

WITTEN PARAMETER IN THE $SU(2)$ -GLUEDYNAMICS

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"Relativistic Nuclear Physics and Quantum Chromodynamics"

- Dimension 0: **Local operators**
(much studied in AdS/CFT)

- Dimension 1: **Line operators**



Wilson line

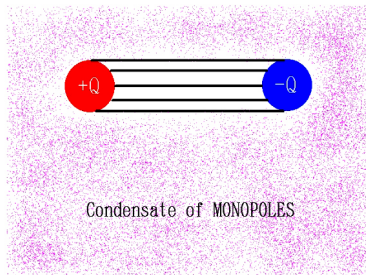
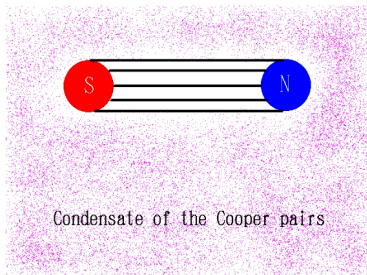


't Hooft line

- Dimension 2: **Surface operators**
(Witten parameter)
- Dimension 3: **Boundaries**

Confinement

– a phenomenon which we can not see free quarks.



The monopole – the source of the magnetic field.

- The Dirac monopole
- The 't Hooft-Polyakov monopole

The value sensitive to monopoles – **Witten parameter**.

Witten parameter

In the $U(1)$ gauge field theory (the electrodynamics) $\implies e^{i\kappa \oint \mathbf{H} \cdot d\mathbf{S}} \equiv 1$

In the $SU(2)$ -gluedynamics this identity does not necessarily hold $\implies e^{i\kappa \sum_k \mathbf{H}_k \cdot d\mathbf{S}_k} \neq 1$

Witten parameter:

The parameter determine flow of the chromomagnetic field through a closed surface:

$$W(S) = \text{Re} \prod_S e^{i\theta_p}$$

We break the symmetry: $SU(2) \rightarrow U(1)$.

The θ_p is got from the plaquette tensor of the gauge field

$$F_p = \hat{1} \cos \theta_p + i n_i \sigma_i \sin \theta_p$$

Phases

- *Confinement:*

$$\langle W_L(C) \rangle \sim e^{-KA(C)}$$

$$\langle W(S) \rangle \sim ? \quad (\text{maybe from volume?})$$

- *Deconfinement:*

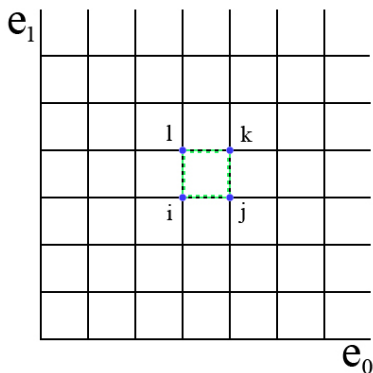
$$\langle W_L(C) \rangle \sim e^{-kp(C)}$$

$$\langle W(S) \rangle \sim ? \quad (\text{maybe from surface?})$$

The lattice calculation

Problems of QFT:

- divergence (UV and IR)
- nonperturbative effects are playing a key role in the QCD



QCD on the lattice:

- vertex - field
 $\psi(x) \rightarrow \psi(x_i)$
- edge (link) - gauge field
 $A_\mu \rightarrow U(L) = e^{ig_0 \int_L A_\mu dx^\mu}$

gauge transformation:

$$U(L) \rightarrow g^{-1}(L_{end}) U(L) g(L_{begin})$$

Wilson: $S_W = \sum_{\text{plaquettes}} S_P$, where $S_P = \beta \left(1 - \frac{1}{N} \text{Re Tr } U_P\right)$

The calculation of the average values

1. The configurations are generated with Boltzmann weight:

$$p(U) d(U) \sim e^{-S_w(U)} d(U)$$

2. The partition function has the form:

$$Z = \int (dU) e^{-S_w(U)}$$

3. The calculation of the average value:

$$\langle A \rangle = Z^{-1} \int (dU) A(U) e^{-S_w(U)} \quad \text{or} \quad \langle A \rangle = \sum_i A(U_i) / N_{\text{conf}}$$

Witten parameter on the lattice

$$W(S) = \text{Re} \prod_S e^{i\theta_p}, \text{ where } \theta_p = \arccos\left(\frac{1}{2} \text{Tr} F_p\right)$$

It used for best statistics:

- Multilevel
 - Multi-hit
- + *HPC with using MPI*

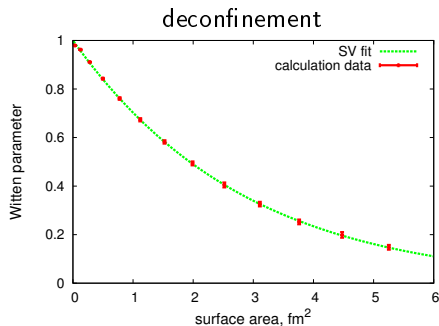
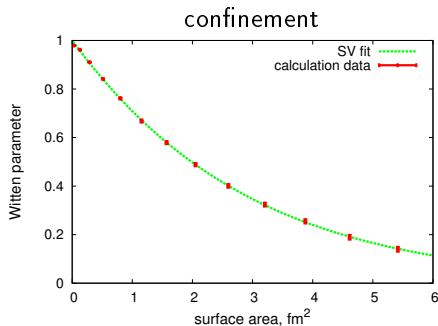
The configurations are prepared in two phases:

- confinement (41^4 , Polyakov loop = 0.0002 ± 0.0006)
- deconfinement ($4 * 30^3$, Polyakov loop = 0.349 ± 0.002)

It has considered the closed surface:

6 plaquettes (cube 1^3) \dots *1014 plaquettes* (cube 13^3)

The physical results



We see:

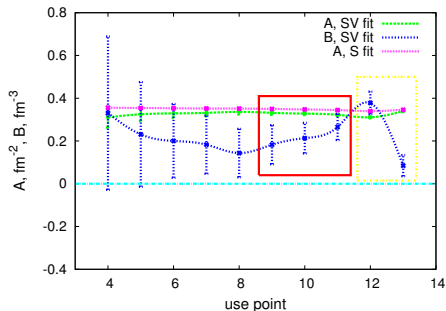
The exponential
behavior

⇒

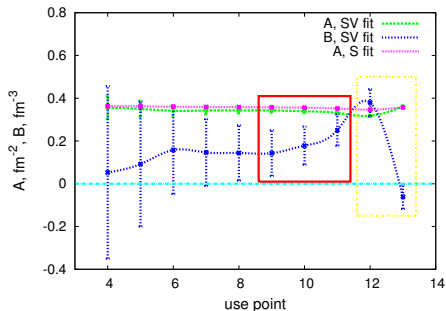
Fit: $W(S, V) = e^{-AS-BV}$

Investigation of the dependency

confinement



deconfinement



The physical results:

- Witten parameter *depend from area surface and volume in two phases.*
- It *isn't the order parameter* for phase transition confinement-deconfinement.

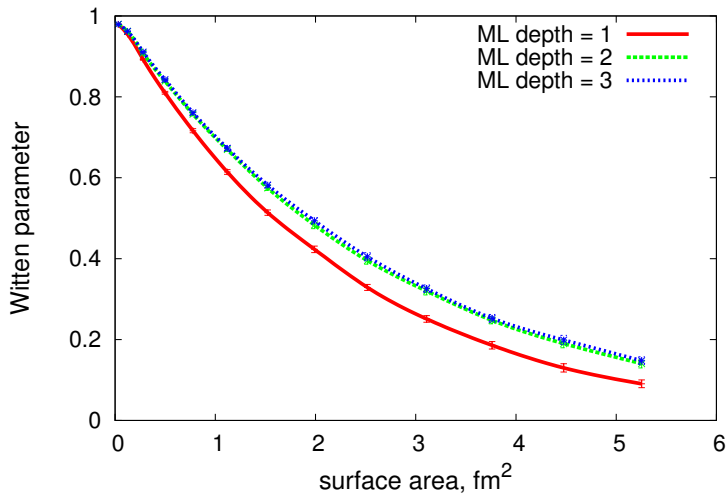
① The physical results:

- Witten parameter *depends from area surface and volume in two phases*.
- It *isn't the order parameter* for phase transition confinement-deconfinement.

② Multilevel algorithm is adapted for calculating a surface operators.

Thank you for attention!

The convergence of MLMH

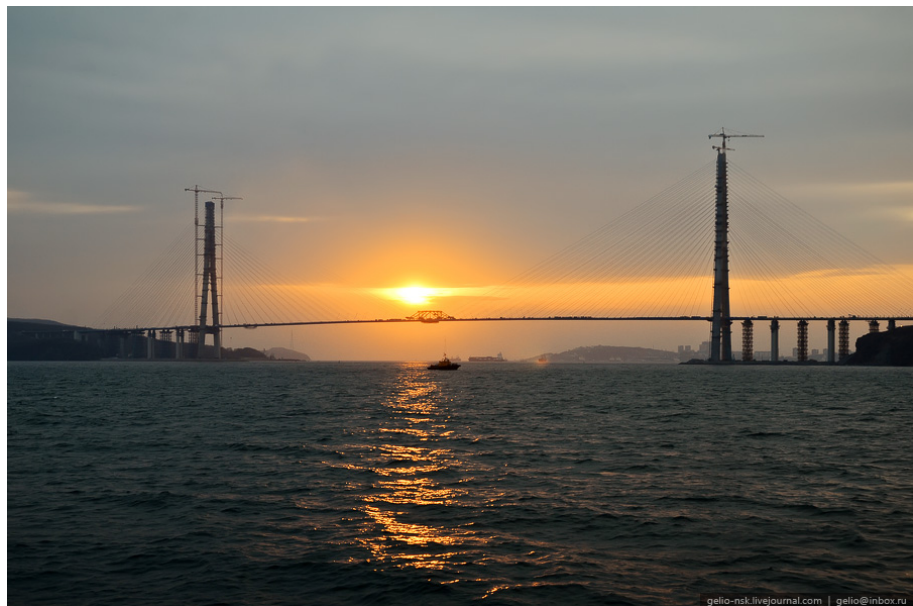


The center of Vladivostok



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HPC with using MPI (Message Passing Interface)



The task of HPC: *The minimization of the calculation time!!!*

$$t = t_{writing} + t_{calc}$$

Parallelism is achieved by calculating Witten parameter at the same time in the different points on the lattice (I use 144 cores for each phase).

1 The generation of configurations

- Complex synchronization (*exchange of boundary conditions*);
- The next configuration depend from current;
- + We can save prepared configurations to use later;
- + It is good for us to understand HPC.

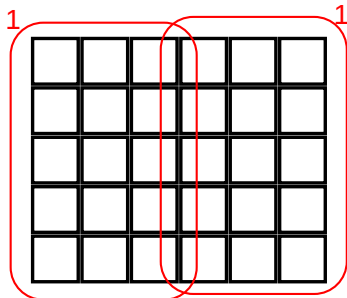
	<i>with HPC</i>	<i>without HPC</i>
<i>write</i>	<i>6 month</i>	<i>1 day</i>
<i>generate</i>	<i>1 day</i>	<i>1 weak</i>

2 The averaging

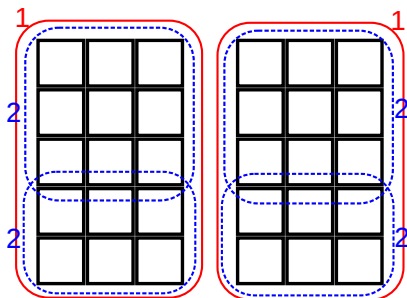
- It is a large load on the NFS;
- + The minimum synchronization.

	<i>with HPC</i>	<i>without HPC</i>
<i>write</i>	<i>1 day</i>	<i>1 hour</i>
<i>calculate</i>	<i>1 weak</i>	<i>1 year</i>

depth equal 1

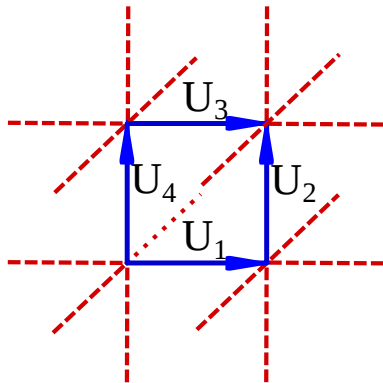


depth equal 2



Multilevel - recursive algorithm, it averages the values from a lower level of recursion.

$$A^{(d)} = \frac{\sum_i a_i^{(d)}}{N}, \quad a_i^{(d)} = \{a^{(d)}\}, \quad a^{(d)} = \sum_j A_j^{(d+1)}, \quad A^{(d=d_{\max})} = A$$



Multi-hit - *the studied value is determined by a bound conditions.*

- We can do it analytically
⇒ *It uses at once*
- We can't do it analytically
⇒ *It average with using the Monte-Carlo algorithm*