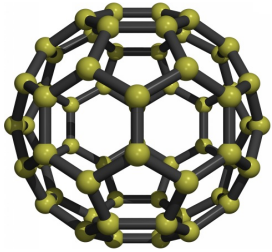


Influence of Vibration and Deformation

C_{60}



of Nuclei

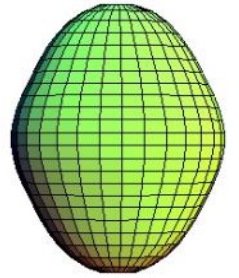
ON the ELLIPTIC FLOW

Peter Filip

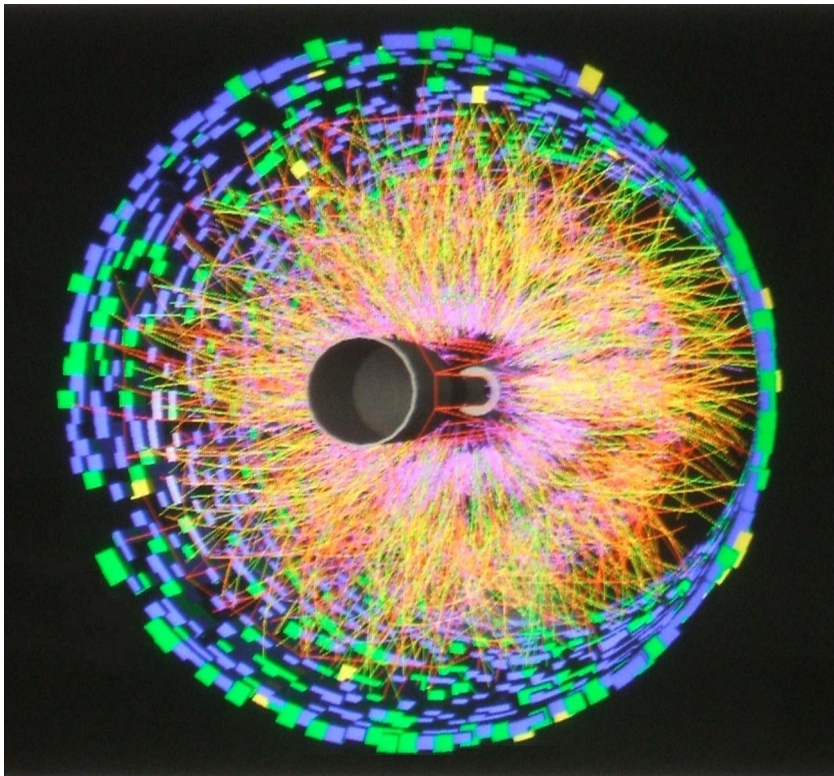
(IP SAS, Bratislava)

XXI Baldin Seminar, 10-15. Sept. Dubna 2012

Sm-154



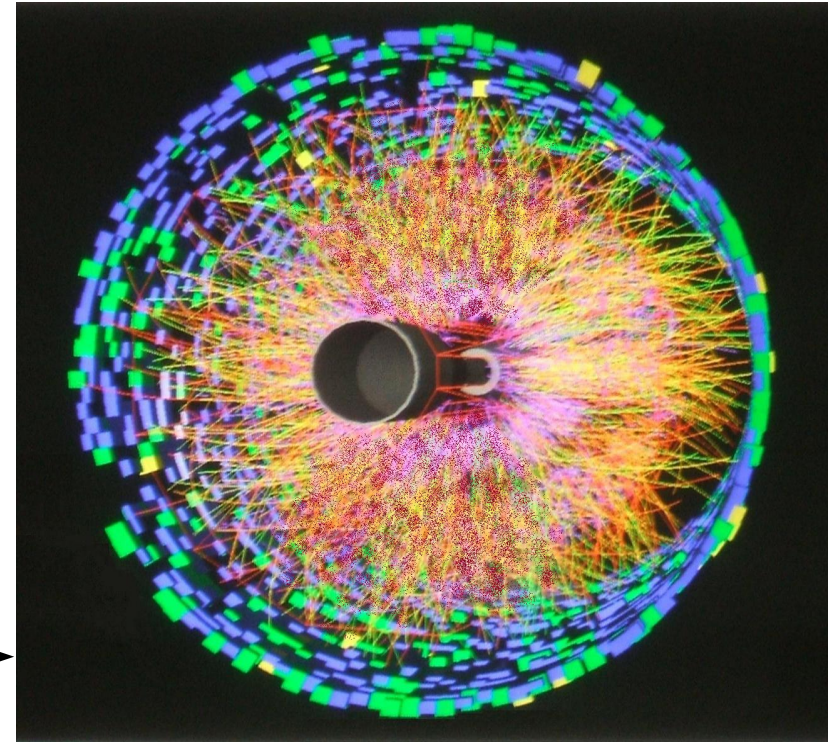
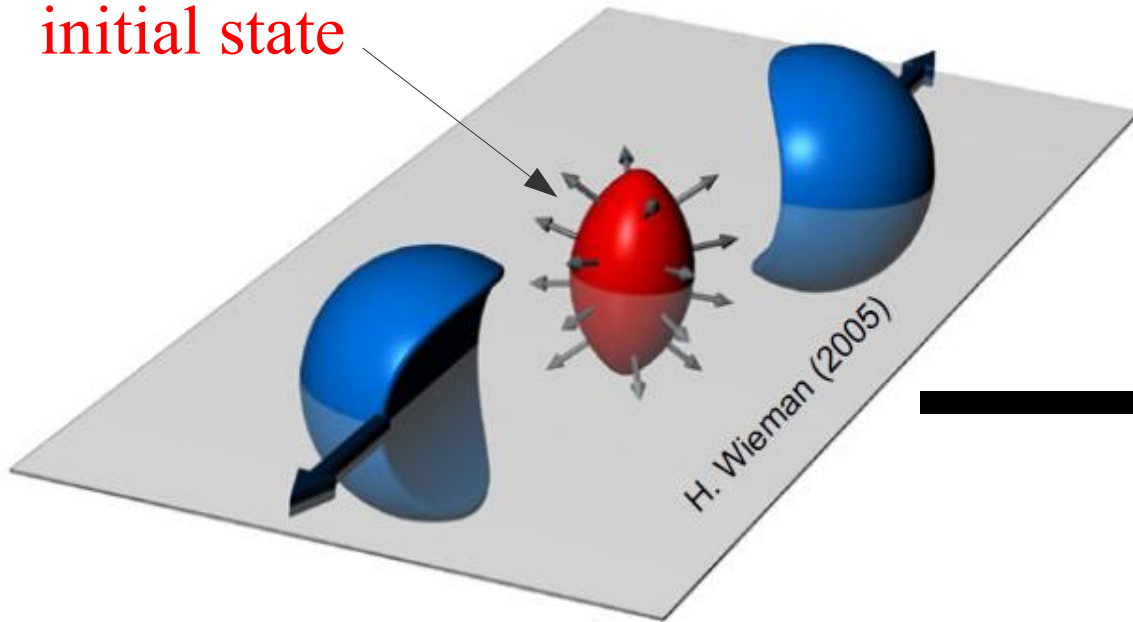
$\beta_2=0.27$ $\beta_4=0.11$



- Elliptic Flow
- Initial eccentricity
- Deformation effects
- GMC simulations
- Vibrations of nuclei
- Summary

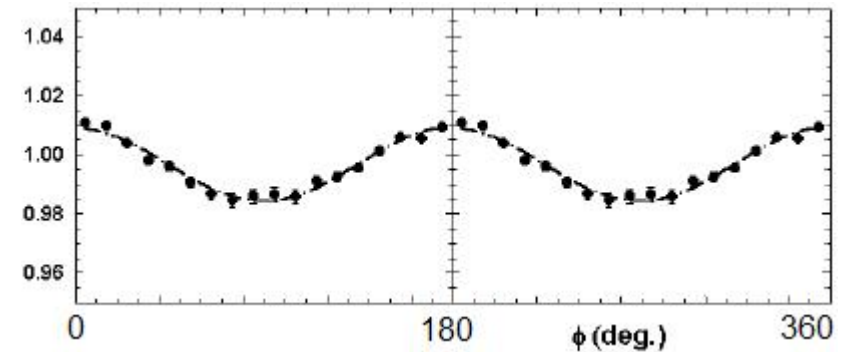
Elliptic Flow: v_2

initial state



Azimuthal asymmetry

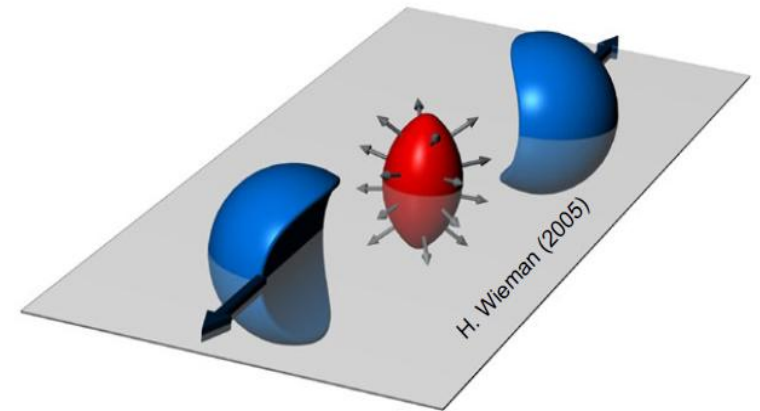
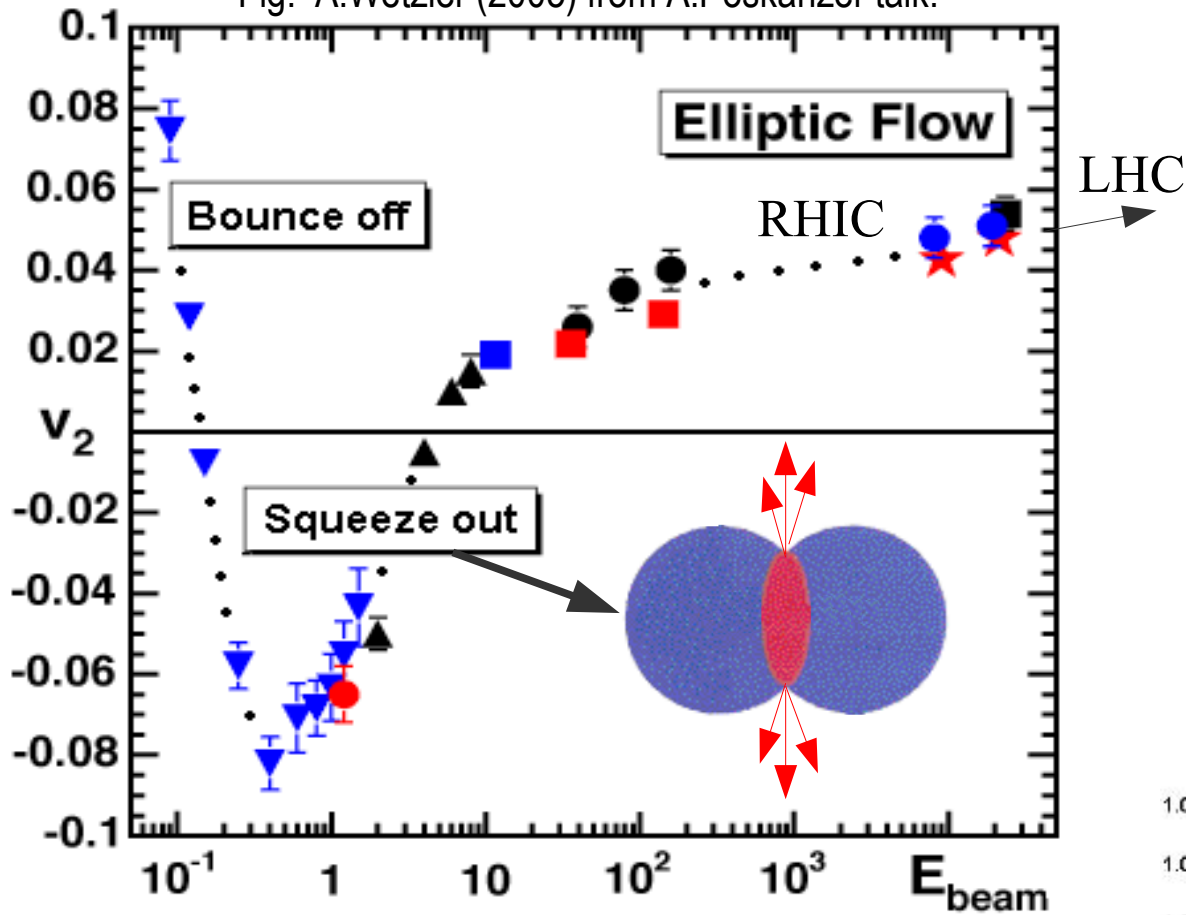
Asymmetry in Spatial distribution leads to
→ asymmetry in **Momentum** distribution



Asymmetry Effect depends on the COLLISION ENERGY

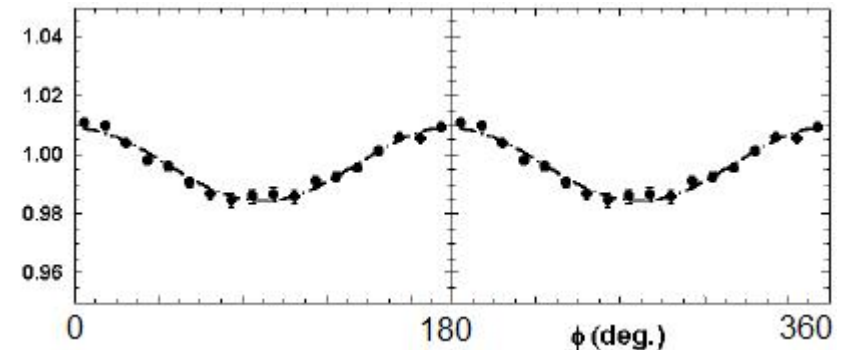
Elliptic Flow: v_2 energy dependence

Fig: A.Wetzler (2005) from A.Poskanzer talk.



Asymmetry strength: v_2

$$dN/d\phi = (N/2\pi)[1 + 2v_2 \cos(2\phi)]$$



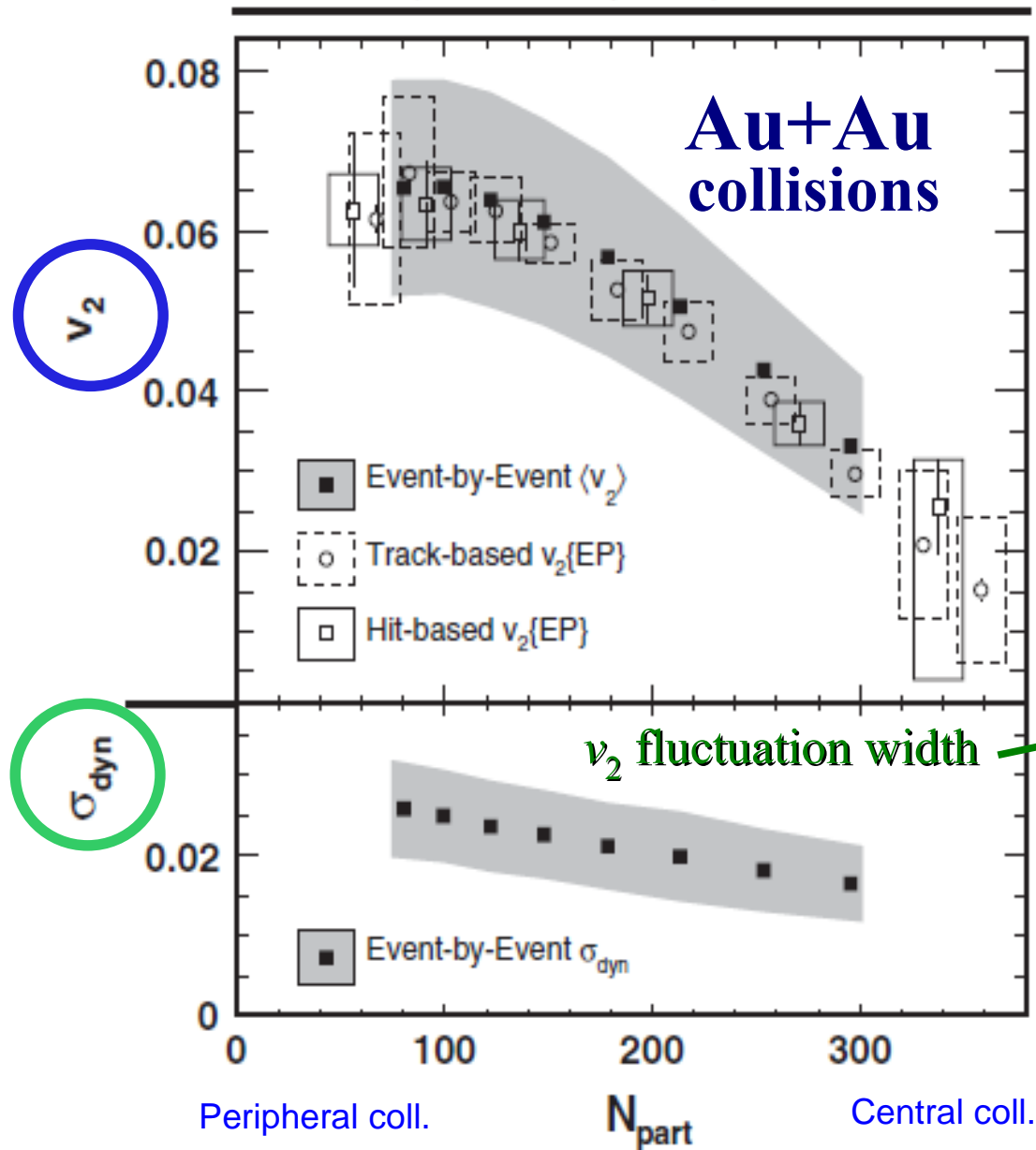
Azimuthal Momentum distribution

v_2 : has **strength** = magnitude $\langle v_2 \rangle$

and **fluctuations**: σ_{v_2}

Elliptic flow v_2 : magnitude & fluctuations

PRL 104, 142301 (2010)



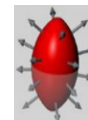
- Magnitude of v_2 decreases
→ for central collisions

- *We believe = assume:*

v_2 fluctuation

comes from: the initial
eccentricity fluctuation

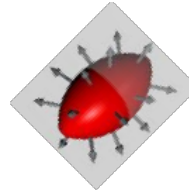
(at given N_{ch} or N_{part})



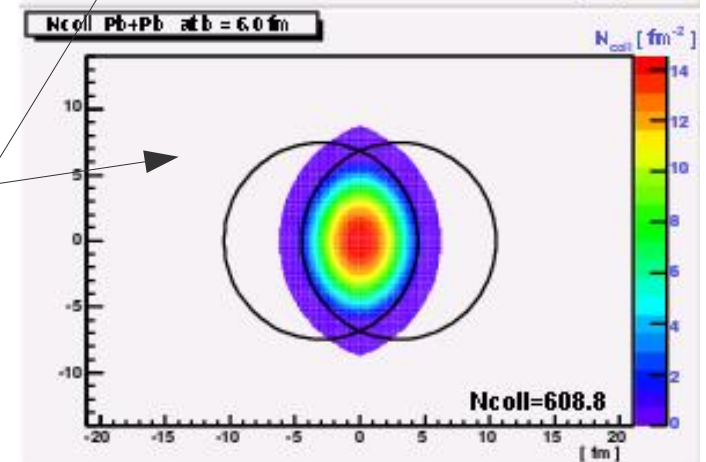
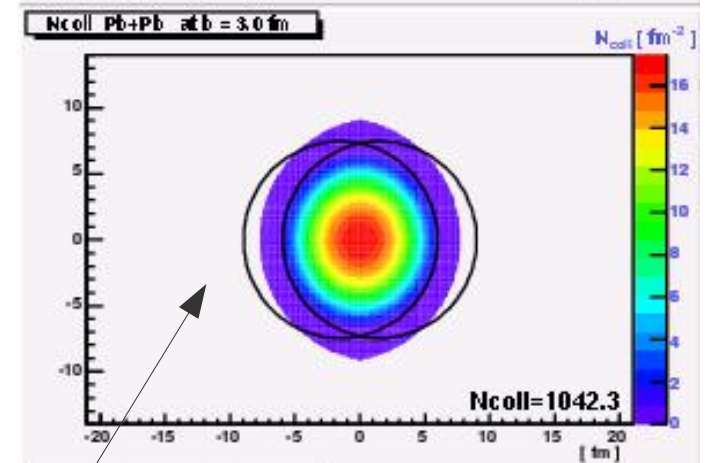
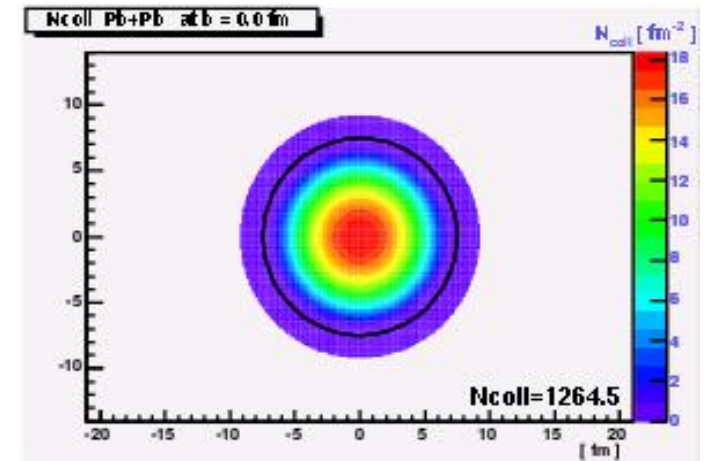
Centrality dependence of initial eccentricity ϵ :

Rotation-invariant formula

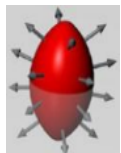
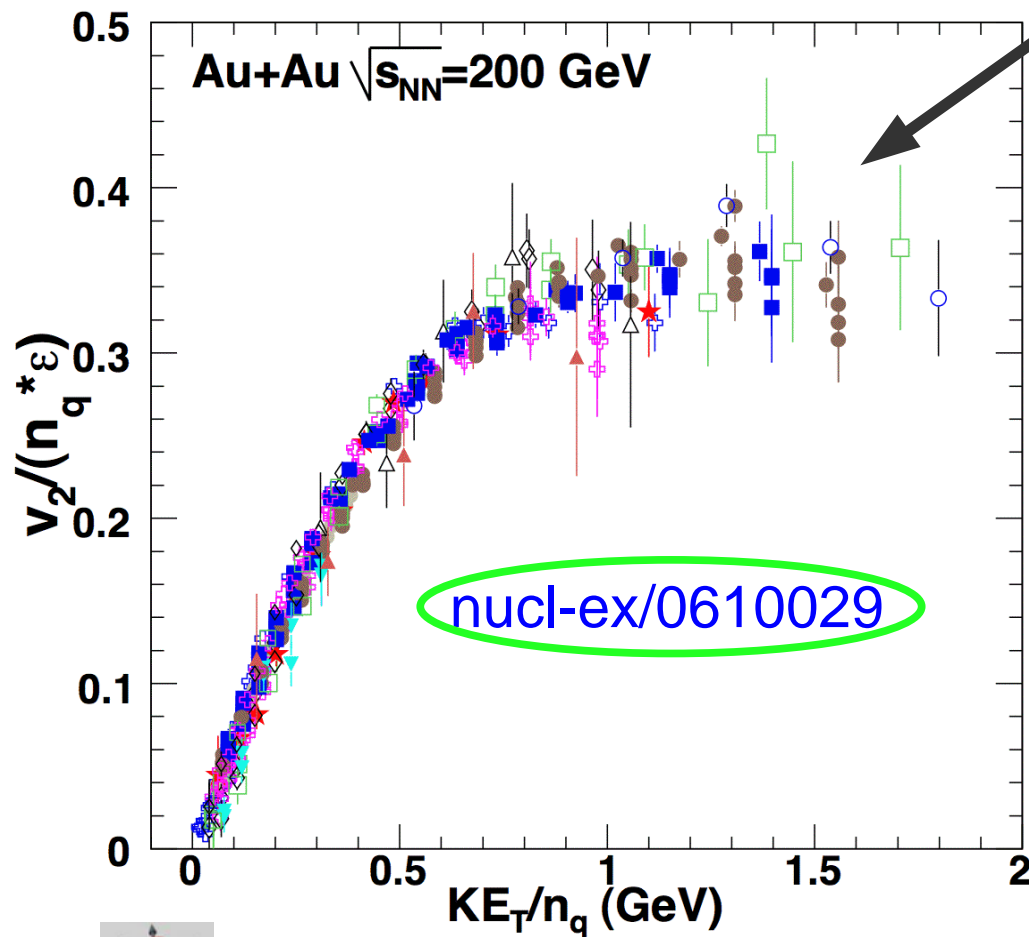
$$\epsilon = \frac{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 + 4\sigma_{xy}^2}}{\sigma_y^2 + \sigma_x^2}$$



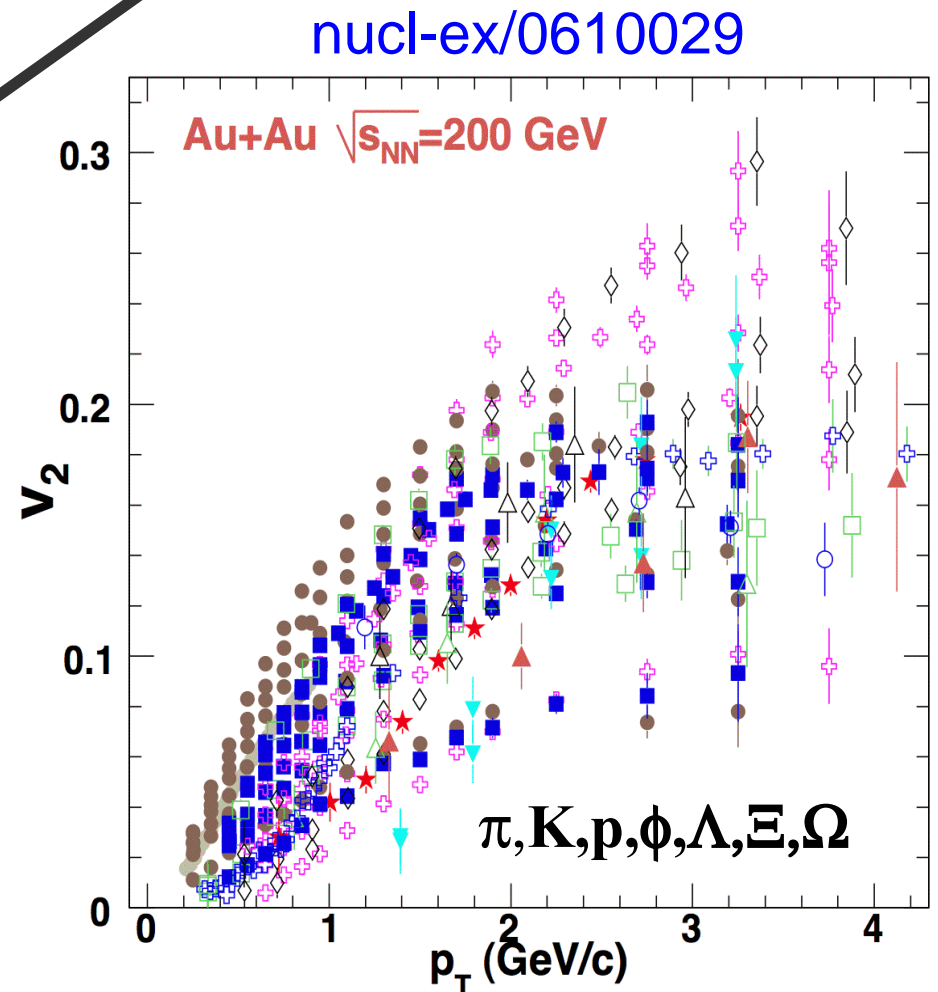
- Eccentricity \rightarrow Elliptic flow is larger for non-central collisions



Elliptic flow at *RHIC*: partonic expansion



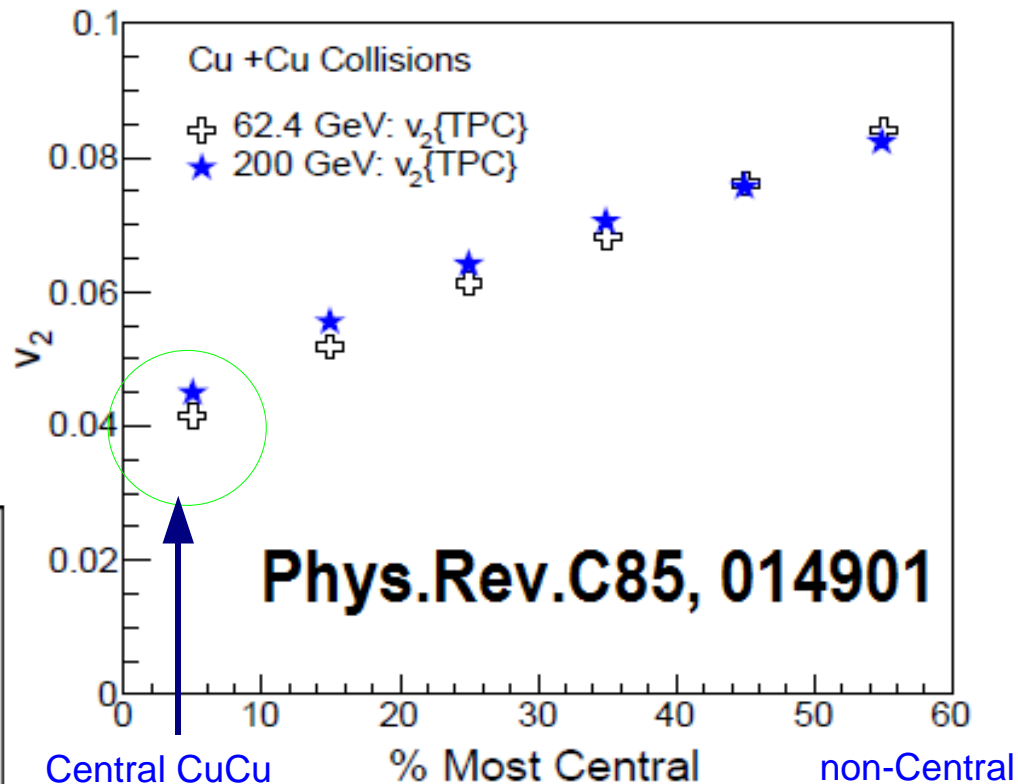
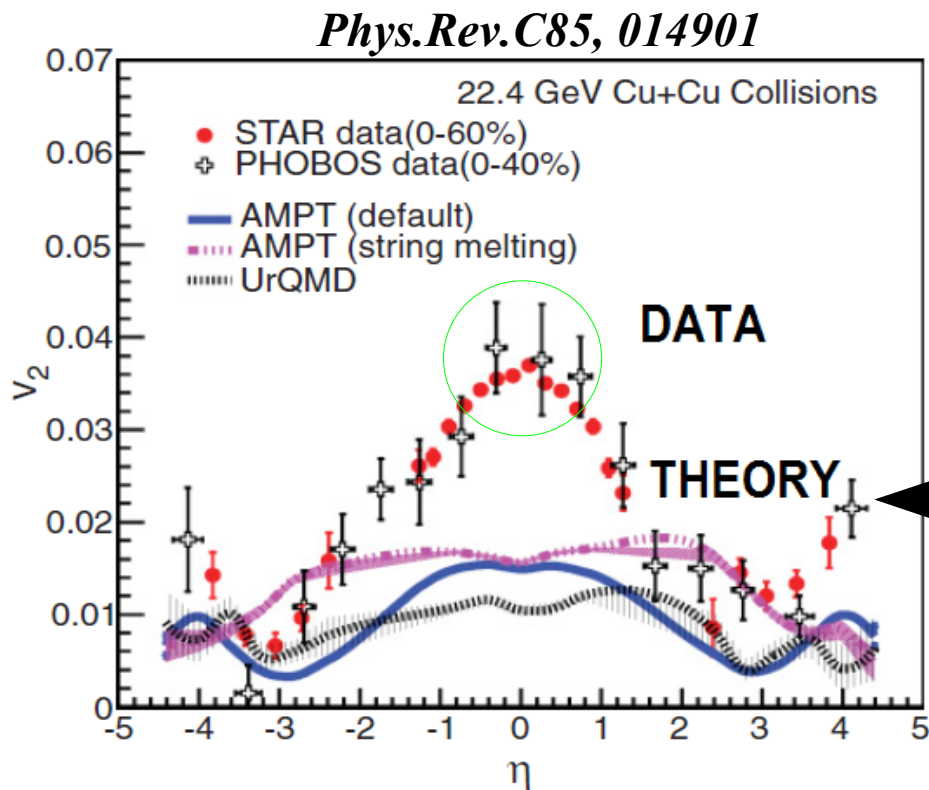
Very nice !



However: Cu+Cu \rightarrow

Elliptic flow v_2 strength in Cu+Cu

- v_2 strength = $\langle v_2 \rangle$
 → average v_2 value
 (at given centrality, η)

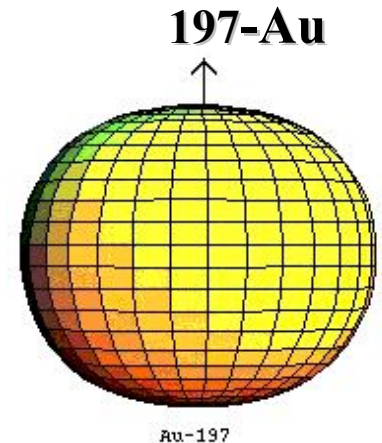


v_2 strength in Cu+Cu
 (RHIC at 22.4 GeV/n)

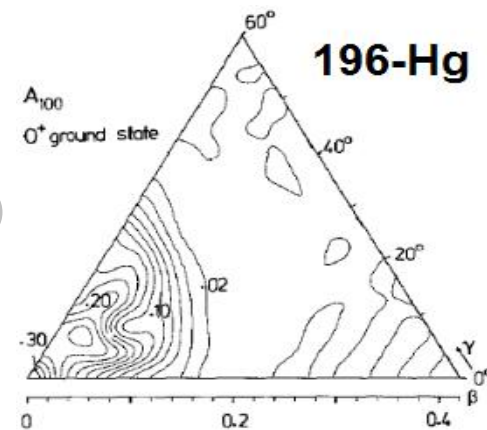
is not understood ...

In this talk

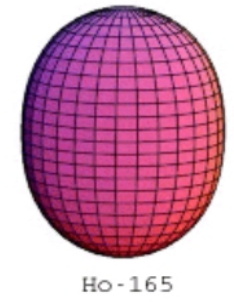
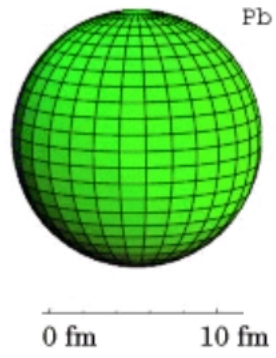
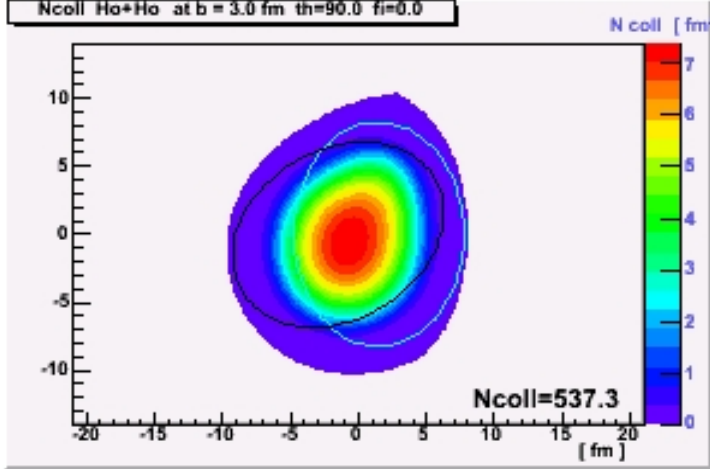
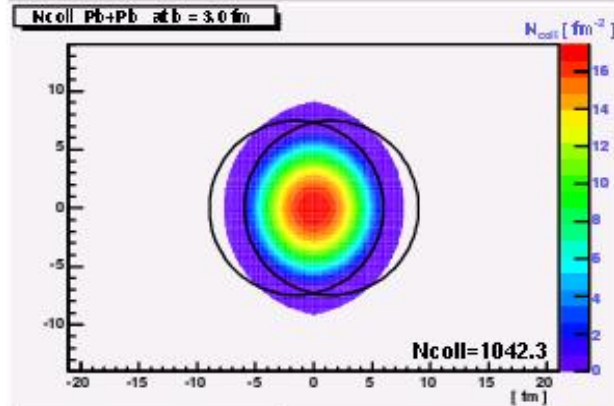
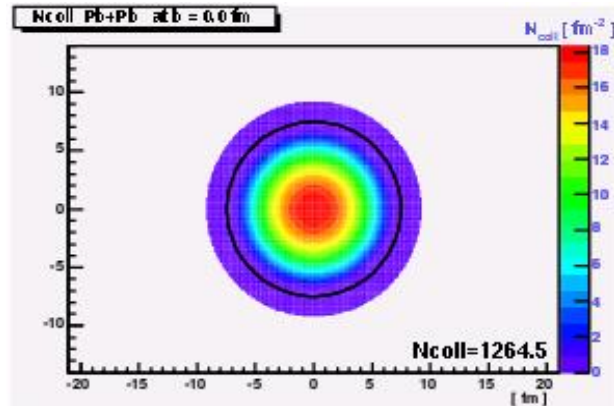
Two initial-state effects:
→ influence Elliptic flow.



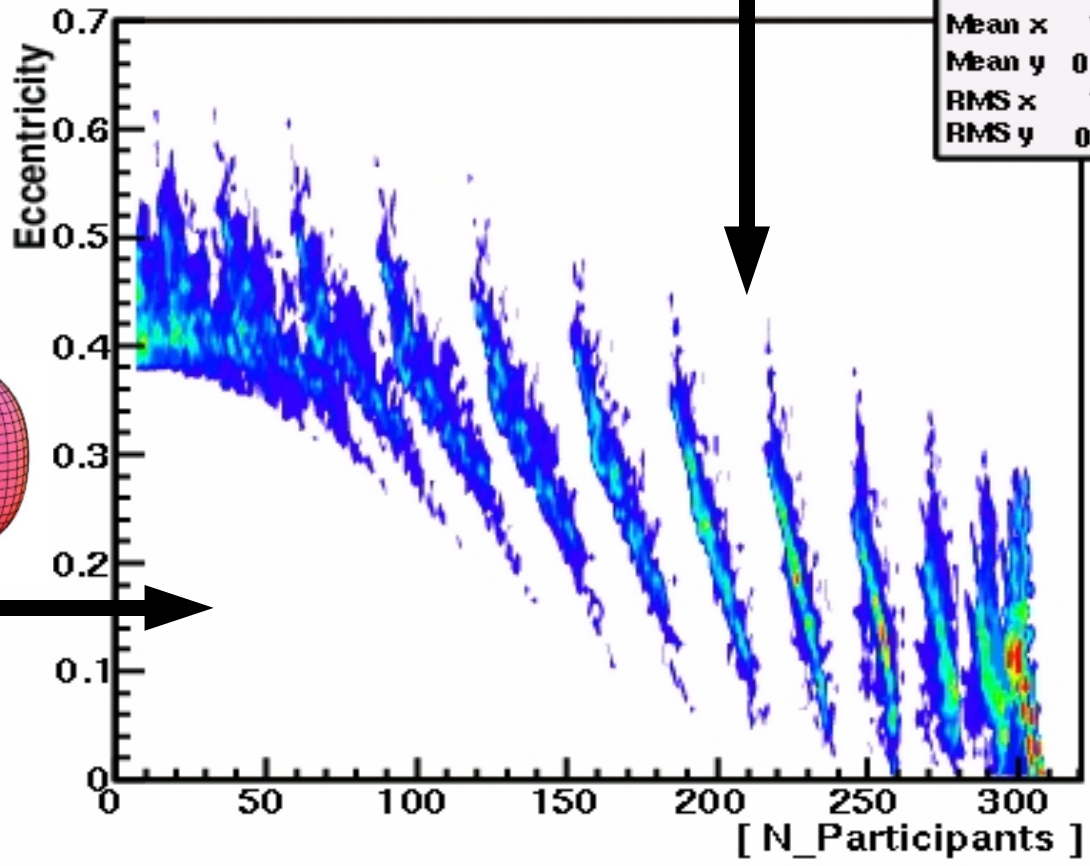
- Ground-state **deformation**
- Ground-state **Vibration** (2012)



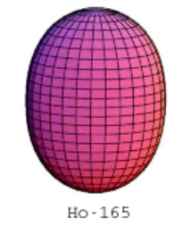
Fluctuating Eccentricity: fixed impact param. [b]



EccPart OGM simulation

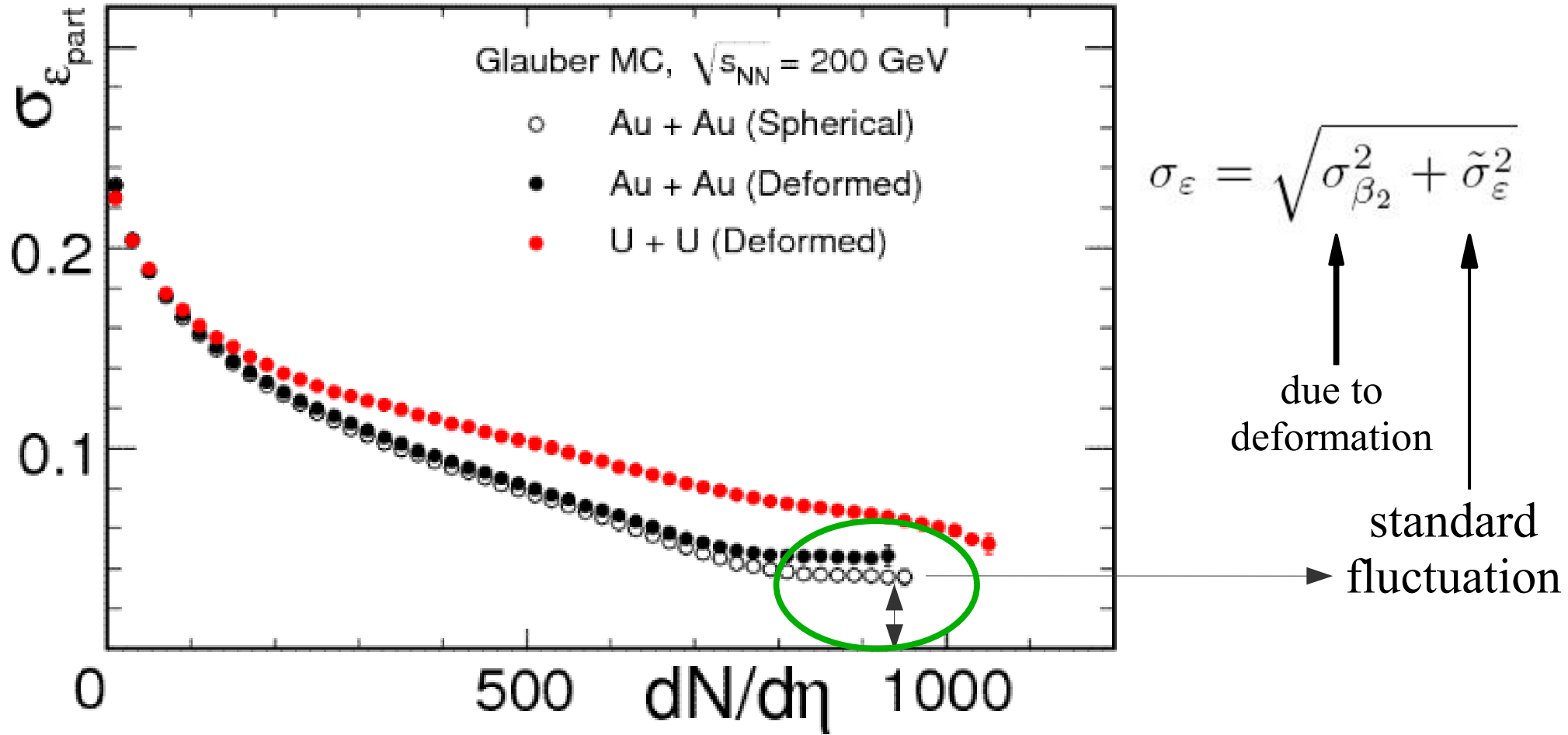


EccPart	
Entries	86090
Mean x	173.3
Mean y	0.2633
RMS x	100.6
RMS y	0.1442



Deformation of nuclei in MC Glauber:

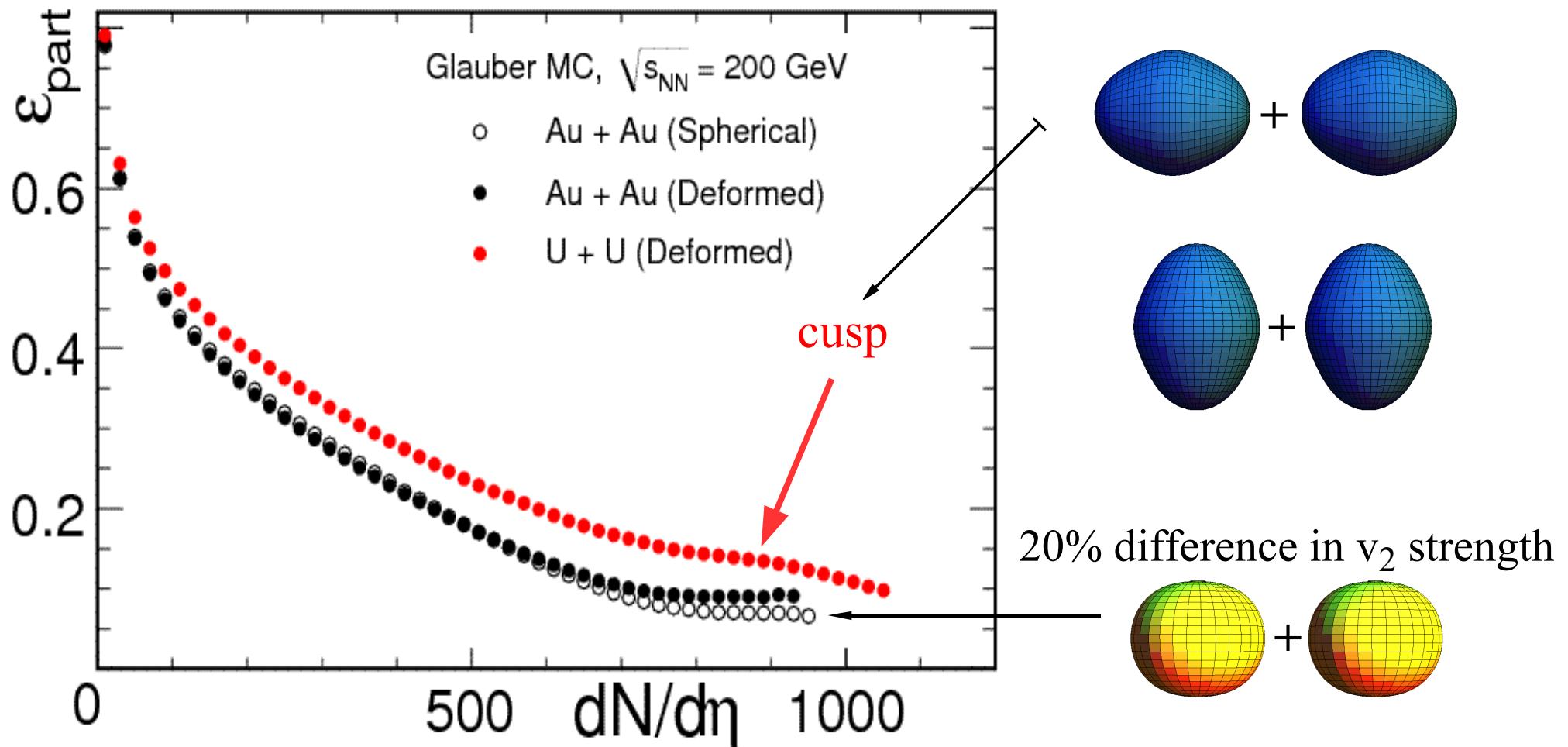
→ **increased Eccentricity fluctuations**



Phys.Rev.C80, 054903: Deformation effects on σ_{v_2} (fluctuation of v_2)

Deformation influence on v_2 strength:

→ **self-orientation effect** in central UU collisions

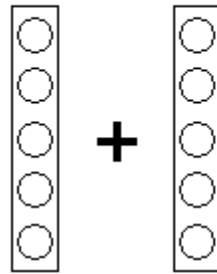
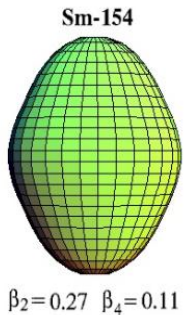


MC Glauber simulation: Phys.Rev.C80, 054903: ϵ CUSP \rightarrow v_2 CUSP

Self-orientation effect:

- for very high N_{ch} multiplicity collisions
 \rightarrow max. binary NN collisions $\rightarrow N_{ch} \rightarrow$ orientation

extremal
case:



NN coll. = 5
N part. = 10

Binary n-n collisions sensitivity to orientation



NN coll. = 25
N part. = 10

$$dN_{ch}/d\eta = (1 - x) \cdot n_{pp} \frac{N_{part}}{2} + x \cdot n_{pp} N_{coll}$$

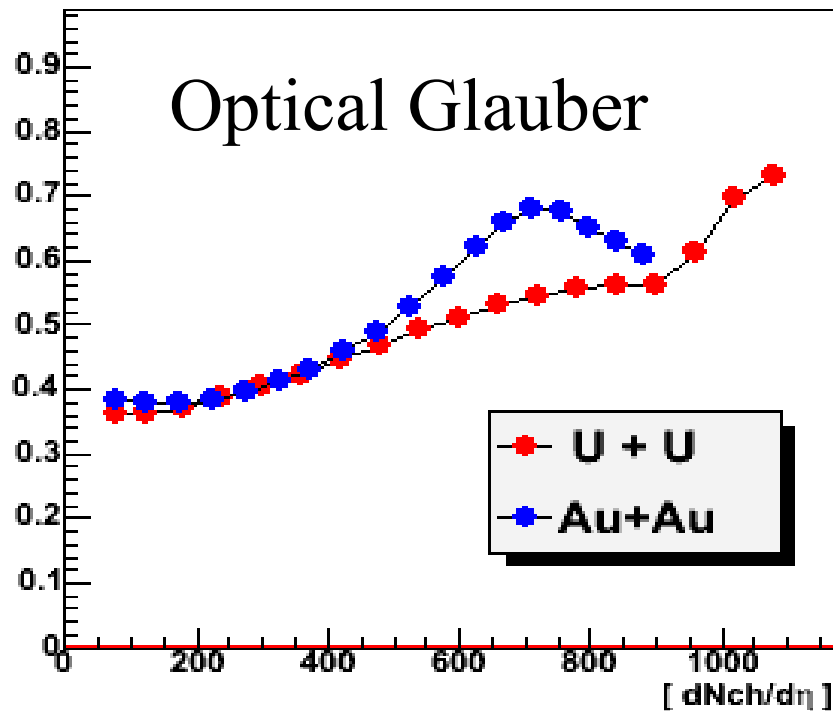
Deformation effects: σ_{v_2}/v_2 [N_{ch}]

Assuming hydrodynamical expansion: σ_{v_2}/v_2

$$\rightarrow \sigma_{v_2}/v_2 \approx \sigma_{\epsilon} / \langle \epsilon \rangle$$

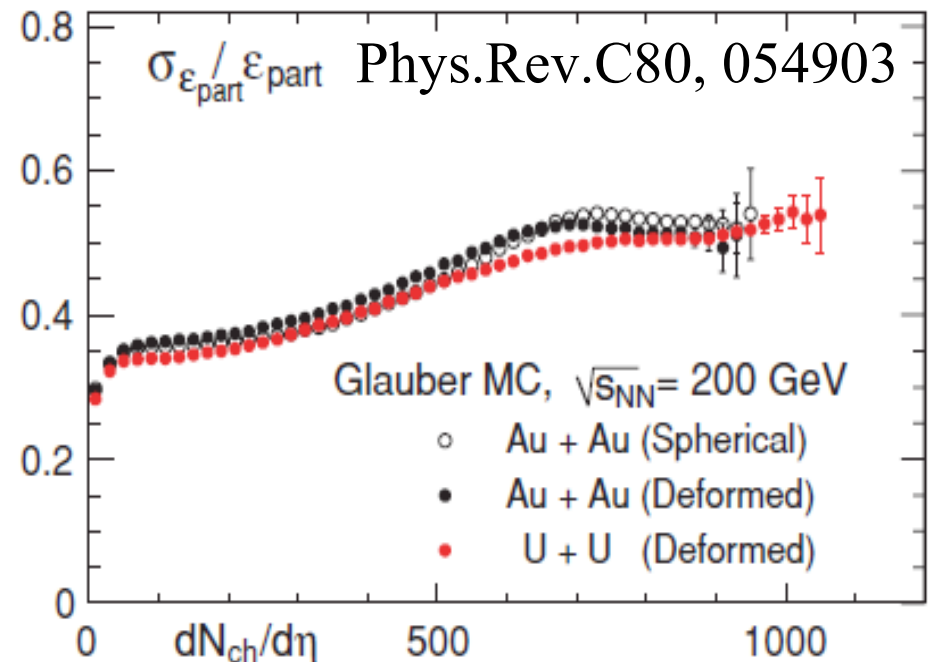
\rightarrow sensitive to deformation of nuclei

$\sigma(\epsilon)/\langle \epsilon \rangle$ [$dN_{ch}/d\eta$]



Optical Glauber Model

FILIP, LEDNICKY, MASUI, AND XU



Full MC Glauber simulation

DEFORMATION OF NUCLEI

*Moller Chart of Nuclides 2000
Quadrupole Deformation*

- Most of nuclei are deformed (including Cu, In, Ho, Au)

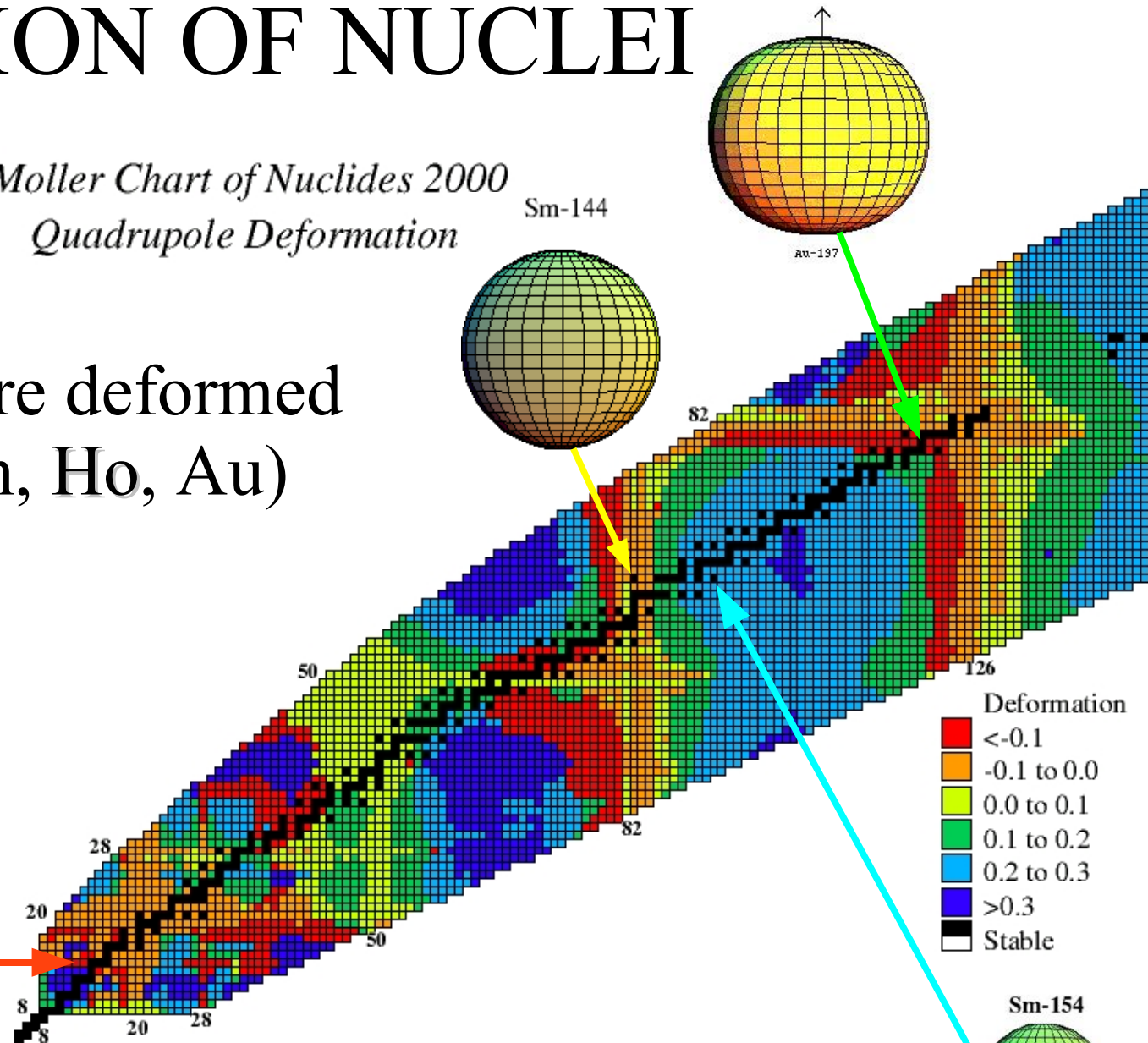
- **Au:** $\beta_2 = -0.13$

- **Ho:** $\beta_2 = 0.30$

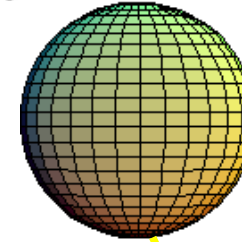
- **Si²⁸:** $\beta_2 = -0.4$



Si-28



Sm-144



Au-197

82

126

28

20

50

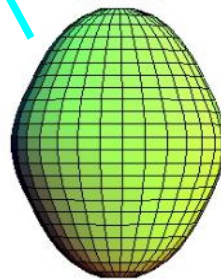
8

20

Si-28

- **Study Nd, Sm, Ho, Si collisions at LHC/Nica/SPS**
→ to verify our understanding of the Elliptic flow.

Sm-154



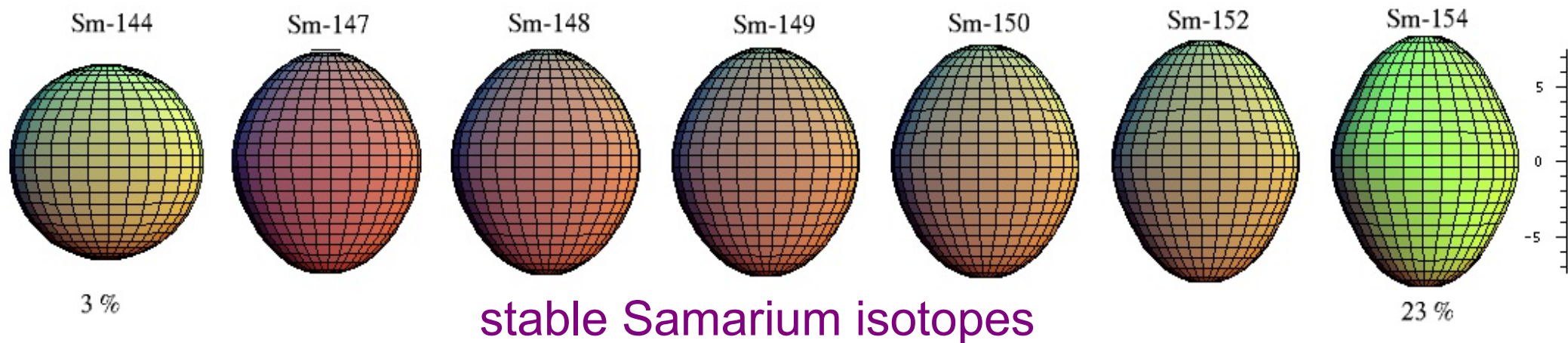
$\beta_2 = 0.27$ $\beta_4 = 0.11$

Suggestion N.1:

Deformation of nuclei: $\rightarrow \beta_2$

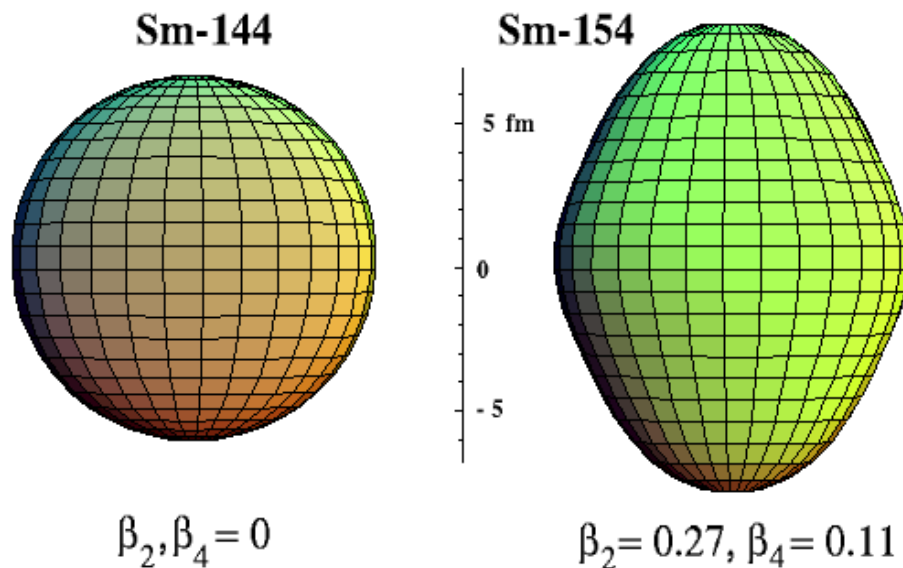
in relativistic Nucleus-Nucleus collisions

\rightarrow may allow a more detailed understanding
of the elliptic flow phenomenon.



Comparing v_2 strength & fluctuations:

→ for spherical & deformed $Sm+Sm$ collisions



$$\sigma_\varepsilon = \sqrt{\sigma_{\beta_2}^2 + \tilde{\sigma}_\varepsilon^2}$$

↑
due to
deformation

↑
all other
fluctuations

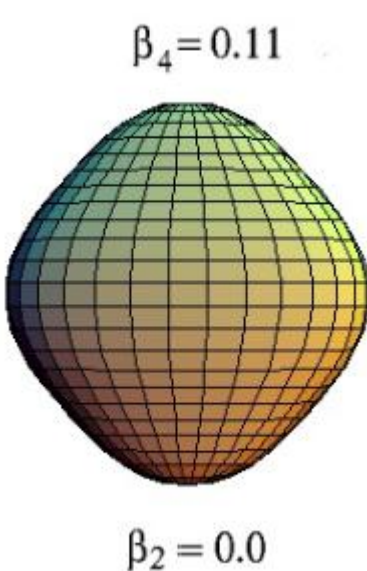
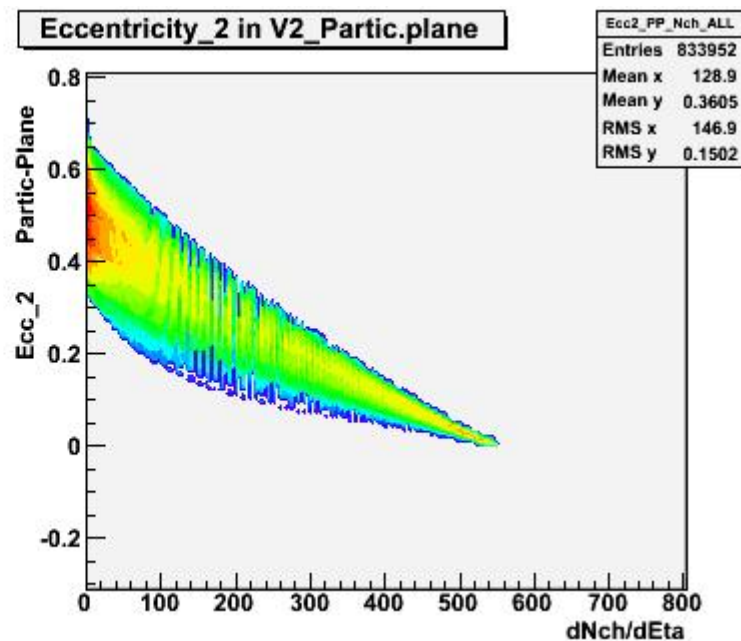
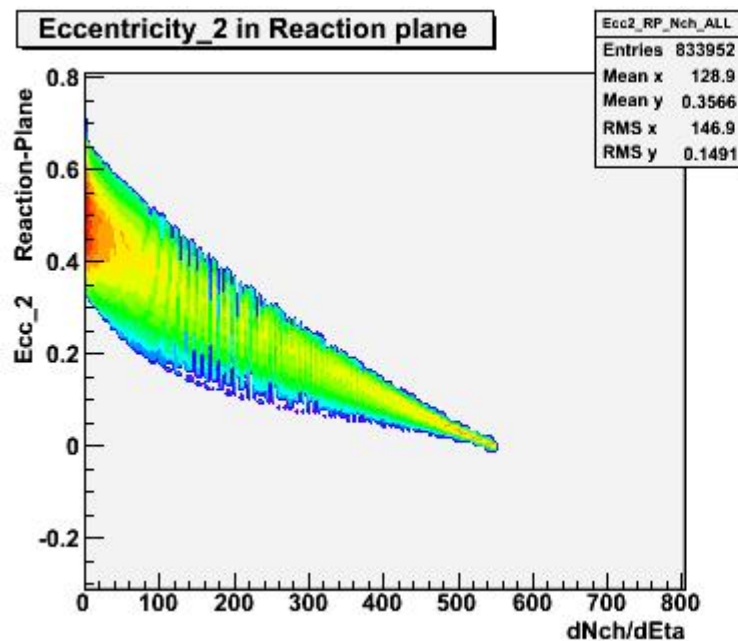
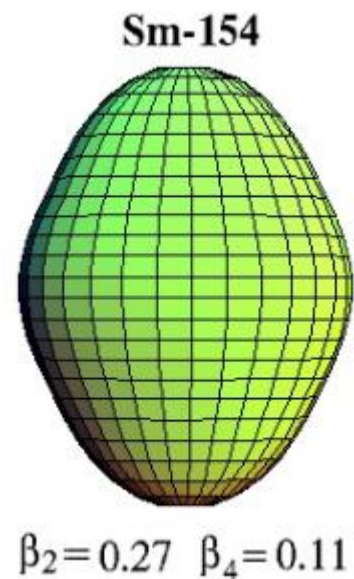
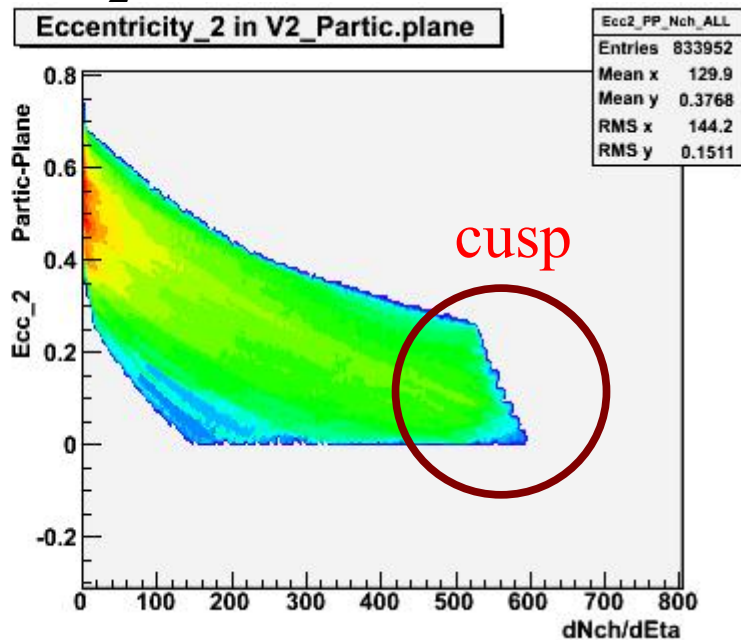
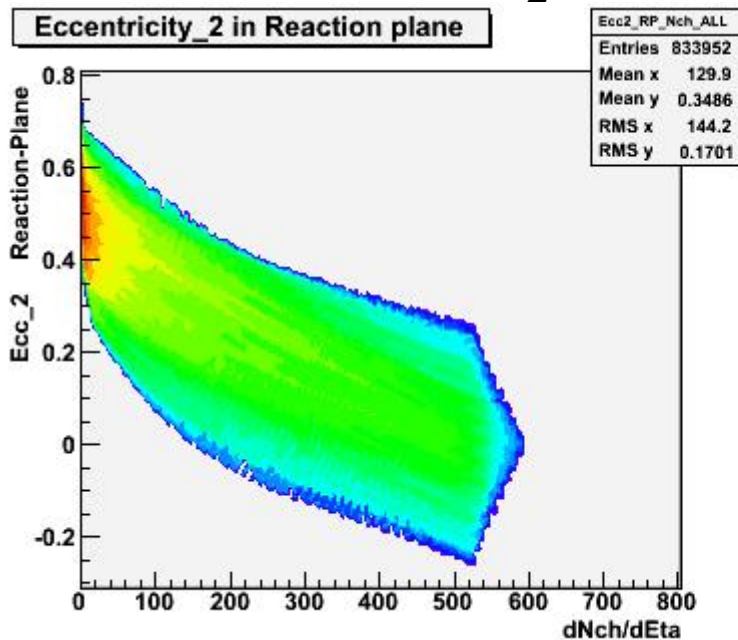
- We know initial eccentricity fluctuation increases for Sm^{154} collisions

→ Experimentally **measured v_2 fluctuation**:

→ **should increase: How much ?**

v_2 eccentricity for Sm+Sm

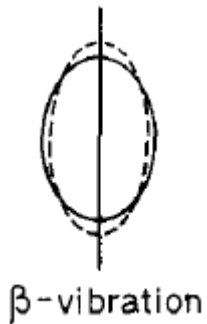
(OGM: larger $\beta_2 \rightarrow$ larger ϵ_2 (v_2) fluctuations)



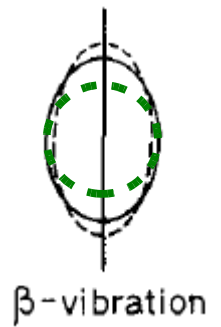
Other initial-state effect:

→ GROUND-STATE nuclear Vibration (ZPV)

- **What is it ? (Zero-Point-Shape-Vibration)**
 - present in deformed nuclei ?
 - present also in spherical nuclei ?
 - Is it stronger or weaker for light nuclei ?
- **Does it really exist ?**
 - has anybody observed Ground-State vibration = GSV ?
(in molecular physics ?)
- **Should we include it in MC simulations of HIC ?**



Bohr and Mottelson on ZPV **Nuclear Structure II**



350

VIBRATIONAL SPECTRA *Ch. 6*

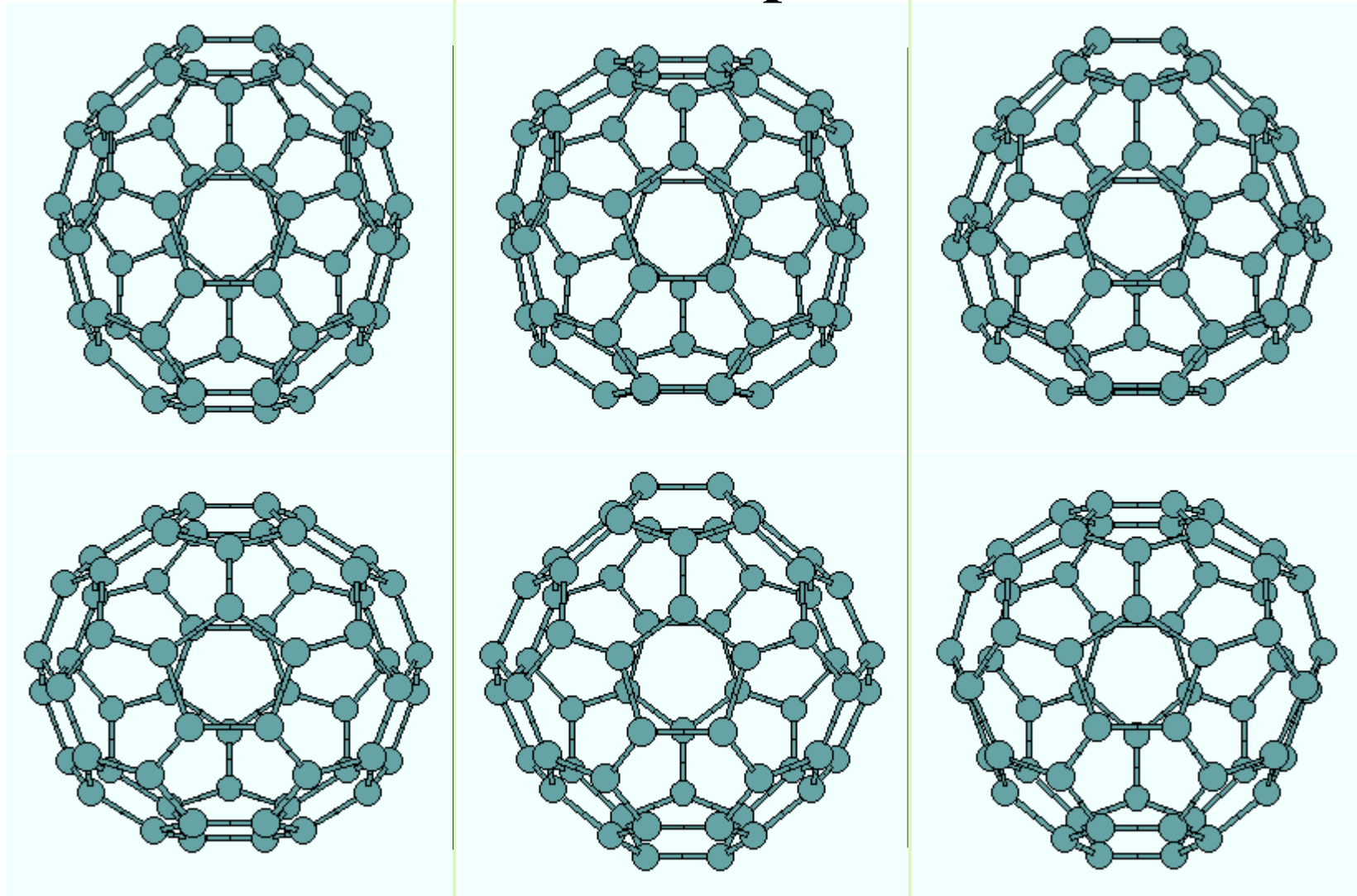
Occurrence of shape oscillations

The study of the low-energy spectra of the even-even nuclei has revealed the systematic occurrence of $I^\pi = 2^+$ and 3^- states with properties suggesting a vibrational interpretation. The collective character of the excitations is implied by the large transition probabilities and by the fact that the properties vary rather smoothly with N and Z . The systematics of the excitation energies of the 2^+ and 3^- modes are shown in Fig. 2-17a, b, Vol. I, pp. 196–197, and Fig. 6-40, p. 560, respectively. The transition probabilities are an order of magnitude larger than the single-particle unit, and if interpreted as shape oscillations, as in Eq. (6-65), correspond to zero-point amplitudes β_2 and β_3 typically of order 0.2 (see Fig. 4-5)

- nuclei do oscillate = vibrate in the ground state
- **amplitude is comparable to static deformation β_2**

Molecular Physics: C_{60}

Ground-state-shape vibration



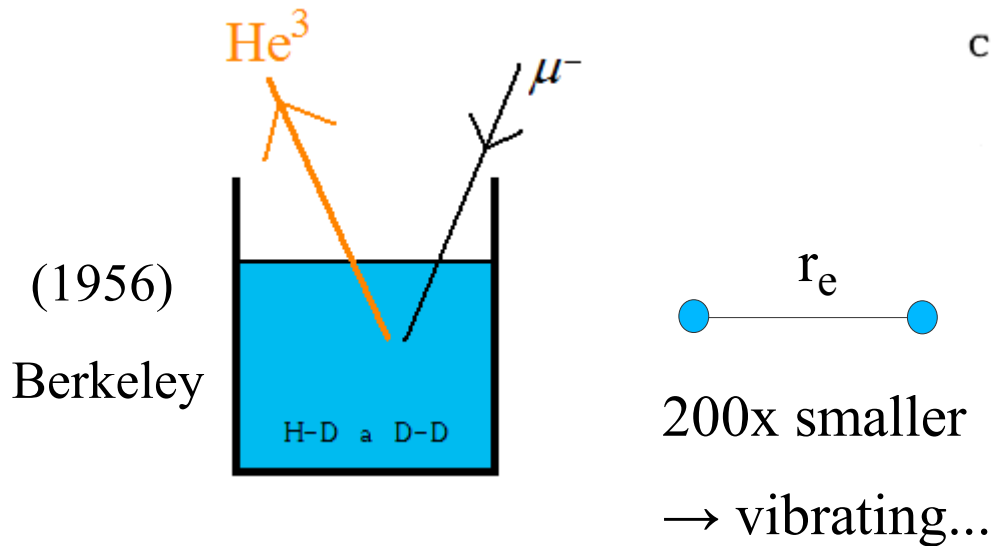
Quadrupole vibration: β_2

Hexa-decapole vibration: β_4

Octupole vibration: β_3

From → J.Ménendez and J.Page:“Vibrational spectroscopy of C_{60} ” **Nuclei may vibrate similarly!**

Molecular Ground-state VIBRATION important in: μ -cF



μ^- meson serves as a *catalyst*

The reaction rate $T \sim 2.5 \times 10^{-6}$ sec can be written

$$1/T = A |\Psi(0)|^2$$

where reaction constant is $A \sim 10^{-22}$ cm³/sec

$\Psi(0)$ is value of wave function at zero separation

CATALYSIS OF NUCLEAR REACTION

J. D. JACKSON
Princeton University
($p+d+\mu^-$) → He^3

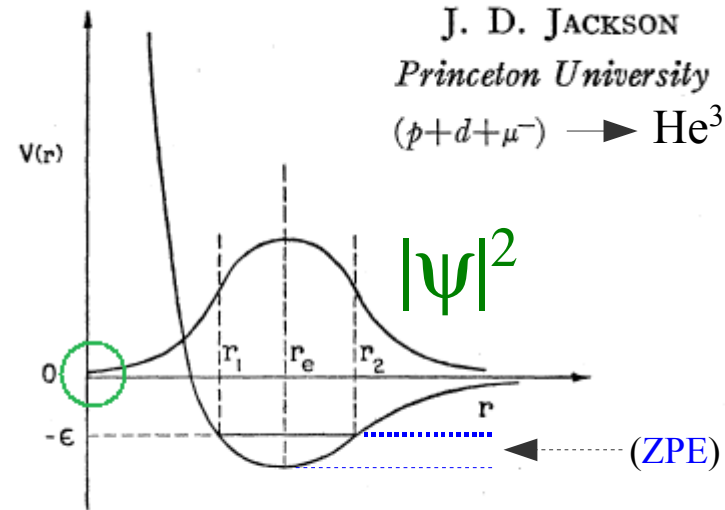


FIG. 1. Nuclear potential energy curve in μ -mesonic hydrogen molecule, and ground-state vibrational wave function for the

$$\Psi_{\text{vib}}(x) = (\alpha/\pi)^{1/4} \exp\left[-\frac{1}{2}\alpha(x-x_0)^2\right]$$

PHYSICAL REVIEW

VOLUME 106, NUMBER 2

APRIL 15, 1957

Catalysis of Nuclear Reactions between Hydrogen Isotopes by μ^- Mesons

J. D. JACKSON*

Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

(Received January 10, 1957; revised manuscript received February 4, 1957)

Bohr and Mottelson on ZP Vibration

Nuclear Structure II

The vibrational wave functions $\varphi_n(\alpha)$ have the form

$$\varphi_n(\alpha) = (2\pi)^{-1/4} (2^n n! \alpha_0)^{-1/2} H_n\left(2^{-1/2} \frac{\alpha}{\alpha_0}\right) \exp\left\{-\frac{1}{4} \frac{\alpha^2}{\alpha_0^2}\right\} \quad (6-14)$$

where H_n is the n th Hermite polynomial ($H_0(x)=1$, $H_1(x)=2x$)

while α_0 is the zero-point amplitude

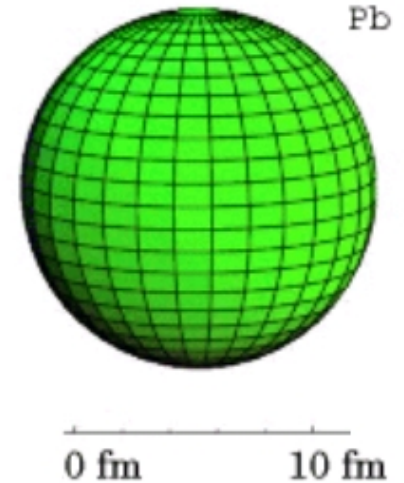
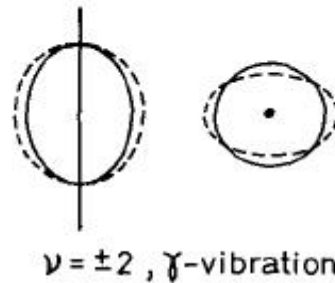
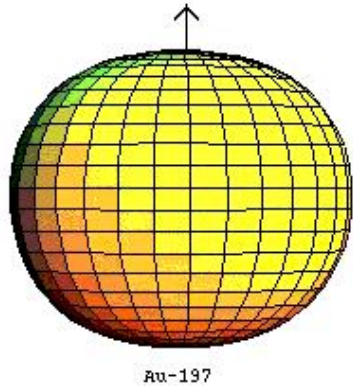
$$\alpha_0 \equiv \langle n=0 | \alpha^2 | n=0 \rangle^{1/2} = \left(\frac{\hbar}{2D\omega}\right)^{1/2} \quad \leftarrow \text{depends on Nucleus}$$

→ Ground-state vibration = Quantum effect !

→ well known from Molecular physics: C_{60} and μCF

^{197}Au (RHIC,GSI) & ^{207}Pb (SPS,LHC) → shape vibrations...

How large they are ?



SHOULD WE INCLUDE

(? Cu+Cu ?)

SHAPE VIBRATION

into Elliptic Flow (eccentricity) SIMULATIONS ?

→ Does it affect v_2 physics at BNL / LHC ?

Frequency of Shape vibrations:

$$E_n = \hbar\omega \left(n + \frac{1}{2} \right)$$

- For molecules: $f \approx 10^{12-13}$ Hz (microwave)
- For Nuclei e.g. ^{196}Hg : $E_{\text{ZPE}} \approx 2$ MeV

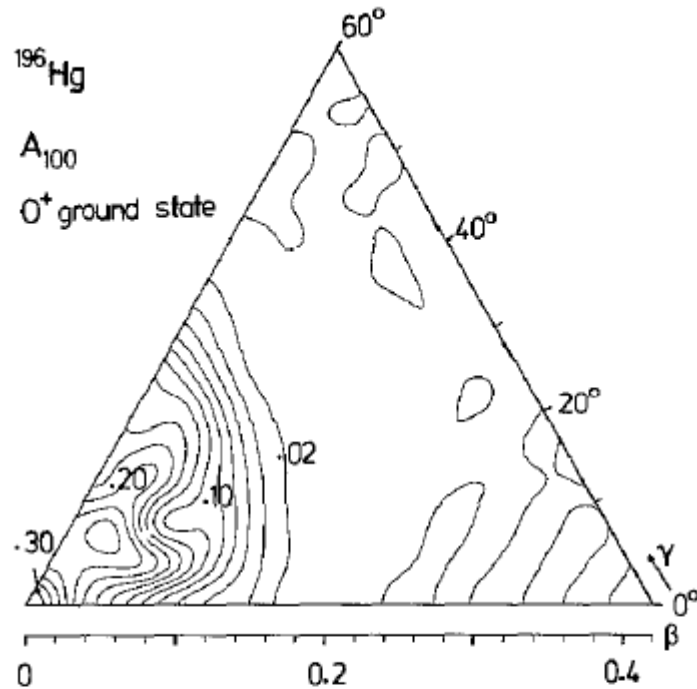
$$\mathbf{f_v \approx 5 * 10^{20} \text{ Hz}}$$

- Comparing to HIC initial overlap time:

$$1 / f_v \approx 2 * 10^{-21} \text{ s} \quad \gg \quad 2 * 10^{-23} \text{ s} = T_{\text{init}} \approx 6 \text{ fm/c}$$

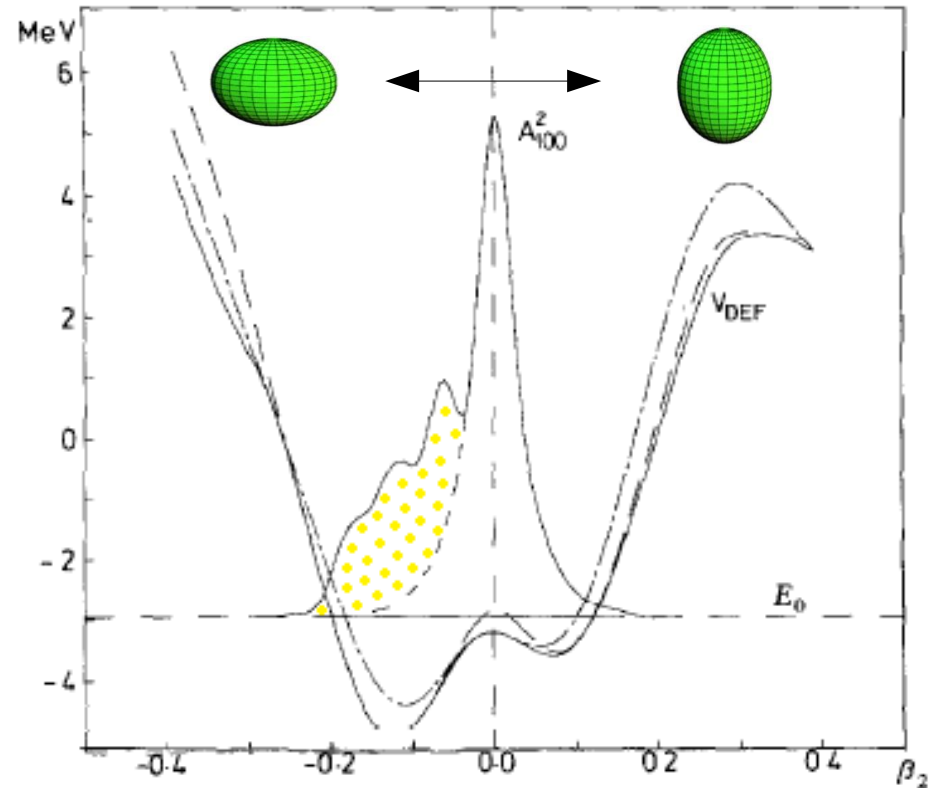
- Initial overlap is fast \rightarrow collision in frozen vibrational state

Ground-state wave function: ^{196}Hg



Contour plot of the ground-state wave function $A_{100}(\beta, \gamma)$ for ^{196}Hg . Remark the pronounced maximum of the wave function for $\beta = 0$. The ground-state energy relative to the deformation potential V_{def} (fig. 2) is $E_0 = -2.959$ MeV and the rms values are $\beta_{\text{rms}} = 0.126$ and $\gamma_{\text{rms}} = 38^\circ$.

Nucl.Phys.A403 (1983) p.263



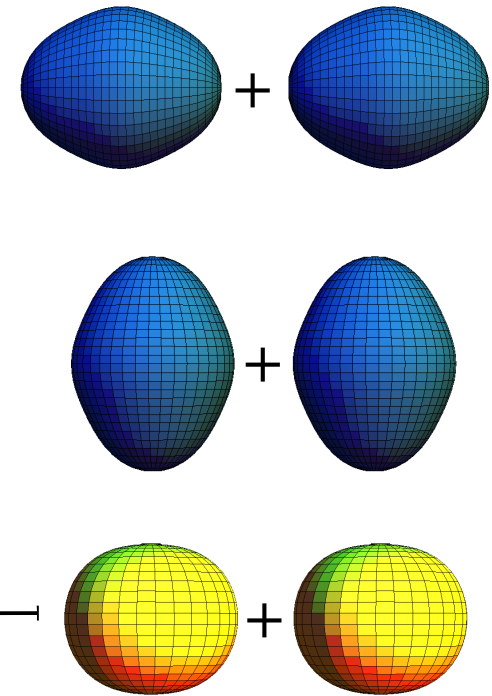
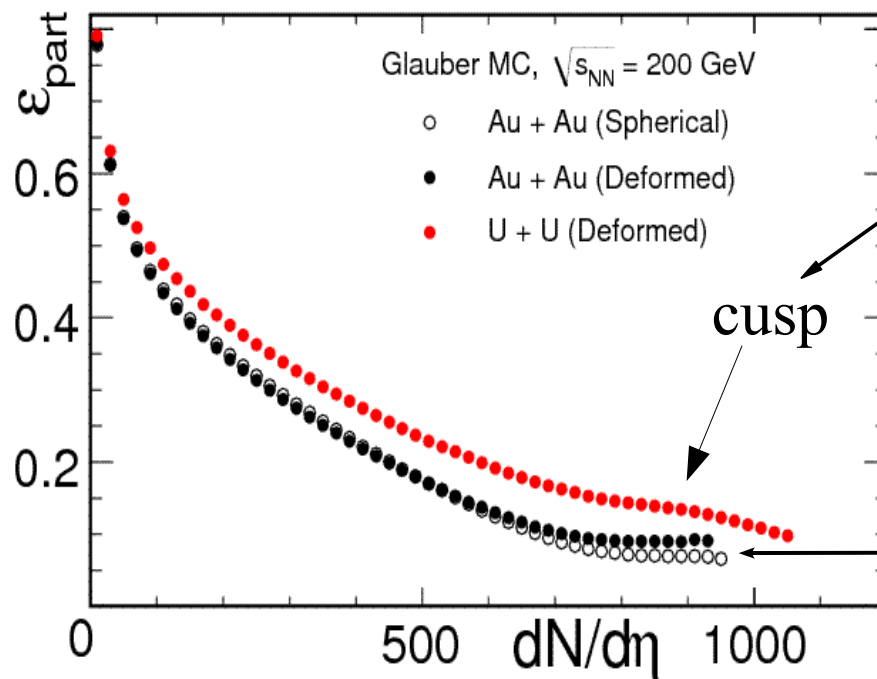
deformation energy V_{def} for different values of the hexadecapole parameter $\beta_4 = 0.0$, $\beta_4 = -0.036$, $\beta_4 = \beta_{4 \text{ min}}$. ground-state wave function A_{100}^2 is also shown E_0 is ground-state energy

Fluctuating β_2 parameter !!! ($\langle \beta_2 \rangle = -0.13$)

→ ^{197}Au similar behavior (proton hole in ^{198}Hg).

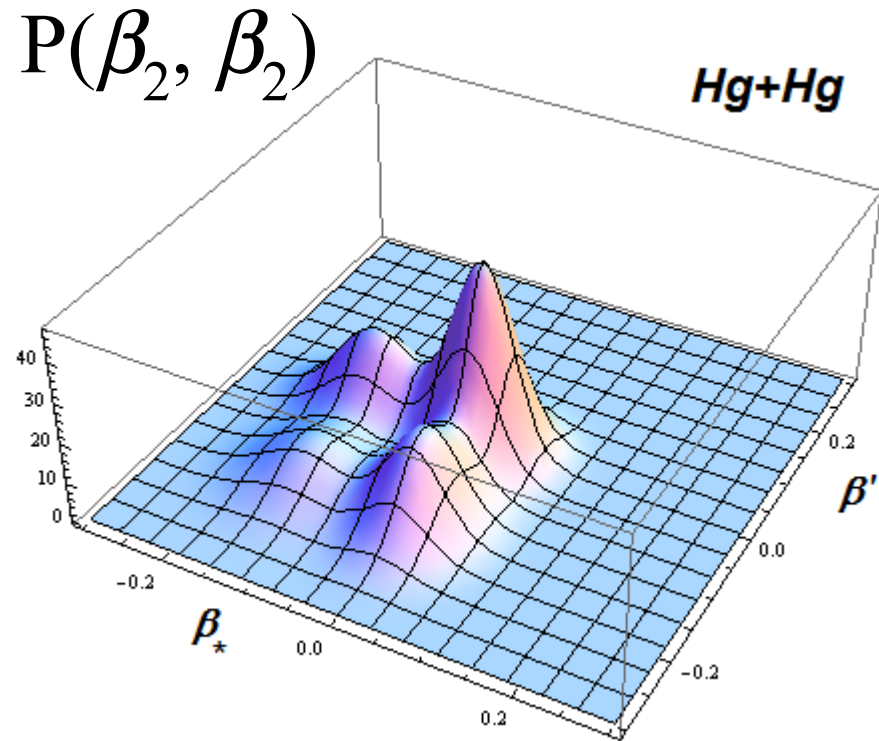
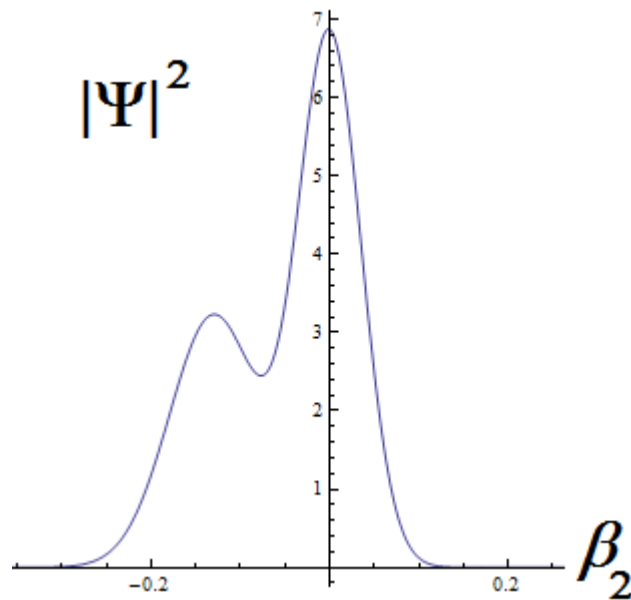
ZPVibration of deformed nuclei in RHIC

- enhanced self-orientation (CUSP) effect in central UU ?
- influencing AuAu eccentricity ?



- in Phys.Rev.C80, 054903 (ZPE vibrations were neglected)
 - need to be studied to obtain a correct $\langle \text{initial state} \rangle$

Collisions of deformed vibrating nuclei:

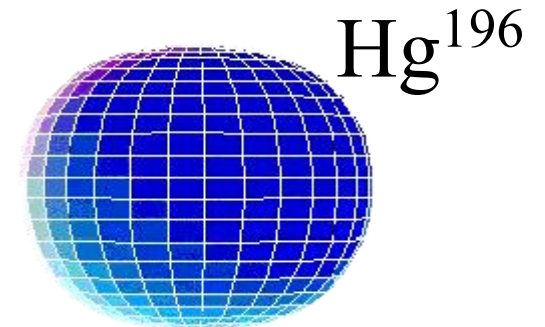


- Collision probability distribution of Hg shapes:

→ spherical + spherical (40%)

→ spherical + deformed (45%)

→ deformed + deformed (15%)

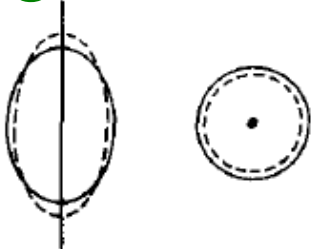


Conclusion:

- Quadrupole vibration of the colliding nuclei

→ expected in Ground state

other (β_3) ZP vibrations possible: 



$\nu=0, \beta$ -vibration
 $\delta R \propto (3 \cos^2 \theta - 1) \cos \omega t$

- Amplitudes up to $|\beta - \beta_0| \approx 0.2-0.3$ (for deformed nuclei)

in frozen vibrational state: Ho,U may have large $\beta_2 \approx 0.45$

→ STRONGER CUSP EFFECT in Ultra-Central Collisions

- During A+A collision: vibration is frozen

Vibrational properties of Nuclei

(comparing $B(E2)$ transition probability with Q_0)

→ some nuclei **Vibrate**: Ca, Fe, Ni, Zn, Ge, Kr, Sr, Pd, Cd, **Sn**

→ some nuclei **do Not vibrate**: Zr, Nd, Sm, Gd, Dy, Er, **W**, Os

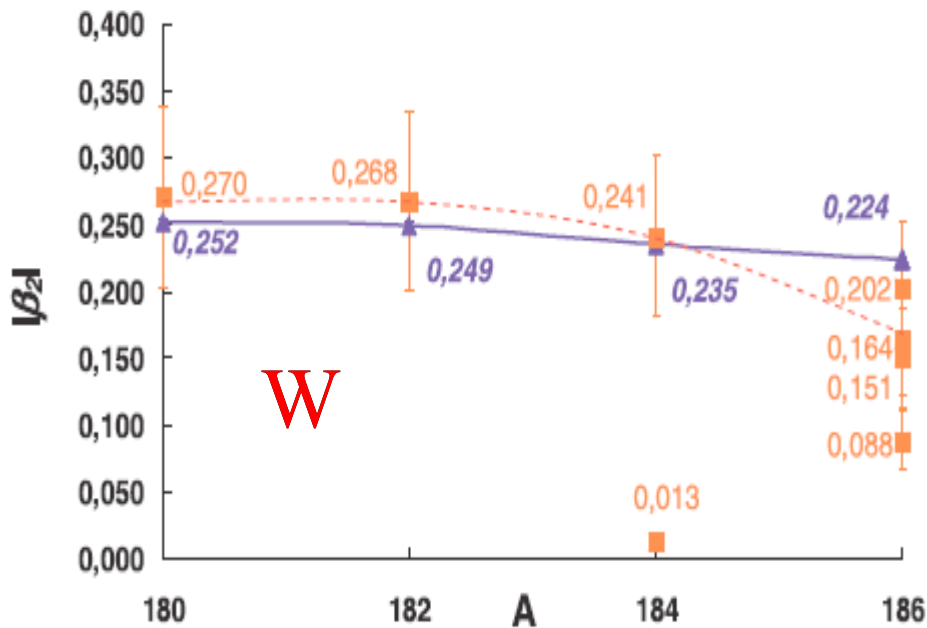


Fig. 1. Comparison of $|\beta_2|$ values obtained from $B(E2) \uparrow$ data (blue triangles, solid line) and from Q data (red squares, dotted line) for W

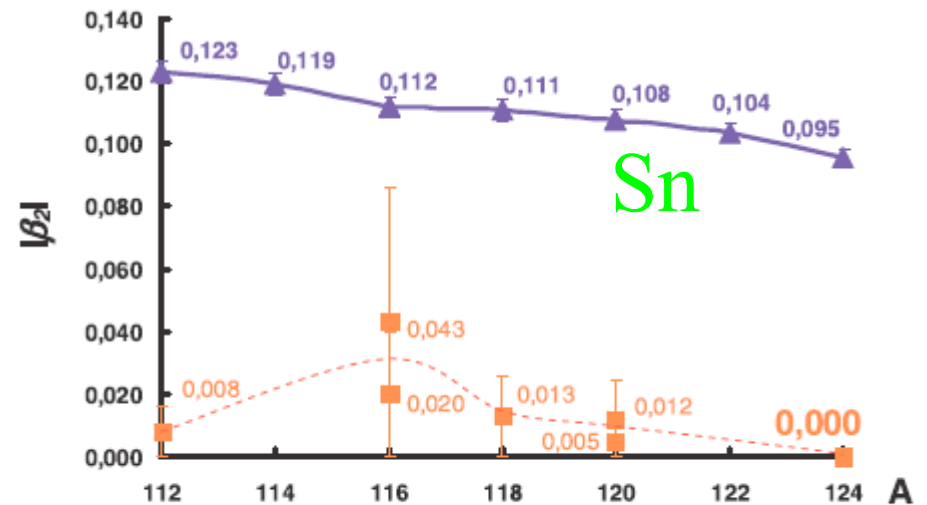


Fig. 2. Comparison of $|\beta_2|$ values obtained from $B(E2) \uparrow$ data and from Q data for Sn ("group 2"). ^{124}Sn case ($|\beta_2|_Q = 0$) is indicated

Publication: I. Boboshin et al.

“Investigation of quadrupole deformation of nucleus and its surface dynamic vibrations”

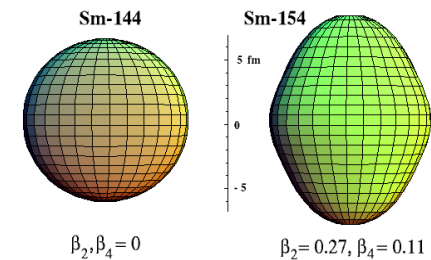
International Conf. on Nuclear Data for Science and Technology 2007 (DOI: 10.1051/ndata:07103)

Summary:

- Ground-state vibration + deformation of nuclei can influence initial state → Elliptic Flow v_2 in HIC.
- Enhancement of eccentricity “cusp” is possible
→ in Ultra-central collisions of prolate vibrating nuclei (U+U)
→ eccentricity in Au+Au may be affected.

- Comparing v_2 in ($^{144}\text{Sm}+^{144}\text{Sm}$) and ($^{154}\text{Sm}+^{154}\text{Sm}$)

→ was suggested for R-HIC



- Collisions of **Cd** (vibrating) nuclei and **Nd, Sm** (non-vibrating) nuclei
→ may clarify influence of vibrations

THANK YOU
for
Your kind ATTENTION

Backup Slide

- We **expect** ^{114}Cd to be strongly vibrating ($\langle\beta_2\rangle = +0.16$)
- We **think** ^{148}Sm , ^{146}Nd do Not vibrate ($\langle\beta_2\rangle = +0.16$)
- ^{63}Cu properties ?