Nonperturbative QCD and transition form factors

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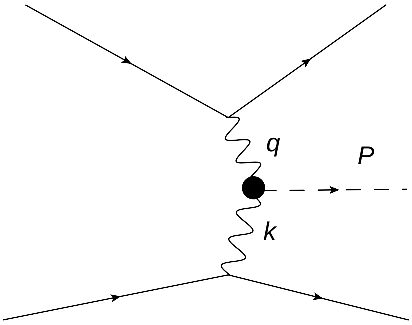
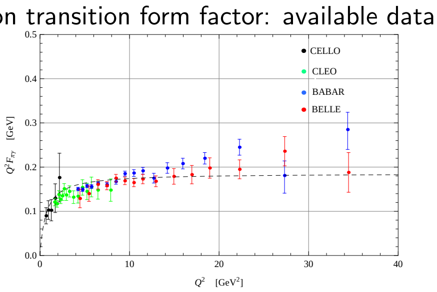
Outline

Transition form factors



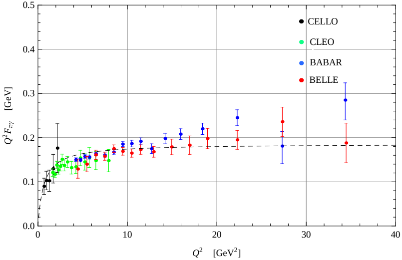
π^0 TFF: theoretical and experimental status



- Standart approach to transition form factors (TFF):
  - at large Q^2 - light cone distributions; QCD factorization predicts $Q^2 F_{\pi\gamma} = \sqrt{2} f_\pi, f_\pi = 0.13\text{GeV}$ [Lepage,Brodsky'79]
  - $Q^2 = 0$ - real photons- constraint of axial anomaly: $F_{\pi\gamma}(0) = \frac{1}{2\sqrt{2}\pi^2 f_\pi}$; the slope of the curve at $Q^2 = 0$
  - in between - Brodsky-Lepage interpolation formula[Brodsky,Lepage'81]: $F_{\pi\gamma}(Q^2) = \frac{1}{2\sqrt{2}\pi^2 f_\pi} \frac{1}{1+Q^2/(4\pi^2 f_\pi^2)}$.

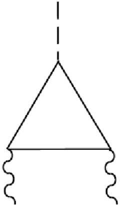
π^0 TFF: theoretical and experimental status

Pion transition form factor: available data



- Our approach - axial anomaly at all Q^2: complementary to factorization and valid even if it is violated.

Anomaly sum rule



The matrix element for the transition of the axial current $J_{\alpha 5}$ with momentum $p = k + q$ into two real or virtual photons with momenta k and q is:

$$T_{\alpha\mu\nu}(k,q) = \int d^4x d^4y e^{(ikx+iqy)} \langle 0 | T\{J_{\alpha 5}(0) J_\mu(x) J_\nu(y)\} | 0 \rangle; \quad (1)$$

Kinematics:

$$k^2 = 0, Q^2 = -q^2$$

The VVA triangle graph amplitude can be presented as a tensor decomposition

$$
\begin{aligned}
T_{\alpha\mu\nu}(k,q) \;=\;& F_1\ \varepsilon_{\alpha\mu\nu\rho}k^\rho + F_2\ \varepsilon_{\alpha\mu\nu\rho}q^\rho \\
&+\ F_3\ k_\nu \varepsilon_{\alpha\mu\rho\sigma}k^\rho q^\sigma + F_4\ q_\nu \varepsilon_{\alpha\mu\rho\sigma}k^\rho q^\sigma \\
&+\ F_5\ k_\mu \varepsilon_{\alpha\nu\rho\sigma}k^\rho q^\sigma + F_6\ q_\mu \varepsilon_{\alpha\nu\rho\sigma}k^\rho q^\sigma ,
\end{aligned}
\tag{2}
$$

$F_j = F_j(p^2, k^2, q^2; m^2),\ p = k+q.$
Dispersive approach to axial anomaly leads to [Hořejší,Teryaev'95]:

$$
\int_{4m^2}^{\infty} A_3(s,Q^2;m^2)ds = \frac{1}{2\pi}N_c C^{(a)}\ , \tag{3}
$$

$$
A_3 \equiv \frac{1}{2}Im(F_3 - F_6),\ \ N_c = 3;
$$

$$
\begin{aligned}
C^{(3)} &= \tfrac{1}{\sqrt{2}}(e_u^2 - e_d^2) = \tfrac{1}{3\sqrt{2}}, \\
C^{(8)} &= \tfrac{1}{\sqrt{6}}(e_u^2 + e_d^2 - 2e_s^2) = \tfrac{1}{3\sqrt{6}}, \\
C^{(0)} &= \tfrac{1}{\sqrt{3}}(e_u^2 + e_d^2 + e_s^2) = \tfrac{2}{3\sqrt{3}}.
\end{aligned}
\tag{4}
$$

$$\int_{4m^2}^{\infty} A_3(s, Q^2; m^2) ds = \frac{1}{2\pi} N_c C^{(a)} \tag{5}$$

- Holds for any Q^2 and any m^2.
- It has neither α_s corrections (Adler-Bardeen theorem) nor non-perturbative corrections (t'Hooft's consistency principle).
- Exact nonperturbative relation – powerful tool.

ASR and meson contributions

Saturating the l.h.s. of the 3-point correlation function (1) with the resonances and singling out their contributions to ASR (5) we get the (infinite) sum of resonances with appropriate quantum numbers:

$$\pi \sum f_M^a F_{M\gamma} = \int_{4m^2}^{\infty} A_3(s, Q^2; m^2) ds = \frac{1}{2\pi} N_c C^{(a)}, \tag{6}$$

where the coupling (decay) constants f_M^a:

$$\langle 0 | J_{\alpha 5}^{(a)}(0) | M(p) \rangle = i p_\alpha f_M^a \ , \tag{7}$$

and form factors $F_{M\gamma}$ of the transitions $\gamma\gamma^* \to M$ are:

$$\int d^4 x e^{ikx} \langle M(p) | T\{J_\mu(x) J_\nu(0)\} | 0 \rangle = \epsilon_{\mu\nu\rho\sigma} k^\rho q^\sigma F_{M\gamma} \tag{8}$$

- Sum of finite number of resonances decreasing $F_{M\gamma}^{\mathrm{asymp}}(Q^2) \propto \frac{f_M}{Q^2}$- infinite number of states are needed to saturate ASR (collective effect). [Y.K.,A.Oganesian,O.Teryaev'10]

Isovector channel: π^0

$J^{(3)}_{\mu 5} = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d)$, $C^{(3)} = \frac{1}{\sqrt{2}}(e_u^2 - e_d^2) = \frac{1}{3\sqrt{2}}$.
For practical purposes let's use QHD and describe the higher resonances by continuum.

- π^0 + higher contributions ("continuum"):

$$\pi f_\pi F_{\pi\gamma}(Q^2) + \int_{s_0}^{\infty} A_3(s, Q^2; m^2) = \frac{1}{2\pi} N_c C^{(3)}. \qquad (9)$$

The spectral density $A_3(s, Q^2; m^2)$ can be calculated from VVA triangle diagram:

$$A_3(s, Q^2; m^2) = \frac{1}{2\sqrt{2}\pi} \frac{1}{(Q^2+s)^2} \left(Q^2 R + 2m^2 \ln \frac{1+R}{1-R} \right), \qquad (10)$$

where $R(s, m) = \sqrt{1 - \frac{4m^2}{s}}$, m is a mass of quark. Then the pion TFF:

$$F_{\pi\gamma}(Q^2) = \frac{1}{2\sqrt{2}\pi^2 f_\pi} \frac{s_0}{s_0 + Q^2} (R_0 - \frac{2m^2}{s_0} \ln \frac{1+R_0}{1-R_0}), \qquad (11)$$

$R_0 = R(s_0, m)$.

- $m = 0$:

$$F_{\pi\gamma}(Q^2) = \frac{1}{2\sqrt{2}\pi^2 f_\pi} \frac{s_0}{s_0 + Q^2} \qquad (12)$$

Considering the limit $Q^2 \to \infty$ and relying the QCD factorization prediction for $Q^2 F_{\pi\gamma} = \sqrt{2} f_\pi$

$$s_0 = 4\pi^2 f_\pi^2 = 0.67\, GeV^2$$

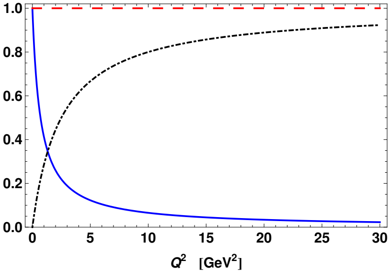
– fits perfectly the value extracted from SVZ (two-point) QCD sum rules $s_0 = 0.7\, GeV^2$ [Shifman,Vainshtein,Zakharov'79].

– reproduces BL interpolation formula[Brodsky,Lepage'81]:

$$F_{\pi\gamma}^{\mathrm{BL}}(Q^2) = \frac{1}{2\sqrt{2}\pi^2 f_\pi} \frac{1}{1 + Q^2/\left(4\pi^2 f_\pi^2\right)}. \qquad (13)$$

Derived from QHD [Radyushkin'96], now we related it to anomaly at all Q^2.

Corrections interplay



- The full integral is exact

$$\frac{1}{2\pi} = \int_0^\infty A_3(s, Q^2)ds = I_\pi + I_{cont}$$

- The continuum contribution $I_{cont} = \int_{s_0}^\infty A_3(s, Q^2)ds$ may have perturbative as well as power corrections.
- $\delta I_\pi = -\delta I_{cont}$: small relative correction to continuum – due to exactness of ASR – must be compensated by large relative correction to the pion contribution!

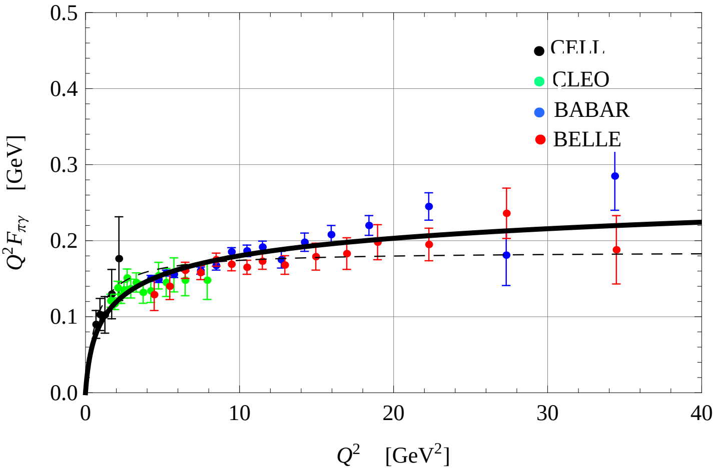
Possible corrections to A_3

- Perturbative two-loop corrections to spectral density A_3 are zero [Jegerlehner&Tarasov'06]
- Nonperturbative corrections to A_3 are possible: vacuum condensates, instantons, short strings.
- General requirements for the correction $\delta I = \int_{s_0}^{\infty} \delta A_3(s, Q^2) ds$: $\delta I = 0$
  - at $s_0 \to \infty$ (the continuum contribution vanishes),
  - at $s_0 \to 0$ (the full integral has no corrections),
  - at $Q^2 \to \infty$ (the perturbative theory works at large Q^2),
  - at $Q^2 \to 0$ (anomaly perfectly describes pion decay width).

$$\delta I = \frac{1}{2\sqrt{2}\pi} \frac{\lambda s_0 Q^2}{(s_0 + Q^2)^2} (\ln \frac{Q^2}{s_0} + \sigma), \tag{14}$$

$$\delta F_{\pi\gamma} = \frac{1}{\pi f_\pi} \delta I_\pi = \frac{1}{2\sqrt{2}\pi^2 f_\pi} \frac{\lambda s_0 Q^2}{(s_0 + Q^2)^2} (\ln \frac{Q^2}{s_0} + \sigma). \tag{15}$$

Correction vs. experimental data



CELLO+CLEO+BABAR: $\lambda = 0.14$, $\sigma = -2.43$, $\chi^2/n.d.f. = 1.08$

	$\frac{\chi^2}{n.d.f.}$	$\delta I \neq 0 : \frac{\chi^2}{n.d.f.}$	λ	σ
CELLO+CLEO+BABAR+BELLE	2.32	1.01	0.12	-2.53
CELLO+CLEO+BABAR	2.92	1.08	0.14	-2.43
CELLO+CLEO+BELLE	1.09	0.46	0.066	-3.11
BABAR	4.87	1.29	0.19	-2.42
BELLE	0.84	0.40	0.14	-2.89

- Although the BELLE data themselves may be described without the correction, but they do not also exclude its possibility. Unless the BABAR data will be disproved, the need for correction remains.

TFFs of η and η': experiment

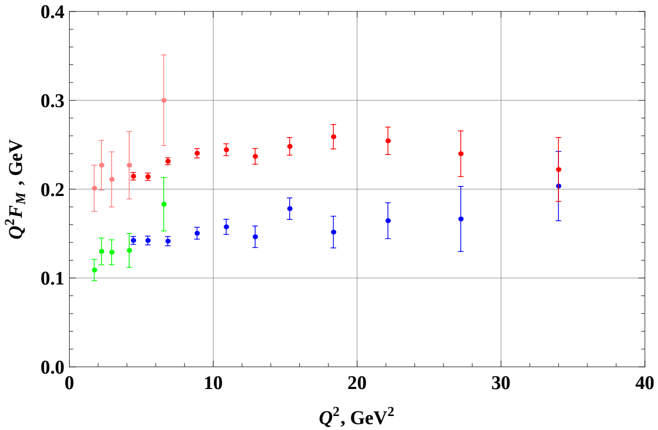


Figure: Experimental data on transition form factors: η (CLEO-green, BABAR-blue), η' (CLEO-pink, BABAR-red)

Octet channel (η,η')

$$J^{(8)}_{\alpha 5} = \frac{1}{\sqrt{6}}(\bar{u}\gamma_\alpha\gamma_5 u + \bar{d}\gamma_\alpha\gamma_5 d - 2\bar{s}\gamma_\alpha\gamma_5 s),$$

$$\int_{4m^2}^{\infty} A_3(s,Q^2;m^2)ds = \frac{1}{2\pi}N_c C^{(8)}, \qquad (16)$$

$$C^{(8)} \equiv \frac{1}{\sqrt{6}}(e_u^2+e_d^2-2e_s^2) = \frac{1}{3\sqrt{6}}$$

ASR in the octet channel:

$$f^8_\eta F_{\eta\gamma}(Q^2) + f^8_{\eta'}F_{\eta'\gamma}(Q^2) = \frac{1}{2\sqrt{6}\pi^2}\frac{s_0}{s_0+Q^2}\left(R_0 - \frac{4m^2}{3s_0}\ln\frac{1+R_0}{1-R_0}\right), \quad (17)$$

$R_0 = \sqrt{1-\frac{4m_s^2}{s_0}}$.

- Significant mixing.
- η' decays into two real photons, so it should be taken into account explicitly along with η meson.

Large Q^2

$$Q^2 F^{as}_{\eta\gamma} = 2(C^{(8)} f^8_\eta + C^{(0)} f^0_\eta) \int_0^1 \frac{\phi^{as}(x)}{x} dx, \tag{18}$$

$$Q^2 F^{as}_{\eta'\gamma} = 2(C^{(8)} f^8_{\eta'} + C^{(0)} f^0_{\eta'}) \int_0^1 \frac{\phi^{as}(x)}{x} dx, \tag{19}$$

$\phi^{as}(x) = 6x(1-x)$. Then ASR at $Q^2 \to \infty$:

$$\begin{aligned} 4\pi^2((f^8_\eta)^2 + (f^8_{\eta'})^2 + 2\sqrt{2}[f^8_\eta f^0_\eta + f^8_{\eta'} f^0_{\eta'}]) = \\ s_0(R_0 - \frac{4m^2}{3s_0} \ln \frac{1+R_0}{1-R_0}) \end{aligned} \tag{20}$$

$Q^2 = 0$

ASR takes the form:

$$f_\eta^8 F_{\eta\gamma}(0) + f_{\eta'}^8 F_{\eta'\gamma}(0) = \frac{1}{2\sqrt{6}\pi^2}\frac{s_0}{s_0+Q^2}\left(R_0 - \frac{4m^2}{3s_0}\ln\frac{1+R_0}{1-R_0}\right), \qquad (21)$$

where

$$F_{M\gamma}(0) = \sqrt{\frac{4\Gamma_{M\to\gamma\gamma}}{\pi\alpha^2 m_M^3}}.$$

Mixing

Octet-singlet basis (of currents):

$$J^{(8)}_{\mu 5}=\frac{1}{\sqrt{6}}(\bar{u}\gamma_\mu\gamma_5 u+\bar{d}\gamma_\mu\gamma_5 d-2\bar{s}\gamma_\mu\gamma_5 s),\, J^{(0)}_{\mu 5}=\frac{1}{\sqrt{3}}(\bar{u}\gamma_\mu\gamma_5 u+\bar{d}\gamma_\mu\gamma_5 d+\bar{s}\gamma_\mu\gamma_5 s). \tag{22}$$

$$\langle 0|J^{(a)}_{\alpha 5}(0)|M(p)\rangle = ip_\alpha f^a_M\ , \tag{23}$$

Matrix of decay constants (23)

$$\mathbf{F}=\begin{pmatrix} f^8_\eta & f^8_{\eta'} \\ f^0_\eta & f^0_{\eta'} \end{pmatrix} \tag{24}$$

- *Octet-singlet (SU(3)) mixing scheme*: $f^8_\eta f^0_\eta + f^8_{\eta'} f^0_{\eta'} = 0.$

$$\mathbf{F}=\begin{pmatrix} f_8\cos\theta & f_8\sin\theta \\ -f_0\sin\theta & f_0\cos\theta \end{pmatrix}. \tag{25}$$

For quark-flavour basis one explores the definitions of axial currents with decoupled light and strange quark composition:

$$J^q_{\mu 5} = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\alpha\gamma_5 u + \bar{d}\gamma_\alpha\gamma_5 d), \ J^s_{\mu 5} = \bar{s}\gamma_\alpha\gamma_5 s, \tag{26}$$

$$\begin{pmatrix} J^8_{\mu 5} \\ J^0_{\mu 5} \end{pmatrix} = \mathbf{V}(\alpha) \begin{pmatrix} J^q_{\mu 5} \\ J^s_{\mu 5} \end{pmatrix}, \ \mathbf{V}(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}, \tag{27}$$

where $\tan\alpha = \sqrt{2}$.

- *Quark-flavour mixing scheme*: [Feldmann,Kroll,Stech'97]

$$f^q_\eta f^s_\eta + f^q_{\eta'} f^s_{\eta'} = 0.$$

$$\mathbf{F_{qs}} = \begin{pmatrix} f_q\cos\phi & f_q\sin\phi \\ -f_s\sin\phi & f_s\cos\phi \end{pmatrix}. \tag{28}$$

Additional constraint - $R_{J/\Psi}$.
The radiative decays $J/\Psi \to \eta(\eta')\gamma$ are dominated by non-perturbative gluonic matrix elements, and the ratio of the decay rates $R_{J/\Psi} = (\Gamma(J/\Psi) \to \eta'\gamma)/(\Gamma(J/\Psi) \to \eta\gamma)$ can be expressed as follows [Novikov'79]:

$$R_{J/\Psi} = \left| \frac{\langle 0 \mid G\widetilde{G} \mid \eta' \rangle}{\langle 0 \mid G\widetilde{G} \mid \eta \rangle} \right|^2 \left(\frac{p_{\eta'}}{p_\eta} \right)^3, \tag{29}$$

where $p_{\eta(\eta')} = M_{J/\Psi}(1 - m^2_{\eta(\eta')}/M^2_{J/\Psi})/2$.

$$\partial_\mu J^8_{\mu 5} = \frac{1}{\sqrt{6}}(m_u \overline{u}\gamma_5 u + m_d \overline{d}\gamma_5 d - 2m_s \overline{s}\gamma_5 s), \tag{30}$$

$$\partial_\mu J^0_{\mu 5} = \frac{1}{\sqrt{3}}(m_u \overline{u}\gamma_5 u + m_d \overline{d}\gamma_5 d + m_s \overline{s}\gamma_5 s) + \frac{1}{2\sqrt{3}} \frac{3\alpha_s}{4\pi} G\widetilde{G}. \tag{31}$$

$$R_{J/\Psi} = \left(\frac{f^8_{\eta'} + \sqrt{2} f^0_{\eta'}}{f^8_\eta + \sqrt{2} f^0_\eta} \right)^2 \left(\frac{m_{\eta'}}{m_{\eta'}} \right)^4 \left(\frac{p_{\eta'}}{p_\eta} \right)^3. \tag{32}$$

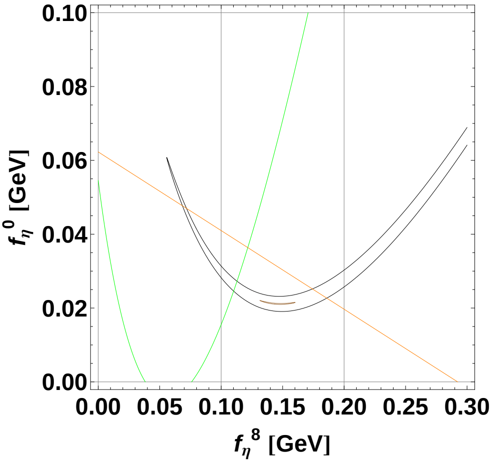
From experiment this ratio is: $R_{J/\Psi} = 4.67 \pm 0.15$ [PDG 2012].

$$f_\eta^8 F_{\eta\gamma}(Q^2) + f_{\eta'}^8 F_{\eta'\gamma}(Q^2) = \frac{1}{2\sqrt{6}\pi^2}\frac{s_0}{s_0+Q^2}(R_0 - \frac{4m^2}{3s_0}\ln\frac{1+R_0}{1-R_0}), \quad (33)$$

$$4\pi^2((f_\eta^8)^2 + (f_{\eta'}^8)^2 + 2\sqrt{2}[f_\eta^8 f_\eta^0 + f_{\eta'}^8 f_{\eta'}^0]) = s_0(R_0 - \frac{4m^2}{3s_0}\ln\frac{1+R_0}{1-R_0}), \quad (34)$$

$$f_\eta^8 F_{\eta\gamma}(0) + f_{\eta'}^8 F_{\eta'\gamma}(0) = \frac{1}{2\sqrt{6}\pi^2}\frac{s_0}{s_0+Q^2}(R_0 - \frac{4m^2}{3s_0}\ln\frac{1+R_0}{1-R_0}), \quad (35)$$

$$R_{J/\Psi} = \left(\frac{f_{\eta'}^8 + \sqrt{2}f_{\eta'}^0}{f_\eta^8 + \sqrt{2}f_\eta^0}\right)^2 \left(\frac{m_{\eta'}}{m_{\eta'}}\right)^4 \left(\frac{p_{\eta'}}{p_\eta}\right)^3. \quad (36)$$



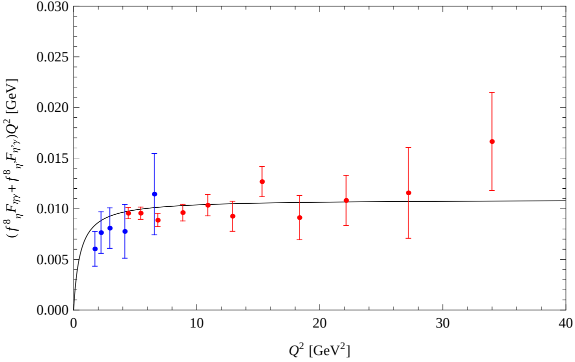
Black; brown curves: $\chi^2/n.d.f. = 1; 0.85$,
green curve: $f_\eta^8 f_\eta^0 + f_{\eta'}^8 f_{\eta'}^0 = 0$,
orange curve: $f_\eta^q f_\eta^s + f_{\eta'}^q f_{\eta'}^s = 0$.

Octet-siglet scheme: mixing parameters

$$\mathbf{F} = \begin{pmatrix} f_\eta^8 & f_{\eta'}^8 \\ f_\eta^0 & f_{\eta'}^0 \end{pmatrix} = \begin{pmatrix} 0.112 \pm 0.005 & -0.029 \pm 0.003 \\ 0.026 \pm 0.005 & 0.101 \pm 0.007 \end{pmatrix} GeV \quad (37)$$

$$f_8/f_\pi = 0.88 \pm 0.04, \quad f_0/f_\pi = 0.81 \pm 0.05, \quad \theta = -15.2^\circ \pm 0.5^\circ$$

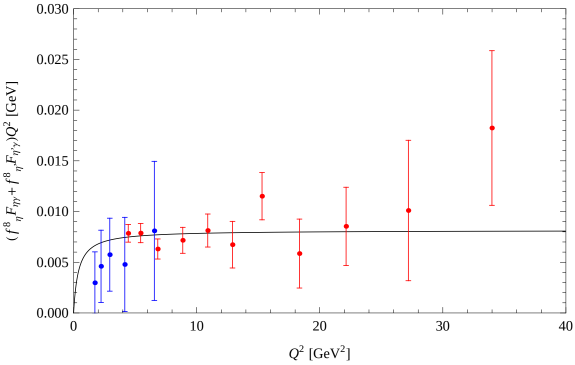
ASR in octet channel and data: octet-singlet mixing scheme



Quark-flavour mixing scheme: mixing parameters

$$f_q/f_\pi = 1.21 \pm 0.09, \quad f_s/f_\pi = 1.66 \pm 0.20, \quad \phi = 38.1^\circ \pm 0.5^\circ$$

ASR in octet channel and data: octet-singlet mixing scheme



Summary

- The manifestation of Axial Anomaly – the Anomaly Sum Rule is an exact NPQCD tool that does not require QCD factorization.
- Axial anomaly for virtual photons is a collective effect.
- Anomaly sum rule allows to derive an expression for the pion transition form factor at arbitrary Q^2, giving the proof for the Brodsky-Lepage interpolation formula.
- The BABAR data for the pion TFF require a *log*-like correction to spectral density, while BELLE data do not exclude the possibility of such correction.
- For η, η' mesons our analysis shows that for a large number of mixing schemes ASR allows to extract in a mixing-scheme independent way the decay constants.
- The future improvement of experimental data on transition form factors of η, η' mesons and ratio $R_{J/\psi}$ can determine which mixing scheme is valid.

Thank you for your attention!

Backup

Bose symmetry implies:

$$\begin{aligned} F_1(k,p) &= -F_2(p,k), \\ F_3(k,p) &= -F_6(p,k), \\ F_4(k,p) &= -F_5(p,k). \end{aligned} \tag{38}$$

One can show also that

$$F_6(k,p) = -F_3(k,p) \tag{39}$$

vector Ward identities

$$k^\mu T_{\alpha\mu\nu} = 0, \qquad p^\nu T_{\alpha\mu\nu} = 0 \tag{40}$$

In terms of formfactors, the identities (40) read

$$\begin{aligned} F_1 &= k.p\, F_3 + p^2\, F_4 \\ F_2 &= k^2\, F_5 + k.p\, F_6 \end{aligned} \tag{41}$$

Anomalous axial-vector Ward identity for the amplitude (2) is [Adler'69]:

$$q^\alpha T_{\alpha\mu\nu}(k,p) = 2m T_{\mu\nu}(k,p) + \frac{1}{2\pi^2}\varepsilon_{\mu\nu\rho\sigma} k^\rho p^\sigma \tag{42}$$

Backup

$$T_{\mu\nu}(k,p) = G\ \varepsilon_{\mu\nu\rho\sigma}k^\rho p^\sigma \tag{43}$$

where G is the relevant form factor. In terms of form factors, eq.(42) reads

$$F_2 - F_1 = 2mG + \frac{1}{2\pi^2} \tag{44}$$

For the form factors F_3, F_4 and G one may write unsubtracted dispersion relations

$$F_j(q^2) = \frac{1}{\pi}\int_{4m^2}^{\infty}\frac{A_j(t)}{t-q^2}dt, \qquad j=3,4 \tag{45}$$

$$G(q^2) = \frac{1}{\pi}\int_{4m^2}^{\infty}\frac{B(t)}{t-q^2}dt$$

Backup

From (39) and (41) it is easy to see that for the considered kinematical configuration one has

$$F_2 - F_1 = (p^2 - q^2)F_3 - p^2 F_4 \tag{46}$$

Using now (45) and taking into account that the imaginary parts of the relevant formfactors satisfy non-anomalous Ward identities, in particular

$$(p^2 - t)A_3(t) - p^2 A_4(t) = 2mB(t) \tag{47}$$

one gets finally

$$F_2 - F_1 - 2mG = \frac{1}{\pi}\int_{4m^2}^{\infty} A_3(t)dt \tag{48}$$

Comparing eq.(48) with (44) one may thus observe that the occurrence of the axial anomaly is equivalent to a “sum rule”

$$\int_{4m^2}^{\infty} A_3(t; p^2, m^2)dt = \frac{1}{2\pi} \tag{49}$$

(which must hold for an arbitrary m and for any p^2 in the considered region).