# Microscopic Pion- Nucleon and Nucleus Scattering at Intermediate Energies

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## Abstract

**The pion – nucleon and nucleus scattering at intermediate energies on the basis of meson exchange theory are studied at intermediate energies. The interaction is described by the scalar-isoscalar sigma- meson and the vector-isovector rho-meson. The calculated results show good agreement with the corresponding experimental available data at these energies and compared with other theoretical model of Kisslinger.**

## Introduction

As is well known the quantum-chromo-dynamics (QCD) is the fundamental theory of strong interactions and it has quarks and gluons as degrees of freedom. Due to the lack and absence of a clear and sharp quark-quark or quark-gluon picture of interaction, and also the difficulties of the extrapolation for the information coming from different energy domains, which did not incorporate in a natural way in one theory [1] man is enforced to use mesons as natural and appropriate degrees of freedom to describe nuclear reactions and also in nuclear structure theory at intermediate energy range. The pion meson occurs in three isotopic spin states $\pi^+$ , $\pi^-$ , $\pi^0$ and their quark structure are $u\bar{d}$, $d\bar{u}$ and ( $u\bar{u}$ - $d\bar{d}$ )/ $\sqrt{2}$ respectively. As is well known the nucleon has isotopic spin 1/2 and two isotopic forms proton and neutron. The quark structures of these forms are (uud) and (ddu) respectively. Due to these above structures of the pion and nucleon, the interaction between them has both attractive and repulsive parts. The attractive part in this work will be represented by the exchange of σ-meson and the repulsive part will be represented by the ρ-meson. The paper is organized as follows: in section 2 the mathematical formulation is given. In section 3 the calculations of the interaction of pion with nucleon and $^{40}Ca$

nucleus are given. The calculations of pion- nucleon and pion- Ca nucleus scattering show good agreement with experimental data and the model represented here is successful for describing these types of soft nuclear reactions.

## Mathematical Formulation

We have used an optical potential to describe the scattering of pion from nucleon and $^{40}Ca$ nucleus within a semi relativistic treatment of Dirac equation. In terms of the one-boson-meson-exchange theory (OBME) the real part of this optical potential can be expressed in the form,

$$V_{\pi N}(r) = V\sigma(r) + V\rho(r) \tag{1}$$

Where $V\sigma(r) = - J_{\sigma}(r)\, \gamma_1^0\, \gamma_2^0$ , $V\rho(r) = J_{\rho}(r)\, \gamma_1^0\, \gamma_2^0\, \gamma_1^{\mu}\, \gamma_2^{\mu}$ and the subscripts 1 and 2 refer to the pion and the nucleon respectively and the $\gamma's$ are the usual Dirac matrices. The J's functions are the functions represent the exchanged mesons fields usually taken as suitable Yukawa functions [2]. The form of the motion of the pion-nucleon has been corrected for the center of mass motion of the two particles with different masses. This is supported by the suggestion that the interacting particles move under the influence of a harmonic oscillator which enable us to deal with the two- body wave function as a product of a separate relative and center of mass wave functions. We denote by $m_1$ and $\mathbf{r_1}$ the pion mass and its position vector respectively, while $m_2$ and $\mathbf{r_2}$ are the nucleon mass and its position vector. It is convenient to transform our coordinate system to the relative r and the center of mass R since we intend to use harmonic oscillator wave functions where,

$$r = \sqrt{2}\,\frac{m_1 r_1 - m_2 r_2}{m_1 + m_2} \quad , \quad R = \sqrt{2}\,\frac{m_1 r_1 + m_2 r_2}{m_1 + m_2} \tag{2}$$

**Since the interacting particles are assumed to move under the influence of a harmonic oscillator potential , then the radial wave functions of the relative motion are the normalized Laugurre functions $R_{nl}(r)$ ,**

$$R_{nl}(r) = \left[ \frac{(n\,)2}{\Gamma\left(n+l+\frac{3}{2}\right)} \right]^{1/2} (1/b)^3\, (r/b)^{2l}\, \exp(-r/b)^{\,2}$$

$$\int L \left(\frac{r}{b}\right) r\ \mathbf{dr} \tag{3}$$

**In the two inequal masses pion-nucleon system the size parameters are defined in relative and center of mass system as follows,**

$$b_r = \sqrt{\frac{\hbar\,(m_1 + m_2)}{m_1\, m_2\, \omega}} \qquad , \qquad b_R = \sqrt{\frac{\hbar}{(m_1 + m_2)\omega}} \tag{4}$$

From these basic equations we can write directly the real part of the one-boson-exchange potential in the form,

$$V_{\pi N}(r,R) = \left(-1 + \frac{M_1\left(2n+l+\frac{3}{2}\right)\hbar\omega}{M_2(M_2+M_1)} + \frac{\left(2N+L+\frac{3}{2}\right)\hbar\omega}{(M_2+M_1)}\right) < R_{nl}(r)|\, J_s(r)|R_{nl}(r) > \; +$$

$$< R_{nl}(r)|\, J_v(r)|R_{nl}(r) > - (1/2)$$

$$\left(\frac{M_1}{M_1+M_2}\right)^2 \omega^2 < R_{nl}(r)|\, r^2 J_s(r)|R_{nl}(r) > \; + \frac{\hbar^2}{4M_2^2}[j(j+1) - l(l+1) - S_2(S_2+1)]$$

$$< R_{nl}(r)\left|\frac{1}{r}\,\frac{dJ_s}{dr}\right| R_{nl}(r) > \; +$$

$$+ \frac{\hbar^2}{4M_2^2}[j(j+1) - l(l+1) - S_2(S_2+1)] < R_{nl}(r)\left|\frac{1}{r}\,\frac{dJ_v}{dr}\right| R_{nl}(r) >$$

$$+ \frac{\hbar^2}{2\,M_2\,(M_1+M_2)} < R_{NL}(R)\left|\frac{d}{dR}\right| R_{NL}(R) >< R_{nl}(r)\left|\frac{dJ_s}{dr}\right| R_{nl}(r) > +$$

$$+ \frac{\hbar^2}{2\,M_2\,(M_1+M_2)} < R_{NL}(R)\left|\frac{d}{dR}\right| R_{NL}(R) > \; < R_{nl}(r)\left|J_s\;\frac{d}{dr}\right| R_{nl}(r) > -$$

$$- \frac{\hbar^2}{4M_2^2} < R_{nl}(r) \,|\, \frac{dJ_s}{dr}\,\frac{d}{dr} \,|\, R_{nl}(r) > - \frac{\hbar^2}{4M_2^2} < R_{nl}(r) \,|\, \frac{dJ_v}{dr}\,\frac{d}{dr} \,|\, R_{nl}(r) > - \frac{\omega^2}{2}$$

$$< R_{NL}(R)|\, R^2\, |R_{NL}(R) > \; < R_{nl}(r)|\, J_s(r)|R_{nl}(r) > \tag{5}$$

The relation between the reduced potential U(r) and the the interacting potential between the two particles can be stated as,

$$U(r) = (2\mu/\hbar^2)\, V_{\pi N}(r),$$

*where $\mu$ is the reduced mass of the two interacting particles.*

We now deal with the imaginary part of the potential which we will take its mathematical shape as taken in the real part but with mudulating parameter β to account for the absorption part in the reaction and the value of this parameter will be $0 < \beta < 1$ . Our optical potential for the pion interaction will take the form,

$$V_{\pi N}^{opt}(\mathrm{r}) = V_{\pi N}(1 + i\,\beta\,) \qquad (6)$$

We state both of the elastic differential and total cross sections as follows,

$$\frac{d\sigma}{d\Omega} = |f(k,\theta)|^{2} = (1/k^{2})\ \sum_{0}^{\infty}(2l+1)^{2}\sin^{2}\delta_{l}\,(k)P_{l}^{2}(\cos\theta) \qquad \textbf{(7)}$$

where $P_{l}^{2}(\cos\theta)$ are Legendre polynomials and the relation between the potential U(r) and the phase shift δ can be written by,

$$\tan(\delta_l\,(k)) \cong (\tan(_{\delta_l\,(k)}))\ \ (in\ the\ first\ Born\ approximation\ [2])$$

$$= -k\,(\int_r r\ j_l(kr)\ \mathrm{U(r)}\ j_l\ (kr)\,dr$$

Where $j_l\ (kr)\ are$ the spherical Bessel functions.

As to the meson functions used we have used associated generalized Yukawa function [2] . The parameterization parameters for both the exchanged mesons used were as follows:

mass of the σ meson = 309. 4955 MeV and its coupling constant = 2.385 and its cut off parameter = 1.7 GeV

mass of the ρ meson = 769 MeV and its coupling constant = 0.917 and its cut off parameter = 1.4 GeV

In addition, the mass of the pion meson was taken equal to 139. 569 MeV and the nucleon mass is equal to 938.9265 MeV

The separation energy ħω of the harmonic oscillator is taken by the phenomenological formula [3],

$$\hbar\omega = 1.85 + 41.55/\,A^{1/3}$$

**To calculate the cross section of the pion interaction with the $^{40}Ca$ nucleus, we use the impulse approximation and the total interacting potential can be written approximately in the form,**

$$V_{\pi A}(r) \cong \sum_{N=1}^{A} V_{\pi N}(r) \tag{8}$$

**where A is the mass number of the nucleus.**

# Results and Discussion

**The results we obtained for the elastic scattering of Pion with nucleon and Ca nucleus differential cross sections show good agreement with the corresponding experimental data using the method of exchange mesons to describe the scattering Process .**

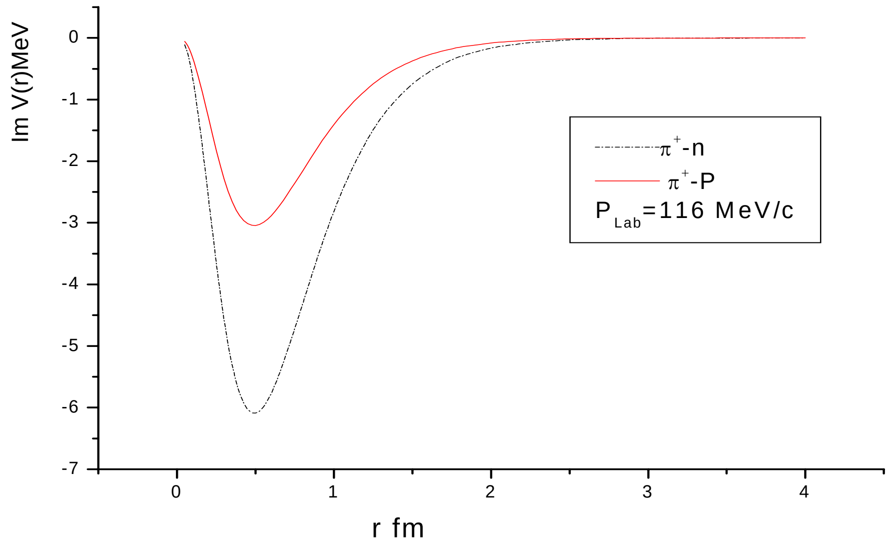


**Fig 1:Imaginary potential of $\pi$+ interaction with both nucleons**

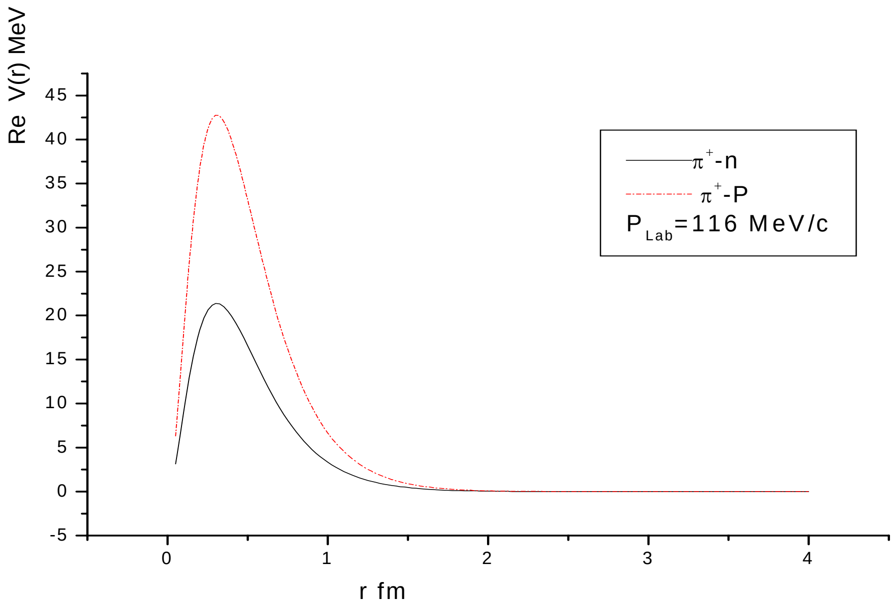


**Fig 2 : Real potential of $\pi^+$ with both nucleons**

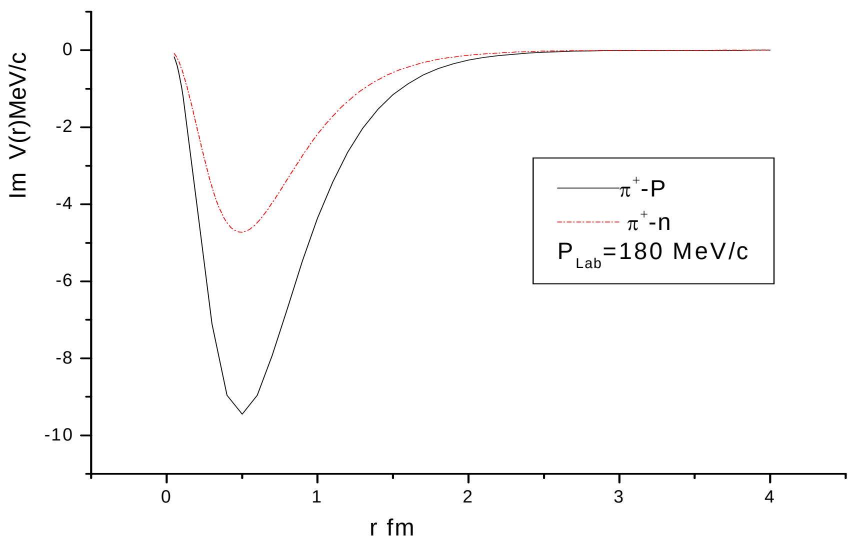


**Fig 3 :Imaginary potential of $\pi$+ interaction with both nucleons**

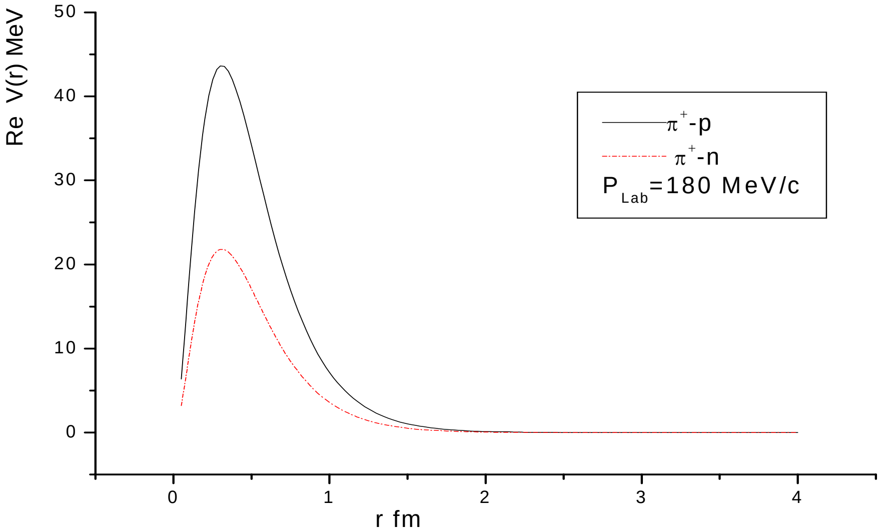


**Fig 4 : Real potential of $\pi^+$ with both nucleons**

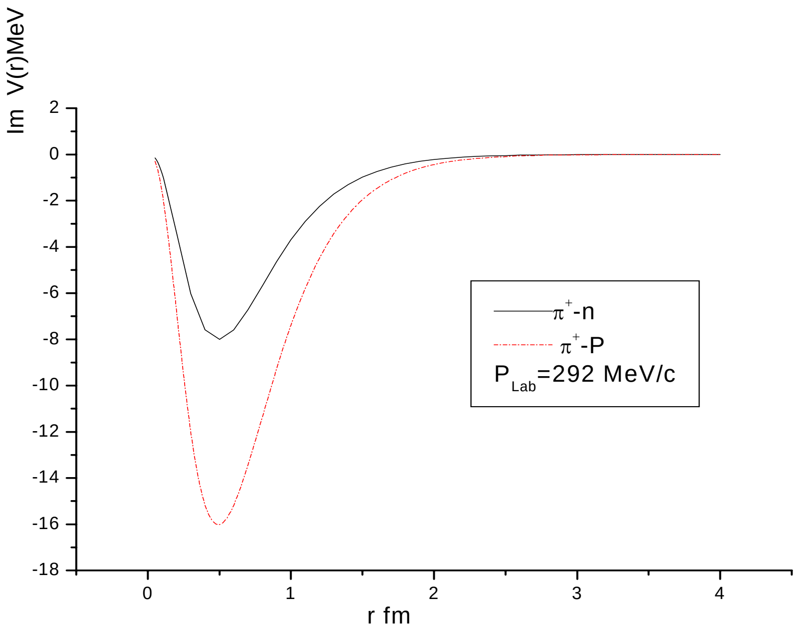


**Fig 5: Imaginary potential of $\pi$+ interaction with both nucleons**

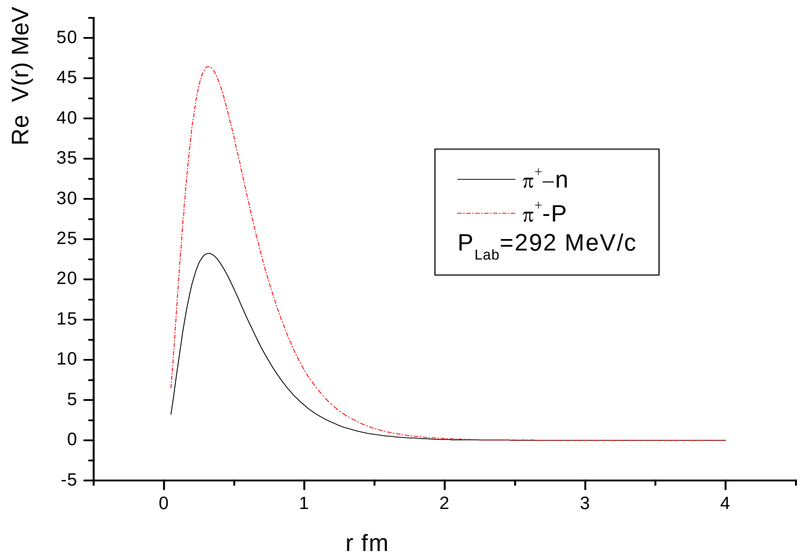


**Fig 6 : Real potential of $\pi^+$ with both nucleons**

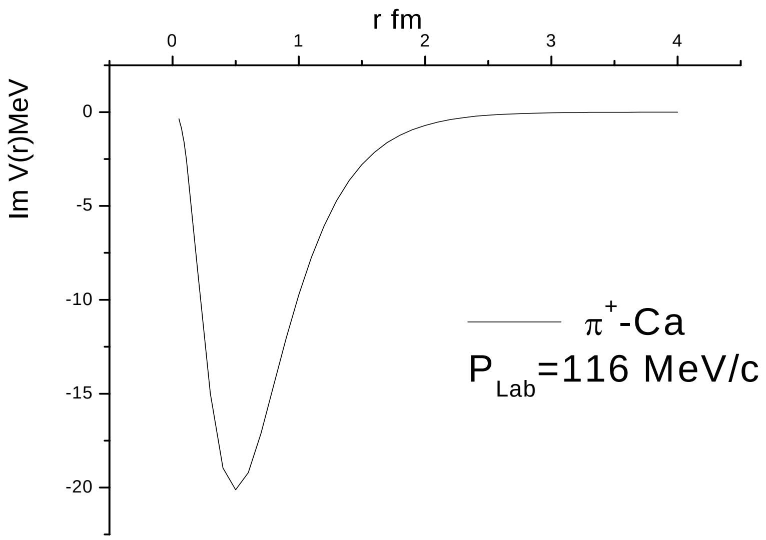


**Fig 7:Imaginary potential of $\pi^+$ with $^{40}$Ca nucleus**

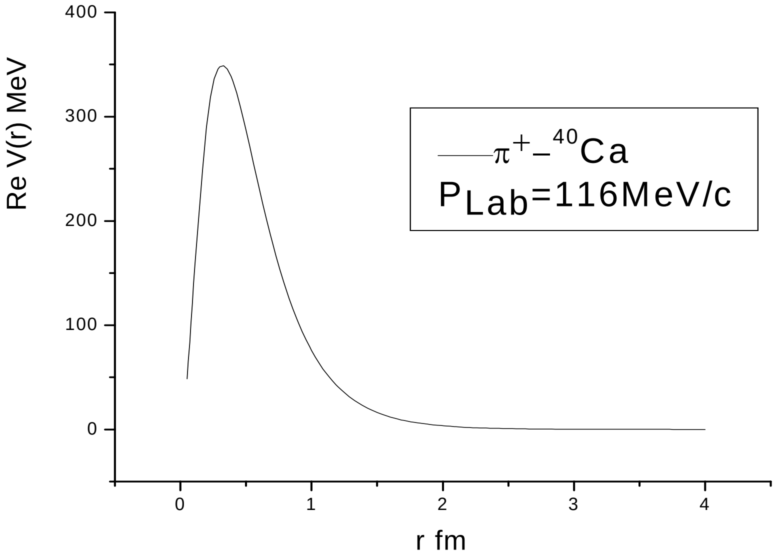


**Fig 8 : Real potential of $\pi^+$ with $^{40}$Ca nucleus**

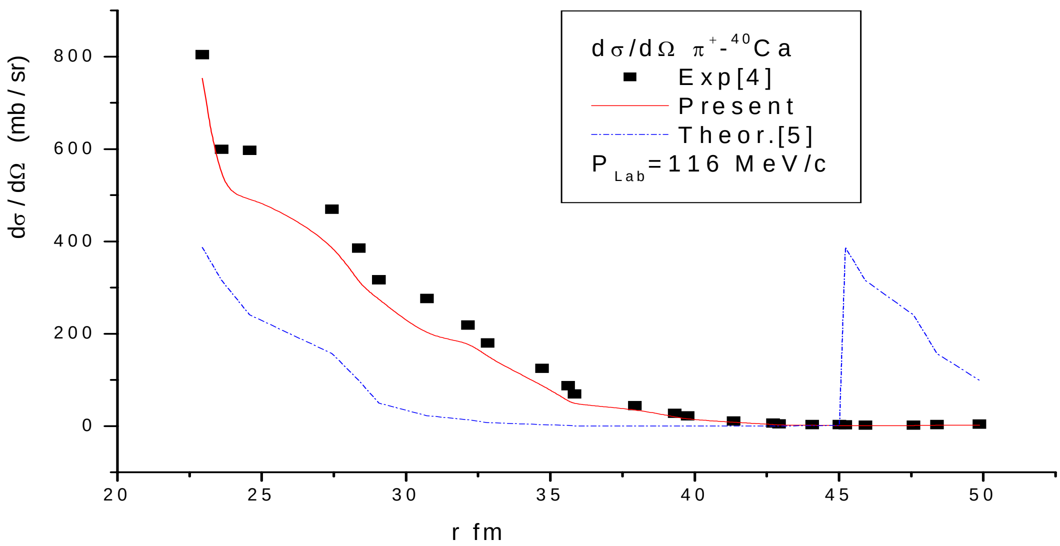


**Fig 9: Diffrential Cross section of $\pi^+$ with $^+$Ca**

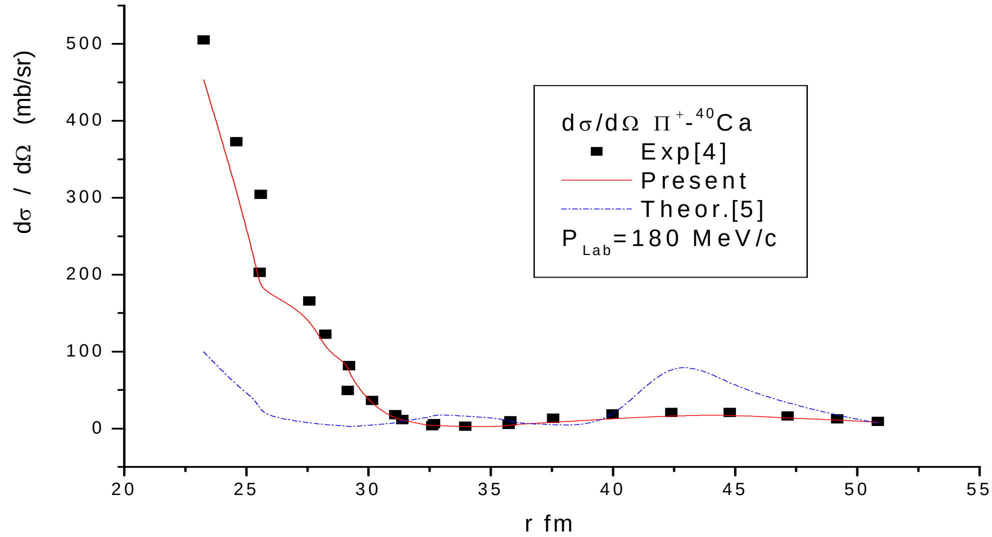


**Fig 10: Diffrential Cross section of $\pi^+$ with $^+$Ca**

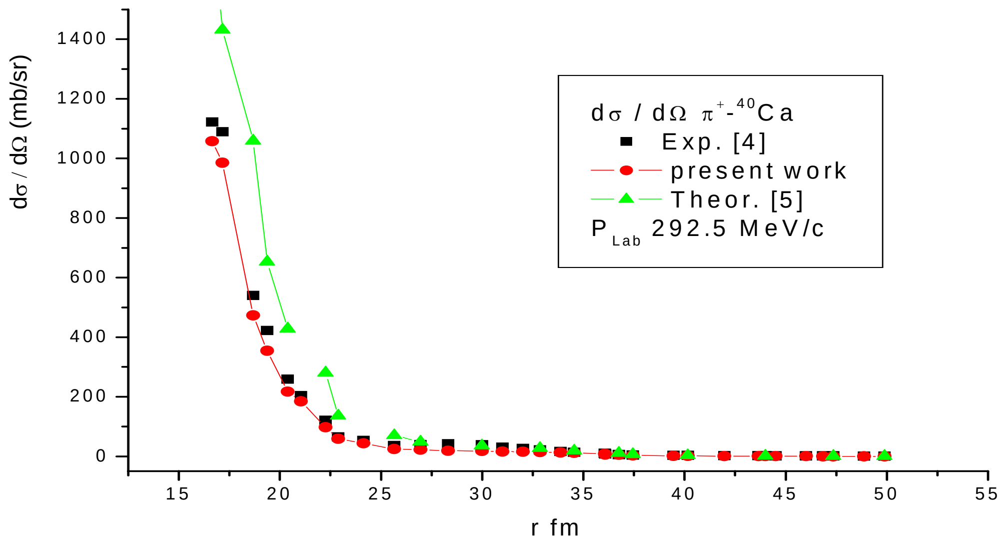


**Fig 11: Diffrential Cross section of $\pi^+$ with $^+$Ca**

## References


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