAR ON HIGH ENERGY PHYSICS PROBLEMS «RELATIVISTIC NUCLEAR PHYSICS & QUANTUM CHROMODYNAMICS»

September 12, 2012

Introduction

The top quark plays a special role in Standard Model and it is an object of intensive research at LHC. Being a short-living particle (due to the open channels with W-boson on mass shell), it may be considered on an equal footing with ordinary hadron resonances. The dressed propagator can be obtained as a result of Dyson summation of self-energy insertions or, equivalently, by solving the Dyson–Schwinger equation. As for top quark, its vertex violates parity, so γ^5 takes part in this process, and it leads to nonstandard form of resonance factor, as we shall see below.

The form of fermion resonance with parity violation was discussed earlier. In particular, in A.E. Kaloshin, V.P. Lomov, Phys. Atom. Nucl. **69**, 541 (2006) were written general formulas for dressed propagator with the use of the off-shell basis. The paper B.A. Kniehl, A. Sirlin, Phys.Rev. **D77**, 116012 (2008) was devoted to extension of the concept of pole mass and width to the case of the parity violation. The obtained dressed fermion propagator was written in a boson-like form without separation of the positive and negative energy poles. It is difficult to compare this general expression with the standard Breit–Wigner form, in particular to recognize there the on-shell decay width.

Standard Breit–Wigner formula in QFT

To obtain Breit–Wigner-like formula in QFT one needs to solve the Dyson–Schwinger equation for the dressed propagator,

$$G = G_0 + G_0 \Sigma G, \quad \text{or} \quad G^{-1} = G_0^{-1} - \Sigma. \tag{1}$$

For bosons one has

$$G_0 = \frac{1}{m_0^2 - s - \mathrm{i}\,\varepsilon} \quad \text{and} \quad G = \frac{1}{m_0^2 - s - \Sigma(s)} \sim \frac{1}{m^2 - s - \mathrm{i}\,\Gamma m},$$

if Σ has imaginary part, the dressed propagator G should be compared with relativistic Breit–Wigner formula for renormalization. For fermions all is similar:

$$G_0 = \frac{1}{\hat{p} - m_0} \quad \text{and} \quad G = \frac{1}{\hat{p} - m_0 - \Sigma(p)},$$

but to make this procedure more transparent, it is convenient to pass to off-shell projection operators.

Let's define off-shell projection operators as follows:

$$\Lambda^{\pm} = \frac{1}{2}\Big(1 \pm \frac{\hat{p}}{W}\Big), \qquad (2)$$

where $W = \sqrt{p^2}$ is invariant mass or rest-frame energy.
In this basis dressing looks like

$$G_0 = \frac{1}{\hat{p} - m_0} = \Lambda^{+}\frac{1}{W - m_0} + \Lambda^{-}\frac{1}{-W - m_0} \Rightarrow$$
$$\Rightarrow G = \Lambda^{+}\frac{1}{W - m_0 - \Sigma_1(W)} + \Lambda^{-}\frac{1}{-W - m_0 - \Sigma_2(W)}, \qquad (3)$$

where the self-energy is also decomposed in this basis

$$\Sigma(p) = A(p^2) + \hat{p}B(p^2) \equiv \Lambda^{+}\Sigma_1(W) + \Lambda^{-}\Sigma_2(W).$$

The positive energy pole should be compared with Breit–Wigner formula

$$\frac{1}{W - m_0 - \Sigma_1(W)} \sim \frac{1}{W - m + \mathrm{i}\,\Gamma/2}. \qquad (4)$$

Fermion Resonance with Parity Violation

In case of parity violation the projection basis (2) must be supplemented by elements with γ^5, it is handy to choose the basis as

$$\mathcal{P}_1 = \Lambda^+, \quad \mathcal{P}_2 = \Lambda^-, \quad \mathcal{P}_3 = \Lambda^+\gamma^5, \quad \mathcal{P}_4 = \Lambda^+\gamma^5. \tag{5}$$

Now the decomposition of a self-energy or a propagator has four terms

$$S = \sum_{M=1}^{4} S_M \mathcal{P}_M, \tag{6}$$

where coefficients S_M are followed by obvious symmetry properties

$$S_2(W) = S_1(-W), \quad S_4(W) = S_3(-W).$$

With the use of decomposition (6), the Dyson–Schwinger equation (1) is reduced to the set of equations for scalar coefficients

$$S_M = (S_0)_M - \Sigma_M, \quad M = 1, \ldots, 4. \tag{7}$$

Considering the self-energy Σ as a known value, we obtain the dressed propagator

$$G = \sum_{M=1}^{4} G_M \mathcal{P}_M,$$

where the coefficients G_M are

$$G_1 = \frac{S_2}{\Delta}, \quad G_2 = \frac{S_1}{\Delta}, \quad G_3 = -\frac{S_3}{\Delta}, \quad G_4 = -\frac{S_4}{\Delta}, \tag{8}$$

and $\Delta = S_1 S_2 - S_3 S_4$.

In spite of simple answer (7), it is inconvenient because the positive and negative energy poles are not separated, compare with formula

$$G = \Lambda^{+} \frac{1}{W - m_0 - \Sigma_1(W)} + \Lambda^{-} \frac{1}{-W - m_0 - \Sigma_2(W)}.$$

We want to obtain the analog of this formula for the parity non-conservation case.

Spectral Representation of Propagator

In order to obtain the analog of above mentioned formula in case of parity violation, we use the spectral representation of inverse propagator

$$S = \lambda_1 \Pi_1 + \lambda_2 \Pi_2, \tag{9}$$

where Π_k are projectors, satisfying the eigenstate problem

$$S\Pi_k = \lambda_k \Pi_k. \tag{10}$$

Let's write the dressed propagator $S(p)$ as

$$S = \sum_{M=1}^{4} S_M \mathcal{P}_M,$$

with arbitrary coefficients and will look for the matrix Π in the same form with coefficients a_M.

It is easy to find that eigenvalues λ_i are roots of the equation

$$\lambda^2 - \lambda(S_1 + S_2) + (S_1 S_2 - S_3 S_4) = 0, \qquad (11)$$

and solution of (10) is

$$\Pi_i = \mathcal{P}_1 a_1^i + \mathcal{P}_2 a_2^i - \frac{S_3}{S_1 - \lambda_i} a_2^i \mathcal{P}_3 - \frac{S_4}{S_2 - \lambda_i} a_1^i \mathcal{P}_4$$

with arbitrary coefficients a_1, a_2.
In order (10) to be a projector, $\Pi^2 = \Pi$, we need only one additional condition

$$a_2 = 1 - a_1.$$

After it the orthogonality property $\Pi_1 \Pi_2 = \Pi_2 \Pi_1 = 0$ defines a_1 coefficient

$$a_1^1 = \frac{S_2 - \lambda_1}{\lambda_2 - \lambda_1}, \quad a_1^2 = -\frac{S_2 - \lambda_2}{\lambda_2 - \lambda_1}.$$

As result we have the projectors

$$\Pi_1 = \frac{1}{\lambda_2 - \lambda_1}\Big((S_2 - \lambda_1)\mathcal{P}_1 + (S_1 - \lambda_1)\mathcal{P}_2 - S_3\mathcal{P}_3 - S_4\mathcal{P}_4\Big),$$
$$\Pi_2 = \frac{1}{\lambda_1 - \lambda_2}\Big((S_2 - \lambda_2)\mathcal{P}_1 + (S_1 - \lambda_2)\mathcal{P}_2 - S_3\mathcal{P}_3 - S_4\mathcal{P}_4\Big), \tag{12}$$

with desired properties:

- $S\Pi_k = \lambda_k \Pi_k$,
- $\Pi_k^2 = \Pi_k$,
- $\Pi_1\Pi_2 = \Pi_2\Pi_1 = 0$,
- $\Pi_1 + \Pi_2 = 1$.

The dressed propagator $G(p)$ is obtained by reversing of equation (9)

$$G = \frac{1}{\lambda_1}\Pi_1 + \frac{1}{\lambda_2}\Pi_2. \tag{13}$$

The determinant $\Delta(W)$ of S is

$$\Delta(W) = S_1 S_2 - S_3 S_4 = (W - m_0 - \Sigma_1)(-W - m_0 - \Sigma_2) - \Sigma_3\Sigma_4,$$

where $\Sigma_i(W)$ are self-energy components in the basis. Free propagator has poles at points $W = m_0$ and $W = -m_0$, the dressed one has them at $W = m$ and $W = -m$. On the other hand, $\Delta(W)$ is equal to product of eigenvalues

$$\Delta(W) = \lambda_1(W)\lambda_2(W). \tag{14}$$

Top Quark as Resonance

Consider the dressing of top quark in SM. The main one-loop contribution to self-energy arises from Wb intermediate state

$$\Sigma(p) = -\,\mathrm{i}\, g^2 |V_{\mathrm{tb}}|^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \gamma^\mu (1-\gamma^5) \frac{\hat{p} - \hat{k} + m_{\mathrm{b}}}{(p-k)^2 - m_{\mathrm{b}}^2} \times \\ \times \gamma^\nu (1-\gamma^5) \frac{g_{\mu\nu} - k_\mu k_\nu / m_{\mathrm{W}}^2}{k^2 - m_{\mathrm{W}}^2}, \quad (15)$$

and generates only kinetic term

$$\Sigma(p) = \hat{p}(1-\gamma^5)\Sigma_0(W^2). \quad (16)$$

Its decomposition in the basis (5) has the following coefficients:

$$\Sigma_1 = W\Sigma_0(W^2),\ \Sigma_2 = -W\Sigma_0,\ \Sigma_3 = -W\Sigma_0,\ \Sigma_4 = W\Sigma_0.$$

As a preliminary, let us forget about renormalization of self-energy and calculate the eigenvalues

$$\lambda_{1,2} = -m \pm W\sqrt{1 - 2\Sigma_0(W^2)}.$$

In analogy with OMS scheme let's subtract the real part of self-energy at resonance point

$$\lambda_{1,2} = -m \pm W\sqrt{1 - 2\big(\Sigma_0(W^2) - \mathrm{Re}\,\Sigma_0(m^2)\big)}.$$

As a result we have rather unusual resonance factor

$$\frac{1}{\lambda_1(W)} = \frac{1}{W\sqrt{1 + \mathrm{i}\,\dfrac{\Gamma}{m}} - m}, \tag{17}$$

which only at $\Gamma/m \ll 1$ returns to standard Breit–Wigner form,

$$\frac{1}{\lambda_1(W)} \simeq \frac{1}{W - m + \mathrm{i}\,W\dfrac{\Gamma}{2m}} \quad \text{at } \Gamma/m \ll 1.$$

Let's suppose that self-energy does not have imaginary part. We put:

- Σ_1 has zero of second order at $W = m$
- Σ_3 has zeroes at $W = m$ and $W = -m$.

The Σ_2 and Σ_4 are defined by substitution $W \to -W$, so the OMS renormalization in this case is

$$\Sigma_1^{\mathrm{r}}(W) = \Sigma_1(W) - \Sigma_1(m) - \Sigma_1'(m)(W - m),$$
$$\Sigma_2^{\mathrm{r}}(W) = \Sigma_1^{\mathrm{r}}(-W),$$
$$\Sigma_3^{\mathrm{r}}(W) = -W\Big(\Sigma_0(W^2) - \Sigma_0(m^2)\Big),$$
$$\Sigma_4^{\mathrm{r}}(W) = \Sigma_3^{\mathrm{r}}(-W).$$

Eigenvalues in OMS scheme are

$$\lambda_{1,2}(W) = -mK \pm WK\sqrt{d}, \text{ where } d = 1 - 2\widetilde{\Sigma}/K \tag{18}$$

and $K = 1 + 2m^2\Sigma_0'(m^2)$, $\widetilde{\Sigma} = \Sigma_0(W^2) - \Sigma_0(m^2)$.
Let us write down the eigenvalues in vicinity of $W = m$

$$\lambda_1(W) = W - m + o(W - m),$$
$$\lambda_2(W) = -2mK - (W - m) + o(W - m),$$

and in vicinity of $W = -m$

$$\lambda_1(W) = -2mK - (-W - m) + o(-W - m),$$
$$\lambda_2(W) = -W - m + o(-W - m).$$

Projectors on eigenstates have the form

$$\Pi_1 = \mathcal{P}_1 \frac{\sqrt{d} + (1 - \widetilde{\Sigma}/K)}{2\sqrt{d}} + \mathcal{P}_2 \frac{\sqrt{d} - (1 - \widetilde{\Sigma}/K)}{2\sqrt{d}} - \mathcal{P}_3 \frac{\widetilde{\Sigma}/K}{2\sqrt{d}} + \mathcal{P}_4 \frac{\widetilde{\Sigma}/K}{2\sqrt{d}},$$
$$\Pi_2 = \mathcal{P}_1 \frac{\sqrt{d} - (1 - \widetilde{\Sigma}/K)}{2\sqrt{d}} + \mathcal{P}_2 \frac{\sqrt{d} + (1 - \widetilde{\Sigma}/K)}{2\sqrt{d}} + \mathcal{P}_3 \frac{\widetilde{\Sigma}/K}{2\sqrt{d}} - \mathcal{P}_4 \frac{\widetilde{\Sigma}/K}{2\sqrt{d}}, \tag{19}$$

and dressed propagator is

$$G(p) = \frac{m_0 + \hat{p} - \hat{p}(1 + \gamma^5)\widetilde{\Sigma}/K}{K(W^2 d - m_0^2)}.$$

The expressions for eigenvalues and projectors may be simplified in vicinity of $W^2 = m^2$, where $\widetilde{\Sigma}(W) \ll 1$ and we take into account only linear in $\widetilde{\Sigma}$ terms

$$\lambda_{1,2}(W) = K(-m \pm W) \mp W\widetilde{\Sigma}(W^2),$$

$$\Pi_1 = \mathcal{P}_1 - \mathcal{P}_3\frac{\widetilde{\Sigma}}{2K} + \mathcal{P}_4\frac{\widetilde{\Sigma}}{2K} = \Lambda^+ - \frac{\widetilde{\Sigma}(W^2)}{2K}\frac{\hat{p}\gamma^5}{W},$$

$$\Pi_2 = \mathcal{P}_2 + \mathcal{P}_3\frac{\widetilde{\Sigma}}{2K} - \mathcal{P}_4\frac{\widetilde{\Sigma}}{2K} = \Lambda^- + \frac{\widetilde{\Sigma}(W^2)}{2K}\frac{\hat{p}\gamma^5}{W}.$$

Let's consider the case when the self-energy $\Sigma(W)$ acquire the imaginary part. The formulas for eigenvalues and projectors, (18) and (19), remain the same, but in this case

$$\widetilde{\Sigma}(W^2) = \Sigma_0(W^2) - \mathrm{Re}\,\Sigma_0(m^2), \quad \text{and} \quad K = 1 + 2m^2(\mathrm{Re}\,\Sigma_0)'(m^2).$$

Resonance factor $1/\lambda_1$ in vicinity of $W = m$ practically coincides with naive expression (17)

$$\frac{1}{\lambda_1(W)} = \frac{1}{K\Big(W\sqrt{1 - 2\widetilde{\Sigma}/K} - m\Big)} \approx \frac{1}{K\Big(W\sqrt{1 + \mathrm{i}\,\frac{\Gamma(W)}{KW}} - m\Big)}, \tag{20}$$

if to introduce the energy-dependent width $\Gamma(W) = -2W\,\mathrm{Im}\,\Sigma_0(W^2)$.

At small Γ resonance factor returns to standard form

$$\frac{1}{\lambda_1(W)} \simeq \frac{1}{W - m + \mathrm{i}\,\Gamma(W)/2} \quad \text{at } W \simeq m,\ \Gamma/m \ll 1.$$

Using the same approximations in projectors, we can write down a parametrization of dressed propagator in vicinity of $W = m$:

$$G = \frac{1}{W - m + \mathrm{i}\,\Gamma(W)/2}\Big(\mathcal{P}_1 + \mathrm{i}\,\frac{\Gamma(W)}{4KW^2}\hat{p}\gamma^5\Big) + \\ + \frac{1}{-2mK - (W - m) - \mathrm{i}\,\Gamma(W)/2}\Big(\mathcal{P}_2 - \mathrm{i}\,\frac{\Gamma(W)}{4KW^2}\hat{p}\gamma^5\Big). \quad (21)$$

Pole Scheme and Spectral Representation

The pole renormalization scheme for fermion with parity non-conservation have been considered in detail in work B.A. Kniehl, A. Sirlin, Phys.Rev. **D77**, 116012 (2008). We will consider the pole scheme on the base of spectral representation. In this case it is sufficient to renormalize the single pole contribution $1/\lambda_1(W)$. It simplifies essentially the algebraic procedure and clarifies some aspects.

The inverse propagator has the form

$$S(p) = \hat{p} - m_0 - \Sigma(p) = \\ = \hat{p} - m_0 - \left(A(p^2) + \hat{p}B(p^2) + C(p^2)\gamma^5 + \hat{p}\gamma^5 D(p^2)\right). \quad (22)$$

In CP-symmetric theory $C(p^2) = 0$.

In terms of scalar functions the eigenvalues and corresponding projectors (12) have the form

$$\lambda_1(W) = -m_0 - A(W^2) + WR(W^2),$$
$$\lambda_2(W) = \lambda_1(-W),$$
$$\Pi_1(W) = \frac{1}{2}\Big[1 - \gamma^5 \frac{C(W^2)}{WR(W^2)} + \frac{\hat{p}}{W}\Big(\frac{1 - B(W^2)}{R(W^2)} - \gamma^5 \frac{D(W^2)}{R(W^2)}\Big)\Big],$$
$$\Pi_2 = \Pi_1(-W),$$

where we have introduced the notation

$$R(W^2) = \sqrt{\big(1 - B(W^2)\big)^2 - D^2(W^2) + C^2(W^2)/W^2}.$$

Let's $\lambda_1(W_1) = 0$, where $W_1 = M_p - \mathrm{i}\,\Gamma_p/2$:

$$-m_0 - A(W_1^2) + W_1 R(W_1^2) = 0.$$

Real part of this equality allows to get rid of m_0 in dressed propagator

$$S(p) = \hat{p} - \Big(\tilde{A}(p^2) + \hat{p}B(p^2) + \gamma^5 C(p^2) + \hat{p}\gamma^5 D(p^2)\Big),$$
$$\tilde{A}(p^2) = A(p^2) - A(W_1^2) + \big(W_1 R(W_1^2)\big).$$

The imaginary part of (23),

$$\mathrm{Im}\,\Big(- A(W_1^2) + W_1 R(W_1^2)\Big) = 0$$

gives relation between Γ_p and self-energy at pole point. In particular, in case of parity conservation it reduces to the obvious relation

$$\mathrm{Im}\,\Big(W_1 - \big(A(W_1^2) + W_1 B(W_1^2)\big)\Big) = 0, \quad \text{or} \quad \frac{\Gamma_p}{2} = -\,\mathrm{Im}\,\Sigma_1(W_1^2).$$

Let's introduce wave function renormalization constants connecting bare and renormalized fields

$$\Psi = Z^{1/2}\Psi^{\rm r}, \quad \bar{\Psi} = \bar{\Psi}^{\rm r}\bar{Z}^{1/2}.$$

In case of parity violation $Z^{1/2}$, $\bar{Z}^{1/2}$ are matrices

$$Z^{1/2} = \alpha + \beta\gamma^5, \quad \bar{Z}^{1/2} = \bar{\alpha} + \bar{\beta}\gamma^5.$$

Renormalized inverse propagator

$$\begin{aligned} S^{\rm r}(p) = (\bar{\alpha} + \bar{\beta}\gamma^5)\Big[\hat{p} - \big(\tilde{A} + \hat{p}B + \gamma^5 C + \hat{p}\gamma^5 D\big)\Big](\alpha + \beta\gamma^5) = \\ = I\big[-\tilde{A}(\alpha\bar{\alpha} + \bar{\beta}\beta) - C(\bar{\alpha}\beta + \bar{\beta}\alpha)\big] + \\ + \hat{p}\big[(1-B)(\alpha\bar{\alpha} - \beta\bar{\beta}) - D(\bar{\alpha}\beta - \bar{\beta}\alpha)\big] + \\ + \gamma^5\big[-C(\bar{\alpha}\alpha + \bar{\beta}\beta) - \tilde{A}(\bar{\alpha}\beta + \bar{\beta}\alpha)\big] + \\ + \hat{p}\gamma^5\big[-D(\bar{\alpha}\alpha - \bar{\beta}\beta) + (1-B)(\bar{\alpha}\beta - \bar{\beta}\alpha)\big] \end{aligned} \tag{23}$$

allows to obtain the renormalized components of self-energy.

Looking at first term in spectral representation, we see that renormalization is divided into two parts: renormalization of eigenvalue and projector.
For stable fermion there is a physical requirement for projector. The projector at point $W = m$ has form

$$\Pi_1^{\mathrm{r}}(m) = \frac{1}{2}\Big[1 - \gamma^5 c + \frac{\hat{p}}{m}(b - \gamma^5 d)\Big],$$

where parameters b, d and c are related by $b^2 - d^2 + c^2 = 1$.
However, if $c \neq 0$, $d \neq 0$ then $\Pi_1^{\mathrm{r}}(m)$ do not commutate with spin projector, what leads to spin flip for fermion on mass shell.
Therefore there are requirements for renormalization of a stable fermion:

$$C^{\mathrm{r}}(m^2) = 0, \quad D^{\mathrm{r}}(m^2) = 0. \tag{24}$$

For unstable fermion, when pole is at point $W_1 = M_p - \mathrm{i}\,\Gamma_p/2$, there is some arbitrariness. The simplest generalization of (24) consists in:

$$C^{\mathrm{r}}(W_1^2) = 0, \quad D^{\mathrm{r}}(W_1^2) = 0. \tag{25}$$

The same relations arise from a principle, suggested in B.A. Kniehl, A. Sirlin, Phys.Rev. **D77**, 116012 (2008): the chiral components should have poles with unit absolute value of residue.

A few words about the relation between renormalization constants $Z^{1/2}$, $\bar{Z}^{1/2}$. The pseudo-hermiticity condition

$$\bar{Z}^{1/2} = \gamma^0 \big(Z^{1/2}\big)^{\dagger} \gamma^0, \tag{26}$$

is traditionally used in literature, which is reduced to $\bar{\alpha} = \alpha^*$, $\bar{\beta} = -\beta^*$. However, as it was noted in D. Espriu, J. Manzano, P. Talavera, Phys. Rev. **D66**, 076002 (2002), one should refused from this condition, if self-energy has absorptive parts. The same is seen from our renormalized propagator (23).

Assuming pseudo-hermiticity we calculate $D^{\mathrm{r}}(W^2)$ thus:

$$D^{\mathrm{r}}(W^2) = |\alpha|^2 \Big\{ D(W^2)\Big(1 + \frac{|\beta|^2}{|\alpha|^2}\Big) - \\ - \big(1 - B(W^2)\big)\Big(\frac{\beta}{\alpha} + \frac{\beta^*}{\alpha^*}\Big)\Big\}. \quad (27)$$

Because $D(W^2)$ and $B(W^2)$ contain physically different contributions we cannot provide the condition $D^{\mathrm{r}}(W_1^2) = 0$ for complex self-energy. So, the pseudo-hermiticity condition seems to be too restrictive for parity violating theory.

Let's consider below the case of CP conservative theory when component $C(p^2) = 0$. In order to avoid CP violation under renormalization it is necessary to require (see (23))

$$\bar{\alpha}\beta + \bar{\beta}\alpha = 0. \tag{28}$$

The pseudo-hermiticity condition (26) leads to (28) in case of real α, β (stable fermion). However, for resonance one have to refuse from pseudo-hermiticity, (26).

Putting into account the condition (28) the renormalized inverse propagator becomes

$$S^{\mathrm{r}} = \alpha\bar{\alpha}\Big\{ - \tilde{A}(W^2)(1-x^2) + \\ + \hat{p}\Big[(1-B(W^2))(1+x^2) - D(W^2)2x\Big] + \\ + \hat{p}\gamma^5\Big[- D(W^2)(1+x^2) + (1-B(W^2))2x\Big]\Big\}, \qquad (29)$$

where α, $\bar{\alpha}$ and $x = \beta/\alpha$ are complex numbers.
The condition at pole $D^{\mathrm{r}}(W_1^2) = 0$ defines

$$x \equiv \frac{\beta}{\alpha} = \frac{1 - B_1 - R_1}{D_1},$$

where $B_1 = B(W_1^2)$, $D_1 = D(W_1^2)$, $R_1 = R(W_1^2)$.

Substituting that into $S^{\rm r}$, taking out common factor and denoting it by Z we get

$$\begin{aligned}S^{\rm r} &= Z\Big\{-\tilde{A}(W^2)+\\ &+\hat{p}\Big[(1-B(W^2))\frac{1-B_1}{R_1}-D(W^2)\frac{D_1}{R_1}\Big]+\\ &+\hat{p}\gamma^5\Big[-D(W^2)\frac{1-B_1}{R_1}+(1-B(W^2))\frac{D_1}{R_1}\Big]\Big\}=\\ &=\hat{p}-\Sigma^{\rm r},\end{aligned} \tag{30}$$

where renormalized components are given by

$$\begin{aligned}\tilde{A}^{\rm r}(W^2) &= Z\tilde{A}(W^2),\\ B^{\rm r}(W^2) &= 1-Z\Big[\big(1-B(W^2)\big)\frac{1-B_1}{R_1}-D(W^2)\frac{D_1}{R_1}\Big],\\ D^{\rm r}(W^2) &= Z\Big[D(W^2)\frac{1-B_1}{R_1}-\big(1-B(W^2)\big)\frac{D_1}{R_1}\Big].\end{aligned}$$

To determine Z factor we consider renormalized eigenvalue $\lambda_1^{\mathrm{r}}(W)$, its derivative at $W = W_1$ has to equal 1. It is easy to check that

$$R^{\mathrm{r}}(W^2) = \sqrt{(1 - B^{\mathrm{r}}(W^2))^2 - (D^{\mathrm{r}}(W^2))^2} = ZR(W),$$

and

$$\lambda_1^{\mathrm{r}}(W) = Z\lambda_1(W).$$

If to require $(\lambda_1^{\mathrm{r}})'(W_1) = 1$ it gives

$$Z = \frac{1}{R(W_1^2) + 2W_1^2 R'(W_1^2) - 2W_1 A'(W_1^2)}. \qquad (31)$$

In case of unstable fermions, the right hand side of (31) is, generally speaking, complex. If we define

$$\lambda^{\mathrm{r}}_{1,2}(W) = |Z|\lambda_{1,2}(W), \tag{32}$$

we have the renormalized propagator with $\lambda_i(W)$ satisfying the Schwartz principle,

$$\lambda^{\mathrm{r}}_i(W^*) = \left(\lambda^{\mathrm{r}}_i(W)\right)^*. \tag{33}$$

So, λ^{r}_i has zeroes at complex conjugate points W_1, W_1^* with unit absolute value of residues.

Conclusions

- We studied in detail the dressing of fermion propagator in the case of the parity non-conservation. We found the representation of propagator where the positive and negative energy poles are separated from each other. The spectral representation also allows to perform pole renormalization in a simple and compact way.
- In case of parity violation the resonance factor (20) differs from Breit–Wigner-like formula. The reason is that in presence of γ^5 the Dyson summation of the self-energy insertions in a propagator takes another form. But in case of SM vertex the self-energy contains only the kinetic term and the obtained resonance factor $1/\lambda_1(W)$ returns to the standard form for small width $\Gamma/m \ll 1$.

Conclusions

- The possibility to see a deviation is related with projectors (12). One sees that Π_k do not commutate with spin projectors $(1 \pm \gamma^5 \hat{s})/2$ and this fact can lead to non-trivial spin properties.
- It is possible to generalize the spectral representation for matrix case, when the coefficients in (6) are matrices, and to use it for mixing problem with parity violation.

Multiplication properties of $\mathcal{P}$ basis

	$\mathcal{P}_1$	$\mathcal{P}_2$	$\mathcal{P}_3$	$\mathcal{P}_4$
$\mathcal{P}_1$	$\mathcal{P}_1$	0	$\mathcal{P}_3$	0
$\mathcal{P}_2$	0	$\mathcal{P}_2$	0	$\mathcal{P}_4$
$\mathcal{P}_3$	0	$\mathcal{P}_3$	0	$\mathcal{P}_1$
$\mathcal{P}_4$	$\mathcal{P}_4$	0	$\mathcal{P}_2$	0

Table : Multiplication table for $\mathcal{P}$-basis

Taking into account the expression for Z, we obtain following formulae for renormalized components

$$
\begin{aligned}
A^{\mathrm{r}}(W^2) &= Z\tilde{A}(W^2) = \\
&= Z\Big[A(W^2) - A(W_1^2) + W_1 R(W_1^2)\Big], \\
B^{\mathrm{r}}(W^2) &= Z\Big\{-2W_1 A'(W_1^2) + 2W_1^2 R'(W_1^2) + \\
&+ \big(B(W^2) - B(W_1^2)\big)\frac{1 - B(W_1^2)}{R(W_1^2)} + \\
&+ \big(D(W^2) - D(W_1^2)\big)\frac{D(W_1^2)}{R(W_1^2)}\Big\}, \\
D^{\mathrm{r}}(W^2) &= Z\Big\{\big(D(W^2) - D(W_1^2)\big)\frac{1 - B(W_1^2)}{R(W_1^2)} + \\
&+ \big(B(W^2) - B(W_1^2)\big)\frac{D(W_1^2)}{R(W_1^2)}\Big\}.
\end{aligned}
$$