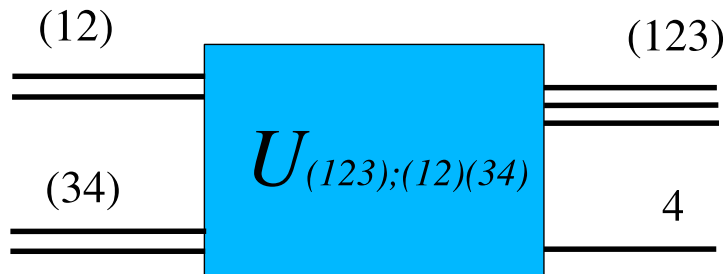


N.B. Ladygina

DIFFERENTIAL CROSS SECTIONS AND ANALYZING POWERS IN
THE $dd \rightarrow {}^3\text{He} n$ REACTION AT INTERMEDIATE ENERGIES

- $dd \rightarrow {}^3\text{He} n(tp)$ is considered at the deuteron kinetic energy between 200 MeV and 520 MeV.
- The theoretical model is suggested to describe differential cross sections and polarization observables.
- The calculation results for the differential cross sections are presented at the energies of 300-520 MeV.
- The vector and tensor analyzing powers are given at 200 MeV .



$$U_{dd \rightarrow {}^3He n} = \delta(2E_d - E_h - E_n) \mathcal{J}$$

The matrix element of the transition operator U is a combination of two terms:

$$\langle n^3He | U | dd \rangle = \frac{1}{\sqrt{6}} [\langle 4(123)_s | U | (12)_s (34)_s \rangle - \langle 1(234)_s | U | (12)_s (34)_s \rangle]$$

The initial state includes two antisymmetrized two-nucleon states

$$(12)_s = \frac{1}{\sqrt{2}} [(12) - (21)]$$

and the final state contains an antisymmetrized three-nucleon state

$$(123)_s = \frac{1}{\sqrt{6}} [(123) - (213) + (231) - (321) + (312) - (132)]$$

- Following P.Grassberger, W.Sandhas, Nucl.Phys.2, 181 (1967) we define transition operator as

$$U_{\beta\alpha}(z) = (1 - \delta_{\beta\alpha})(z - H_{\alpha}) + \sum_{ik \notin \beta} T_{ik}(z)G_0(z)U_{ik,\alpha}(z) + \sum_{ik \notin \beta} V_{\alpha}\delta_{\alpha,ik}$$

α, β are two-cluster partitions of the four-particle state.

- Reaction $dd \rightarrow {}^3He n$ corresponds to the case of
 $\alpha = (ij)(kl) = (12)(34)$,
 $\beta = (ijk) = (123)$ [or (234)]

- Interaction $V_{\alpha(\beta)}$ is defined by two-nucleon potential:

$$V_{\alpha} \equiv V_{(ij)(kl)} = V_{ij} + V_{kl}$$

$$V_{\beta} \equiv V_{(ijk)} = V_{ij} + V_{jk} + V_{ik}$$

$$U_{\beta\alpha}(z) = (1 - \delta_{\beta\alpha})(z - H_\alpha) + \sum_{ik \notin \beta} T_{ik}(z)G_0(z)U_{ik,\alpha}(z) + \sum_{ik \notin \beta} V_\alpha \delta_{\alpha,ik}$$

The channel Hamiltonian H_α is defined as a sum of the free particles Hamiltonian H_0 and the interaction potential V_α

$$H_\alpha = H_0 + V_\alpha$$

$T_{ij}(z)$ is a two-body transition operator, which obeys the Lippmann-Schwinger equation

$$T_{ij}(z) = V_{ij} + V_{ij}G_0(z)T_{ij}.$$

As usually, $G_0(z)$ is a resolvent of the four-nucleon kinetic energy operator

$$G_0(z) = (z - H_0)^{-1}$$

Iterating AGS-equations up to I-order terms over T one obtains

$$U_{(123),(12)(34)} \approx (z - H_{12}) + T_{14}(z) + T_{24}(z)$$

$$U_{(234),(12)(34)} \approx (z - H_{34}) + T_{13}(z) + T_{14}(z)$$

Since the initial and final states are antisymmetrized, the contributions of the T_{24} and T_{14} matrix elements are equal to each other. It also concerns T_{13} and T_{14} matrix elements in the exchange contribution.

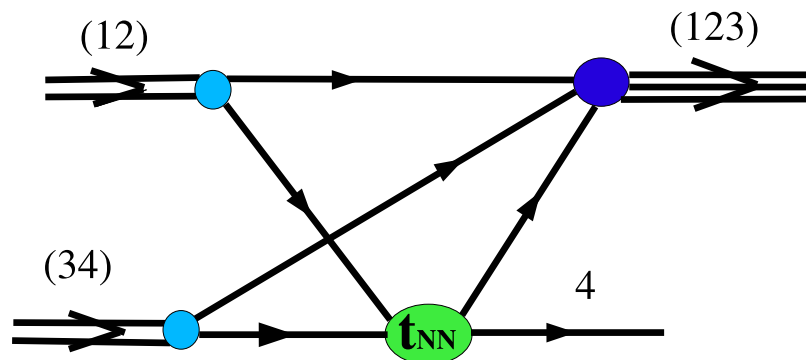
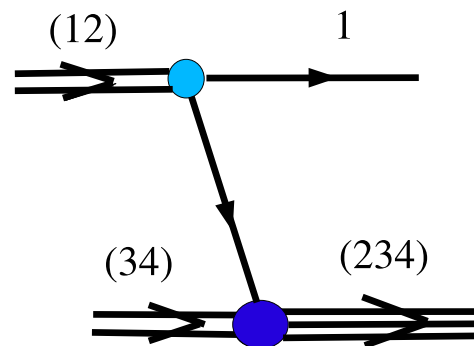
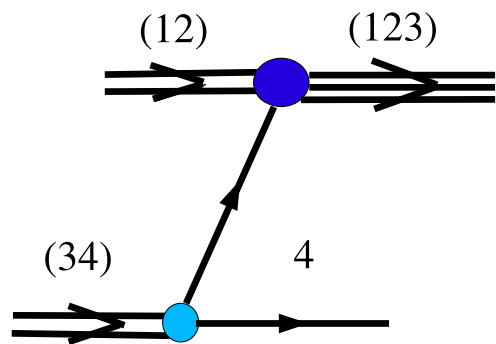
$$\langle 4(123)_s | T_{24} | (12)_s (34)_s \rangle = \langle 4(123)_s | T_{14} | (12)_s (34)_s \rangle$$

$$\langle 1(234)_s | T_{13} | (12)_s (34)_s \rangle = \langle 1(234)_s | T_{14} | (12)_s (34)_s \rangle$$

As consequence, we derive only three contributions into the reaction amplitude

$$\begin{aligned} \langle n^3 He | U | dd \rangle = & \frac{1}{\sqrt{6}} [\langle 1(234)_s | z - H_{12} | (12)_s (34)_s \rangle - \\ & \langle 4(123)_s | z - H_{34} | (12)_s (34)_s \rangle + 2 \langle 4(123)_s | T_{14}^{sym} | (12)_s (34)_s \rangle] \end{aligned}$$

The nucleon-nucleon T -matrix is antysymmetrized: $T_{14}^{sym} = (1 - P_{14})T_{14}$



Deuteron wave function

M. Lacombe et al. (1981) Phys.Lett.B101, 139

R.Machleidt (2001) Phys. Rev.C 63, 024001

The deuteron wave function in the standard form is used:

$$\begin{aligned} & \langle \vec{P}_d/2 + \vec{p}_0, m_p; \vec{P}_d/2 - \vec{p}_0, m_n | \Phi(\vec{P}_d M_d) \rangle = \\ & \sum_{L=0,2} u_L(p_0) \langle \frac{1}{2} m_p \frac{1}{2} m_n | 1 M_s \rangle \langle L M_L 1 M_s | 1 M_d \rangle Y_L^{M_L}(\hat{p}_0) , \end{aligned}$$

where \hat{p}_0 is the unit vector in \vec{p}_0 direction, and m_p , m_n are the proton and neutron spin projections, respectively.

The radial part of the DWF is presented by $u_0(p_0)$ and $u_2(p_0)$, which describe S and D components of the deuteron wave function:

$$u_0(p) = \sqrt{\frac{2}{\pi}} \sum_i \frac{C_i}{\alpha_i^2 + p^2} , \quad u_2(p) = \sqrt{\frac{2}{\pi}} \sum_i \frac{D_i}{\beta_i^2 + p^2}$$

${}^3\text{He}$ wave function

V.Baru, J.Haidenbauer, C.Hanhart, and J.A.Niskanen, Eur.Phys.J. A16, 437 (2003)

The parametrized wave function of a three-nucleon system is used as a ${}^3\text{He}$ wave function. This wave function is fully antisymmetrized

$$|\Psi\rangle = |\psi[(12)3]\rangle + |\psi[(23)1]\rangle + |\psi[(31)2]\rangle$$

and defined in terms of the nucleon pair and spectator momentum:

$$\begin{aligned} \langle \vec{p}_0 \vec{q} \nu | \Psi \rangle &= \langle \vec{p}_0 \vec{q} \nu | \psi[(12)3] \rangle + \langle \vec{p}_0 \vec{q} \nu | \vec{p}_{23} \vec{q}_1 \nu_{23} \rangle \langle \vec{p}_{23} \vec{q}_1 \nu_{23} | \psi[(23)1] \rangle + \\ &+ \langle \vec{p}_0 \vec{q} \nu | \vec{p}_{31} \vec{q}_2 \nu_{31} \rangle \langle \vec{p}_{31} \vec{q}_2 \nu_{31} | \psi[(31)2] \rangle \end{aligned}$$

The kinematical variables: pair relative momentum \vec{p}_0 and spectator momentum \vec{q} are defined as:

$$\begin{aligned} \vec{p}_0 &= \frac{\vec{p}_1 - \vec{p}_2}{2}, & \vec{q} &= \vec{p}_3 - \frac{\vec{P}}{3}, & \vec{P} &= \vec{p}_1 + \vec{p}_2 + \vec{p}_3 = \vec{P}_h \\ \vec{p}_{ij} &= \frac{\vec{p}_i - \vec{p}_j}{2}, & \vec{q}_k &= \vec{p}_k - \frac{\vec{P}}{3} \end{aligned}$$

The radial part of the three-nucleon wave function is presented as a sum of two terms each of them has a separable form:

$$\langle p_0 q \nu | \Psi \rangle = v_1^\nu(p_0) w_1^\nu(q) + v_2^\nu(p_0) w_2^\nu(q),$$

where $v_\lambda^\nu(p_0)$, $w_\lambda^\nu(q)$ are defined as:

$$v^\nu(p_0) = \sum_{n=1}^5 \frac{a_n^\nu}{p_0^2 + (m_n^\nu)^2}, \quad w^\nu(q) = \sum_{n=1}^5 \frac{b_n^\nu}{q^2 + (M_n^\nu)^2}$$

The wave function was derived by fitting the full Faddeev wave function obtained with CD Bonn and Paris NN-potentials. Two sets of the coefficients a_n^ν , b_n^ν , m_n^ν , and M_n^ν exist.

Quantum numbers of the partial waves included into the definition of the three-body wave function. T, S, \mathcal{L}, J refer to the isospin, spin, orbital momentum and total angular momentum of the NN-subsystem. l is the relative orbital momentum of the spectator and K is the channel spin.

ν	Label	Subsystem	\mathcal{L}	S	J^π	T	K	l
1	1s_0S	1s_0	0	0	0^+	1	1/2	0
2	3s_1S	3s_1	0	1	1^+	0	1/2	0
3	3s_1D	3s_1	0	1	1^+	0	3/2	2
4	3d_1S	3d_1	2	1	1^+	0	1/2	0
5	3d_1D	3d_1	2	1	1^+	0	3/2	2

Nucleon-Nucleon t -matrix

W.G.Love, M.A.Franey, Phys.Rev.C24, 1073 (1981)

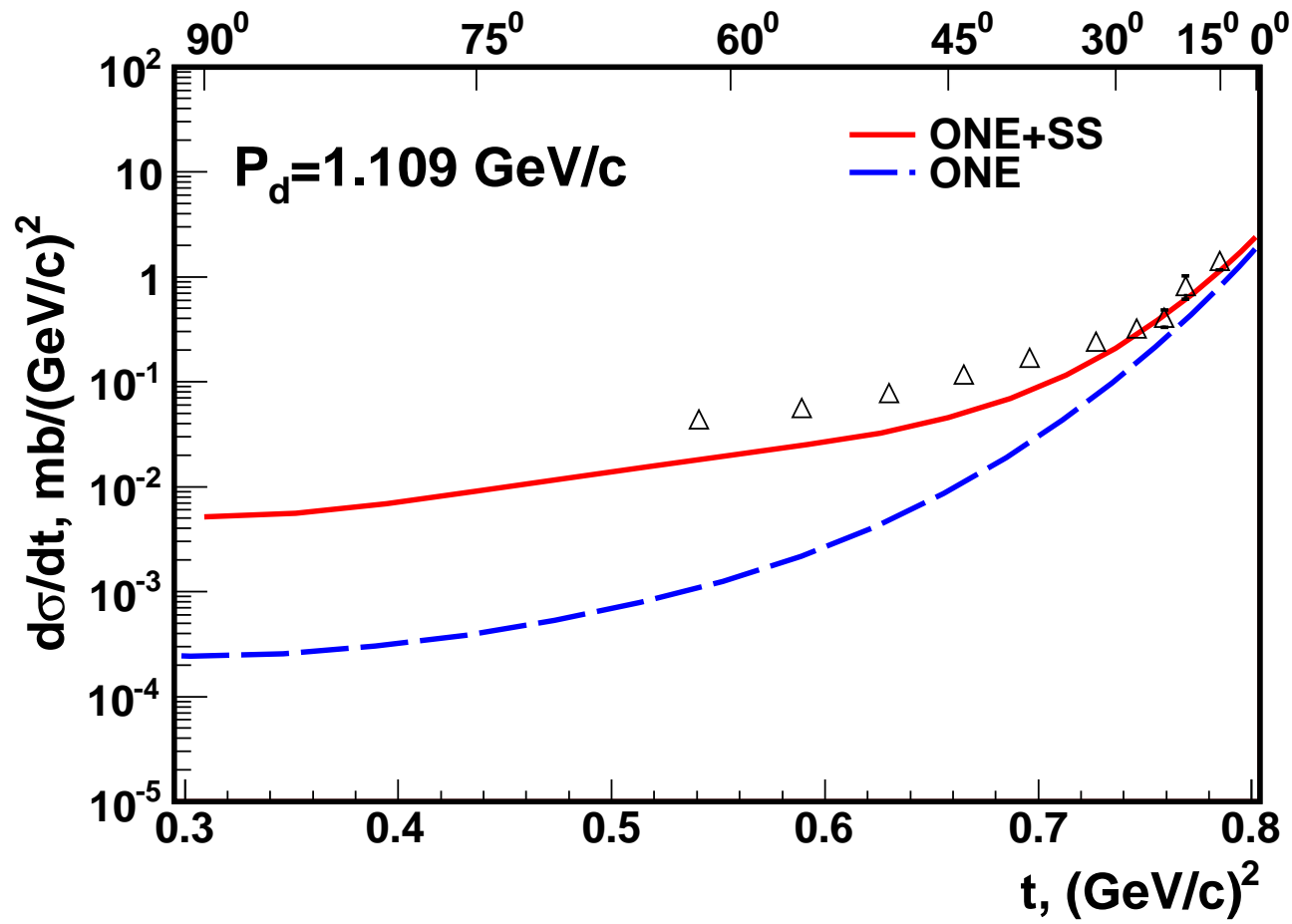
N.B.Ladygina,nucl-th/0805.3021

$$\langle \kappa' m'_1 m'_2 | t | \kappa m_1 m_2 \rangle = \langle \vec{\kappa}' m'_1 m'_2 | A + B(\vec{\sigma}_1 \hat{N}^*)(\vec{\sigma}_2 \hat{N}^*) + C(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \hat{N}^* + D(\vec{\sigma}_1 \hat{q}^*)(\vec{\sigma}_2 \hat{q}^*) + F(\vec{\sigma}_1 \hat{Q}^*)(\vec{\sigma}_2 \hat{Q}^*) | \vec{\kappa} m_1 m_2 \rangle$$

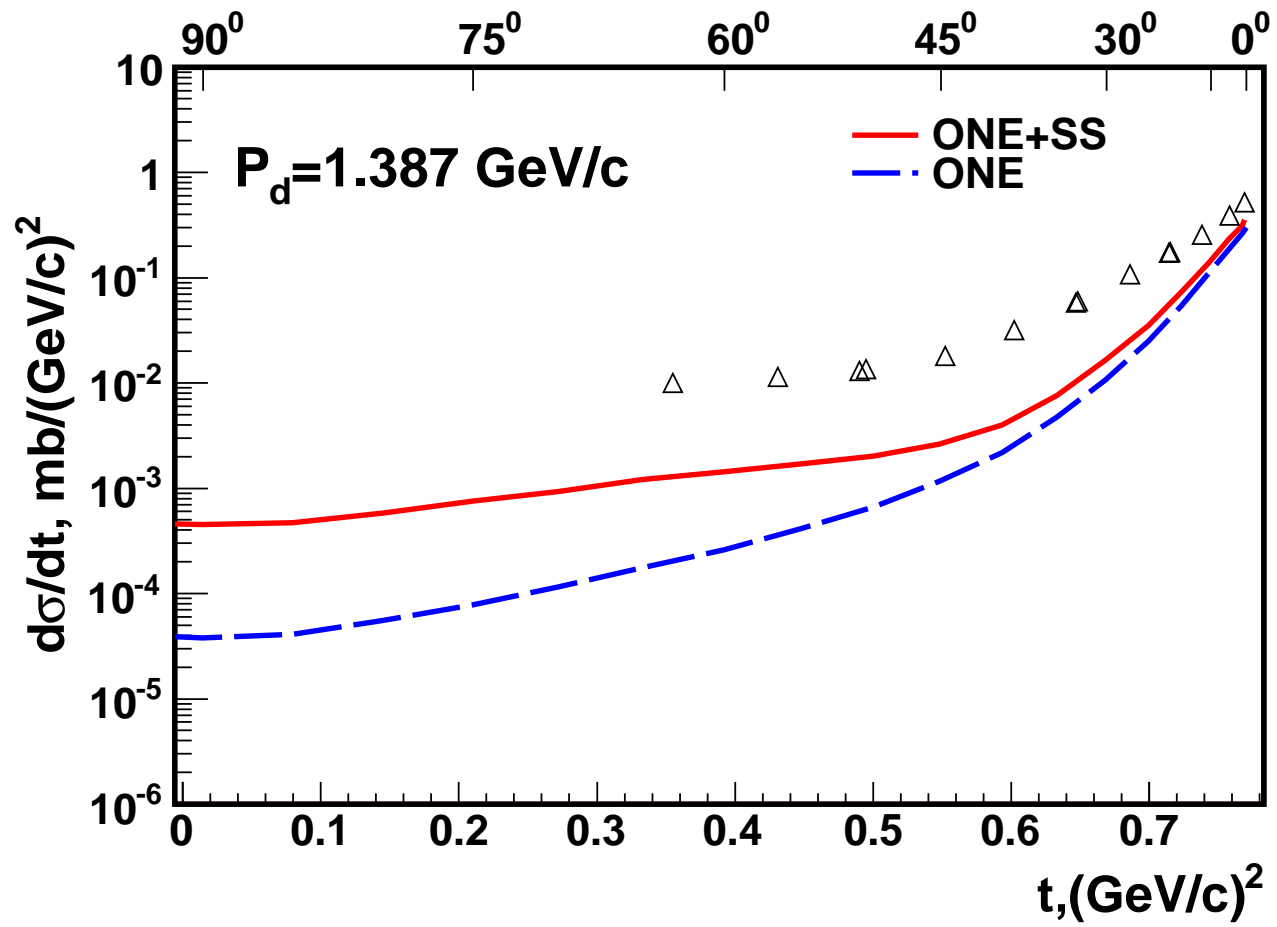
where the orthonormal basis is combinations of the nucleons relative momenta in the initial $\vec{\kappa}$ and final $\vec{\kappa}'$ states

$$\hat{q}^* = \frac{\vec{\kappa} - \vec{\kappa}'}{|\vec{\kappa} - \vec{\kappa}'|}, \quad \hat{Q}^* = \frac{\vec{\kappa} + \vec{\kappa}'}{|\vec{\kappa} + \vec{\kappa}'|}, \quad \hat{N}^* = \frac{\vec{\kappa} \times \vec{\kappa}'}{|\vec{\kappa} \times \vec{\kappa}'|}$$

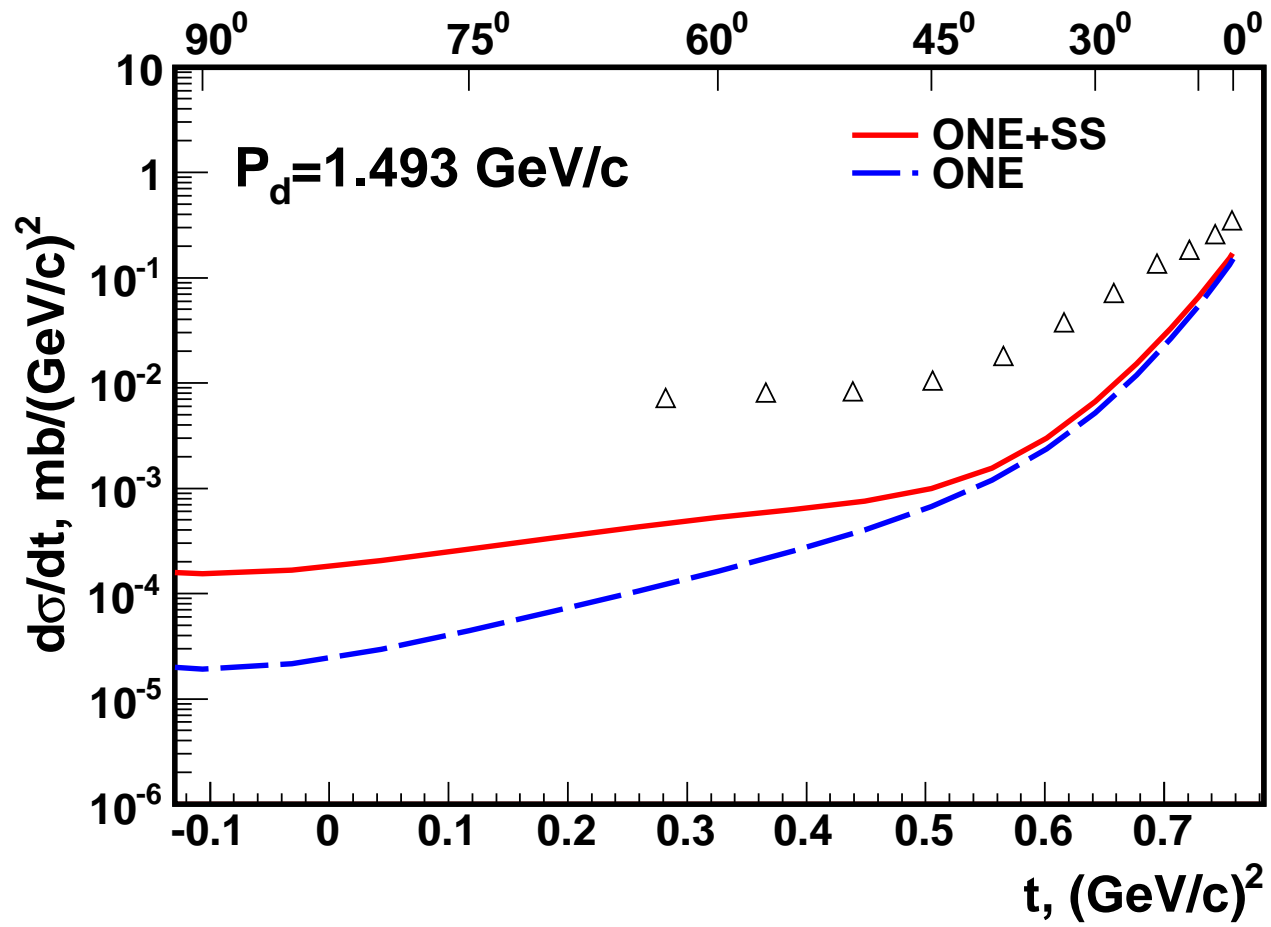
$$\langle \vec{p}' p'_3; m' m'_3 | t | \vec{p} p_3; m m_3 \rangle \sim \langle \kappa' m'_1 m'_2 | W_{1/2}^\dagger(\vec{p}') W_{1/2}^\dagger(\vec{p}'_3) t W_{1/2}(\vec{p}) W_{1/2}(\vec{p}_3) | \kappa m_1 m_2 \rangle$$



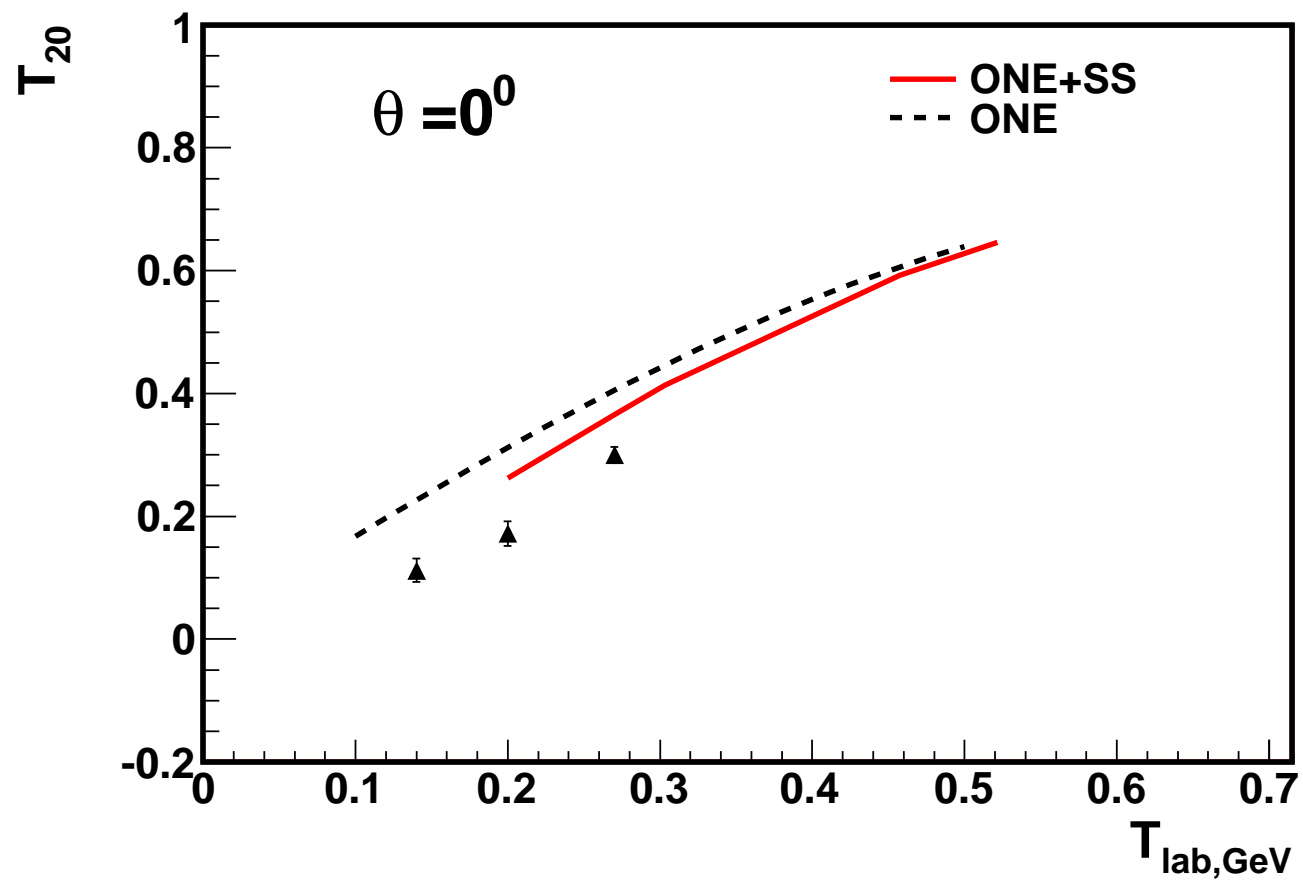
▲ - G.Bizard, et al., Phys.Rev.C 22, p.1632, 1980



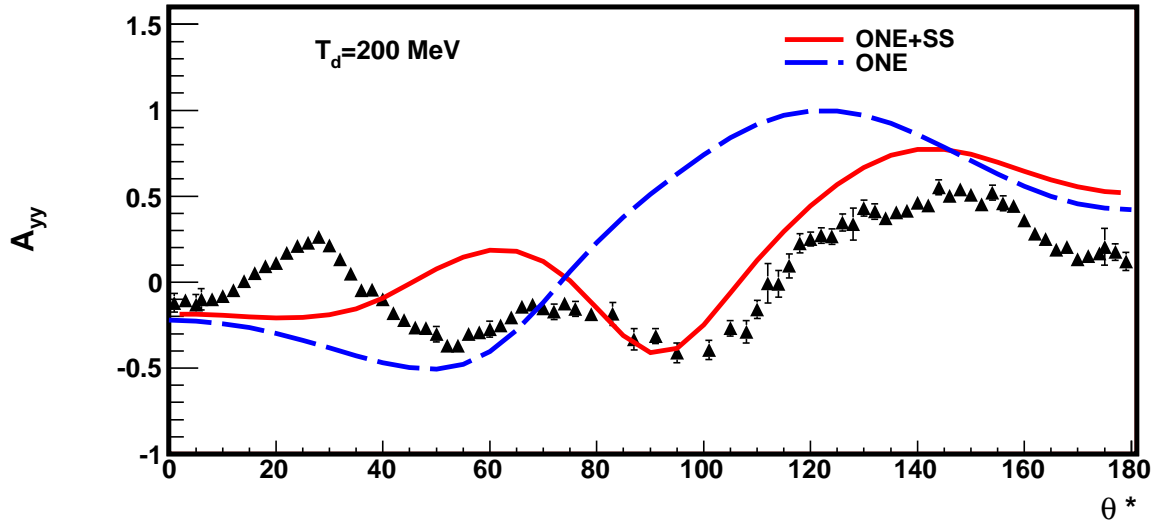
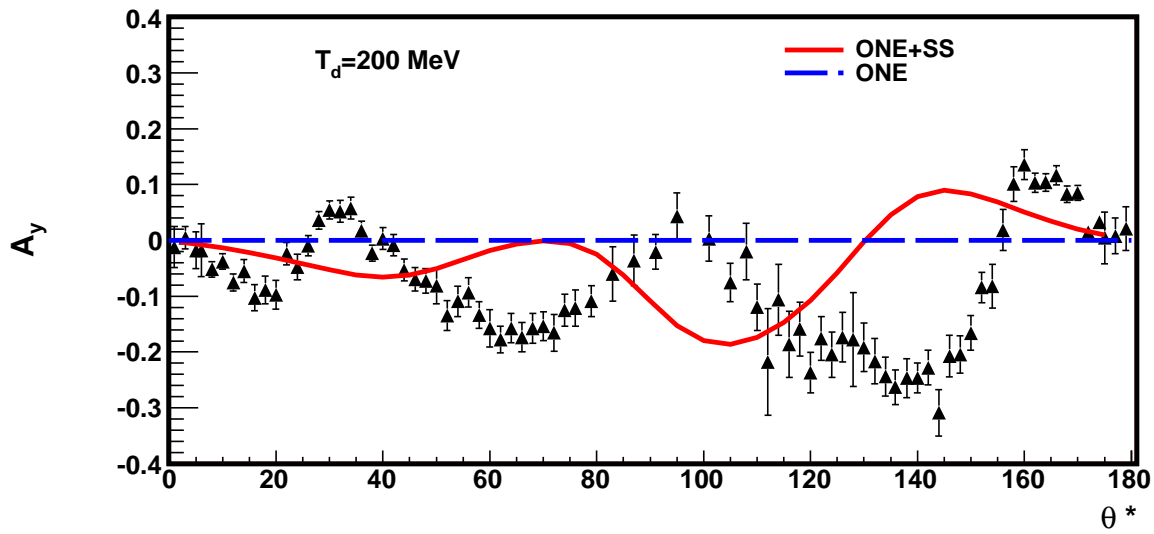
▲ - G.Bizard, et al., Phys.Rev.C 22, p.1632, 1980



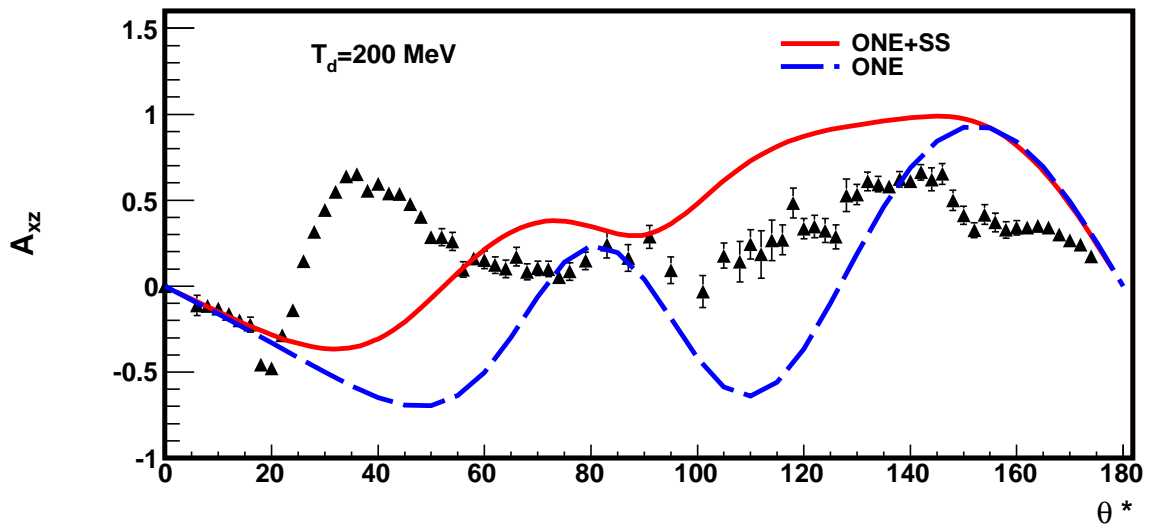
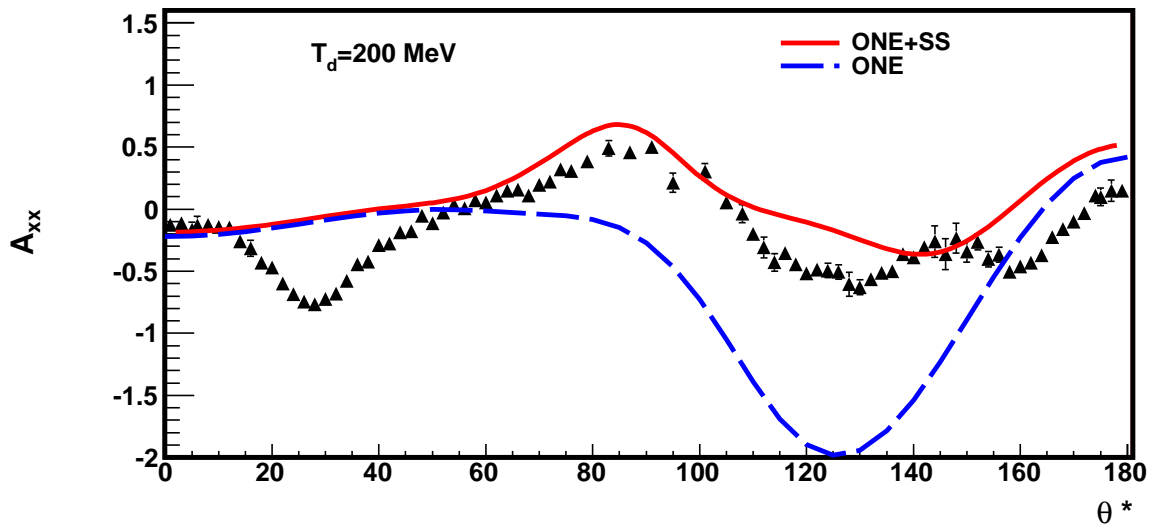
▲ - G.Bizard, et al., Phys.Rev.C 22, p.1632, 1980



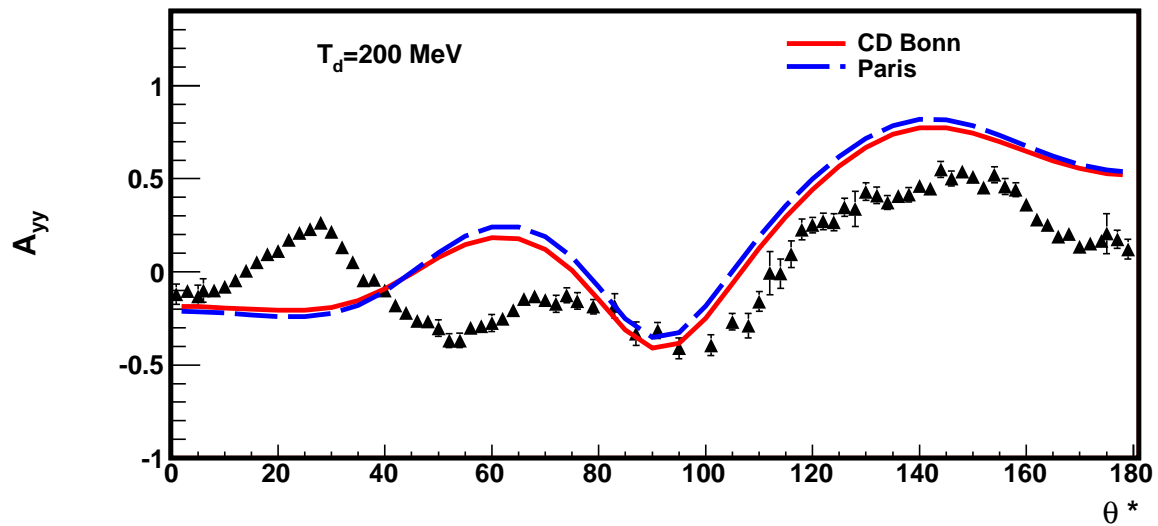
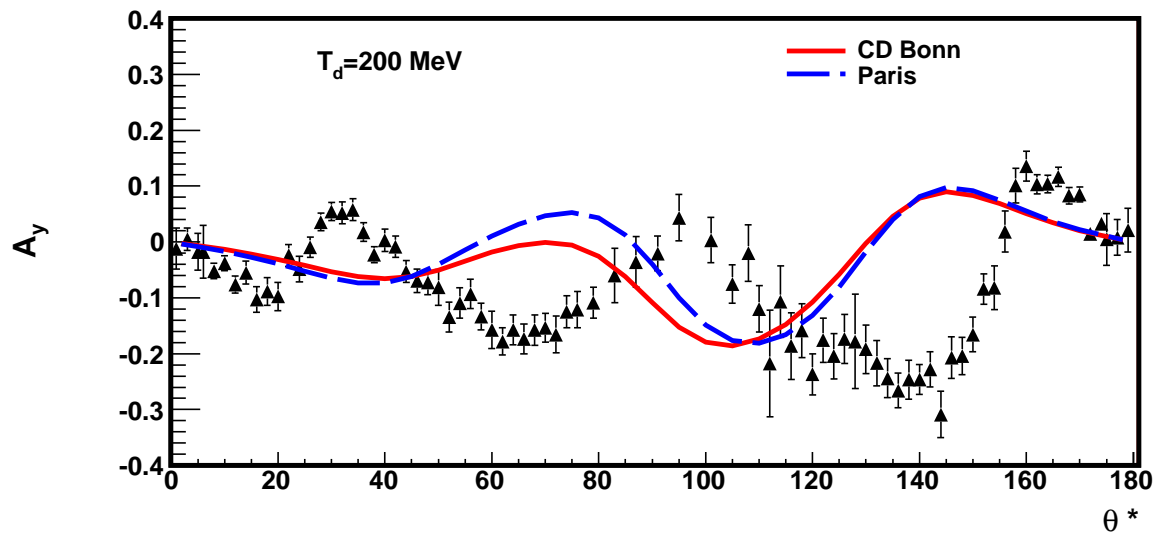
▲ - V.P. Ladygin et al., Phys. Lett. B598 (2004) 47;
V.P. Ladygin et al., Phys.Atom.Nucl.69 (2006) 1271.



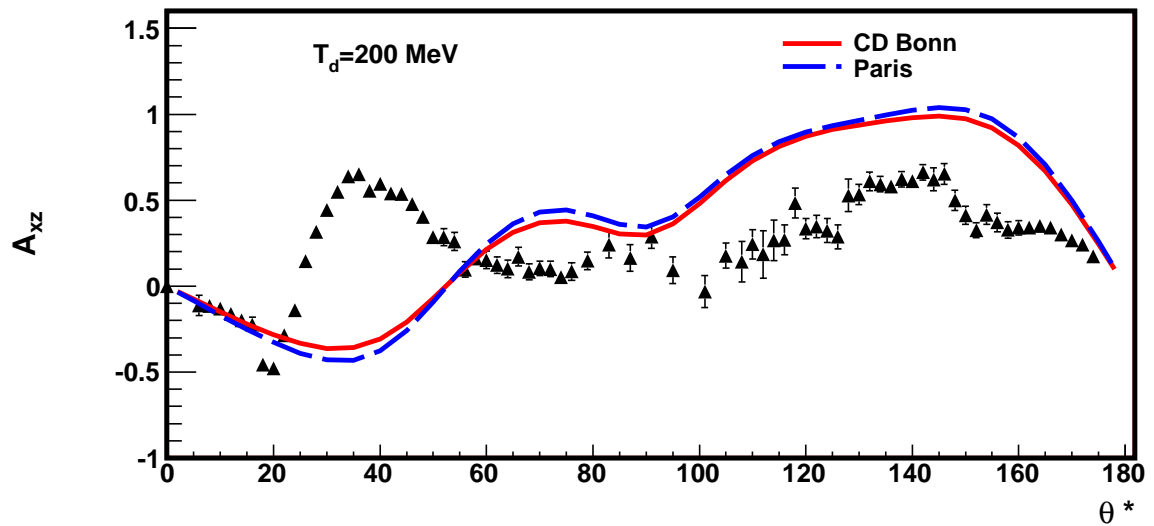
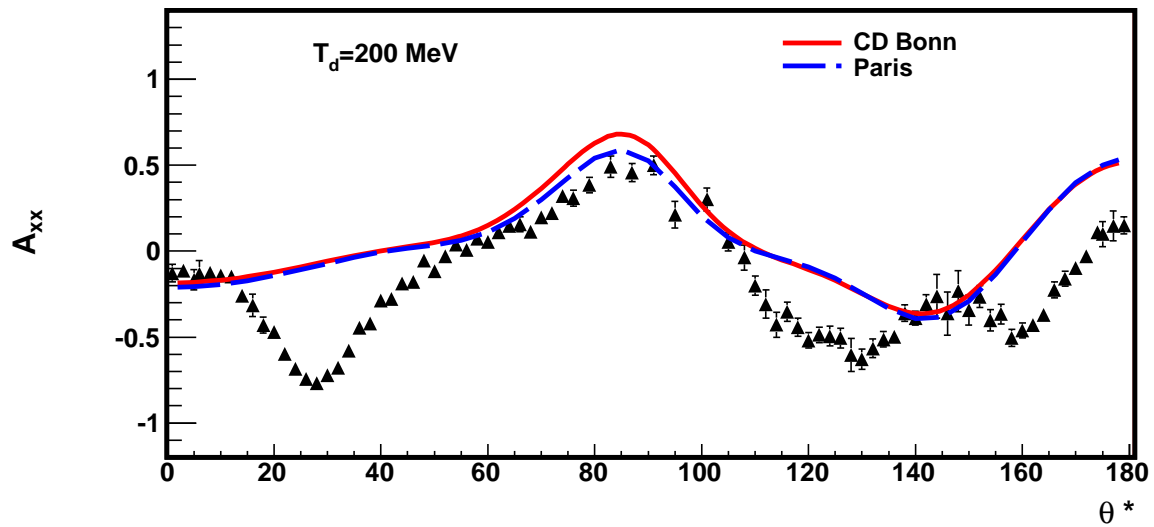
▲ - A.K. Kurilkin et al., Int.J.Mod.Phys. A24 (2009) 526-529



▲ - A.K. Kurilkin et al., Int.J.Mod.Phys. A24 (2009) 526-529



▲ - A.K. Kurilkin et al., Int.J.Mod.Phys. A24 (2009) 526-529



▲ - A.K. Kurilkin et al., Int.J.Mod.Phys. A24 (2009) 526-529

Conclusions

- $dd \rightarrow {}^3\text{He} n$ reaction is considered for the deuteron energies between 200 MeV and 520 MeV.
- The theoretical model for description of this process is suggested. This model is based on the multiple scattering expansion formalism taking relativistic kinematics and relativistic spin theory into account.
- The one-nucleon-exchange and single-scattering reaction mechanisms have been included into consideration.
- A reasonable agreement between the theoretical predictions and experimental data was obtained both for differential cross section and analyzing powers.