Infrared Confinement

and

Meson Spectrum

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QCD at Large Distances

- ♣ QCD behavior at large distances is an active field of research because many novel behaviors are expected at energies **below 1 GeV (**IR region**)**.

- ♣ Understanding of a number of phenomena (quark confinement, QCD running coupling, etc.) requires a correct description of hadron dynamics in the IR region.

- ♣ The PT cannot be used effectively in the IR region and it is required either to supply some phenomenological models, or to use some non-PT methods.

- ♣ One of the fundamental parameters of nature, the QCD effective coupling, can provide a continuous interpolation between the **asymptotical free** state, where PT works well, and the **hadronization regime**, where non-PT techniques must be employed. (e.g. Yu.L.Dokshitzer et al.,1996).

Aim:

A self-consistent and physically meaningful prediction of the QCD effective charge in the IR regime remains actual in particle physics.

We determine the QCD effective charge α_s in the low-energy (below ~1 GeV) region by exploiting the hadron spectrum.

- ♣ Consider a relativistic QF model with infrared (IR) confinement
- ♣ Determine the meson masses by Bethe-Salpeter-type eqs.
- ♣ Adjust the model parameters by fitting some meson masses.
- ♣ Estimate $\hat{\alpha}_s$ in the low-energy domain below ~1 GeV.
- ♣ Extract a specific IR-fixed point $\hat{\alpha}_S^0 = \hat{\alpha}_s(0)$.
- ♣ Estimate intermediate-heavy meson masses (0.7<M<9.5GeV).

Confinement

Confinement and dynamical symmetry breaking are crucial features of QCD.

Color confinement is the result of strong interaction with higher-order terms and requires use of PT techniques.

However, in the hadron scales (~ 200 MeV ~ 1 Fermi) QCD becomes non-PT.

Moreover, there is **no analytic proof** that QCD should be color confining.

The reason for confinement may be somewhat complicated.

Some different explanations of confinement:

Analytic Confinement (AC) [e.g., H.Leutwyler 1980; G.V.Efimov et al. 1995]
IR Confinement [e.g., C.S.Fischer, R.Alkofer, L.von Smekal 2002]
Confinement in lattice MC simulations [e.g., C.D.Roberts 1994; F. Lenz 2004]
Confinement in string theory in higher-D [e.g., R.Alkofer, J.Greensite, 2007]

IR-finite Propagators

i) A.G.Williams et. al. [2001]

$$p^2 \tilde{D}(p^2) \xrightarrow[|p|\to 0]{} 0$$

ii) Schwinger-Dyson Eqs. + lattice QCD

$$\tilde{D}(p^2) \xrightarrow[|p|\to 0]{} const \neq 0$$

iii) IR confined:

$$\tilde{D}(p^2) \sim \frac{1}{p^2} = \int_0^\infty ds\, e^{-sp^2} \to \int_0^{1/\Lambda^2} ds\, e^{-sp^2} = \frac{1 - \exp(-p^2/\Lambda^2)}{p^2}$$

QCD Effective Coupling

The polarization of QCD vacuum causes **two opposite effects**, the color charge ***g*** is **screened** by the virtual quark-antiquark pairs and **antiscreened** by the polarization of virtual gluons.

The competition of these effects results in a variation of the physical coupling $\alpha_s = g^2 / 4\pi$ under changes of distance **1/Q.**

THEORY: QCD predicts a dependence of $\alpha_{s(Q)}$ on energy scale **Q**. This ***dependence*** is described theoretically by the RG equations.

EXPERIMENT: but its ***actual value*** must be obtained from experiment. It is well determined experimentally at relatively high energies Q >2 GeV.

Some measurements of α_s at intermediate energies.

Process	Q (GeV)	α_s	Reference
τ decays	1.78	0.330 ± 0.014	S. Bethke (2009)
$Q\bar{Q}$ states	4.1	0.239 ± 0.012	S. Davies (2003)
Υ decays	4.75	0.217 ± 0.021	A. Penin (1998)
$Q\bar{Q}$ states	7.5	0.1923± 0.0024	S. Bethke (2009)
Υ decays	9.46	0.184 ± 0.015	S. Bethke (2009)
e^+e^- jets	14.0	0.170 ± 0.021	P. A. M. Fernandez (2002)

$$\alpha_s(2 < Q < 180) \to measured$$
$$\alpha_s(Q \to \infty) \to 0$$

$$\alpha_s(Q < 1) \to ?$$

D.Zwanziger 1992
Y.Simonov 2001
A.Williams 2001

$$\alpha_s(Q << 1) \to \alpha_s^0$$

The Model

- Consider a relativistic quantum-field model of quark-gluon interaction.

[G.G., PRD79 (2009); PRD81 (2010); Phys.Part.Nucl.(2012)]

$$L = -\frac{1}{4}\left(F^{A}_{\mu\nu} - g f^{ABC} A^{B}_{\mu} A^{C}_{\nu}\right)^{2} + \sum_{f}\left(\bar{q}^{a}_{f}\left[\gamma_{\alpha}\partial_{\alpha} - m_{f} + g\Gamma^{\alpha}_{C} A^{C}_{\alpha}\right]^{ab} q^{b}_{f}\right)$$

$$F^{B}_{\mu\nu} \equiv \partial_{\mu} A^{B}_{\nu} - \partial_{\nu} A^{B}_{\mu} \qquad \Gamma^{\alpha}_{C} \doteq i\gamma_{\alpha} t^{C}$$

- Partition Functional written in terms of quark and gluon variables

$$Z = \iint \delta\bar{q}\,\delta q \int \delta A \;\exp\left\{-\int dx\, L[\bar{q}, q, A]\right\}$$

Assumptions (in **hadronization region**):

- ♣ Quark and gluon propagators are infrared confined functions.
- ♣ The coupling remains weak (<1). Ladder BSE is sufficient to estimate the meson masses with reasonable accuracy.

- The quark and gluon propagators are IR-confined functions in Euclidean space

$$\tilde{S}(\hat{p}) = \frac{1}{-i\hat{p}+m} = \frac{i\hat{p}+m}{p^2+m^2} = (i\hat{p}+m)\cdot\int_0^\infty dt\, e^{-t\cdot(p^2+m^2)}$$

$$\tilde{S}_{IR}(\hat{p}) = (i\hat{p}+m)\cdot\int_0^{1/\Lambda^2} dt\, e^{-t\cdot(p^2+m^2)} = \frac{i\hat{p}+m}{p^2+m^2}\left(1-e^{-(p^2+m^2)/\Lambda^2}\right)$$

$$D(x) = \frac{1}{4\pi^2 x^2} = \int_0^\infty ds\, e^{-s x^2} \quad \Rightarrow \quad \tilde{D}(p) = \frac{1}{p^2}$$

$$D_{IR}(x) = \int_{\Lambda^2/4}^\infty ds\, e^{-s x^2} = \frac{e^{-x^2\Lambda^2/4}}{4\pi^2 x^2} \quad \Rightarrow \quad \tilde{D}_{IR}(p) = \frac{1-e^{-p^2/4\Lambda^2}}{p^2}$$

Λ - IR confinement scale (~MeV)

$\Lambda \to 0$: deconfinement

Quark-Antiquark Bound States

- Leading-order contributions to quark-antiquark bound state

$$Z_{(\bar{q}q)} = \iint \delta\bar{q}\,\delta q\, \exp\left\{ -\left(\bar{q}\, S^{-1} q\right) + \frac{g^2}{2} \left\langle \left(\bar{q}\,\Gamma A q\right)\left(\bar{q}\,\Gamma A q\right) \right\rangle_D \right\}$$

$$\left\langle (\bullet) \right\rangle_D \doteq \int \delta A \;\; e^{-\frac{1}{2}\left(A D^{-1} A\right)} (\bullet)$$

- Allocate one-gluon exchange between colored currents

$$L_{qq} = \frac{g^2}{2} \sum_{f_1 f_2} \iint dx_1 dx_2\, J^B_{\mu\, f_1 f_2}(x_1, x_2) D^{BC}_{\mu\nu}(x_1, x_2) J^C_{\nu\, f_1 f_2}(x_2, x_1),$$

$$J^B_{\mu\, f_1 f_2}(x_1, x_2) \equiv i\, \bar{q}_{f_1}(x_1)\, \gamma_\mu t^B q_{f_2}(x_2).$$

- Isolate color-singlet combination $\left(t^A\right)^{ij}\delta^{AB}\left(t^B\right)^{i'j'} = \frac{4}{9}\delta^{ii'}\delta^{jj'} - \frac{1}{3}\left(t^A\right)^{ii'}\left(t^A\right)^{jj'}$

- Perform Fierz transformation **(J = S, P, A, V, T)**

$$(i\gamma_\mu)\delta^{\mu\nu}(i\gamma_\nu) = \sum_J C_J \Gamma_J \Gamma_J, \quad C_J = \left\{1, 1, \frac{1}{2}, -\frac{1}{2}, 0\right\}, \quad \Gamma_J = \left\{I, i\gamma_5, \gamma_\mu, \gamma_5\gamma_\mu, \frac{i[\gamma_\nu\gamma_\mu]}{2}\right\}$$

- Go to centre-of-masses frame (due to different quark masses)

- Orthonormalized system: {U_Q} with quantum numbers Q={n,l, …}:

$$\delta(x-y) = \sum_J U_J(x) U_J(y), \qquad \delta_{JJ'} = \int dx\, U_J(x) U_{J'}(x)$$

- Local quark currents and vertices with given quantum numbers

$$J_{QJf_1f_2}(x) \equiv \bar{q}_{f_1}(x) V_{JQ}(\overleftrightarrow{\partial}) q_{f_2}(x), \qquad V_{JQ}(\overleftrightarrow{\partial}) \equiv i^l \int dy \sqrt{D(y)}\, \Gamma_J U_Q(y)\, e^{\frac{y}{2}\overleftrightarrow{\partial}}$$

- Diagonalization on colorless quark currents

$$L_{qq} = \frac{g^2}{2} \sum_N \int dx\, J_N^+(x)\, J_N(x), \qquad N \equiv \{Q\ J\ \ f_1\ \ f_2\}$$

- Gaussian representation: a new path integration over auxiliary fields B:

$$e^{g^2 (J_N^+ J_N)} = \iint \delta B_N^+\, \delta B_N \exp\left\{-\sum_N (B_N^+ B_N) + g \sum_N [(B_N^+\ J_N) + (J_N^+\ B_N)]\right\}$$

- Explicit path-integration over quark variables and write the effective action

$$S_{eff}[B] = -\frac{1}{2}(B_N B_N) + Tr\left\{\ln[1 + g\,(B_N V_N)\, S]\right\}$$

- Hadronization Ansatz: B_N fields are identified as meson fields with given quantum numbers N

• Generating functional can be rewritten in terms of meson field variables. Isolate all quadratic (kinetic part) field configurations:

$$Z_N = \int \prod_N \delta B_N \exp\left\{ -\frac{1}{2}(B_N [1 + g^2 Tr(V_N S V_N S)] B_N) + W_{resid}[B_N] \right\}$$

• Diagonalization of the quadratic part is equivalent to the solution of the ladder Bethe-Salpeter equation on the orthonormalized system {U_N}

$$g^2 Tr(V_N S\, V_{N'} S) = \left(U_N\, \lambda\, U_{N'} \right) = \lambda_N(-p^2)\; \delta^{JJ'} \delta^{QQ'}$$

• Symmetric Bethe-Salpeter kernel is defined: [G.G., PRD81 (2010)]

$$\lambda_J(-p^2) = \frac{4 g^2 C_J}{9} \int \frac{d^4 k}{(2\pi)^4} \{V(k)\}^2 \cdot Tr\left\{ \Gamma_J\, \tilde{S}\left(\hat{k} + \mu_1 \hat{p}\right) \Gamma_J\, \tilde{S}\left(\hat{k} - \mu_2 \hat{p}\right) \right\}$$

Meson Mass Equation

- Renormalization: $$U_{REN}(x) \equiv \sqrt{-\dot{\lambda}_N(M_N^2)} \cdot U_N(x)$$

$$\left\langle U_N \middle| 1+\lambda_N(-p^2) \middle| U_N \right\rangle = \left\langle U_N \middle| 1+\lambda_N(M_N^2) - \dot{\lambda}_N(M_N^2)(p^2+M_N^2) \middle| U_N \right\rangle$$
$$= \left\langle U_{REN} \middle| (p^2+M_N^2) \middle| U_{REN} \right\rangle$$

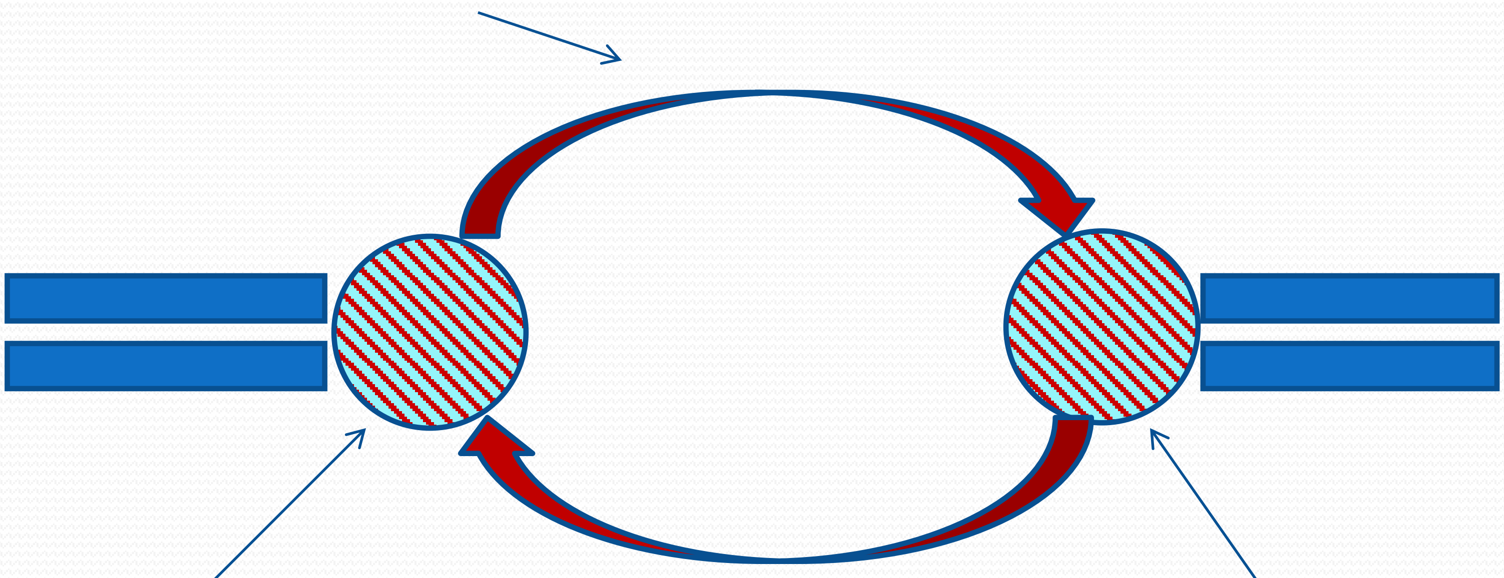
In relativistic quantum-field theory a stable bound state of “n” massive particles shows up as a pole in the S-matrix with a center of mass energy.

- The meson mass may be derived from the equation:

$$1+\lambda_N(M_N^2) = 0 \;, \qquad p^2 = -M_N^2 \qquad [Eq.(1)]$$

$$1 = \alpha_s \cdot \lambda_J(M^2) = \frac{8\alpha_s C_J}{3\pi^3}\int d^4k\; V_J(k)\cdot\Pi_J(p,k)\cdot V_J(-k), \qquad (p^2 = -M_J^2)$$

$$\Pi_J(k) = -\frac{1}{4!}Tr\left\{\Gamma_J\,\tilde{S}\left(\hat{k}+\xi_1\hat{p}\right)\Gamma_J\,\tilde{S}\left(\hat{k}-\xi_2\hat{p}\right)\right\}$$



$$V_J(k) = \int dx\,\sqrt{D(x)}\,U_J(x)\,e^{ikx}$$

$$V_J(-k) = \int dx\,\sqrt{D(x)}\,U_J(x)\,e^{-ikx}$$

Spectra of q-qbar bound states

Consider orthonormal basis functions:

$$U_{n\ell_\mu}(x,a) \approx T_{\ell_\mu}(ax) \cdot L_n^{l+1/2}(a^2x^2) \cdot \sqrt{D(x)} \cdot e^{-ax^2},$$

$$\sum_\mu \int dx \left[U_{nl_\mu}(x,a)\right]^2 = 1$$

1) Extract intermediate values the effective coupling in the interval ~ 3<M<10 GeV from a smooth interpolation of PDG-2010 data

2) Then, by fitting some meson masses (from PDG-2012) we solve the quark masses {m_u, m_s, m_c, m_b} at given Λ

3) Having fixed the model parameters, we estimate masses of conventional (P and V) mesons.

4) For light mesons we solve the inverse problem – to estimate α(M) by fitting meson masses.

Analytic results

Regge-type behaviour of the square masses:

$$M_l^2 \approx M_0^2 + (l+1)\cdot const \qquad for \;\; l \geq 3 \quad and \;\; J = V$$

Vector mesons are heavier than pseudoscalars at same quark contents:

$$1 \approx C_J \cdot \exp(M_J^2)\cdot(M_J^2 + const)$$

$$1 = C_P > C_V = 1/2$$

$$\Longrightarrow \quad M_P^2 < M_V^2$$

The coupling is bounded from above:

$$\alpha_s(M) = 1/\lambda_J(M^2) \leq \alpha_s^{\max} < \infty$$

Finite behaviour of the running coupling at origin:

$$\alpha_s(0) \leq \alpha_s^{\max} < \infty$$

Numerical results for "P" and "V" meson masses

$J^{PC} = 0^{-+}$ $J^{PC} = 1^{--}$

P-mesons	PDG-2010	Our estim.	V-mesons	PDG-2010	Our estim.
D	1870	1892	ρ	770	771
D_s	1970	1998	K*	892	893
η_c	2980	3042	D*	2010	1961
B	5279	5117	D_s*	2112	2079
B_s	5370	5232	J/ψ	3097	3097
B_c	6286	6238	B*	5325	5168
η_b	9389	9384	Υ	9460	9461

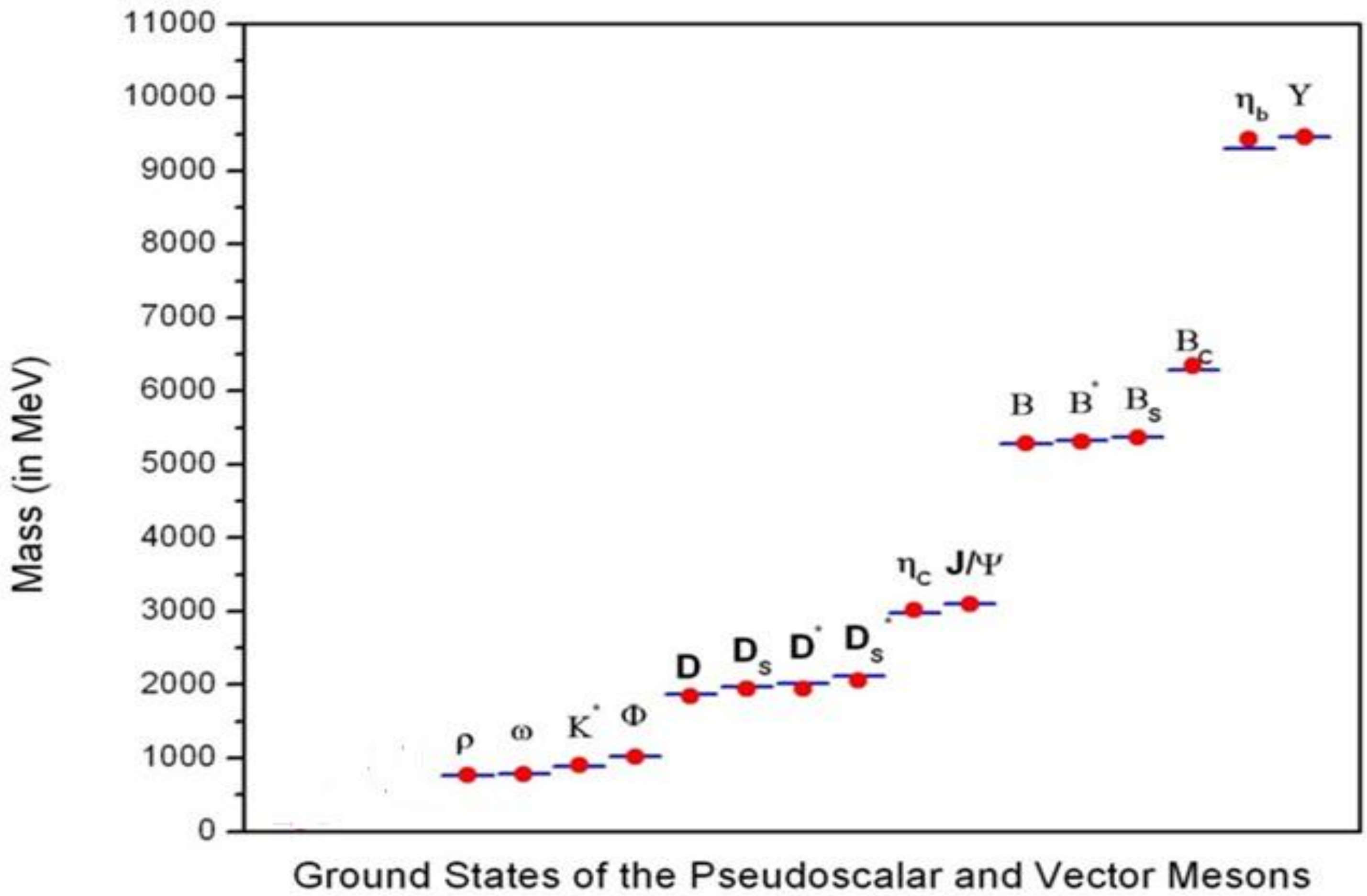
$\Lambda \approx 220 MeV$

$m_{ud} \approx 247.2 MeV$ $m_s \approx 432.5 MeV$

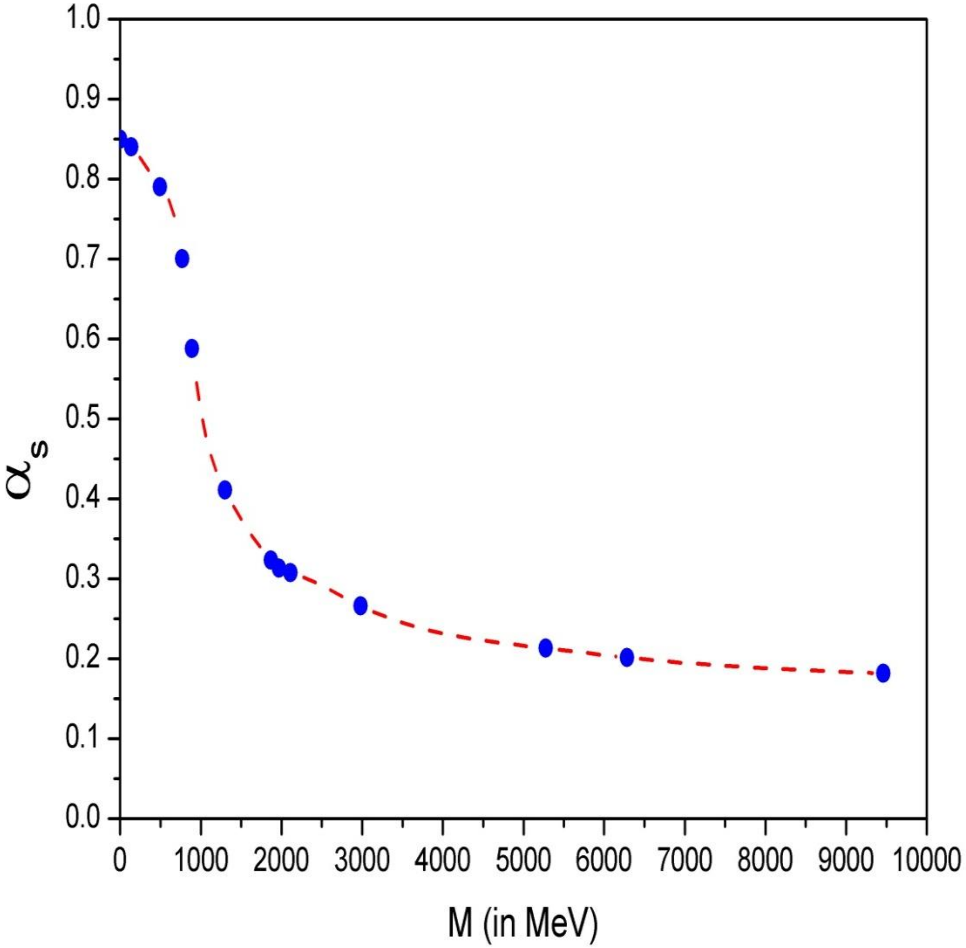
$m_c \approx 1544.5 MeV$ $m_b \approx 4740.9 MeV$

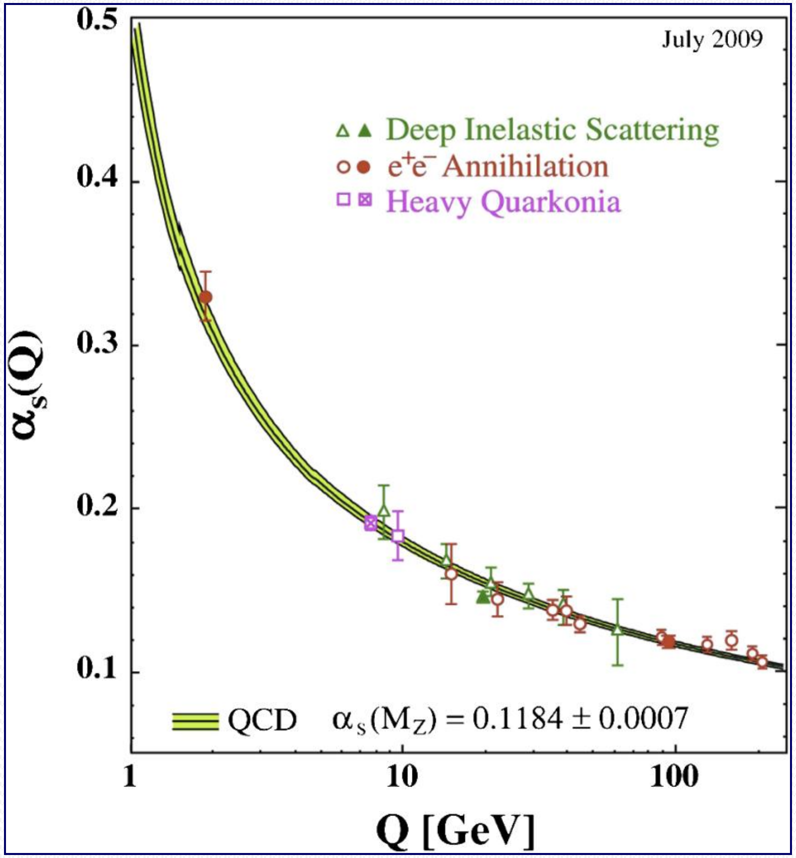
Conventional Meson Mass Estimates

|relative errors| < **2%**



Predicted Behaviors of QCD Running Coupling





We revealed a new IR-fixed point

$$\alpha_s(0) = \alpha_s^{\max} = \frac{3\pi}{16\ln(2)} \approx 0.8498 \quad \rightarrow \quad \alpha_s^{\max}/\pi = \frac{3}{16\ln(2)} \approx 0.2705$$

Our result is in reasonable agreement with often-quoted estimates:

$$\alpha_s^0/\pi = 0.19 - 0.25 \quad [S.Godfrey\ 1985],$$

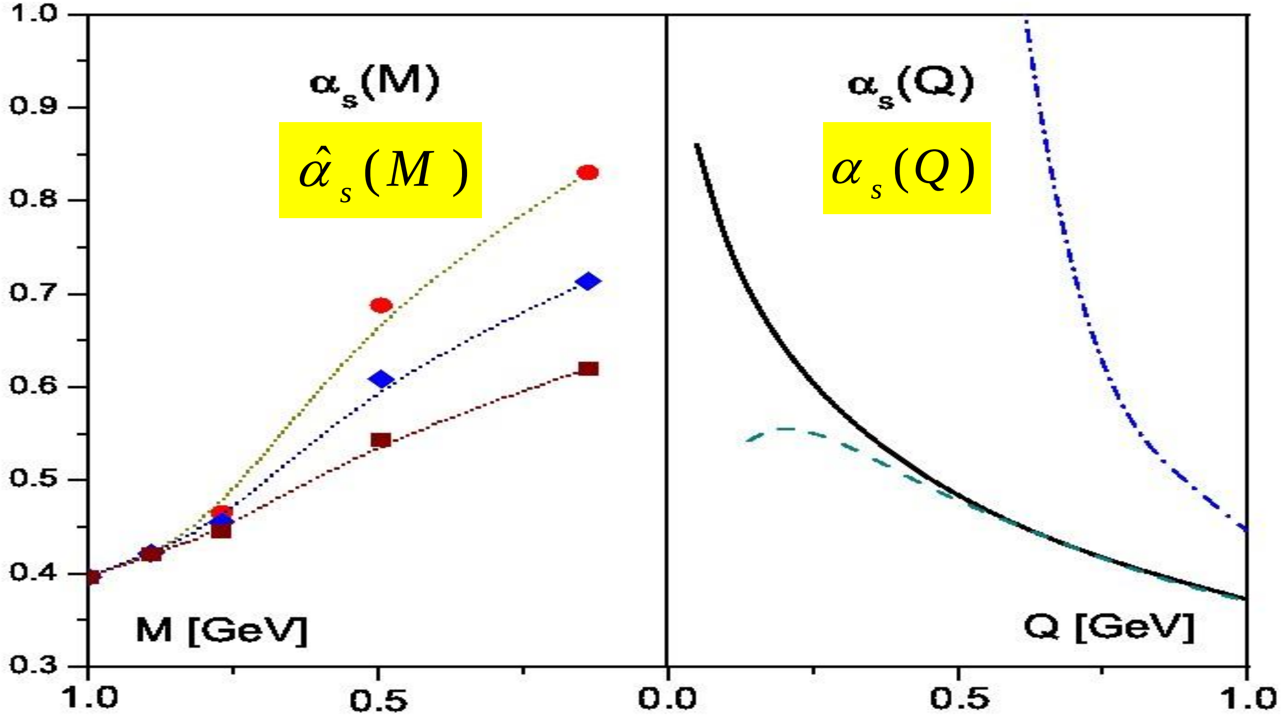
$$\alpha_s^0/\pi = 0.265 \quad [T.Zhang\ 1991],$$

$$\alpha_s^0/\pi = 0.26 \quad [F.Halzen\ 1993],$$

$$\left\langle \alpha_s^0/\pi \right\rangle_{1GeV} = 0.2 \quad [M.Baldicchi\ 2008]$$

Our estimates of $\hat{\alpha}_s(M)$ at different values of Λ

compared with
PT (dot-dashed),
3-loop analytic coupling (solid),
massive 1-loop analytic coupling (dashed)



G.Ganbold, PRD81, 094008 (2010)

[M.Baldicci et al. 2008]

Summary:

- ♣ The conventional meson spectrum may be reasonably described in the framework of a simple relativistic quantum-field model of quark-gluon interaction based on infrared **confinement**.

- ♣ The behaviour of the QCD running coupling in the low-energy region (below 1 GeV) may be explained reasonably by using the meson data.

- ♣ Despite its model origin, the approximations used, and questions about the very definition of the coupling in the IR region, our approach exhibits a new, independent, and specific **IR-finite behavior of QCD coupling**.

- ♣ The model is able to address simultaneously different sections of the low-energy particle physics. Consideration can be extended to other problems (baryons, exotic mesons, glueballs, baryons, mixed and multiquark states, hadronic decay processes, …etc.).