Baldin seminar – Dubna –2012

Subtractions in Exclusive Vector Meson Electroproduction

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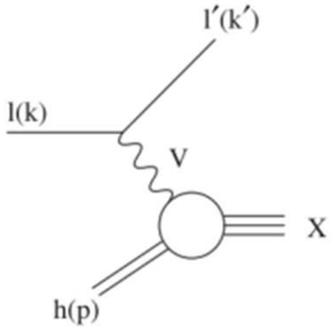
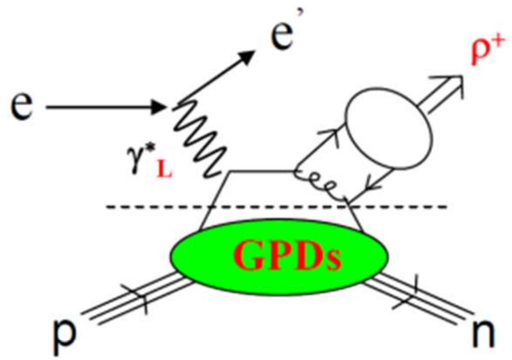
Outline

- Introduction: factorization, GPD, DD
- Relation between GPD and DD
- Holographic sum rule
- Radon transform
- LO photon GPD investigation
- Qualitative analysis of model corrections to ρ° cross section.

Factorization in perturbative QCD

- Due to asymptotic freedom interactions between quarks and gluons on small distances or high energies are well described by perturbative QCD
- Nonperturbative behavior on high distances can be phenomenologically parameterized by parton density functions or generalized parton distributions
- Hadron cross section is expressed by convolution of partonic densities with pertutbative cross section





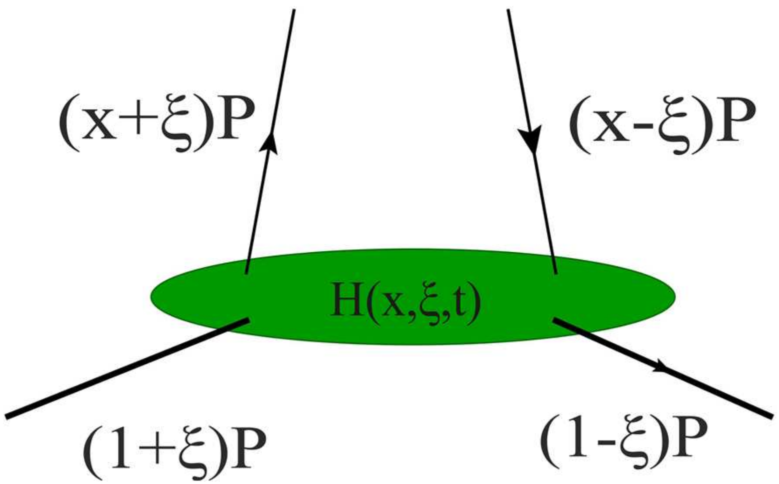
Deeply inelastic scattering

$$\frac{d\sigma}{dX} = \sum_{j,k} \int_{\hat{X}} f_j(x_1, Q) f_k(x_2, Q) \, \frac{d\hat{\sigma}_{jk}(Q)}{d\hat{X}} \, F(\hat{X} \to X; Q)$$

Deeply Virtual Compton Scattering $A_{hadronic}(\xi, Q^2) = \int H(x, \xi) A_{partonic}(x, \xi, Q^2) dx$

Collinear GPDs

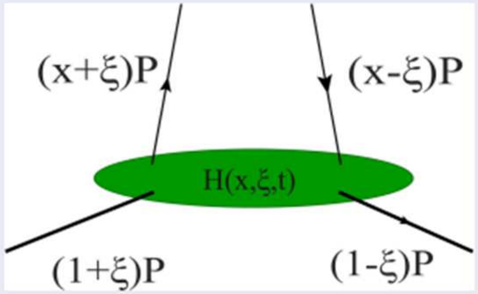
$$
\begin{aligned}
F^q &= \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p'|\, \bar{q}(-\tfrac{1}{2}z)\,\gamma^+ q(\tfrac{1}{2}z)\,|p\rangle \Big|_{z^+=0,\,\mathbf{z}=0} \\
&= \frac{1}{2P^+}\left[H^q(x,\xi,t)\,\bar{u}(p')\gamma^+ u(p) + E^q(x,\xi,t)\,\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m} u(p) \right]
\end{aligned}
$$



Relation between GPD and DD

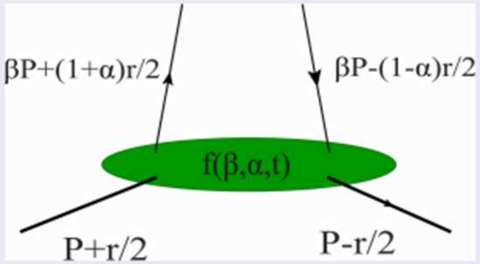
Generalized parton distributions

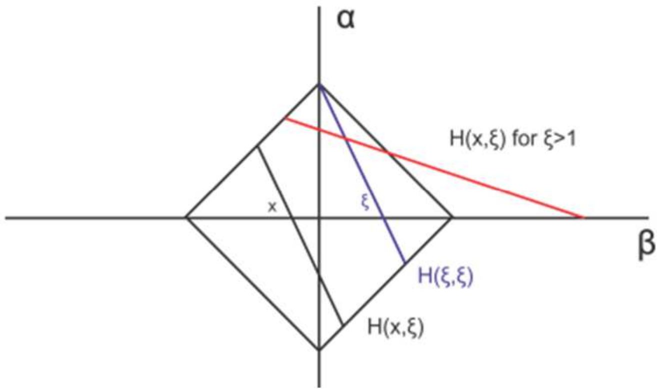
$$\langle p'|\bar{\psi}_a(-z/2)\hat{z}\psi_a(z/2)|p\rangle$$
$$= \bar{u}(p')\hat{z}u(p)\int_{-1}^{1} e^{-i\tilde{x}(Pz)}\, H_a(\tilde{x},\xi;t)\, d\tilde{x} + O(r) \text{ term } + O(z^2)$$



Double distributions

$$\langle P - r/2|\bar{\psi}_a(-z/2)\hat{z}\psi_a(z/2)|P + r/2\rangle$$
$$= \bar{u}(p')\hat{z}u(p)\int_{-1}^{1} dx \int_{-1+|x|}^{1-|x|} e^{-ix(Pz)-i\alpha(rz)/2}\, f_a(x,\alpha;t)\, d\alpha$$
$$+O(r) \text{ terms } + O(z^2),$$





$$H(\tilde{x},\xi;t) = \int_{-1}^{1} dx \int_{-1+|x|}^{1-|x|} \delta(x + \xi\alpha - \tilde{x})\, f(x,\alpha;t)\, d\alpha$$

Forms of Double Distributions

One DD representation

$$\frac{H(z,\xi)}{z} = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy f(x,y)\delta(z-x-\xi y)$$

Two DD representation

$$H(z,\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x,y)+\xi G(x,y))\delta(z-x-\xi y)$$

Connection

$$F(\beta,\alpha) = \beta * f(\beta,\alpha)$$

$$G(\beta,\alpha) = \alpha * f(\beta,\alpha)$$

D-term

Consider matrix element for a hadron

$$< P - r/2|\bar{\psi}(-z/2)\hat{z}\psi(z/2)|P + r/2 >=$$
$$(Pz) \int_{-1}^{1} d\beta \int_{-1+|\alpha|}^{1-|\alpha|} e^{-i\beta(Pz)-i\alpha(rz)} f(\beta,\alpha) d\alpha + (rz) \int_{-1}^{1} e^{-i\alpha(rz)} D(\alpha) d\alpha$$

- Where the second concerns only dependence of (rz)

What means:
$$H(z,\xi) = \int_{-1}^{1} d\alpha \int_{-1+|\alpha|}^{1-|\alpha|} d\beta (F(\beta,\alpha) + \xi\delta(\beta)D(\alpha))\delta(z - \beta - \alpha\xi)$$

Which is particular case for:
$$H(z,\xi) = \int_{-1}^{1} d\alpha \int_{-1+|\alpha|}^{1-|\alpha|} d\beta (F(\beta,\alpha) + \xi G(\beta,\alpha))\delta(z - \beta - \alpha\xi)$$

- So D-term is a particular form of DD $G(\beta,\alpha)$ concentrated only on the line $\beta = 0$ or in terms of GPD in the area $|x| < |\xi|$
- Mathematically necessity of D-term arises from polynomiality condition for $H(x,\xi)$ (Moments $\int_{-1}^{1} H(x,\xi)x^n dx$ must be maximum $n+1$ power of ξ)

M. V. Polyakov and C. Weiss, Phys.Rev. D60, 114017 (1999)

Holographic sum rule

$$A(\xi,t) = \int_{-1}^{1} dx \frac{H^{(+)}(x,\xi,t)}{x+\xi-i\epsilon}$$

The crucial part of DVCS and proton GPD part of the meson electroproduction

Dispersion relations with subtraction

$$P\int_{-1}^{1} \frac{H(x,\xi)}{x-\xi} dx = P\int_{-1}^{1} \frac{H(x,x)}{x-\xi} dx + \Delta(\xi)$$

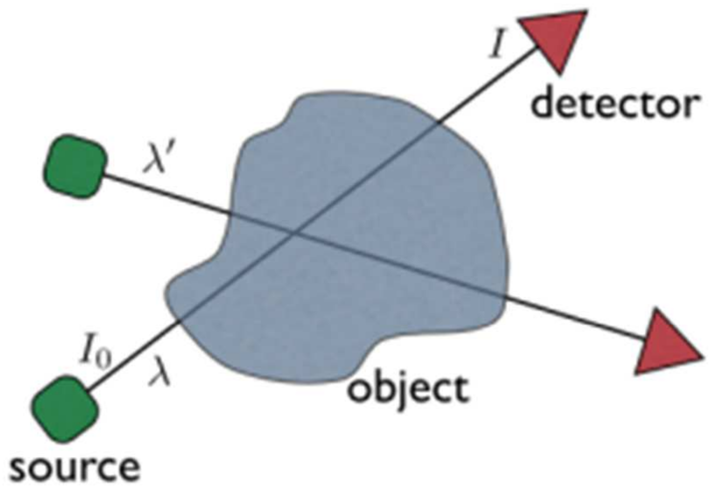
$$P\int_{-1}^{1} \frac{H(x,\xi)-H(x,x)}{x-\xi} dx = \Delta = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \frac{G(\beta,\alpha)}{\alpha-1}$$

Statement of holographic sum rule:

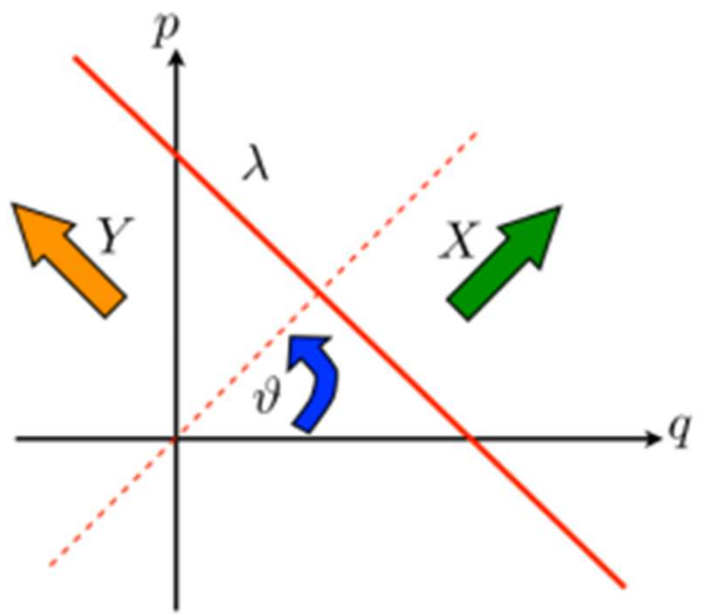
Information about exclusive amplitude concentrated only on line $x = \pm\xi$!

O. Teryaev, arXiv:hep-ph/0510031 (2005).
I. Anikin and O. Teryaev, Phys.Rev. D76, 056007 (2007)

Radon transform



$$I(s) = I_0 \exp\left(-\int_0^s \mu(r)\,\mathrm{d}r\right)$$



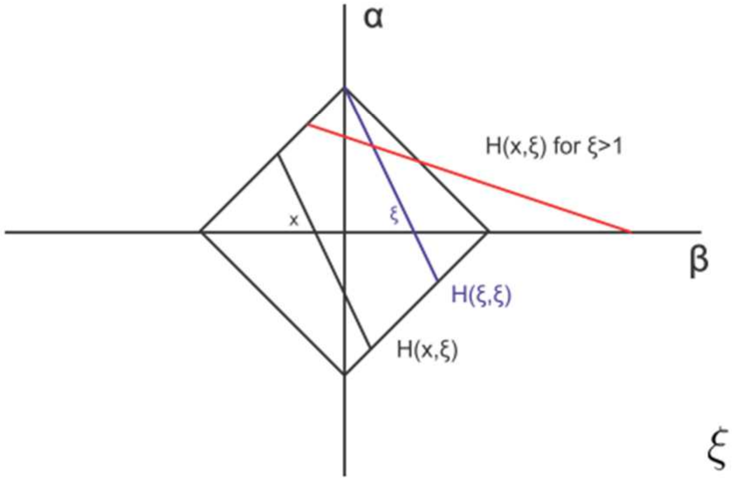
$$R(\lambda) = R(X, \vec{\xi}) = \int_{R^2} f(\vec{x})\delta(X - \vec{x}\vec{\xi})dx$$

Classical and quantum aspects of tomography.
Paolo Facchi, Marilena Ligabo arXiv:1001.5169v1 [math-ph]

Radon transform

$$H(z,\xi) = \int\limits_{-1}^{1} dx \int\limits_{|x|-1}^{1-|x|} dy \delta(x+\xi y - z) f(x,y)$$

$$R(p,\vec{\xi}) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy f(x,y)\delta(p-\vec{x}\vec{\xi})$$

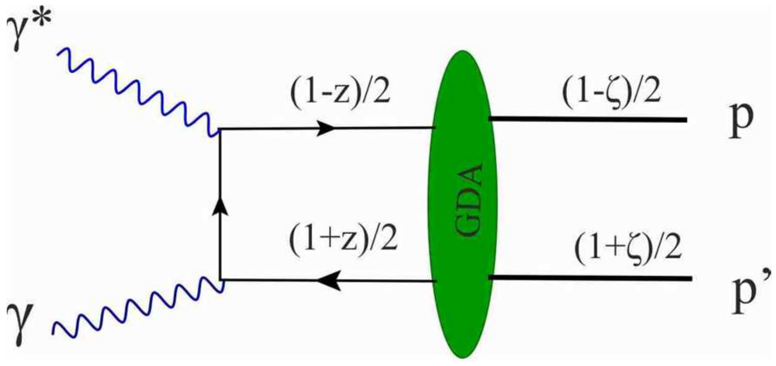


$$\xi = tg\phi, z = p/cos\phi; H(z,\xi) = R(p,\vec{\xi})|cos\phi|$$

$$f(x,y) = -\frac{1}{2\pi^2} \int\limits_{-\infty}^{\infty} \frac{dz}{z^2} \int\limits_{-\infty}^{\infty} d\xi (H(z+x+y\xi,\xi) - H(x+y\xi,\xi))$$

O.V. Teryaev. Crossing and Radon tomography for generalized parton distributions.
Phys.\ Lett.\ B {\bf 510} (2001) 125 [http://arxiv.org/abs/hep-ph/0102303v2]

Generalized Distribution Amplitudes



Typical example where GDAs are used is $\gamma\gamma^* \to 2p$. GDA can be expressed by crossed channel GPD.

$$H(z,\xi) = sign(\xi)\Phi(\frac{z}{\xi},\frac{1}{\xi})$$

Expression of GPD in the unphysical region |ξ |>1 via GDA was derived in the paper below

*O.V.Teryaev. Crossing and Radon tomography for generalized parton distributions,"
Phys.\ Lett.\ B {\bf 510} (2001) 125 [http://arxiv.org/abs/hep-ph/0102303v2]

Photon GPDs

$$T^{\mu\nu\alpha\beta}(\Delta_T = 0) = \frac{1}{4} g_T^{\mu\nu} g_T^{\alpha\beta} W_1 + \frac{1}{8}\left(g_T^{\mu\alpha} g_T^{\nu\beta} + g_T^{\nu\alpha} g_T^{\mu\beta} - g_T^{\mu\nu} g_T^{\alpha\beta}\right) W_2 + \frac{1}{4}\left(g_T^{\mu\alpha} g_T^{\nu\beta} - g_T^{\mu\beta} g_T^{\alpha\nu}\right) W_3$$

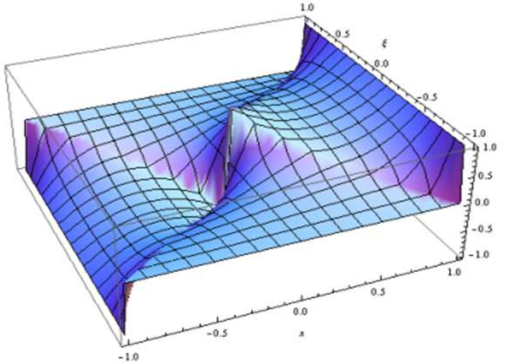


Figure: $H_1^q(z,\xi)$

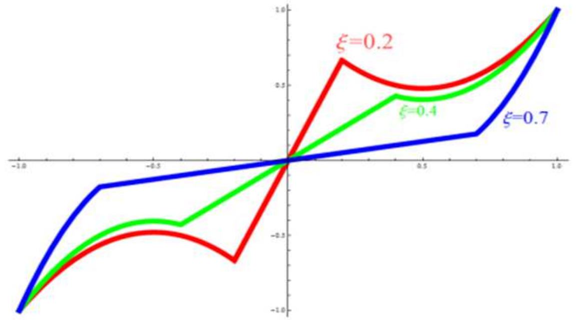


Figure: $H_1^q(z,\xi)$ for ζ =0.2,0.4,0.7

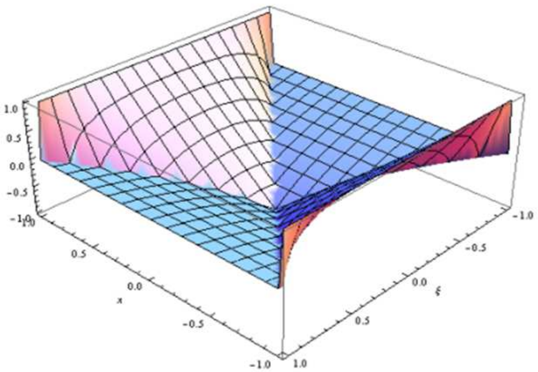


Figure: $H_3^q(x,\xi)$

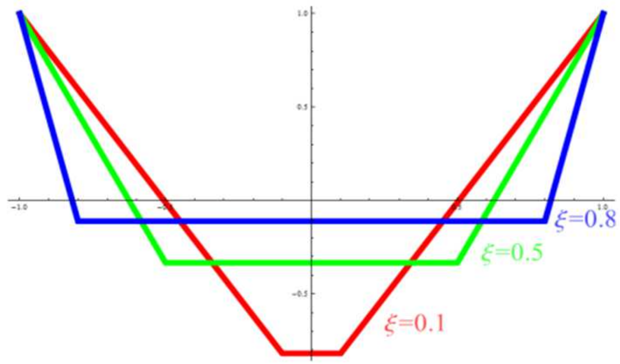


Figure: $H_3^q(x,\xi)$ for ξ =0.2,0.4,0.7

S. Friot, B. Pire and L. Szymanowski, Phys. Lett. B **645** (2007) 153
[http://arxiv.org/abs/hep-ph/0611176v2].

Photon GDAs

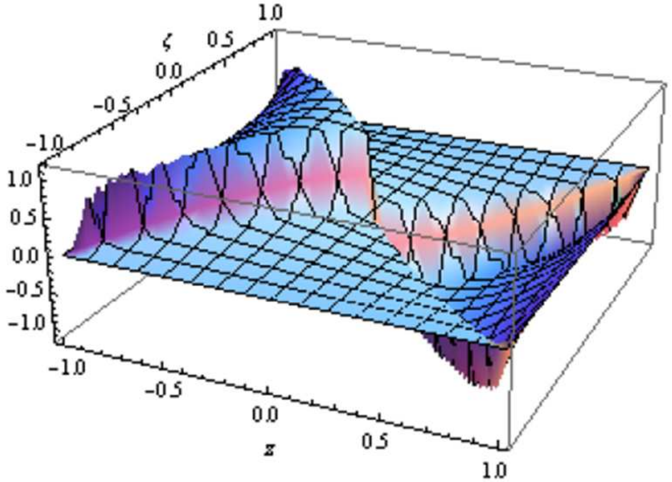


Figure: $\Phi_1^q(z,\zeta)$

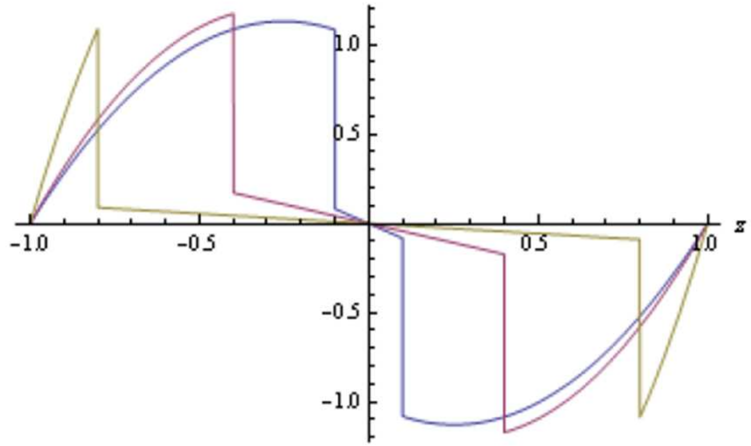
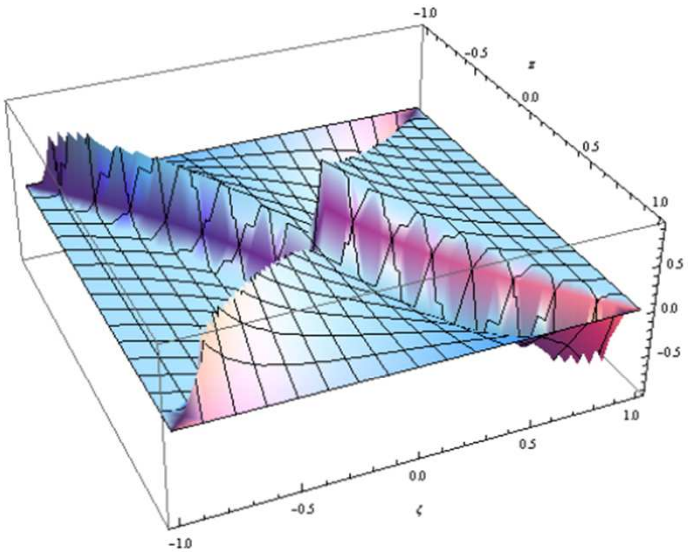


Figure: $\Phi_1^q(z,\zeta)$ for ζ =0.1,0.4,0.8



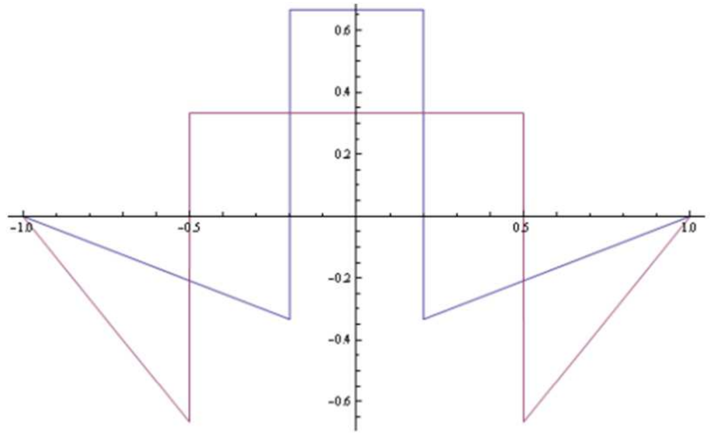


Figure: $\Phi_3^q(z,\zeta)$ for ζ =0.2,0.5

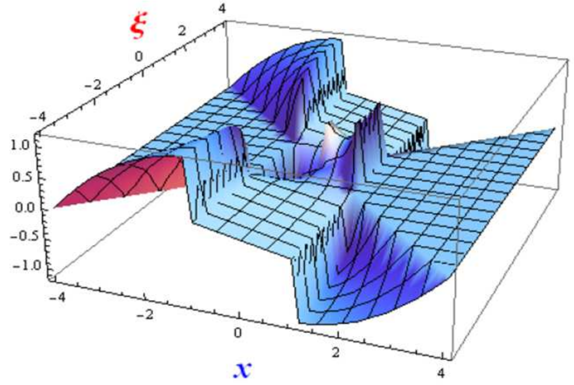
M. El Beiyad, B. Pire, L. Szymanowski and S. Wallon, Phys. Rev. D **78** (2008) 034009 [http://arxiv.org/abs/0806.1098v1].

Extension of photon GPD

Applying mentioned relation

$$H(z,\xi) = sign(\xi)\Phi(\frac{z}{\xi},\frac{1}{\xi})$$

we extended photon GPDs to the unphysical region by GDAs



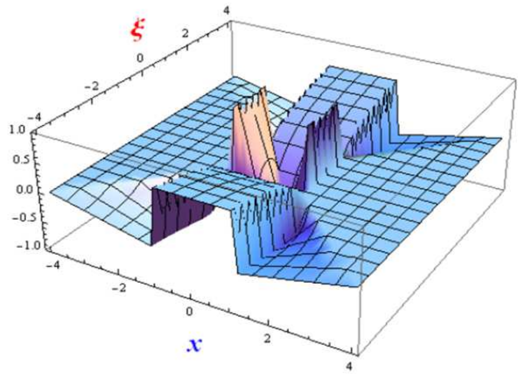
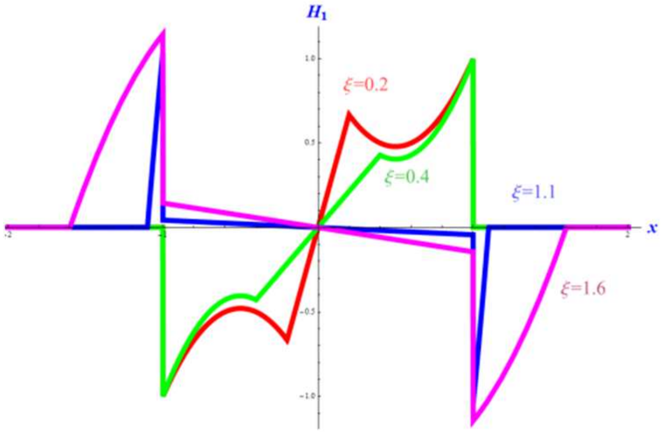
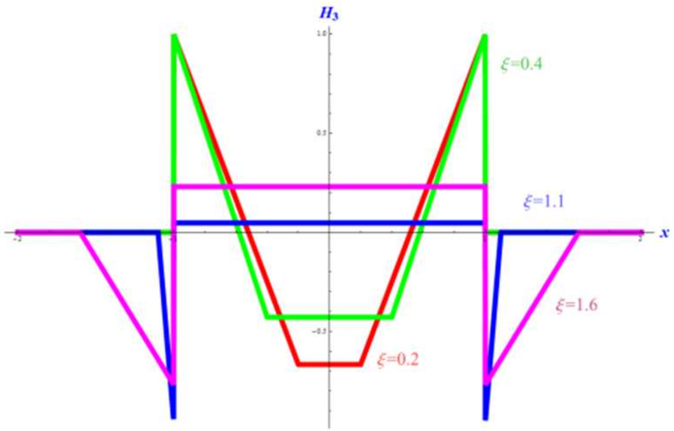


Figure: $H_1(x,\xi)$

Figure: $H_3(x,\xi)$





I.R. Gabdrakhmanov, O.V.Teryaev, arXiv:1204.6471 [hep-ph], 10.1016/j.physletb.2012.08.041

Holographic sum rule applied to photon GPD

$$P\int_{-1}^{1}\frac{H(x,\xi)-H(x,x)}{x-\xi}dx=\Delta\equiv\int_{-1}^{1}\frac{D(\alpha)}{\alpha-1}$$

O. V. Teryaev, "Analytic properties of hard exclusive amplitudes," arXiv:hep-ph/0510031.
I. V. Anikin and O. V. Teryaev, "Dispersion relations and QCD factorization in hard reactions," Fizika B **17** (2008) 151

$$\int_{-\xi}^{\xi}\frac{H_1(x,x)-H_1(x,\xi)}{x-\xi}dx=\underline{-2\ln 2}\text{ for }|\xi|>1 \qquad \int_{-\xi}^{\xi}\frac{H_3(x,x)-H_3(x,\xi)}{x-\xi}dx=\underline{0}\text{ for }|\xi|>1$$

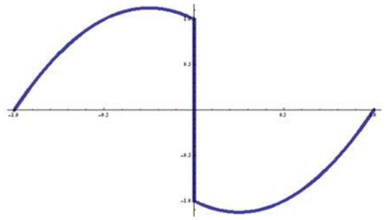
$$\int_{-1}^{1}\frac{H_1(x,x)-H_1(x,\xi)}{x-\xi}dx=\underline{-2\ln 2}\text{ for }|\xi|<1 \qquad \int_{-1}^{1}\frac{H_3(x,x)-H_3(x,\xi)}{x-\xi}dx=\underline{0}\text{ for }|\xi|<1$$

I.R. Gabdrakhmanov, O.V.Teryaev, arXiv:1204.6471 [hep-ph], 10.1016/j.physletb.2012.08.041

Deriving photon Double Distribution

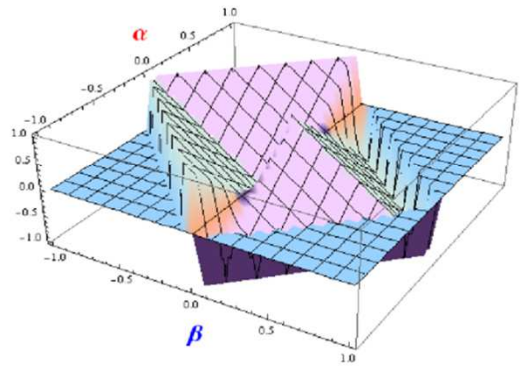
D-term can be obtained from GDA: $D(\alpha) = \Phi(\alpha, 0)$

$$D_1(\alpha) \;=\; (|\alpha| - 1)(2|\alpha| + 1)Sgn(\alpha)$$



Having D-term we can subtract corresponding $H_D(x, \xi)$ and then apply inverse Radon tranform Corresponding DD for $D_1(\alpha)$ is

$$F_{1D}(\beta, \alpha) = [2(1 - |\beta| - |\alpha|) - 1 + \delta(\alpha)]sgn(\beta)$$



In full analogy we obtain $F_{3D}(\beta, \alpha) = \delta(\alpha) - 1$

$$\int_{-1}^{1} \frac{D_1(\alpha)}{\alpha - 1} d\alpha = \underline{2\ln 2}$$

$$D_3(\alpha) = \Phi_3(\alpha, 0) = 0$$

$$\int_{-1}^{1} \frac{D_3(\alpha)}{\alpha - 1} d\alpha = \underline{0}$$

I.R. Gabdrakhmanov, O.V.Teryaev, arXiv:1204.6471 [hep-ph], 10.1016/j.physletb.2012.08.041

Gauge transformations of DD

$$\langle p'|\bar{\psi}\left(-\frac{z}{2}\right)\gamma\cdot z\psi\left(\frac{z}{2}\right)|p\rangle = (2P\cdot z)\int_{-1}^{1}dx\int_{|x|-1}^{1-|x|}dy e^{-ixPz-iy\Delta z/2}F(x,y,\Delta^2)$$
$$+(\Delta\cdot z)\int_{-1}^{1}dx\int_{|x|-1}^{1-|x|}dy e^{-ixPz-iy\Delta z/2}G(x,y,\Delta^2);$$

Double distributions defined through Fourier image of matrix element.
So one can make integration by parts and obtain

$$-i\int_{-1}^{1}d\beta\int_{1-|\beta|}^{-1+|\beta|}d\alpha e^{-i(\beta z^- P^+ + \alpha z^- \frac{\Delta^+}{2})}\left(\frac{\partial F(\beta,\alpha)}{\partial\beta}+\frac{\partial G(\beta,\alpha)}{\partial\alpha}\right)$$

+Integration on the border of support area

$$F(\beta,\alpha)\to F(\beta,\alpha)+\frac{\partial\chi(\beta,\alpha)}{\partial\alpha}$$

$$G(\beta,\alpha)\to G(\beta,\alpha)-\frac{\partial\chi(\beta,\alpha)}{\partial\beta}$$

Gauge transformations of photon DD

$$F(\beta,\alpha) = \beta * f(\beta,\alpha) \qquad \chi(\beta,\alpha) \qquad F_D(\beta,\alpha)$$
$$G(\beta,\alpha) = \alpha * f(\beta,\alpha) \qquad \longleftrightarrow \qquad D(\alpha)$$

One DD representation

$$f_1(\beta,\alpha) = \frac{\delta(\alpha)}{|\beta|} - 1 + 2\delta(\beta)(1-|\alpha|) + \delta(\beta)\frac{D_1(\alpha)}{\alpha}$$

D - term was evaluated directly, $f_1(\beta,\alpha)$ and $F_{1D}(\beta,\alpha)$ was evaluated semi numerically semi intuitively

Different representations in 2 DD approach

$$F_{1D}(\beta,\alpha) = [2(1-|\beta|-|\alpha|) - 1 + \delta(\alpha)]sgn(\beta)$$

$$D_1(\alpha) = (|\alpha|-1)(2|\alpha|+1)sgn(\alpha)$$

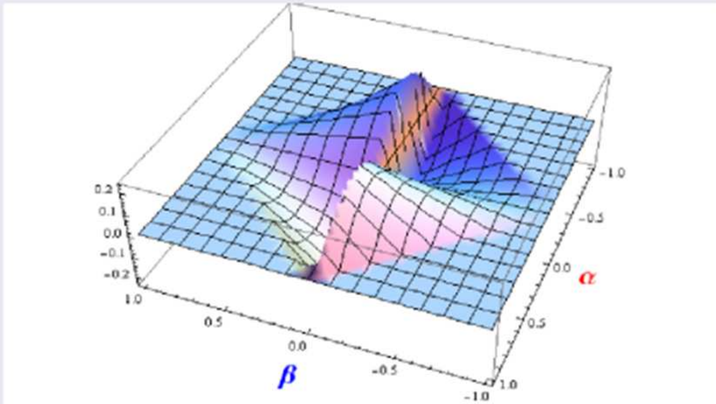
$$\chi(\beta,\alpha)$$
$$\Downarrow$$

$$F_1(\beta,\alpha) = \delta(\alpha) * sgn(\beta) - \beta$$

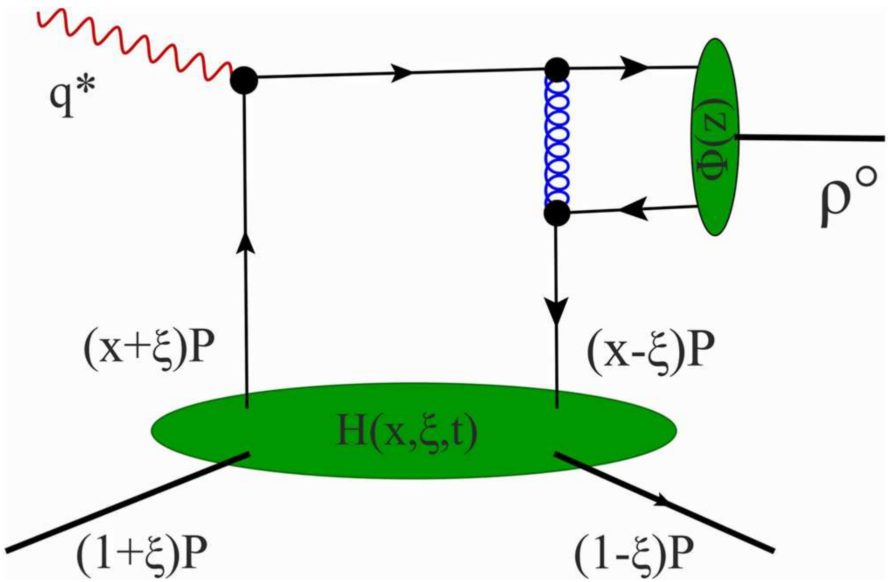
$$G_1(\beta,\alpha) = -\alpha + 2\delta(\beta)(1-|\alpha|)\alpha + \delta(\beta)D_1(\alpha)$$

Transforming function χ_1

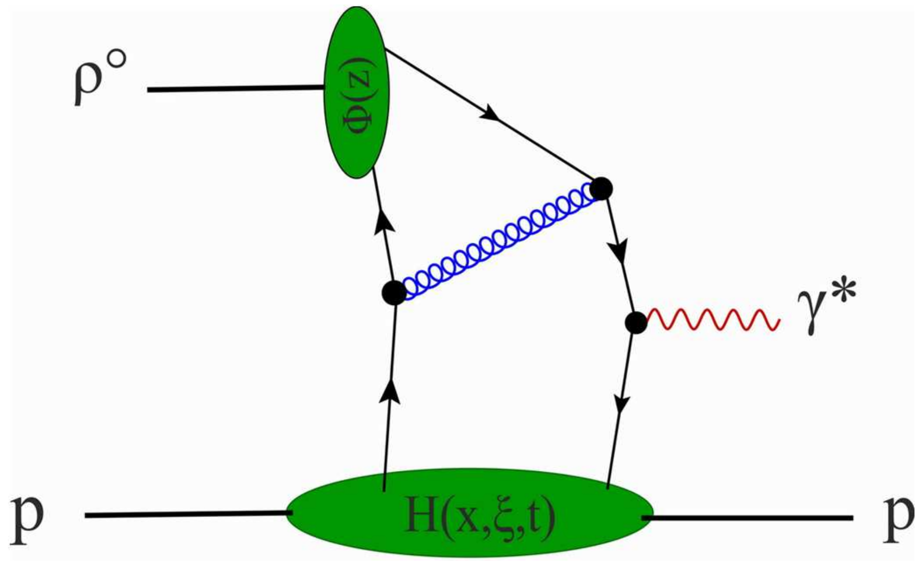
$$\chi_1(\beta,\alpha) = -\alpha * sgn(\beta)(1-|\beta|-|\alpha|)$$



ρ° meson electroproduction



Exclusive ρ° meson electroproduction



Drell-Yann ρ° p scattering

GK GPD parametrization

GPDs are constructed from standard DD anzatz:

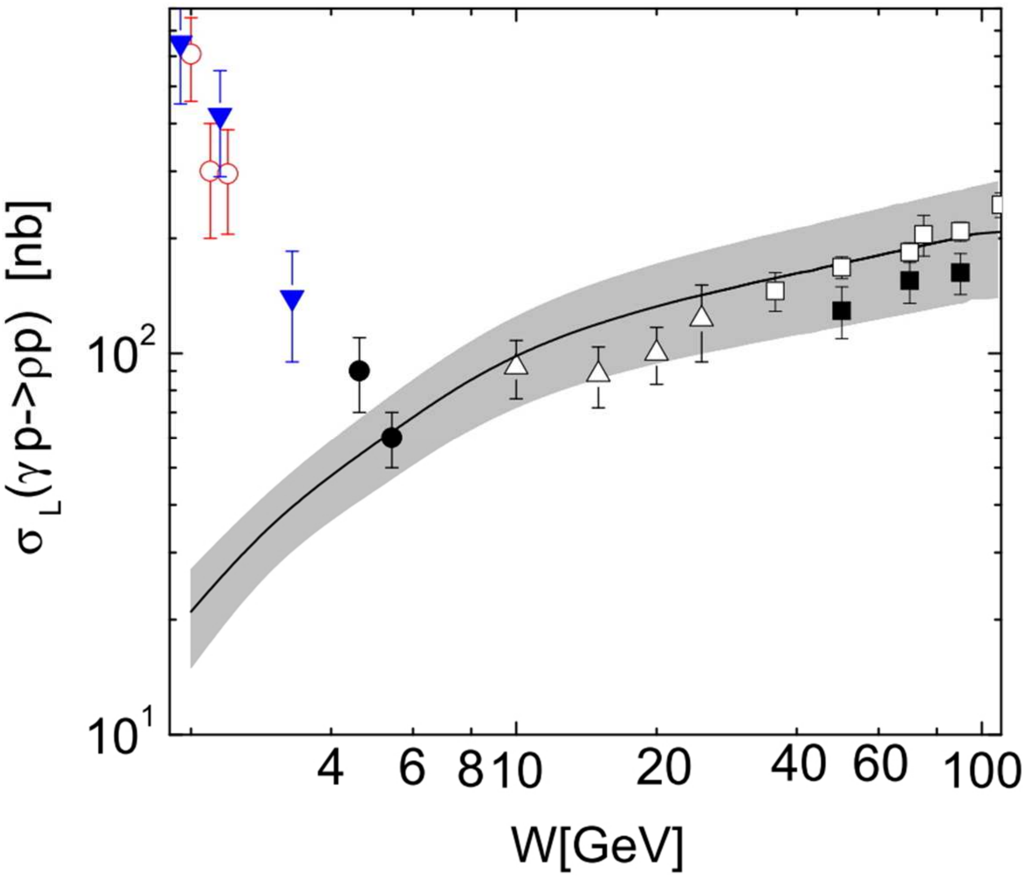
$$f_i(\beta,\alpha,t') = e^{(b_i+\alpha_i' \ln(1/|\beta|))t'} \boxed{h_i(\beta)} \frac{\Gamma(2n_i+2)}{2^{2n_i+1}\,\Gamma^2(n_i+1)} \frac{[(1-|\beta|)^2-\alpha^2]^{n_i}}{(1-|\beta|)^{2n_i+1}}$$

$$h_i(\beta) = \beta^{-\delta_i}\,(1-\beta)^{2n_i+1} \sum_{j=0}^{3} c_{ij}\,\beta^{j/2}$$

$$H_i(\bar{x},\xi,t') = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\,\delta(\beta+\xi\alpha-\bar{x})\,f_i(\beta,\alpha,t')$$

S.V. Goloskokov, Nucl.Phys.Proc.Suppl. 219-220 (2011) 185-192
S.V. Goloskokov, P. Kroll, Euro. Phys. J. C50, (2007) 829-842.
S.V. Goloskokov, P. Kroll, Euro. Phys. J. C53, (2008) 367-384

GK GPD parametrization



S.V. Goloskokov, Nucl.Phys.Proc.Suppl. 219-220 (2011) 185-192
S.V. Goloskokov, P. Kroll, Euro. Phys. J. C50, (2007) 829-842.
S.V. Goloskokov, P. Kroll, Euro. Phys. J. C53, (2008) 367-384

Subtraction corrections

Standard form D-term with the use of the results of the Chiral Quark-Soliton model :

$$D(\alpha) = -\frac{12}{N_f} z(1-z^2)\Theta(1-|z|)$$

For Nf=5 gives constant subtraction :

$$\Delta = \frac{8\sqrt{2}}{5}$$

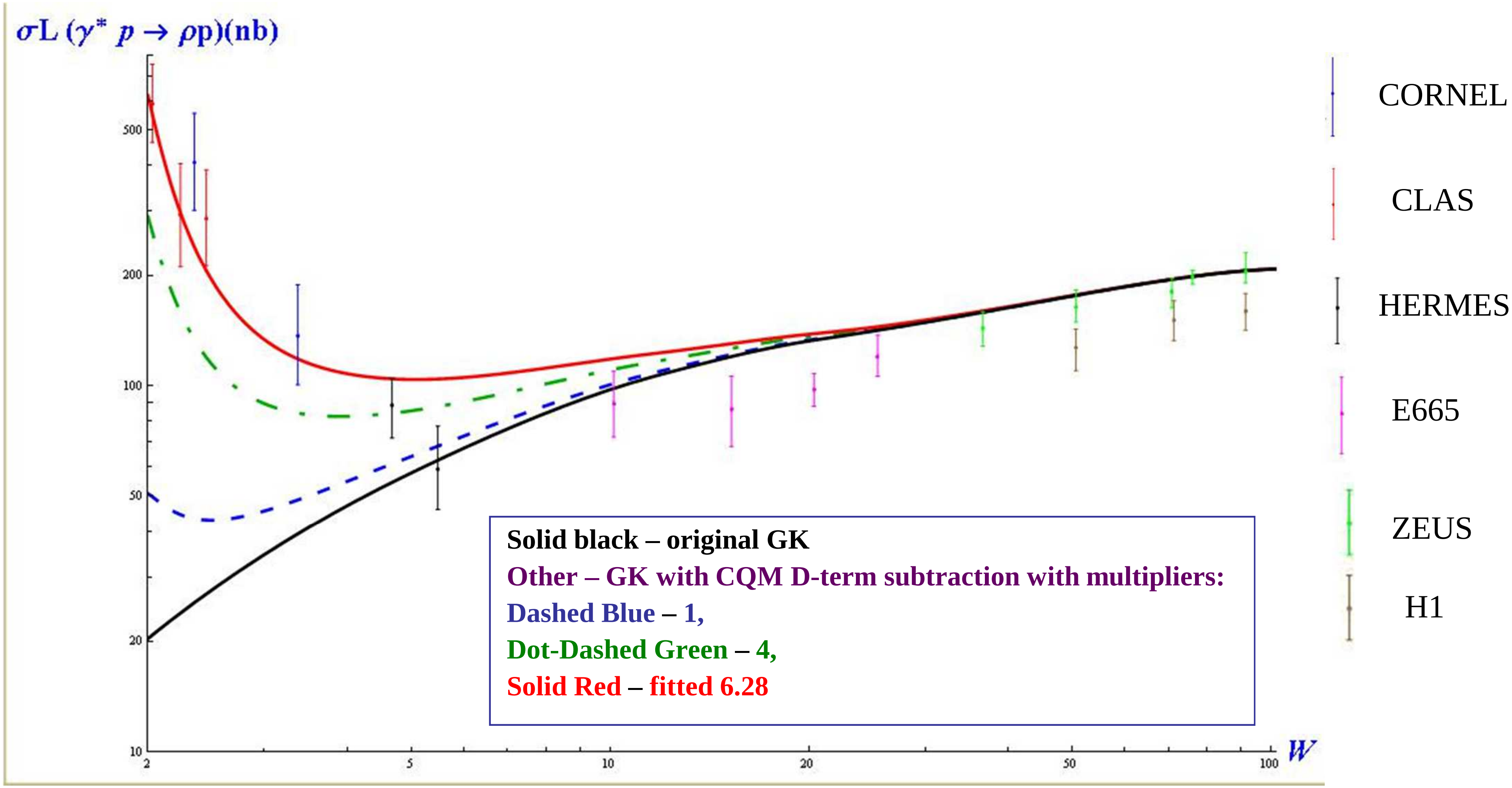
M. V. Polyakov and C. Weiss, Phys. Rev. D 60, 114017 (1999)
V. Y. Petrov, P. V. Pobylitsa, M. V. Polyakov, I. Bornig, K. Goeke and C. Weiss, Phys. Rev. D 57, 4325 (1998)

Qualitative estimations on d-term like corrections on GK model

In assumption of small kT dependence of cross sections ratio:

$$\sigma(W) = \sigma_0(W) \left| \frac{A_{collinear}(W) + \Delta(W)}{A_{collinear}(W)} \right|^2$$

Subtraction corrections



Summary:

- Photon GPDs was extended to the full definition area
- Holographic sum rule was checked for photon GPD, subtraction constants were derived
- Photon DDs was derived using inverse Radon transform
- Preliminary assumptions were made for longitudinal ρ° electroproduction cross section for GK GPD model
- D-term like contributions can explain cross section grow at low W (further calculations (and data) are needed)

Thanks for your attention!