Perturbative Stability of the QCD Predictions for the Ratio $R = F_L / F_T$ and Azimuthal Asymmetry in Heavy-Quark Leptoproduction

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Two novel challenging proposals for heavy quark physics in DIS

(COMPASS, EIC, eRHIC, LHeC...)

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ON HIGH ENERGY PHYSICS PROBLEMS

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Outline:

- Stability of the QCD predictions for $R = F_L / F_T$ and azimuthal $\cos 2\varphi$ asymmetry in heavy quark leptoproduction :
  - ➢ Perturbative stability;
  - ➢ Parametric stability;
  - ➢ Stability under DGLAP evolution
- Analytic hadron-level results for F_L / F_T and $\cos 2\varphi$ asymmetry at low $x \ll 1$
- Application of the stability:
  - ✓ Extraction of the structure functions from (experimentally measured) reduced cross sections;
  - ✓ Determination of the charm content of the proton

References

1. Stability of the cos 2φ asymmetry:

- N.Ya.Ivanov, A.Capella, A.B.Kaidalov, Nucl. Phys. **B** 586 (2000), 382
- N.Ya.Ivanov, Nucl. Phys. **B** 615 (2001), 266
- N.Ya.Ivanov, P.E.Bosted, K.Griffioen, S.E.Rock, Nucl. Phys. **B** 650 (2003), 271
- N.Ya.Ivanov, Nucl. Phys. **B** 666 (2003), 88
- L.N.Ananikyan and N.Ya.Ivanov, Nucl. Phys. **B** 762 (2007), 256
- L.N.Ananikyan and N.Ya.Ivanov, Phys. Rev. **D** 75 (2007), 014010

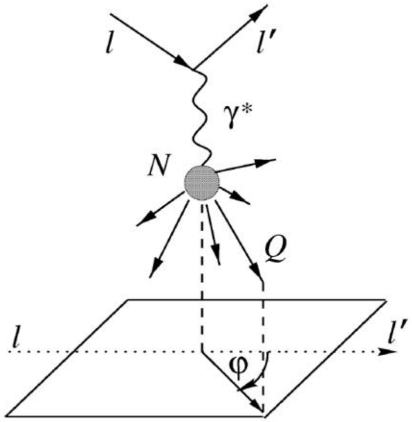
2. Stability of the Calan-Gross ratio $R = F_L / F_T$:

- N.Ya.Ivanov and B.A. Kniehl, Eur. Phys. J. **C** 59 (2009), 647
- N.Ya.Ivanov, Nucl. Phys. **B** 814 (2009), 142
- N.Ya.Ivanov, arXiv:1010.5424 [hep-ph]

1. Definitions and Cross Sections

We consider the Callan-Gross ratio $R = F_L / F_T$ and azimuthal $\cos 2\varphi$ asymmetry, $A = 2xF_A / F_2$, in heavy-quark leptoproduction:

$$l(\ell) + N(p) \to l(\ell - q) + Q(p_Q) + X[\overline{Q}](p_X)$$

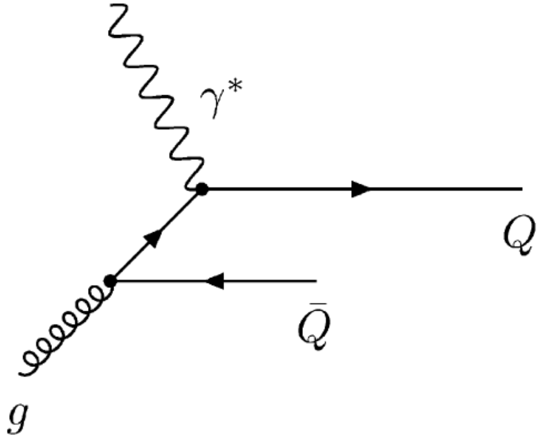


Corresponding cross section is:

$$\frac{\mathrm{d}^3\sigma_{lN}}{\mathrm{d}x\mathrm{d}Q^2\mathrm{d}\varphi} = \frac{\alpha_{em}^2}{xQ^4}\Big\{\left[1+(1-y)^2\right]F_2(x,Q^2) - 2xy^2F_L(x,Q^2)$$
$$+4x(1-y)F_A(x,Q^2)\cos 2\varphi + 4x(2-y)\sqrt{2(1-y)}F_I(x,Q^2)\cos\varphi\Big\}$$

where $F_2(x,Q^2) = 2x(F_T + F_L)$ and x, y, Q^2 are usual DIS observables

The leading mechanism is the photon-gluon fusion



$$\gamma^*(q) + g(k_g) \to Q(p_Q) + \overline{Q}(p_{\bar{Q}})$$

Leveille, Weiler, PRD 24 (1981) 1789
Watson, Z. Phys. C 12 (1982) 123

The NLO contributions are usually presented in the form:

$$\hat{\sigma}_k(z,\lambda,m^2,\mu^2) = \frac{e_Q^2\alpha_{\text{em}}\alpha_s}{m^2}\left\{c_k^{(0,0)}(z,\lambda) + 4\pi\alpha_s\left[c_k^{(1,0)}(z,\lambda) + c_k^{(1,1)}(z,\lambda)\ln\frac{\mu^2}{m^2}\right]\right\} + \mathcal{O}(\alpha_s^2)$$

Laenen, Riemersma, Smith, Neerven, NPB 392 (1993) 162

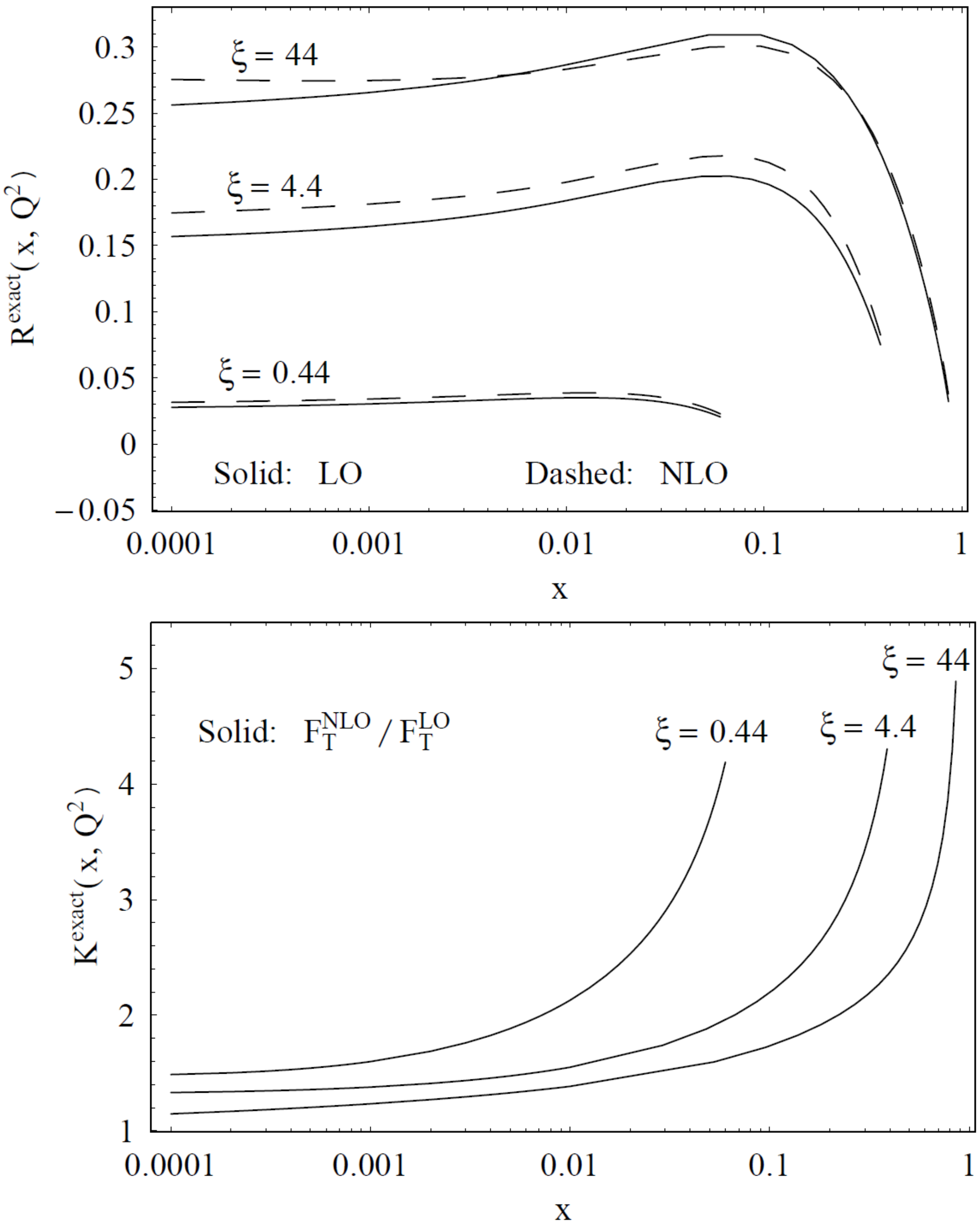
The hadron-level results are:

$$F_k(x,Q^2) = \frac{Q^2}{8\pi^2\alpha_{\text{em}}x}\int_{x(1+4\lambda)}^{1} \mathrm{d}z\, g(z,\mu_F)\hat{\sigma}_k\left(\frac{x}{z},\lambda,\mu\right)$$

$k = T, L, A, I$

$\mu = \mu_R = \mu_F = \sqrt{4m_c^2 + Q^2}$

2. Perturbative Stability of R = F_L / F_T

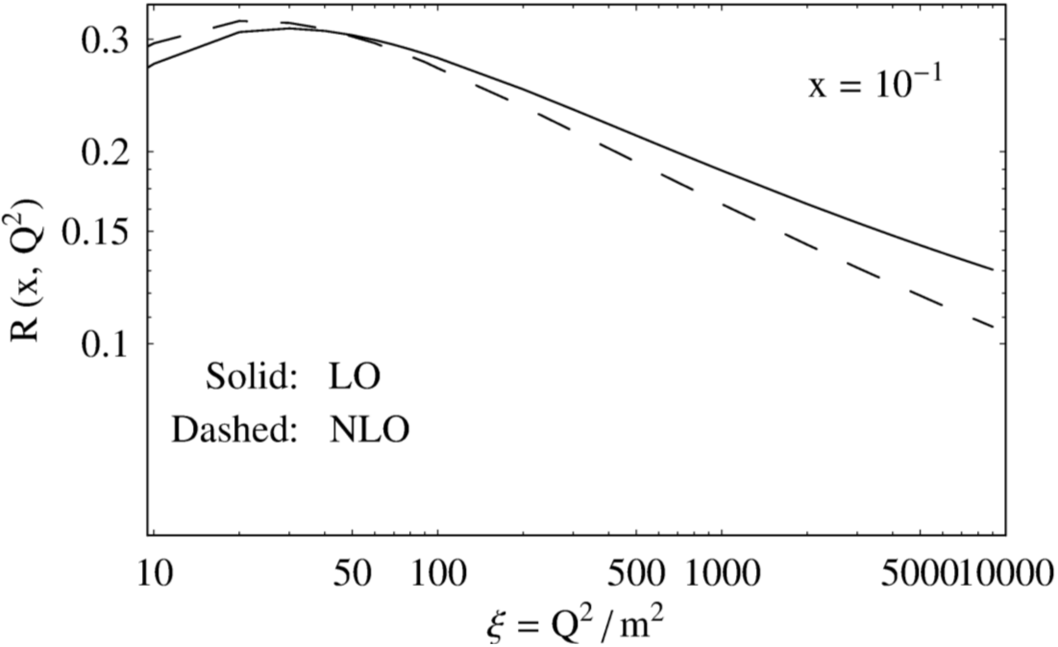


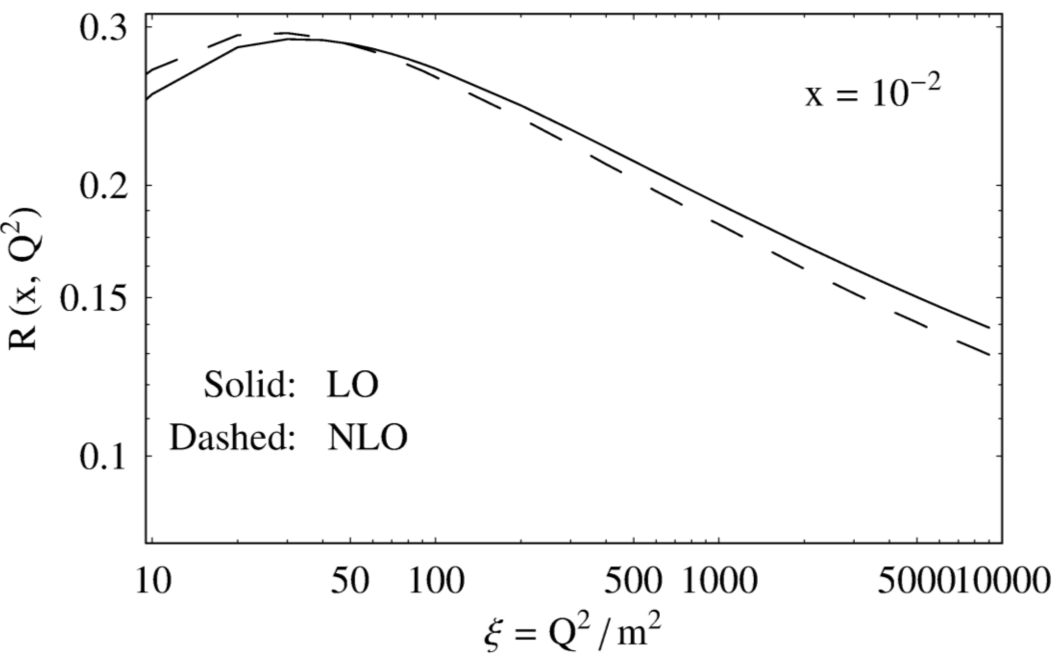
$$\xi = \frac{Q^2}{m^2}$$

$$R(x, Q^2) = \frac{F_L}{F_T}$$

$$K(x, Q^2) = \frac{F_T^{\mathrm{NLO}}}{F_T^{\mathrm{LO}}}$$

2. Perturbative Stability of $R = F_L / F_T$

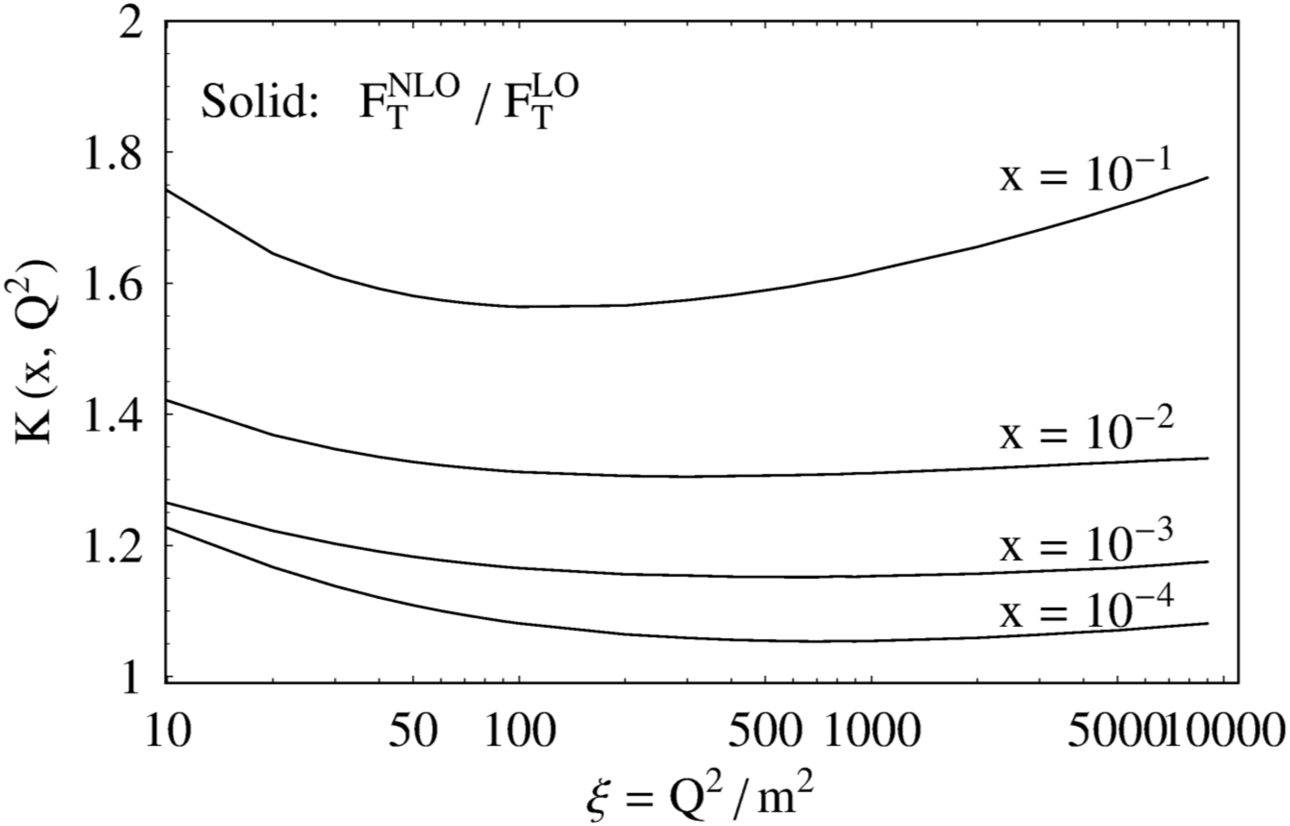




$$\xi = \frac{Q^2}{m^2}$$

$$R(x, Q^2) = \frac{F_L}{F_T}$$

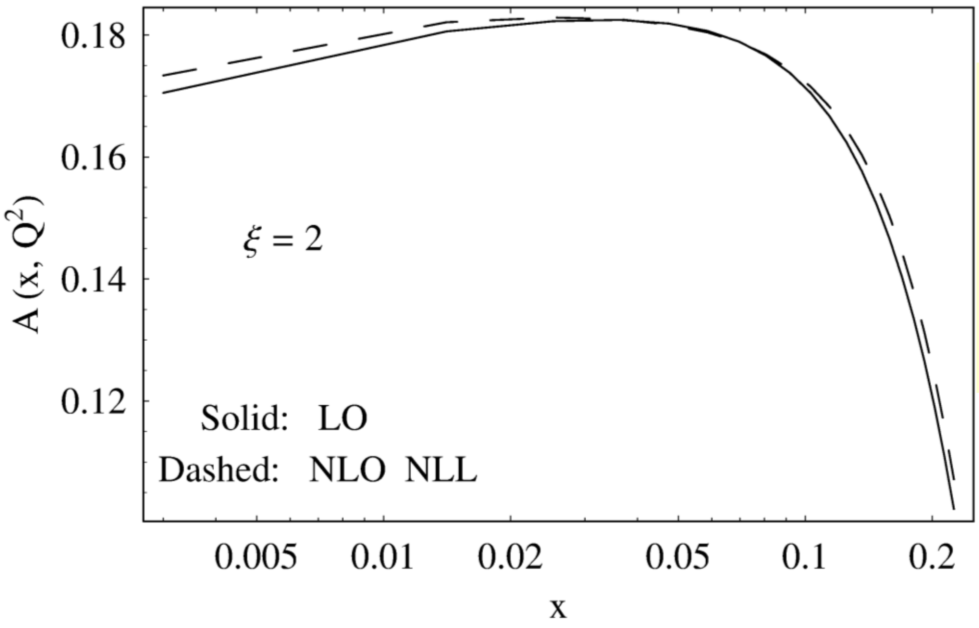
2. Perturbative Instability of F_T

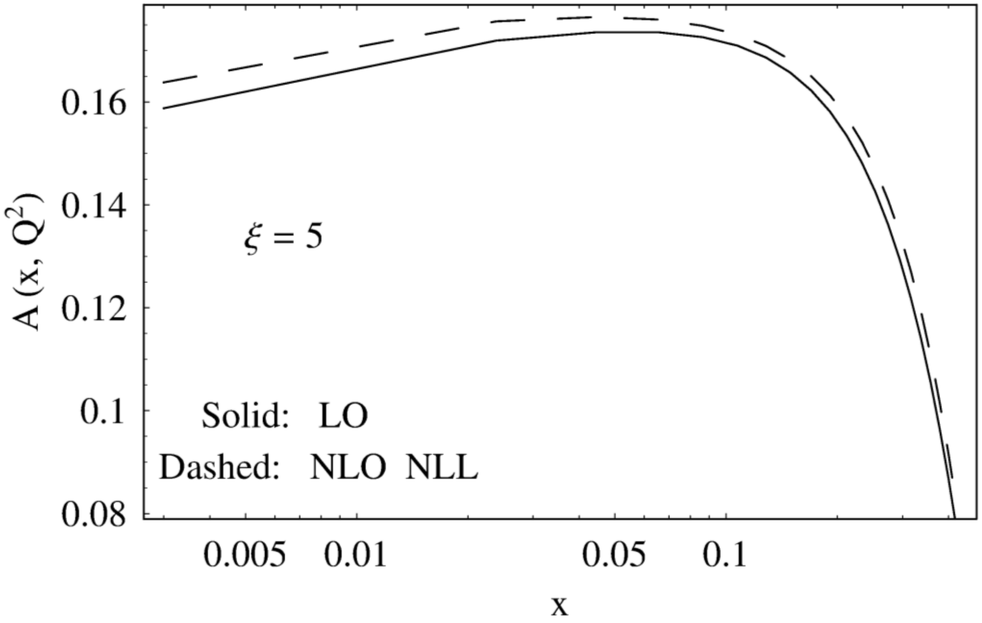


$$\xi = \frac{Q^2}{m^2}$$

$$K(x, Q^2) = \frac{F_T^{NLO}}{F_T^{LO}}$$

3. Perturbative Stability of $A = 2xF_A / F_2$





- Exact NLO results for A are not yet available.
- The soft-gluon (threshold) NLO NLL corrections are presented.

Laenen ,Moch, PR D 59 (1999) 034027
Ivanov, Kniehl, Eur. Phys. J. C 59 (2009) 647

$$\xi = \frac{Q^2}{m^2}$$

$$A(x, Q^2) = 2x\frac{F_A}{F_2}$$

4. Analytic LO Results for $R = F_L / F_T$

The low-*x* behavior of cross sections is determined by the low-*x* asymptotics of the gluon PDF:

$$g(x, Q^2) \xrightarrow{x \to 0} \frac{1}{x^{1+\delta}}$$

We derive compact low-*x* approximation formulae for the ratio

$$R_2(x, Q^2) = 2x\frac{F_L}{F_2} = \frac{R}{1+R}$$

$$R(x, Q^2) = \frac{F_L}{F_T}$$

i.e. the quantity $R_2^{(\delta)}(Q^2)$

$$R_2(x, Q^2) \stackrel{x \to 0}{=} R_2^{(\delta)}(Q^2) + \mathcal{O}(x)$$

4. Analytic LO Results for $R_2 = 2xF_L / F_2$

$$g(x, Q^2) \xrightarrow{x\to 0} \frac{1}{x^{1+\delta}} \qquad \lambda = \frac{m^2}{Q^2}$$

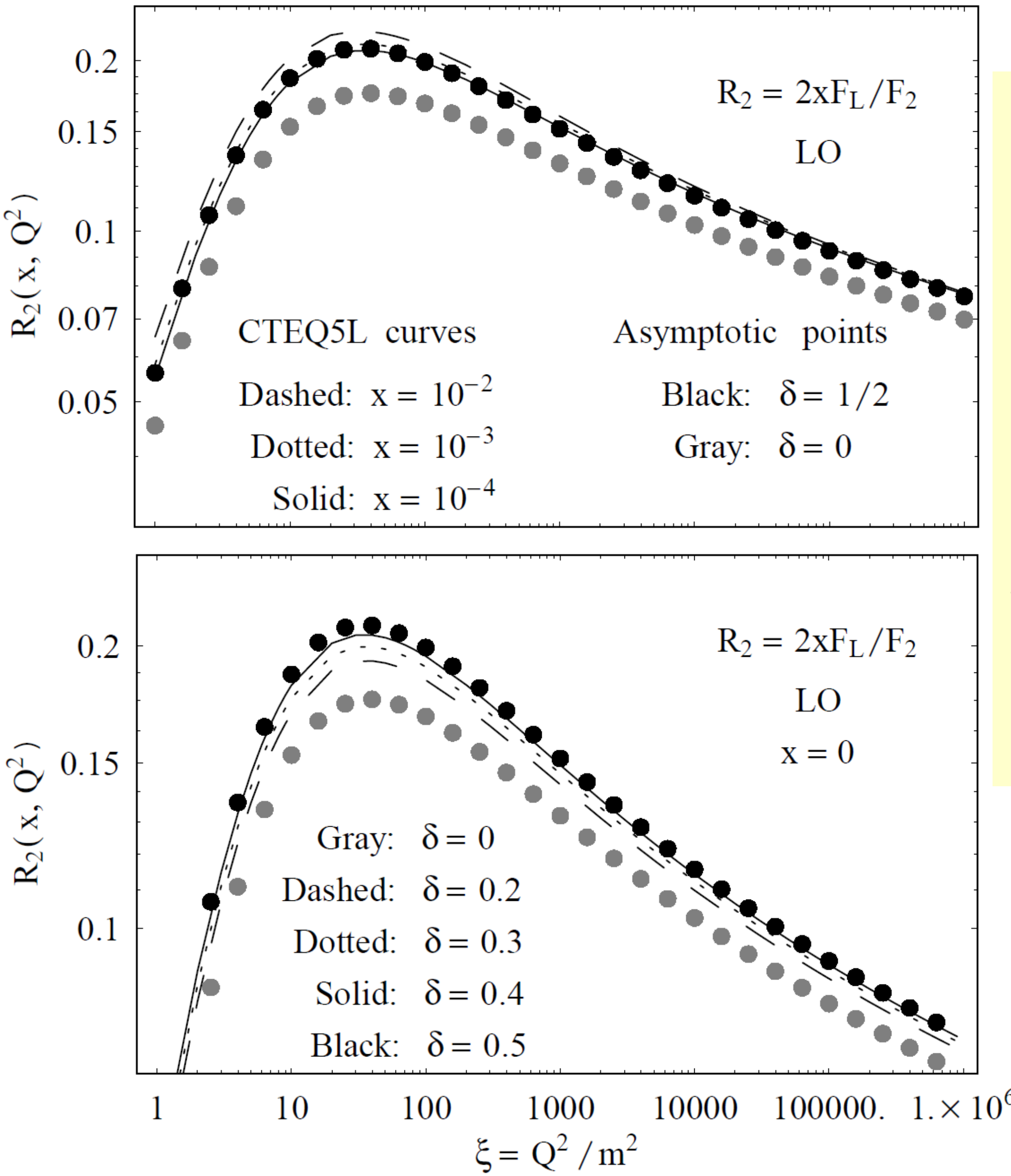
$$R_2^{(\delta)}(Q^2) = \frac{4\left[\frac{2+\delta}{3+\delta}\Phi\left(1+\delta, \frac{1}{1+4\lambda}\right) - (1+4\lambda)\,\Phi\left(2+\delta, \frac{1}{1+4\lambda}\right)\right]}{\left[1+\frac{\delta(1-\delta^2)}{(2+\delta)(3+\delta)}\right]\Phi\left(\delta, \frac{1}{1+4\lambda}\right) - (1+4\lambda)\left(4-\delta-\frac{10}{3+\delta}\right)\Phi\left(1+\delta, \frac{1}{1+4\lambda}\right)}$$

Ivanov, Kniehl, Eur. Phys. J. C 59 (2009) 647

$$\Phi(r, z) = \frac{z^{1+r}}{1+r}\frac{\Gamma(1/2)\,\Gamma(1+r)}{\Gamma(3/2+r)}\,{}_2F_1\left(\frac{1}{2}, 1+r; \frac{3}{2}+r; z\right)$$

$${}_2F_1(a, b; c; z) = \frac{\Gamma(c)}{\Gamma(a)\,\Gamma(b)}\sum_{n=0}^{\infty}\frac{\Gamma(a+n)\,\Gamma(b+n)}{\Gamma(c+n)}\frac{z^n}{n!}$$

6. Stability under DGLAP evolution



- QCD predictions for $R_2^{(\delta)}(Q^2)$ depend weakly on δ practically in the entire region of Q^2 for $0.2 < \delta < 0.9$
- Hadron-level predictions for $R_2^{(\delta)}(Q^2)$ are stable not only under NLO corrections to the partonic cross sections, but also under the DGLAP evolution of the gluon PDF

$$g(x, Q^2) \xrightarrow{x \to 0} \frac{1}{x^{1+\delta}}$$

7. Applications: Extraction of Structure Functions

Usually, it is the so-called ”reduced cross section” that can directly be measured in DIS experiments:

$$\tilde{\sigma}(x,Q^2) = \frac{1}{1+(1-y)^2}\frac{xQ^4}{2\pi\alpha_{\rm em}^2}\frac{{\rm d}^2\sigma_{lN}}{{\rm d}x{\rm d}Q^2} = F_2(x,Q^2) - \frac{2xy^2}{1+(1-y)^2}F_L(x,Q^2)$$

$$R_2(x,Q^2) = 2x\frac{F_L}{F_2} = \frac{R}{1+R} \qquad R(x,Q^2) = \frac{F_L}{F_T}$$

We propose to use the following expression with the quantity $R_2^{(\delta)}(Q^2)$ calculated in LO approximation:

$$\tilde{\sigma}(x,Q^2) = F_2(x,Q^2)\left[1 - \frac{y^2}{1+(1-y)^2}R_2(x,Q^2)\right]$$

7. Applications: Values of $F_2^c\ (x,Q^2)$ extracted from the HERA measurements

Q^2 (GeV^2)	x ($\times 10^{-3}$)	y	$\tilde{\sigma}^c$	Error (%)	F_2^c(NLO) H1	F_2^c(LO) $\delta = 0.5$	F_2^c(LO) $\delta = 0.3$
12	0.197	0.600	0.412	18	0.435 ± 0.078	0.435 ± 0.078	0.434 ± 0.078
12	0.800	0.148	0.185	13	0.186 ± 0.024	0.185 ± 0.024	0.185 ± 0.024
25	0.500	0.492	0.318	13	0.331 ± 0.043	0.331 ± 0.043	0.330 ± 0.043
25	2.000	0.123	0.212	10	0.212 ± 0.021	0.212 ± 0.021	0.212 ± 0.021
60	2.000	0.295	0.364	10	0.369 ± 0.040	0.369 ± 0.040	0.368 ± 0.040
60	5.000	0.118	0.200	12	0.201 ± 0.024	0.200 ± 0.024	0.200 ± 0.024
200	0.500	0.394	0.197	23	0.202 ± 0.046	0.202 ± 0.046	0.202 ± 0.046
200	1.300	0.151	0.130	24	0.131 ± 0.032	0.130 ± 0.031	0.130 ± 0.031
650	1.300	0.492	0.206	27	0.213 ± 0.057	0.213 ± 0.057	0.213 ± 0.057
650	3.200	0.200	0.091	31	0.092 ± 0.028	0.091 ± 0.028	0.091 ± 0.028

8. Applications: Determination of the charm content of the proton

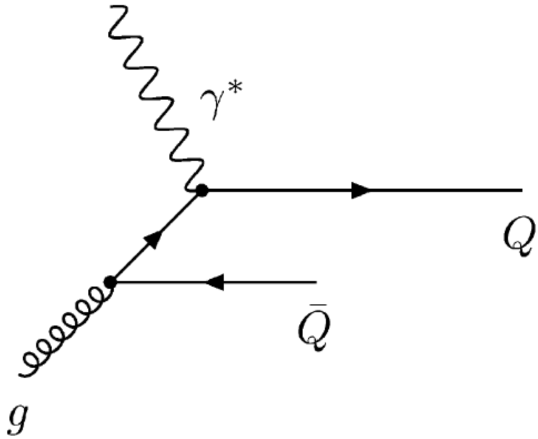
Our approach is based on following observations:

- The ratios $R = F_L / F_T$ and $A = 2xF_A / F_2$ in heavy-quark leptoproduction are perturbatively stable within the FFNS.
- The quantities F_L / F_T and $2xF_A / F_2$ are sensitive to resummation of the mass logarithms of the type $\alpha_s \ln(Q^2 / m^2)$ within the VFNS.

These facts together imply that (future) high-Q^2 data on the ratios $R = F_L / F_T$ and $A = 2xF_A / F_2$ will make it possible to probe the heavy-quark densities in the nucleon, and thus to compare the convergence of perturbative series within the FFNS and VFNS.

Remember that, within the VFNS, the heavy-quark content of the proton is due to resummation of the mass logarithms of the type $\alpha_s \ln(Q^2 / m^2)$ and, for this reason, closely related to behavior of asymptotic perturbative series for high Q^2.

The leading mechanism is the photon-gluon fusion



$$\gamma^*(q) + g(k_g) \to Q(p_Q) + \overline{Q}(p_{\bar{Q}})$$

Leveille, Weiler, PRD 24 (1981) 1789
Watson, Z. Phys. C 12 (1982) 123

$$\hat{\sigma}^{(0)}_{2,g}(z,\lambda) = \frac{\alpha_s}{2\pi}\hat{\sigma}_B(z)\{\left[(1-z)^2+z^2+4\lambda z(1-3z)-8\lambda^2 z^2\right]\ln\frac{1+\beta_z}{1-\beta_z} - [1+4z(1-z)(\lambda-2)]\,\beta_z\},$$

$$\hat{\sigma}^{(0)}_{L,g}(z,\lambda) = \frac{2\alpha_s}{\pi}\hat{\sigma}_B(z)z\{-2\lambda z\ln\frac{1+\beta_z}{1-\beta_z} + (1-z)\,\beta_z\},$$

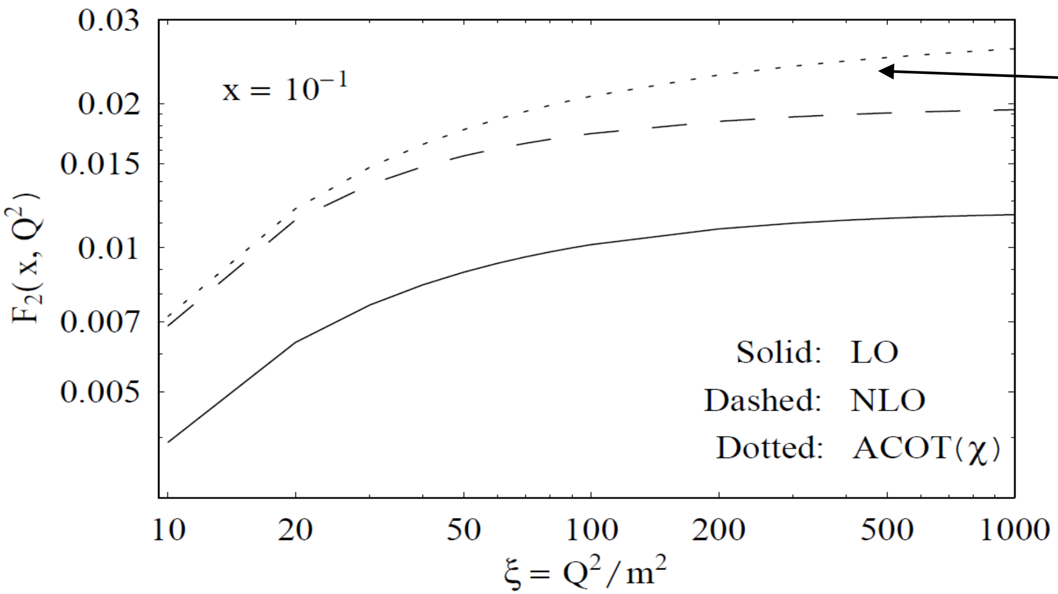
$$\hat{\sigma}^{(0)}_{A,g}(z,\lambda) = \frac{\alpha_s}{\pi}\hat{\sigma}_B(z)z\{2\lambda[1-2z(1+\lambda)]\ln\frac{1+\beta_z}{1-\beta_z} + (1-2\lambda)(1-z)\beta_z\},$$

$$\hat{\sigma}^{(0)}_{I,g}(z,\lambda) = 0$$

$$z = \frac{Q^2}{2q\cdot k_g}, \qquad \lambda = \frac{m^2}{Q^2}, \qquad \beta_z = \sqrt{1-\frac{4\lambda z}{1-z}}$$

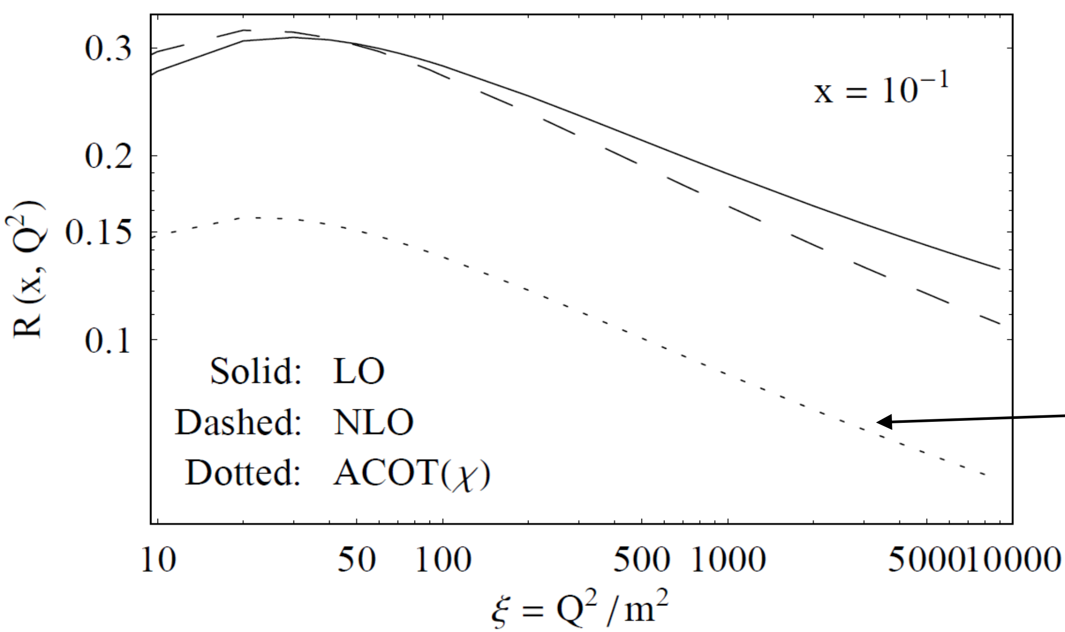
$$\hat{\sigma}_B(z) = \frac{(2\pi)^2 e_Q^2 \alpha_{em}}{Q^2}\,z$$

pQCD Predictions for F_2 and R



Resummation for F_2

For F_2 the NLO and resummation contributions are very close



CTEQ6M PDFs are used for estimates

$$x = 10^{-1}$$

Resummation for $R = F_L / F_T$

Main Conclusions

1. The pQCD predictions for $R = F_L / F_T$ and $A = 2xF_A / F_2$ are sufficiently insensitive (to within ten percent) to:

- radiative NLO corrections;
- standard uncertainties in the QCD input parameters (μ_R, μ_F, Λ_{QCD}, and PDFs);
- DGLAP evolution of the PDFs.

2. The quantities F_L / F_T and $2xF_A / F_2$ are sensitive to resummation of the mass logarithms of the type $\alpha_s \ln(Q^2 / m^2)$.

3. High-Q^2 data on ratios $R = F_L / F_T$ and $A = 2xF_A / F_2$ will make it possible to probe the heavy-quark densities in the nucleon and thus to compare convergence of perturbative series within the FFNS and VFNS.


Thank You!