Multiquark states
in the covariant quark confinement model

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Baldin ISHEPP XXI-2012

Contents

Relativistic Quark Model of Hadrons

G. V. Efimov, M. A. Ivanov, V. E. Lyubovitskij, J. G. Körner, P. Santorelli,. . .

- **Main assumption: hadrons interact via quark exchange only**
- **Interaction Lagrangian**

$$\mathcal{L}_{\rm int} = g_H \cdot H(x) \cdot J_H(x)$$

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- **Main assumption: hadrons interact via quark exchange only**
- **Interaction Lagrangian**

$$\mathcal{L}_{\rm int} = g_H \cdot H(x) \cdot J_H(x)$$

- **Quark currents**

$$J_M(x) = \int dx_1 \int dx_2 \, F_M(x; x_1, x_2) \cdot \bar{q}^a_{f_1}(x_1) \, \Gamma_M \, q^a_{f_2}(x_2) \qquad \text{Meson}$$

$$J_B(x) = \int dx_1 \int dx_2 \int dx_3 \, F_B(x; x_1, x_2, x_3) \qquad \text{Baryon}$$
$$\times \, \Gamma_1 \, q^{a_1}_{f_1}(x_1) \left(q^{a_2}_{f_2}(x_2) C \, \Gamma_2 \, q^{a_3}_{f_3}(x_3) \right) \cdot \varepsilon^{a_1 a_2 a_3}$$

$$J_T(x) = \int dx_1 \ldots \int dx_4 \, F_T(x; x_1, \ldots, x_4) \qquad \text{Tetraquark}$$
$$\times \left(q^{a_1}_{f_1}(x_1) \, C\Gamma_1 \, q^{a_2}_{f_2}(x_2) \right) \cdot \left(\bar{q}^{a_3}_{f_3}(x_3) \, \Gamma_2 C \, \bar{q}^{a_4}_{f_4}(x_4) \right) \cdot \varepsilon^{a_1 a_2 c} \varepsilon^{a_3 a_4 c}$$

Compositeness condition $Z_H = 0$

Salam 1962; Weinberg 1963

- A composite field and its constituents are introduced as elementary particles
- The transition of a composite field to its constituents is provided by the interaction Lagrangian
- The renormalization constant $Z^{1/2}$ is the matrix element between a physical state and the corresponding bare state. If there is a stable bound state which we wish to represent by introducing a quasi-particle H, then elementary particle must have renormalization factor Z equal to zero

$$Z_H^{1/2} = < H_{\rm bare} | H_{\rm dressed} > = 0$$

We use the compositeness condition to determine the hadron-quark coupling constant, e.g. in the case of mesons

$$Z_M = 1 - \tilde{\Pi}'(m_M^2) = 0$$

where $\tilde{\Pi}(p^2)$ is the meson mass operator.

The vertex functions and quark propagators

- **The vertex functions**

$$F_B(x, x_1, \ldots, x_n) = \delta^{(4)}\left(x - \sum_{i=1}^{n} w_i x_i\right) \Phi_H\left(\sum_{i<j}(x_i - x_j)^2\right)$$

where $w_i = m_i / \sum_i m_i$.

- **The quark propagators**

$$S_q(x_1 - x_2) = \int \frac{d^4k}{(2\pi)^4 i} \frac{e^{-ik(x_1 - x_2)}}{m_q - \not{k}}$$

The matrix elements

- The matrix elements are described by a set of the Feynman diagrams which are convolution of the quark propagators and vertex functions.
- Let n, ℓ and m be the number of the propagators, loops and vertices, respectively. In the momentum space the ℓ-loop diagram will be represented as

$$\Pi(p_1, \ldots, p_m) = \int [d^4k]^\ell \prod_{i_1=1}^{m} \Phi_{i_1+n}\left(-K^2_{i_1+n}\right) \prod_{i_3=1}^{n} S_{i_3}(\tilde{k}_{i_3} + v_{i_3})$$

$$K^2_{i_1+n} = \sum_{i_2} (\tilde{k}^{(i_2)}_{i_1+n} + v^{(i_2)}_{i_1+n})^2$$

$\tilde{k}_i$ are linear combinations of the loop momenta k_i

v_i are linear combinations of the external momenta p_i

Infrared confinement

- **Use the Schwinger representation of the propagator:**

$$\frac{m+\not k}{m^2-k^2} = (m+\not k)\int\limits_0^\infty d\alpha\ \exp[-\alpha(m^2-k^2)]$$

- **Choose a simple Gaissian form for the vertex function**

$$\Phi(-K^2) = \exp\left(K^2/\Lambda^2\right)$$

where the parameter Λ characterizes the hadron size.

- **The general expression for the diagram**

$$\Pi(p_1,\ldots,p_m) = \int\limits_0^\infty d^n\alpha \int [d^4k]^\ell\ \Phi\ \exp[-\sum_{i=1}^{n}\alpha_i(m_i^2-(K_i+P_i)^2)]$$

where K_i is the linear combination of the loop momenta and P_i is the linear combination of the external momenta. Φ stands for the numerator product of propagators and vertex functions.

Infrared confinement

- **After doing the loop integrations one obtains**

$$\Pi = \int_0^\infty d^n\alpha \, F(\alpha_1, \ldots, \alpha_n) \,,$$

where F stands for the whole structure of a given diagram.

- **The set of Schwinger parameters α_i can be turned into a simplex by introducing an additional t–integration via the identity**

$$1 = \int_0^\infty dt \, \delta(t - \sum_{i=1}^n \alpha_i)$$

leading to

$$\Pi = \int_0^\infty dt t^{n-1} \int_0^1 d^n\alpha \, \delta\Big(1 - \sum_{i=1}^n \alpha_i\Big) \, F(t\alpha_1, \ldots, t\alpha_n) \,.$$

Infrared confinement

T. Branz, A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Körner and V. E. Lyubovitskij,
Phys. Rev. D81, 034010 (2010)

- We cut off the upper integration at $1/\lambda^2$ and obtain

$$\Pi^c = \int\limits_0^{1/\lambda^2} dt t^{n-1} \int\limits_0^1 d^n\alpha \, \delta\Big(1 - \sum_{i=1}^n \alpha_i\Big) \, F(t\alpha_1, \ldots, t\alpha_n)$$

- By introducing the infrared cut-off one has removed all possible thresholds in the quark loop diagram.

- We take the cut-off parameter λ to be the same in all physical processes.

Infrared confinement

- **We consider the case of a scalar one–loop two–point function:**

$$\Pi_2(p^2) = \int \frac{d^4k_E}{\pi^2} \frac{e^{-s\,k_E^2}}{[m^2 + (k_E + \frac{1}{2}p_E)^2][m^2 + (k_E - \frac{1}{2}p_E)^2]}$$

where the numerator factor $e^{-s\,k_E^2}$ comes from the product of nonlocal vertex form factors of Gaussian form. k_E, p_E are Euclidean momenta ($p_E^2 = -\,p^2$).

- **Doing the loop integration one obtains**

$$\Pi_2(p^2) = \int\limits_0^\infty dt \frac{t}{(s+t)^2} \int\limits_0^1 d\alpha\, \exp\left\{-t\,[m^2 - \alpha(1-\alpha)p^2] + \frac{st}{s+t}\left(\alpha - \frac{1}{2}\right)^2 p^2\right\}$$

A branch point at $p^2 = 4m^2$

Infrared confinement

- By introducing a cut-off in the t–integration one obtains

$$\Pi_2^c(p^2) = \int\limits_0^{1/\lambda^2} dt \frac{t}{(s+t)^2} \int\limits_0^1 d\alpha \; \exp\left\{-t\,[m^2 - \alpha(1-\alpha)p^2] + \frac{st}{s+t}\left(\alpha - \frac{1}{2}\right)^2 p^2\right\}$$

  where the one–loop two–point function $\Pi_2^c(p^2)$ no longer has a branch point at $p^2 = 4m^2$.

- Such a confinement scenario can be realized with only minor changes in our approach by shifting the upper t–integration limit from infinity to $1/\lambda^2$.

- The confinement scenario also allows to include all possible both two-quark and multi-quark resonance states in our calculations.

Subtleties: gauging

In order to guarantee local invariance of the strong interaction Lagrangian one multiplies each quark field $q(x_i)$ in nonlocal quark current $J_H(x)$ with a gauge field exponential:

$$q_i(x_i) \to e^{-ie_{q_1} I(x_i,x,P)}\, q_i(x_i) \quad \text{where} \quad I(x_i, x, P) = \int\limits_x^{x_i} dz_\mu A^\mu(z).$$

The path P connects the end-points of the path integral.
We use the path-independent definition of the derivative of $I(x, y, P)$:

Mandelstam,1962,Terning,1991

$$\lim_{dx^\mu \to 0} dx^\mu \frac{\partial}{\partial x^\mu} I(x, y, P) \;=\; \lim_{dx^\mu \to 0} [I(x + dx, y, P') - I(x, y, P)]$$

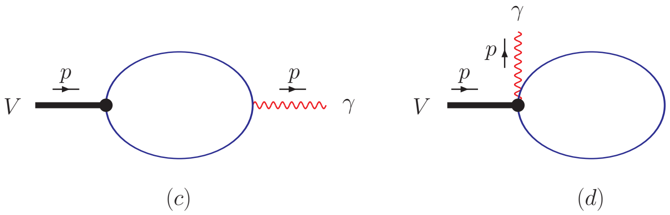
where the path P' is obtained from P by shifting the end-point x by dx.
The definition leads to the key rule

$$\frac{\partial}{\partial x^\mu} I(x, y, P) = A_\mu(x)$$

which in turn states that the derivative of the path integral $I(x, y, P)$ does not depend on the path P originally used in the definition.

Subtleties: gauging

Diagrams describing $\mathbf{V \to \gamma}$ transition:



$$M_c^{\mu\nu}(p) = \int \frac{d^4k}{4\pi^2 i}\Phi_V(-k^2)\,\mathrm{tr}\Big(\gamma^\mu S(k+\tfrac{1}{2}p)\gamma^\nu S(k-\tfrac{1}{2}p)\Big)$$

$$M_d^{\mu\nu}(p) = -\int \frac{d^4k}{4\pi^2 i}(2k+\tfrac{1}{2}p)^\mu \int_0^1 d\alpha\,\Phi_V'\Big(-\alpha(k+\tfrac{1}{2}p)^2-(1-\alpha)k^2\Big) \times \mathrm{tr}\Big(\gamma^\nu S(k)\Big)$$

Subtleties: gauging

If $\mathbf{p = 0}$ then the second diagram maybe transfered to the first one by using integration by parts

$$\int \frac{d^4k}{4\pi^2 i} \frac{\partial}{\partial k^\mu} \Big\{ \Phi_V\big(-k^2\big) \mathrm{tr}\big(\gamma^\nu S(k)\big) \Big\} =$$

$$= \int \frac{d^4k}{4\pi^2 i} \Big\{ -2k^\mu \Phi_V'\big(-k^2\big) \mathrm{tr}\big(\gamma^\nu S(k)\big)$$

$$+\Phi_V\big(-k^2\big) \mathrm{tr}\big(\gamma^\mu S(k) \gamma^\nu S(k)\big) \Big\} = 0.$$

Model parameters

M. A. I., J. G. Körner, S. G. Kovalenko, P. Santorelli and G. G. Saidullaeva, Phys. Rev. D85, 034004 (2012)

Input values for the leptonic decay constants f_H (in MeV) and our least-squares fit values.

	Fit Values	PDG/LAT
f_π	128.7	130.4 ± 0.2
f_K	156.1	156.1 ± 0.8
f_D	205.9	206.7 ± 8.9
f_{D_s}	257.5	257.5 ± 6.1
f_B	191.1	192.8 ± 9.9
f_{B_s}	234.9	238.8 ± 9.5
f_{B_c}	489.0	489 ± 5
f_ρ	221.1	221 ± 1

	This work	PDG/LAT
f_ω	198.5	198 ± 2
f_ϕ	228.2	227 ± 2
$f_{J/\psi}$	415.0	415 ± 7
f_{K^*}	213.7	217 ± 7
f_{D^*}	243.3	245 ± 20
$f_{D_s^*}$	272.0	272 ± 26
f_{B^*}	196.0	196 ± 44
$f_{B_s^*}$	229.0	229 ± 46

Model parameters

Input values for some basic electromagnetic decay widths and our least-squares fit values (in keV).

Process	Fit Values	PDG
$\pi^0 \to \gamma\gamma$	5.06×10^{-3}	$(7.7 \pm 0.4) \times 10^{-3}$
$\eta_c \to \gamma\gamma$	1.61	1.8 ± 0.8
$\rho^\pm \to \pi^\pm\gamma$	76.0	67 ± 7
$\omega \to \pi^0\gamma$	672	703 ± 25
$K^{*\pm} \to K^\pm\gamma$	55.1	50 ± 5
$K^{*0} \to K^0\gamma$	116	116 ± 10
$D^{*\pm} \to D^\pm\gamma$	1.22	1.5 ± 0.5
$J/\psi \to \eta_c\gamma$	1.43	1.58 ± 0.37

Model parameters

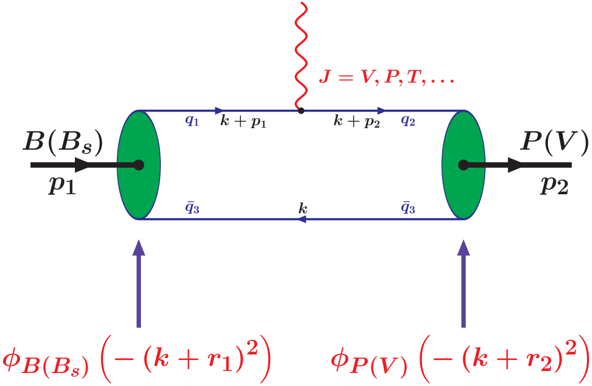
The results of the fit for the values of quark masses m_{q_i}, the infrared cutoff parameter λ and the size parameters Λ_{H_i} (all in GeV).

m_u	m_s	m_c	m_b	λ	
0.235	0.424	2.16	5.09	0.181	GeV

Λ_π	Λ_K	Λ_D	Λ_{D_s}	Λ_B	Λ_{B_s}	Λ_{B_c}	Λ_ρ
0.87	1.04	1.47	1.57	1.88	1.95	2.42	0.61

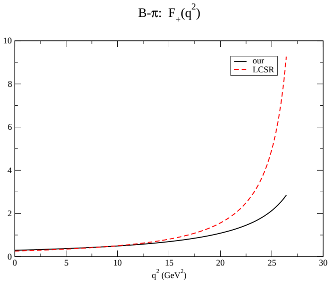
Λ_ω	Λ_ϕ	$\Lambda_{J/\psi}$	Λ_{K^*}	Λ_{D^*}	$\Lambda_{D_s^*}$	Λ_{B^*}	$\Lambda_{B_s^*}$
0.47	0.88	1.48	0.72	1.16	1.17	1.72	1.71

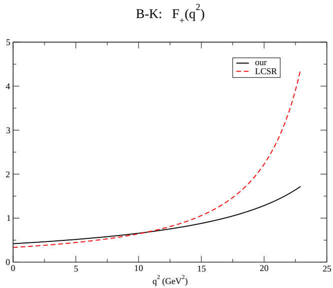
Form factors

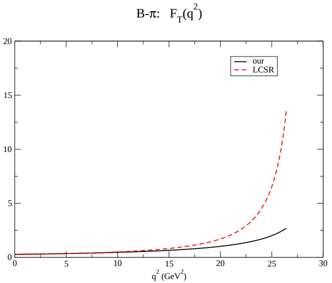


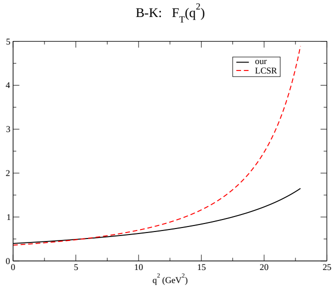
$$r_i = \frac{m_{q_3}}{m_{q_i}+m_{q_3}}\,p_i$$

Form factors









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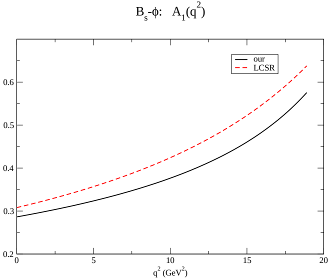
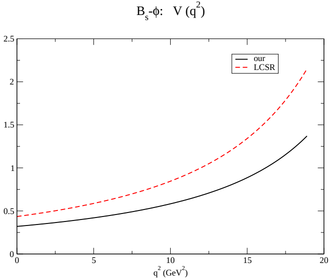
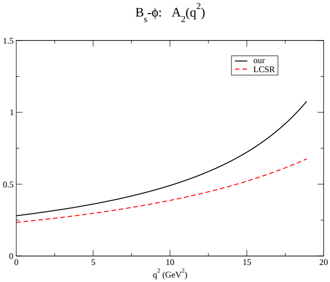
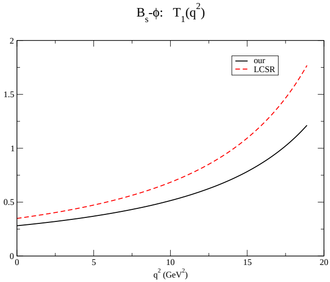
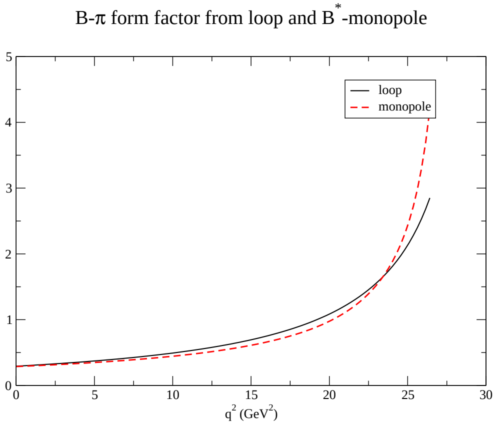








Form factors



$$F^{B\pi}_{\rm VDM}(q^2) = \frac{F^{B\pi}_{+}(0)}{m^2_{B^*} - q^2}.$$

Form factors

	This work	LCSR
$F_+^{B\pi}(0)$	0.29	0.258±0.031
$F_+^{BK}(0)$	0.42	0.335±0.042
$F_T^{B\pi}(0)$	0.27	0.253±0.028
$F_T^{BK}(0)$	0.40	0.359±0.038
$V^{B\rho}(0)$	0.28	0.324±0.029
$V^{BK^*}(0)$	0.36	0.412±0.045
$V^{B_s\phi}(0)$	0.32	0.434±0.035
$A_1^{B\rho}(0)$	0.26	0.240±0.024
$A_1^{BK^*}(0)$	0.33	0.290±0.036
$A_1^{B_s\phi}(0)$	0.29	0.311±0.029
$A_2^{B\rho}(0)$	0.24	0.221±0.023
$A_2^{BK^*}(0)$	0.32	0.258±0.035
$A_2^{B_s\phi}(0)$	0.28	0.234±0.028
$T_1^{B\rho}(0)$	0.25	0.268±0.021
$T_1^{BK^*}(0)$	0.33	0.332±0.037
$T_1^{B_s\phi}(0)$	0.28	0.349±0.033

Nonleptonic B_s decays

- The modes

$$B_s \to D_s^- D_s^+,\, D_s^{*\,-} D_s^+ + D_s^- D_s^{*\,+},\, D_s^{*\,-} D_s^{*\,+}$$

give the largest contribution to $\Delta\Gamma_s \equiv \Gamma_L - \Gamma_H$ for the $B_s - \bar{B}_s$ system.

- The mode $B_s \to J/\psi\phi$ is color–suppressed but it is interesting for the search of possible CP-violating new physics effects in $B_s - \bar{B}_s$ mixing.

- The effective Hamiltonian describing the B_s nonleptonic decays:

$$\mathcal{H}_{\rm eff} = -\frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^\dagger \sum_{i=1}^{6} C_i\, Q_i,$$

$$Q_1 = (\bar{c}_{a_1} b_{a_2})_{V-A} (\bar{s}_{a_2} c_{a_1})_{V-A}, \qquad Q_2 = (\bar{c}_{a_1} b_{a_1})_{V-A}, (\bar{s}_{a_2} c_{a_2})_{V-A},$$
$$Q_3 = (\bar{s}_{a_1} b_{a_1})_{V-A} (\bar{c}_{a_2} c_{a_2})_{V-A}, \qquad Q_4 = (\bar{s}_{a_1} b_{a_2})_{V-A} (\bar{c}_{a_2} c_{a_1})_{V-A},$$
$$Q_5 = (\bar{s}_{a_1} b_{a_1})_{V-A} (\bar{c}_{a_2} c_{a_2})_{V+A}, \qquad Q_6 = (\bar{s}_{a_1} b_{a_2})_{V-A} (\bar{c}_{a_2} c_{a_1})_{V+A},$$

where the subscript $_{V-A}$ refers to the usual left–chiral current $O^\mu_- = \gamma^\mu(1-\gamma^5)$ and $_{V+A}$ to the usual right–chiral one $O^\mu_+ = \gamma^\mu(1+\gamma^5)$.

Nonleptonic B_s decays

Calculated branching ratios (%) of the B_s nonleptonic decays.

Process	This work	PDG
$B_s \to D_s^- D_s^+$	1.65	$1.04^{+0.29}_{-0.26}$
$B_s \to D_s^- D_s^{*\,+} + D_s^{*\,-} D_s^+$	2.40	2.8 ± 1.0
$B_s \to D_s^{*\,-} D_s^{*\,+}$	3.18	3.1 ± 1.4
$B_s \to J/\psi\phi$	0.16	0.14 ± 0.05

Nucleon as three-quark state: Lagrangian

T. Gutsche, M. A. I., J. G. Körner, V. E. Lyubovitskij, P. Santorelli, arXiv:1207.7052 (to appear in PRD)

Lagrangian describing the interaction of proton (antiproton) with its constituents:

$$\mathcal{L}^{p}_{int}(x) = g_N\, \bar{p}(x) \cdot J_p(x) + h.c.$$

The interpolating three-quark current:

$$J_p(x) = \int dx_1 \int dx_2 \int dx_3\, F_N(x; x_1, x_2, x_3)\, J^{(p)}_{3q}(x_1, x_2, x_3)$$

$$J^{(p)}_{3q}(x_1, x_2, x_3) = \Gamma^A \gamma^5\, d^{a_1}(x_1) \cdot [\epsilon^{a_1 a_2 a_3}\, u^{a_2}(x_2)\, C\, \Gamma_A\, u^{a_3}(x_3)]\,.$$

There are two kinds of three-quark currents:

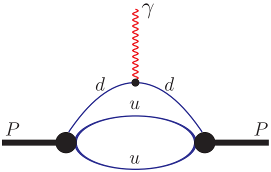
$$\Gamma^A \otimes \Gamma_A = \gamma^\alpha \otimes \gamma_\alpha \quad \text{(vector)} \quad \Gamma^A \otimes \Gamma_A = \frac{1}{2}\sigma^{\alpha\beta} \otimes \sigma_{\alpha\beta} \quad \text{(tensor)}$$

We consider a general linear superposition:

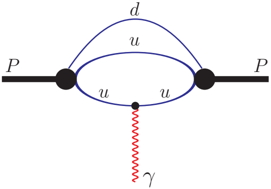
$$J_N = xJ^T_N + (1-x)J^V_N\,, \quad N = p, n$$

with a mixing parameter x $(0 \le x \le 1)$.

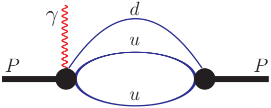
Electromagnetic vertex function of proton



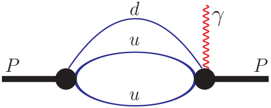
(a)



(b)



(c)



(d)

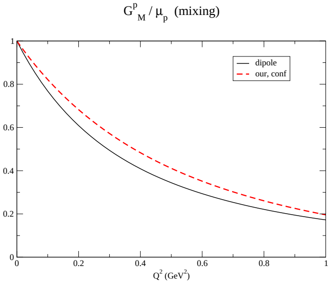
Static properties of nucleons

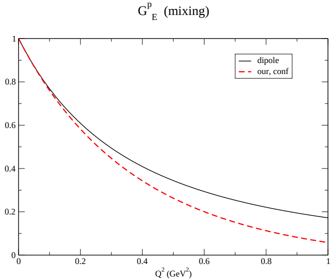
Parameters:

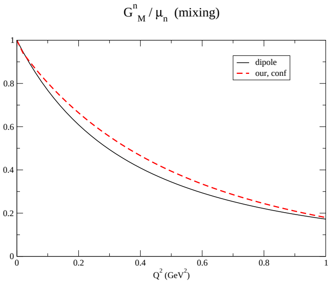
- a superposition of the V– and T–currents of nucleons with $x = 0.8$
- the size parameter of the nucleon we take $\Lambda_N = 0.5$ GeV.

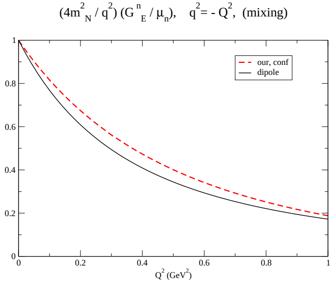
Quantity	Our results	PDG
μ_p (in n.m.)	2.96	2.793
μ_n (in n.m.)	-1.83	-1.913
r_E^p (fm)	0.805	0.8768 ± 0.0069
$< r_E^2 >^n$ (fm^2)	-0.121	-0.1161 ± 0.0022
r_M^p (fm)	0.688	$0.777 \pm 0.013 \pm 0.010$
r_M^n (fm)	0.685	$0.862^{+0.009}_{-0.008}$

Electromagnetic form factors







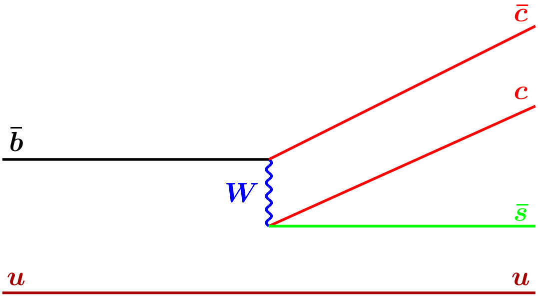


X(3872)-meson as a tetraquark state: Introduction

A narrow charmonium-like state X(3872) was observed in the exclusive decay process:

$$B^+ \to K^+\pi^+\pi^- J/\psi$$

S. K. Choi *et al.* [Belle Collaboration] Phys. Rev. Lett. 91, 262001 (2003)



X(3872)-meson as a tetraquark state: Introduction

- **X-mass is close to $D^0 - D^{*\,0}$ mass threshold:**

$$M_X = 3872.0 \pm 0.6\,(\text{stat}) \pm 0.5\,(\text{syst})\,\text{MeV}$$

$$M_{D^0} + M_{D^{*\,0}} = 3871.81 \pm 0.25\,\text{MeV}$$

- **Its width $\Gamma_X \leq 2.3$ MeV at 90% CL.**

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- **The state was confirmed in B-decays by BaBar experiment**

  B. Aubert *et al.* Phys. Rev. Lett. 93, 041801 (2004)

  and in $p\bar{p}$ production by Tevatron experiments CDF and DØ.

  D. E. Acosta *et al.* [CDF Collaboration] Phys. Rev. Lett. 93, 072001 (2004);
  V. M. Abazov *et al.* [D0 Collaboration] Phys. Rev. Lett. 93, 162002 (2004)

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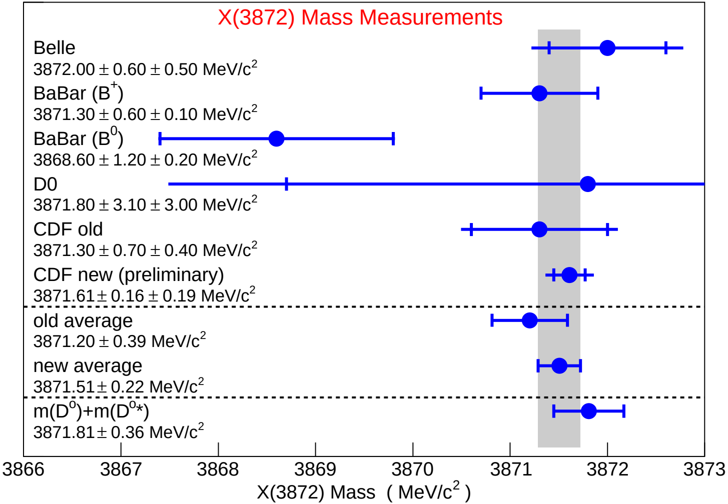

K. Yi [CDF Collaboration] arXiv: 0906.4996.

New average mass: $M_X = 3871.51 \pm 0.22$ MeV

X(3872)-meson as a tetraquark state: Introduction

- **From the observation of decays $\mathbf{X(3872) \to J/\psi\gamma}$ reported by both Belle and BaBar collaborations and from the angular analysis performed by CDF experiment it was shown that the only quantum numbers $\mathbf{J^{PC} = 1^{++}}$ or $\mathbf{2^{-+}}$ are compatible with data.**

K. Abe *et al.*, [Belle Collaboration], arXiv:hep-ex/0505037; hep-ex/0505038
B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. D 74, 071101 (2006)

A. Abulencia *et al.* [CDF Collaboration], Phys. Rev. Lett. 98, 132002 (2007)

- **The observation of decays into $\mathbf{D^0\overline{D}^0\pi^0}$ by Belle and BaBar collaborations allows one to exclude the choice $\mathbf{2^{-+}}$ because the near-threshold decay $\mathbf{X \to D^0\overline{D}^0\pi^0}$ is expected to be strongly suppressed for $\mathbf{J = 2}$.**

G. Gokhroo *et al.* [Belle Collaboration], Phys. Rev. Lett. 97, 162002 (2006)

B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. D 77, 011102 (2008).

- **The quantum numbers of the X(3872) are**

$$J^{PC} = 1^{++}$$

X(3872)-meson as a tetraquark state: Introduction

- **Belle collaboration has reported evidence for the decay mode $X \to \pi^+\pi^-\pi^0 J/\psi$ dominated by the sub-threshold decay $X \to \omega J/\psi$.**

K. Abe *et al.*, [Belle Collaboration], arXiv:hep-ex/0505037,hep-ex/0505038

- **It was found that the branching ratio of this mode is almost the same as of $X \to \pi^+\pi^- J/\psi$ decay:**

$$\frac{\mathcal{B}(X\to J/\psi\pi^+\pi^-\pi^0)}{\mathcal{B}(X\to J/\psi\pi^+\pi^-)} = 1.0 \pm 0.4\,(\mathrm{stat}) \pm 0.3\,(\mathrm{syst}).$$

- **It implies strong isospin violation because the three-pion decay proceeds via intermediate ω-meson with isospin 0 whereas the two-pion decay proceeds via intermediate ρ-meson with isospin 1.**

X(3872)-meson as a tetraquark state: Introduction

- **The two-pion decay via intermediate ρ-meson is very difficult to explain by using an interpretation of the X(3872) as simple $c\bar{c}$ charmonium state with isospin 0.**

- **The possible candidate from $\bar{c}c$-spectroscopy:**

$$\chi_{c_1}(2^3P_1) - \text{state with } J^{PC} = 1^{++}$$

BUT the value of its mass varies from 3925 up to 3953 MeV. Also the decay width calculated in variuos models is too large.

- **The X(3872) IS NOT the pure $\bar{c}c$-state**

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- **The X(3872) IS NOT the pure $\bar{c}c$-state**
- **a molecule bound state $D^0\bar{D}^{*0}$ with small binding energy**
- **a tetraquark state composed from a diquark and antidiquark**
- **threshold cusps**
- **hybrids and glueballs**

X(3872)-meson as a tetraquark state: Introduction

- **An intepretation of the X(3872) as a tetraquark was suggested in**

  L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Phys. Rev. D 71, 014028 (2005)

$$\mathbf{X_q \Longrightarrow [cq]_{S=1}[\bar{c}\bar{q}]_{S=0} + [cq]_{S=0}[\bar{c}\bar{q}]_{S=1}, \qquad (q = u, d)}$$

- **Isospin breaking: the state $\mathbf{X_u}$ breaks isospin symmetry maximally:**

$$\mathbf{X_u = \frac{1}{\sqrt{2}}\Big\{ \underbrace{\frac{X_u + X_d}{\sqrt{2}}}_{I=0} + \underbrace{\frac{X_u - X_d}{\sqrt{2}}}_{I=1} \Big\}.}$$

X(3872)-meson as a tetraquark state: Introduction

- **The physical states are the mixing of X_u and X_d**

$$\begin{aligned} X_l \equiv X_{low} &= X_u \cos\theta + X_d \sin\theta, \\ X_h \equiv X_{high} &= -X_u \sin\theta + X_d \cos\theta. \end{aligned}$$

- **The mixing angle θ is supposed to be found from the known ratio of the two-pion (via ρ) and three-pion (via ω) decay widths.**

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- **The mixing angle θ is supposed to be found from the known ratio of the two-pion (via ρ) and three-pion (via ω) decay widths.**

- **We are aiming to perform independent analysis of the X(3872)-meson considered as a tetraquark state in the framework of the relativistic constituent quark model with infrared confinement.**

X(3872)-meson as a tetraquark state: Lagrangian

S. Dubnicka, A. Z. Dubnickova, M. A. Ivanov and J. G. Körner, Phys. Rev. D 81, 114007 (2010)

- **An effective interaction Lagrangian**

$$\mathcal{L}_{\rm int} = g_X\, X_{q\,\mu}(x) \cdot J^{\mu}_{X_q}(x), \qquad (q = u, d).$$

- **The nonlocal version of the four-quark interpolating current**

$$J^{\mu}_{X_q}(x) = \int dx_1 \ldots \int dx_4\, \delta\Big(x - \sum_{i=1}^{4} w_i x_i\Big)\, \Phi_X\Big(\sum_{i<j}(x_i - x_j)^2\Big)\, J^{\mu}_{4q}(x_1, \ldots, x_4)$$

$$J^{\mu}_{4q} = \frac{1}{\sqrt{2}}\, \varepsilon_{abc}\, [q_a(x_4) C\gamma^5 c_b(x_1)]\, \varepsilon_{dec}\, [\bar{q}_d(x_3)\gamma^{\mu} C\bar{c}_e(x_2)] + (\gamma^5 \leftrightarrow \gamma^{\mu}),$$

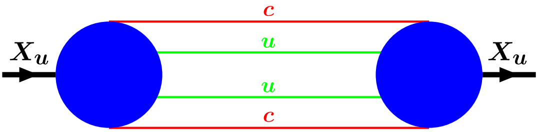
$$w_1 = w_2 = \frac{m_c}{2(m_q + m_c)} \equiv \frac{w_c}{2}, \qquad w_3 = w_4 = \frac{m_q}{2(m_q + m_c)} \equiv \frac{w_q}{2}.$$

Compositeness condition

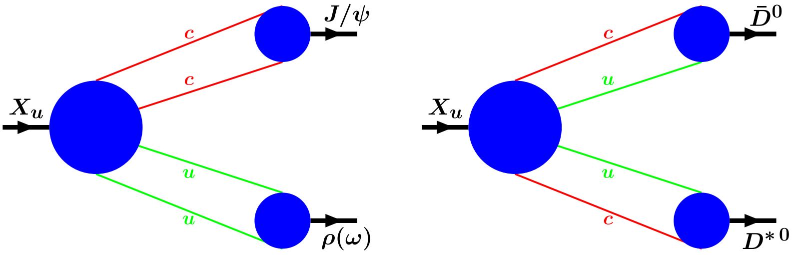
The coupling constant g_X is determined from the compositeness condition

$$Z_X = 1 - \Pi'_X(M_X^2) = 0$$

where $\Pi_X(p^2)$ is the scalar part of the vector-meson mass operator.



Strong off-shell decays



Since the X(3872) lies nearly the respective thresholds in both cases,

$$\begin{aligned} m_X - (m_{J/\psi} + m_\rho) &= -0.90 \pm 0.41\,\mathrm{MeV}, \\ m_X - (m_{D^0} + m_{D^{*0}}) &= -0.30 \pm 0.34\,\mathrm{MeV} \end{aligned}$$

the intermediate $\rho(\omega)$ and D^* mesons should be taken off-shell.

The narrow width approximation

$$\frac{d\Gamma(X \to J/\psi + n\pi)}{dq^2} = \frac{1}{8\, m_X^2\, \pi} \cdot \frac{1}{3} |M(X \to J/\psi + v^0)|^2 \times \frac{\Gamma_{v^0}\, m_{v^0}}{\pi} \frac{p^*(q^2)}{(m_{v^0}^2 - q^2)^2 + \Gamma_{v^0}^2\, m_{v^0}^2} \mathrm{Br}(v^0 \to n\pi),$$

$$\frac{d\Gamma(X_u \to \bar{D}^0 D^0 \pi^0)}{dq^2} = \frac{1}{2\, m_X^2\, \pi} \cdot \frac{1}{3} |M(X_u \to \bar{D}^0 D^{*\,0})|^2 \times \frac{\Gamma_{D^{*\,0}}\, m_{D^{*\,0}}}{\pi} \frac{p^*(q^2)\, \mathcal{B}(D^{*\,0} \to D^0 \pi^0)}{(m_{D^{*\,0}}^2 - q^2)^2 + \Gamma_{D^{*\,0}}^2\, m_{D^{*\,0}}^2},$$

Strong decay widths

- **Two new adjustable parameters:** θ **and** Λ_X.
- **The ratio**

$$\frac{\Gamma(X_u \to J/\psi + 3\,\pi)}{\Gamma(X_u \to J/\psi + 2\,\pi)} \approx 0.25$$

is very stable under variation of Λ_X.

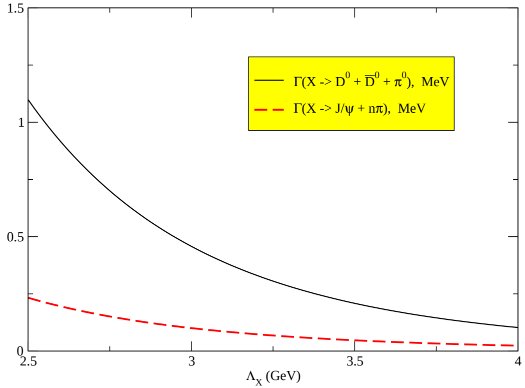
- **Using this result and the central value of the experimental data**

$$\frac{\Gamma(X_{l,h} \to J/\psi + 3\,\pi)}{\Gamma(X_{l,h} \to J/\psi + 2\,\pi)} \approx 0.25 \cdot \left(\frac{1 \pm \tan\theta}{1 \mp \tan\theta}\right)^2 \approx 1$$

gives $\theta \approx \pm 18.4^o$ **for** X_l ("+") **and** X_h ("−")**, respectively.**

- **This is in agreement with the results obtained by both Maiani:** $\theta \approx \pm 20^o$ **and Nielsen:** $\theta \approx \pm 23.5^o$.

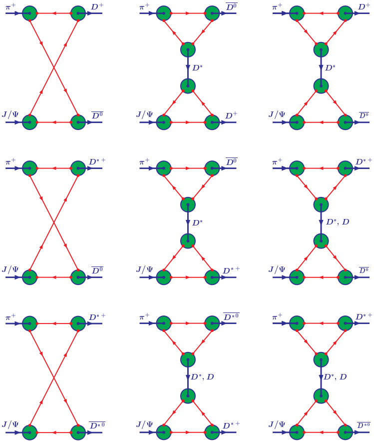
Strong decay widths



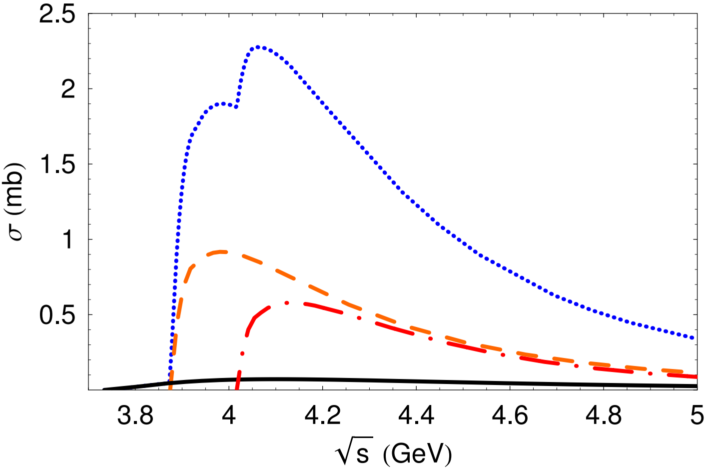
$$\frac{\Gamma(X\to D^0\bar{D}^0\pi^0)}{\Gamma(X\to J/\psi\pi^+\pi^-)} = \begin{cases} 4.5 \pm 0.2 & \text{theor} \\ 10.5 \pm 4.7 & \text{expt} \end{cases}$$

Impact of the X on the J/ψ-dissociation

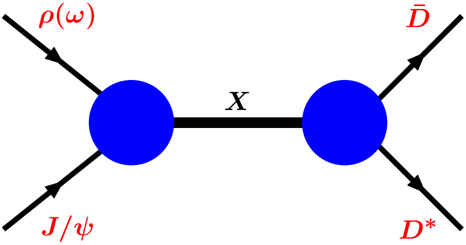
M. A. Ivanov, J. G. Körner and P. Santorelli, Phys. Rev. D 70, 014005 (2004)



Impact of the X on the J/ψ-dissociation



Impact of the X on the J/ψ-dissociation



$$\begin{aligned} & \sigma(\mathrm{J}/\psi + \mathrm{v}^0 \to \mathrm{D} + \bar{\mathrm{D}}^*) + \sigma(\mathrm{J}/\psi + \mathrm{v}^0 \to \bar{\mathrm{D}} + \mathrm{D}^*) \\ = \quad & 2\,(\cos\theta \mp \sin\theta)\,\sigma(\mathrm{J}/\psi + \mathrm{v}^0 \to \mathrm{X_u} \to \bar{\mathrm{D}} + \mathrm{D}^*), \end{aligned}$$

$\mathrm{v}^0 = \rho$ (sign minus); $\mathrm{v}^0 = \omega$ (sign plus).

Impact of the X on the J/ψ-dissociation

$$\sigma(J/\psi + v^0 \to X_u \to \bar{D} + D^*)$$

$$= \frac{1}{16\,\pi\, s} \frac{\lambda^{1/2}(s, m_D^2, m_{D*}^2)}{\lambda^{1/2}(s, m_{J/\psi}^2, m_{v^0}^2)} \frac{1}{9} \sum_{\rm pol} |A|^2 \frac{1}{(s - m_X^2)^2 + \Gamma_X^2 m_X^2},$$

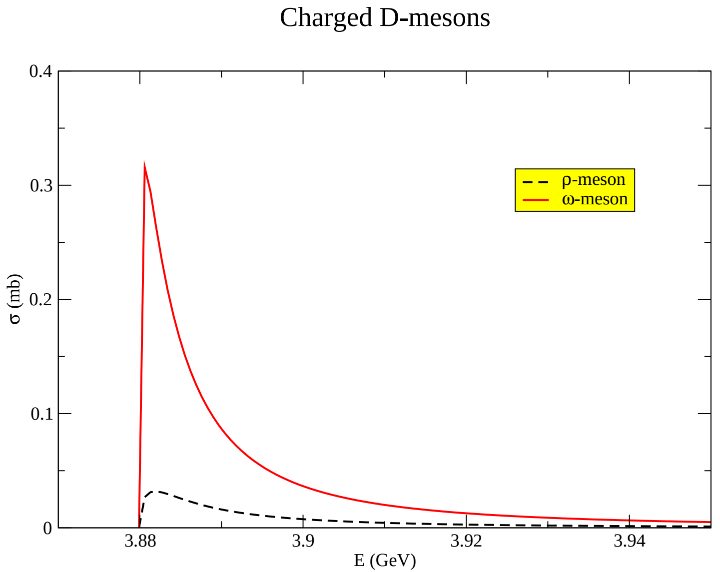
$$A = \varepsilon^{\nu}_{J/\psi} \varepsilon^{\rho}_{v^0} M_{\mu\nu\rho}(J/\psi + v^0 \to X)$$

$$\times \quad \left(- g^{\mu\alpha} + \frac{p^\mu p^\alpha}{m_X^2} \right)$$

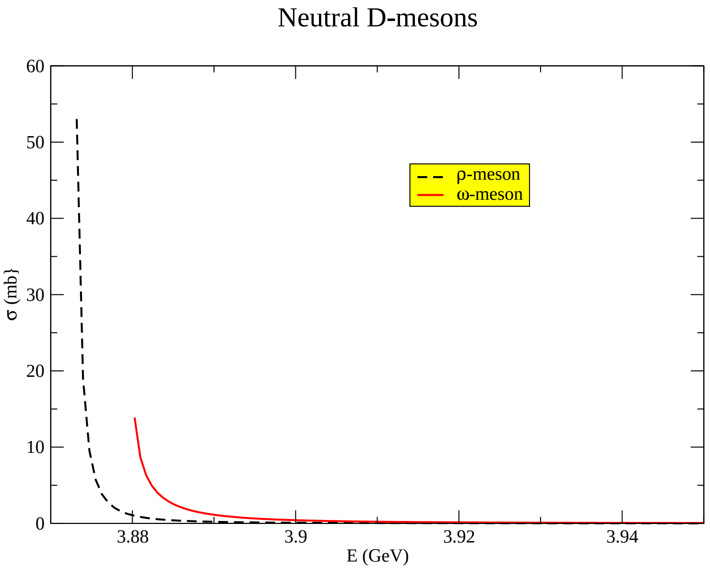
$$\times \quad \varepsilon^{\beta}_{D*} M_{\alpha\beta}(X \to \bar{D} + D^*)$$

Note that $s \geq (m_{D^+} + m_{D^{*+}})^2$ for charged D-mesons and $s > (m_{J/\psi} + m_{v^0})^2$ for neutral D-mesons. In the last case the cross section blows up at $s = (m_{J/\psi} + m_{v^0})^2$ because of the function $\lambda^{1/2}(s, m_{J/\psi}^2, m_{v^0}^2)$ is in the denominator.

Impact of the X on the J/ψ-dissociation

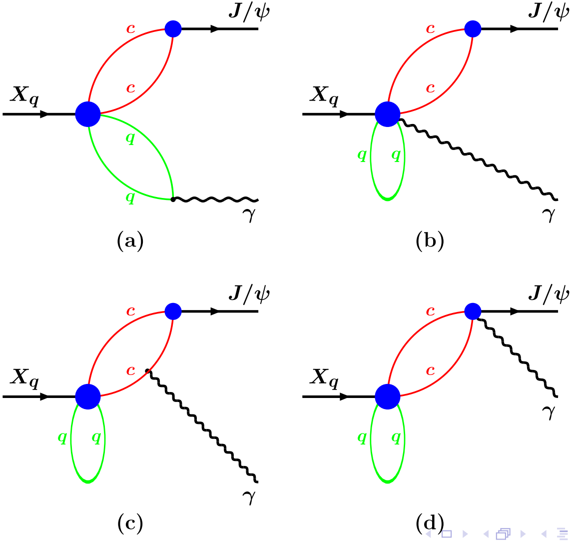


Impact of the X on the J/ψ-dissociation



Radiative X-decay

S. Dubnicka, A. Z. Dubnickova, M. A. Ivanov, J. G. Koerner, P. Santorelli and G. G. Saidullaeva, Phys. Rev. D 84, 014006 (2011)



Radiative X-decay

The on-mass shell conditions

$$\varepsilon_X^\mu p_\mu = 0, \qquad \varepsilon_{J/\psi}^\nu q_{1\nu} = 0, \qquad \varepsilon_\gamma^\rho q_{2\rho} = 0$$

leave us five Lorentz structures:

$$\begin{aligned} T_{\mu\rho\nu}(q_1, q_2) &= \varepsilon_{q_2\mu\nu\rho}(q_1 \cdot q_2)\, W_1 + \varepsilon_{q_1 q_2 \nu\rho} q_{1\mu}\, W_2 + \varepsilon_{q_1 q_2 \mu\rho} q_{2\nu}\, W_3 \\ &+ \varepsilon_{q_1 q_2 \mu\nu} q_{1\rho}\, W_4 + \varepsilon_{q_1 \mu\nu\rho}(q_1 \cdot q_2)\, W_5\, . \end{aligned}$$

Using the gauge invariance condition

$$q_2^\rho T_{\mu\rho\nu} = (q_1 \cdot q_2)\varepsilon_{q_1 q_2 \mu\nu}(W_4 + W_5) = 0$$

one has $W_4 = -W_5$ which reduces the set of independent covariants to four. However, there are two nontrivial relations among the four covariants which can be derived by noting that the tensor

$$T_{\mu[\nu_1\nu_2\nu_3\nu_4\nu_5]} = g_{\mu\nu_1}\varepsilon_{\nu_2\nu_3\nu_4\nu_5} + \mathrm{cycl.}(\nu_1\nu_2\nu_3\nu_4\nu_5)$$

vanishes in four dimensions since it is totally antisymmetric in the five indices $(\nu_1, \nu_2, \nu_3, \nu_4, \nu_5)$.

Radiative X-decay

The two conditions reduce the set of independent covariants to two. This is the appropriate number of independent covariants since the photon transition is described by two independent amplitudes as e.g. by the E1 and M2 transition amplitudes. One has

$$\Gamma(X \to \gamma\, J/\psi) = \frac{1}{12\pi}\frac{|\vec{q}_2|}{m_X^2}\left(|H_L|^2 + |H_T|^2\right) = \frac{1}{12\pi}\frac{|\vec{q}_2|}{m_X^2}\left(|A_{E1}|^2 + |A_{M2}|^2\right),$$

where the helicity amplitudes H_L and H_T are expressed in terms of the Lorentz amplitudes as

$$H_L = i\frac{m_X^2}{m_{J/\psi}}|\vec{q}_2|^2\left[W_1 + W_3 - \frac{m_{J/\psi}^2}{m_X|\vec{q}_2|}W_4\right],$$

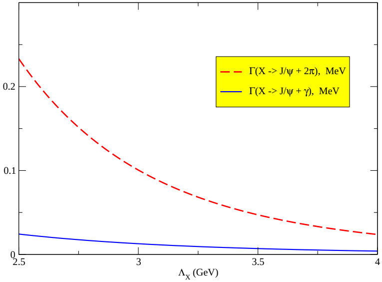
$$H_T = -i m_X|\vec{q}_2|^2\left[W_1 + W_2 - \left(1 + \frac{m_{J/\psi}^2}{m_X|\vec{q}_2|}\right)W_4\right],$$

$$|\vec{q}_2| = \frac{m_X^2 - m_{J/\psi}^2}{2m_X}.$$

The E1 and M2 multipole amplitudes are obtained via

$$A_{E1/M2} = (H_L \mp H_T)/\sqrt{2}.$$

Radiative X-decay



If one takes $\Lambda_X \in (3,4)$ GeV with the central value $\Lambda_X = 3.5$ GeV then our prediction for the ratio of widths reads

$$\left.\frac{\Gamma(X_l \to \gamma + J/\psi)}{\Gamma(X_l \to J/\psi + 2\pi)}\right|_{\rm theor} = 0.15 \pm 0.03$$

which fits very well the experimental data from the Belle Collaboration

$$\frac{\Gamma(X \to \gamma + J/\psi)}{\Gamma(X \to J/\psi\, 2\pi)} = \begin{cases} 0.14 \pm 0.05 & \text{Belle} \\ 0.22 \pm 0.06 & \text{BaBar} \end{cases}$$

Summary and outlook

- **We have presented a refined covariant quark model which includes infrared confinement of quarks.**

- **We have calculated the transition form factors of the heavy $\mathbf{B}$ and $\mathbf{B_s}$ mesons to light pseudoscalar and vector mesons, which are needed as ingredients for the calculation of the semileptonic, nonleptonic, and rare decays of the $\mathbf{B}$ and $\mathbf{B_s}$ mesons. Our form factor results hold in the full kinematical range of momentum transfer.**

- **We have made use of the calculated form factors to calculate the nonleptonic decays $\mathbf{B_s \to D_s \bar{D}_s}$, ... and $\mathbf{B_s \to J/\psi\phi}$, which have been widely discussed recently in the context of $\mathbf{B_s - \bar{B}_s}$–mixing and CP violation.**
- **We have applied our approach to baryon physics by using the same values of the constituent quark masses and infrared cutoff as in meson sector.**

- **We have calculated the nucleon magnetic moments and charge radii and also electromagnetic form factors at low energies.**

Summary and outlook

- **The properties of the $\mathbf{X(3872)}$ as a tetraquark are studied in the framework of a covariant quark model with infrared confinement.**
- **The matrix elements of the off-shell transitions $\mathbf{X \to J/\psi + \rho(\omega)}$ and $\mathbf{X \to D + \bar{D}^*}$ are calculated.**
- **The obtained results are then used to evaluate the widths of the experimentally observed decays $\mathbf{X \to J/\psi + 2\pi(3\pi)}$ and $\mathbf{X \to D^0 + \bar{D}^0 + \pi^0}$.**
- **The possible impact of the $\mathbf{X(3872)}$ on the $\mathbf{J/\psi}$-dissociation process was disscussed.**
- **We have calculated the matrix element of the transition $\mathbf{X \to \gamma + J/\psi}$ and have shown its gauge invariance. We have evaluated the $\mathbf{X \to \gamma + J/\psi}$ decay width and the polarization of the $\mathbf{J/\psi}$ in the decay.**
- **The comparison with available experimental data allows one to conclude that the $\mathbf{X(3872)}$ can be a tetraquark state.**