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Theoretical manifestation of the conformal symmetry in the expressions for the polarized DIS sum rules

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Plan of Presentation

Introduction – Reminding of the concept of Conformal Symmetry

Existing perturbative Quenched U(1) identity $O(E^3)$ result: Coefficient functions of Ellis-Jaffe and Bjorken sum rules coinsides! **Q1** Accident or Not ?

A1

- Not! Statement: true in all orders of PT

Explanation due to Kataev- 2010 derivation of new Crewther-type relation from triangle diagram of Axial - V-V currents for the Green function using first OPE expansion of VV -currents, which define EJ sum rule **SI** in the product with Axial current and comparing with basic **non-singlet** Crewther relation for non-singlet A-V-V triangle Crewther 1972

Positive conclusion of discussion between Kataev 1996 and Crewther 1997

A2

The conformal-invariant result for **DIS** sum rules at $O(A_s^3)$-level the case of $SU(N_c)$ Kataev and Mikhailov 2010, Teor.Mat.Fiz. 2012

Conclusions

1) Possible applications- tests of future higher-order EJ SR analytical calculations- may be of interest for JLAB, HERA, studies

2) Other Comments, including theoretical ones

Conformal Invariance

is valid in the quark-parton model limit, and perturbative quenched QED. It is the symmetry under the following transformations of coordinates :

1. Translations $x'^{\mu} = x^{\mu} + \alpha^{\mu}$ with 4 parameters α^{μ},
2. Scale (or dilaton) transformation $x'^{\mu} = \rho x^{\mu}$ with 1 parameter $\rho > 0$,
3. Special conformal transformations $x'^{\mu} = \frac{x^{\mu} + \beta^{\mu} x^2}{1 + 2\beta x + \beta^2 x^2}$ with 4 parameters β^{μ} and
4. Homogeneous Lorentz transformations $x'^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu}$, which also contain 4 parameters.
5. Consequences are widely studied at present, though in renormalized QFT models the CI is violated- appearance of β-function and the effects of running of the coupling constants- QCD, QED

Bjorken polarized sum rule:

$$Bjp(Q^2) = \int_0^1 \left(g_1^{lp}(x,Q^2) - g_1^{ln}(x,Q^2)\right) dx = \frac{1}{6} g_a C_{Bjp}^{ns}(A_s) \tag{1}$$

Depends from Q^2 through the running of $A_s(Q^2) = \alpha_s(Q^2)/(4\pi)$,
Consider pqQED limit- $\mathrm{C_F} = 1$ $\mathrm{N_F} = 0$, $A = \alpha/(4\pi)$

$$C_{Bjp}^{ns} = 1 - 3A + \frac{21}{2}A^2 - \frac{3}{2}A^3 - \left(\frac{4823}{8} + 96\zeta(3)\right)A^4 + O(A^5) \quad . \tag{2}$$

$O(A^3)$ Larin and Vermaseren (1991);$O(A^4)$- Baikov,Chetyrkin,Kuhn (2010)
Ellis-Jaffe sum rule :

$$EJ^{p(n)}(Q^2) = \int_0^1 g_1^{lp(n)}(x,Q^2)dx = C_{Bjp}^{ns}(A_s(Q^2))\left(\pm\frac{1}{12}a_3 + \frac{1}{36}a_8\right) + C_{EJp}\frac{1}{9}\Delta\Sigma(Q^2) \tag{3}$$

where $a_3 = \Delta u - \Delta d$, $a_8 = \Delta u + \Delta d - 2\Delta s$, Δu, Δd and Δs are the polarized distributions and $\Delta\Sigma$ depends from the scheme choice. In the $\overline{MS}$-scheme it is defined as $\Delta\Sigma = \Delta u + \Delta d + \Delta s$ and $C_{Ejp} = exp\int \frac{\gamma_{SI}(x)}{\beta(x)} dx C_{Ej}^s$.

Note, that in the $\overline{MS}$-scheme the definition of the singlet coefficient function reads (Larin (1994), Larin, van Ritbergen, Vermaseren (1997).

$$C^s_{EJp} = \overline{C}^s_{EJp}/Z^s_5 \quad .$$

In the perturbative quenched QED the result is

$$\overline{C}^s_{EJ} = 1 - 7A + \frac{89}{2}A^2 - \left(\frac{1397}{6} - 96\zeta(3)\right)A^3 + O(A^4) \quad . \tag{4}$$

Z^s_5 is the finite renormalization constants of $\overline{\Psi}\gamma_\mu\gamma_5\Psi$ current, Z^{ns}_5 is the renormalization constant of $\overline{\Psi}\gamma_5(\lambda^a/2)\Psi$-current . They were evaluated in Larin (1994) and Larin and Vermaseren (1991). In the limit of pqQED we have $\gamma_{SI}(x) = 0, \;\; exp \int \gamma_{SI}(x)/\beta(x)dx = 0$ and

$$Z^s_5(pqQED) = Z^{ns}_5(pqQED) = 1 - 4A + 22A^2 + \left(-\frac{370}{3} + 96\zeta_3\right)A^3 + O(A^4) \quad . \tag{5}$$

Nontrivial scheme-independent pqQED (conformal invariant limit of QED)consequence of QCD results for SI and NS coefficient functions of EJ sum rule Kataev (2010) is

$$C_{EJp}(A) = C^s_{EJp}(A) = 1 - 3A + \frac{21}{2}A^2 - \frac{3}{2}A^3 + O(A^4) = C^{ns}_{Bjp}(A) \tag{6}$$

Q1: What is the theoretical explanation ? Q2: Is this result true in all orders of PT ? Q3 : Is there any Q^2-dependence ?

A1: Follow from Crewther-type relations for AVV diagrams in CI limit
A2: Valid in all orders of PT A3: No Q^2 dependence- fixed coupling constant
Proof of A1: Using OPE for

$$T^{ab}_{\mu\alpha\beta}(p,q) = i\int <0|TA_\mu(y)V^a_\alpha(x)V^b_\beta(0)|0> e^{ipx+iqy}dxdy = \delta^{ab}\Delta^{(1-loop)}_{\mu\alpha\beta}(p,q) \quad (7)$$

where $A_\mu = \overline{\psi}\gamma_\mu\gamma_5\psi$ and

$$T^{abc}_{\mu\alpha\beta}(p,q) = i\int <0|TA^a_\mu(y)V^b_\alpha(x)V^c_\beta(0)|0> e^{ipx+iqy}dxdy = d^{abc}\Delta^{(1-loop)}_{\mu\alpha\beta}(p,q) \quad (8)$$

The consideration of the first and second triangle graphs+ the concept of CI give the relations, derived in p-space Kataev 1996 for details Gabdadze and Kataev 1995 and in x-space by Crewther 1972

$$C^{si}_{EJp}(A)\times C^{si}_D(A) = 1 \ \ (1996) \ \ C^{ns}_{Bjp}(A)\times C^{ns}_D(A) = 1 \ \ (1972) \quad (9)$$

where $C^{si}_D(A)$ and $C^{ns}_D(A)$ are defined from taking $q^2\frac{d}{dq^2}$ of $\Pi^{si}(q^2)$ and $\Pi^{ns}(Q^2)$ defined as

$$i\int <0|TA_\mu(x)A_\nu(0)|0> e^{iqx}dx = (q_\mu q_\nu - q^2 g_{\mu\nu})\Pi^{si}(q^2) \quad (10)$$

$$i\int <0|TA^a_\mu(x)A^b_\nu(0)|0> e^{iqx}dx = \delta^{ab}(q_\mu q_\nu - q^2 g_{\mu\nu})\Pi^{ns}(q^2) \ . \quad (11)$$

In the massless limit chiral symmetry is exact, thus
$C^{si}_D(A) = C^{ns}_D(A)$

Thus in the CI limit in all orders of PT

$$C^{si}_{EJp}(A) = C^{ns}_{Bjp}(A) \quad \textit{Kataev}\ 2010 \tag{12}$$

In pqQED with $A = \alpha/(4\pi)$ CI limit we have

$$C_{EJp}(A) = C^{si}_{EJp}(A) = 1 - 3A + \frac{21}{2}A^2 - \frac{3}{2}A^3 = C^{si}_{Bjp}(A) \tag{13}$$

In the case of the $SU(N_c)$ with $A_s = \alpha_s/(4\pi)$

$$\begin{aligned} C^{ns}_{Bjp} &= 1 - 3C_F A_s + \left(\frac{21}{2}C_F^2 - C_F C_A\right)A_s^2 \\ &+ \left[\left(-\frac{3}{2}C_F^3 - 65C_F^2 C_A - \left(\frac{523}{12} - 216\zeta_3\right)C_F C_A^2\right]A_s^3 + O(A_s^4) = C^s_{EJ}(A_s) \end{aligned} \tag{14}$$

Numbers obtained from Crewther relation in Kataev and Mikhailov (2010-2012)
This is the CI predictions for $O(A_s^3)$-approximation of $C^s_{EJp}(A_s)$
In general in the $\overline{MS}$-scheme following Mikhailov (2007)
$C_{Bjp} = 1 + c_1 A_s + c_2 A_s^2 + c_3 A_s^3 + \ldots$ and $c_1 = c_1[0]$, $c_2 = c_2[0] + \beta_0 c_2[1]]$,
$c_3 = c_3[0] + \beta_0^2 c_3[2] + \beta_1 c_3[0,1] + \beta_0 c_3[1]$
In Eq.14 the results are for $c_i[0]$- which respect conformal symmetry

Conclusion

In the CI limit in $SU(N_c)$

1. It is possible to try to get $O(A_s^4)$ for C_{Bjp}- from Kataev, Mikhailov (2012)- strong test for analytical evaluation of Ellis-Jaffe sum rule at A_s^4 ($SU(N_c)$ results may be obtained soon).
2. In this CI limit we get the recover quark-parton model expressions for the ratios of polarized sum rules

$$\frac{EJ^{p(n)}}{Bjp} = \pm\frac{1}{2} + \frac{a_8}{6\ a_3} + \frac{2\Delta\Sigma}{3\ a_3} \tag{15}$$

where $a_3 = \Delta u - \Delta d$, $a_8 = \Delta u + \Delta d - 2\Delta s$, and $\Delta\Sigma = \Delta u + \Delta d + \Delta s$, $a_3 = g_A$, $a_8 = 3a_3 - 4D$

$$\frac{EJ^p}{Bjp} = 1 + \frac{2}{3}\frac{\Delta\Sigma - D}{a_3} \quad \frac{EJ^n}{Bjp} = \frac{2}{3}\frac{\Delta\Sigma - D}{a_3} \quad \frac{EJ^p}{Bjp} - \frac{EJ^n}{Bjp} = 1 \tag{16}$$

3. How to use CFT limit for the Bjorken sum rule in phenomenlogy ?

Use "Principle of Maximal Conformality" (Brodsky and Wu 20111, 2012)

PMC= Generalization of BLM approach to higher level using

Grunberg and Kataev 1991+β-expansion approach Mikhailov, Quarks 2004- JHEP 2007

$$\begin{aligned} C^{ns}_{Bjp} &= 1 - 3C_F A_s(Q^{PMC}) + (\frac{21}{2}C_F^2 - C_F C_A)A_s(Q^{PMC})^2 \\ &+ [(-\frac{3}{2}C_F^3 - 65C_F^2 C_A - (\frac{523}{12} - 216\zeta_3)C_F C_A^2]A_s(Q^{PMC})^3 + O(A_s^4) \end{aligned} \tag{17}$$

Q^{PMC} absorbs proportional to β_0, β_0^2, β_1 terms in c_2 and c_3 BjSR coefficients.