Relativistic theory of nucleon-nucleon interaction

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Introduction

- Study of static and dynamic electromagnetic properties of light nuclei enables us to understand more deeply a nature of strong interactions and, in particular, the nucleon - nucleon interaction.
- Urgency of such researches is connected to a large amount of experimental data, and also with planned new experiments, which will allow to move in region of the large transfer momenta in elastic, inelastic, and deep-inelastic lepton - nucleus reactions.
- At such energies an assumptions of nucleus as a nucleon system is not well justified. For this reason the problems to study in intermediate energy region the nonnucleonic degrees of freedom (Δ-isobars, quarks etc.) and Mesonic Exchange Currents (MEC) are widely discussed.

Introduction

- However, in spite of the significant progress being achieved in this way, the relativistic effects (which ***a priori*** are very important at large transfer momenta) are needed to be included.
- Other actively discussed problem is the extraction of the information about the structure bound nucleons from experiments with light nuclei . Such tasks require to take into account relativistic kinematics of reaction and dynamics of ***NN*** interaction. For this reason const-ruction of covariant approach and detailed analysis of relativistic effects in electromagnetic reactions with light nuclei are very important and interesting.
- *Bethe - Salpeter approach give a possibility to take into account relativistic effects in a consistent way.*

Bethe –Salpeter Formalism

- Let us define full two particle Green Function:

$$G_{\alpha,\beta;\gamma,\delta}(x_1,x_2;y_1,y_2) = -\left\langle 0 \left| T\left[\psi_\alpha(x_1)\psi_\beta(x_2)\bar{\psi}_\gamma(y_1)\bar{\psi}_\delta(y_2)\right]\right\| 0\right\rangle,$$

- Bethe –Salpeter Equation for G:

$$G_{\alpha,\beta;\gamma,\delta}(x_1,x_2;y_1,y_2) = G^{(0)}{}_{\alpha,\beta;\gamma,\delta}(x_1,x_2;y_1,y_2) +$$

$$+i\int\prod_{k=1}^{4} dw_k G^{(0)}{}_{\alpha,\beta;\sigma,\rho}(x_1,x_2;w_1,w_2)\times$$

$$\times K_{\sigma,\rho;\lambda,\omega}(w_1,w_2;w_3,w_4)G_{\lambda,\omega;\gamma,\delta}(w_3,w_4;y_1,y_2),$$

where

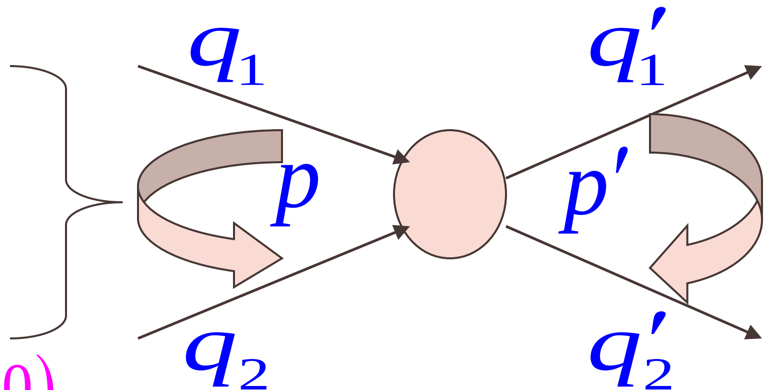
$$G^{(0)}{}_{\alpha,\beta;\gamma,\delta}(x_1,x_2;y_1,y_2) = S'_{F\alpha\gamma}{}^{(1)}(x_1-x_2)S'_{F\beta\delta}{}^{(2)}(y_1-y_2),$$

- Let us make Fourier transformation of $K, G, G^{(0)}$:

$$V\left(p', p; P\right) = \int dx_1 dx_2 dy_1 dy_2 K\left(x_1, x_2; y_1, y_2\right)\times$$
$$\exp\left[iP\left(\frac{x_1+x_2}{2}-\frac{y_1+y_2}{2}\right)+ip'\left(x_1-x_2\right)-ip\left(y_1-y_2\right)\right],$$

where P is total, $p\left(p'\right)$ are relative 4-momentum

$$P = q_1 + q_2 \qquad q_1 = P/2 + p$$
$$p = \left(q_1 - q_2\right) \qquad q_2 = P/2 - p$$



The expressions for G and $G^{(0)}$ are similar.

- Full Green function for two particle system is:

$$G_{\alpha\beta;\gamma\delta}(p',p;P)=S'^{(1)}_{F\alpha\gamma}\left(\frac{P}{2}+p\right)S'^{(2)}_{F\beta\delta}\left(\frac{P}{2}-p\right)\delta^{(4)}(p'-p)+$$
$$+iS'^{(1)}_{F\alpha\varepsilon}\left(\frac{P}{2}+p'\right)S'^{(2)}_{F\beta\lambda}\left(\frac{P}{2}-p'\right)\int\frac{dk}{(2\pi)^4}V_{\varepsilon\lambda;\nu\mu}(p',k;P)G_{\nu\mu;\gamma\delta}(k,p;P).$$

- The full one particle Green function is:

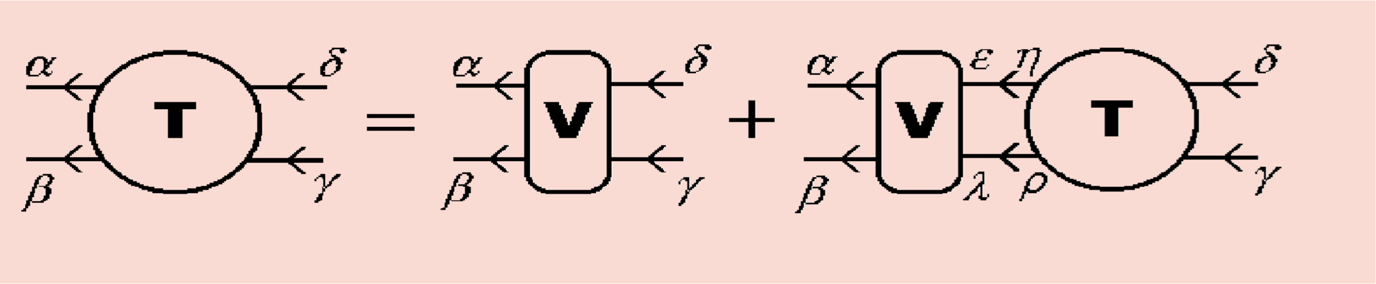
$$S'^{(1)}_{F\alpha\gamma}(p)=\frac{1}{p\cdot\gamma-m-\Sigma(p)+i0}.$$

Mass operator

We will use propagators without mass operator:

$$S'_{F\alpha\beta}(p)\Rightarrow S_{\alpha\beta}(p)$$

T-matrix



- Let us introduce *T*-matrix:

$$GV = G^{(0)}T$$

- The BSE for T-matrix is:

$$T_{\alpha\beta;\delta\gamma}\left(p', p; P\right) = V_{\alpha\beta;\delta\gamma}\left(p', p; P\right) + i\int \frac{dk}{(2\pi)^4} V_{\alpha\beta;\varepsilon\lambda}\left(p', k; P\right) \times$$

$$\times S_{\varepsilon\eta}{}^{(1)}\left(\frac{P}{2}+k\right) S_{\lambda\rho}{}^{(2)}\left(\frac{P}{2}-k\right) T_{\eta\rho;\delta\gamma}\left(k, p; P\right).$$

- Thus a bound state corresponds to a pole in a *T*-matrix at $P^2 = M^2$ (*M* is the mass of the bound state):

$$T_{\alpha\beta;\delta\gamma}\left(p', p; P\right) = \frac{\Gamma_{\alpha\beta}\left(P, p'\right)\bar{\Gamma}_{\gamma\delta}\left(P, p\right)}{P^2 - M^2} + R_{\alpha\beta;\delta\gamma}\left(p', p; P\right),$$

$R_{\alpha\beta;\delta\gamma}\left(p', p; P\right)$ is regular at $P^2 = M^2$ function,

$\Gamma_{\alpha\beta}\left(P, p\right)$ is a vertex function.

Vertex function of BSE

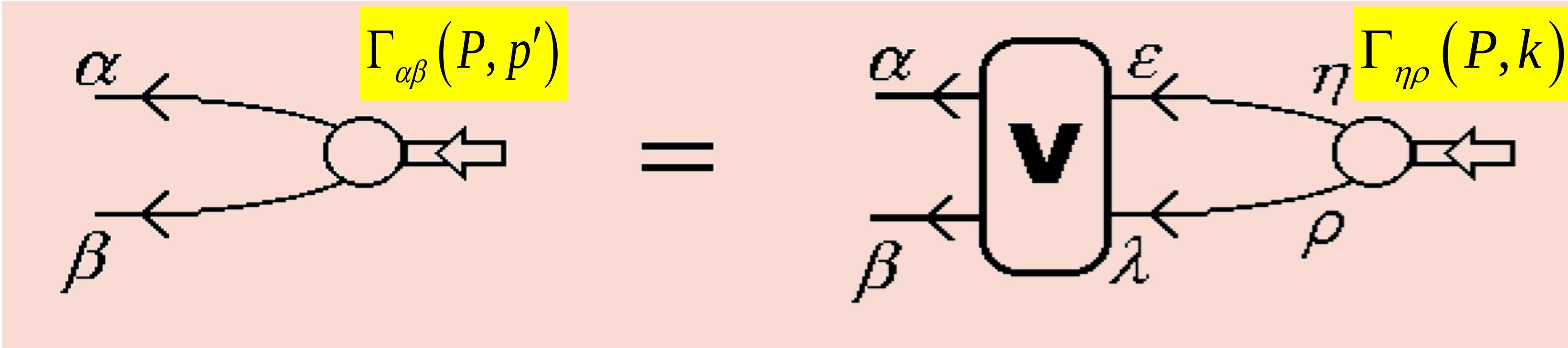
- Let us write the vertex function of BSE:

$$\Phi_{\alpha\beta}(P,p) = \int dx_1 dx_2 \exp\left[iP\left(\frac{x_1+x_2}{2}\right) + ip\left(x_1 - x_2\right)\right] \times$$
$$\times \left\langle 0 \middle| T\left[\psi_\alpha(x_1)\psi_\beta(x_2)\right] \middle| D,P \right\rangle$$

$\left|D,P\right\rangle$ denotes a state of the deuteron with total momenta P.

- One can express it by the BS Amplitude:

$$\Phi_{\alpha\beta}(P,p) = S_{\alpha\lambda}^{(1)}\left(\frac{P}{2}+p\right) S_{\beta\delta}^{(2)}\left(\frac{P}{2}-p\right)\Gamma_{\gamma\delta}(P,p).$$



BSE for vertex function $\Gamma_{\alpha\beta}(P,p')$.

We can obtain the vertex function of BSE using that *T*- matrix for bound state has pole at $P^2 = M^2$:

$$\Gamma_{\alpha\beta}(P,p') = i\int \frac{dk}{(2\pi)^4} V_{\alpha\beta;\varepsilon\lambda}(p',k;P)\times$$

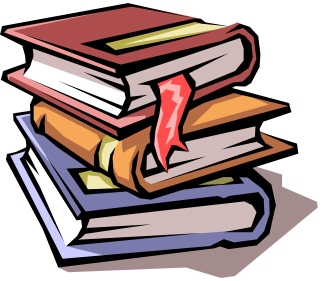
$$\times S_{\varepsilon\eta}{}^{(1)}\left(\frac{P}{2}+k\right) S_{\lambda\rho}{}^{(2)}\left(\frac{P}{2}-k\right)\Gamma_{\eta\rho}(P,k).$$

The normalization condition follows from: $\langle D,P|J_\mu(0)|D,P\rangle = 2iP_\mu$

$$2P_\mu = i\int \frac{dk}{(2\pi)^4}\bar{\Gamma}(P,k)\frac{\partial}{\partial P^\mu}\left[S^{(1)}\left(\frac{P}{2}+k\right)S^{(2)}\left(\frac{P}{2}-k\right)\right]_{P^2=M^2}\Gamma(P,k).$$

BS equation for Amplitude:

$$\Phi_{\alpha\beta}(P,p') = iS_{\alpha\eta}^{(1)}\left(\frac{P}{2}+p'\right)S_{\beta\rho}^{(2)}\left(\frac{P}{2}-p'\right)\int\frac{dk}{(2\pi)^4}V_{\eta\rho;\varepsilon\lambda}(p',k;P)\Gamma_{\varepsilon\lambda}(P,k).$$



1. E.E. Salpeter and H.A.Bethe, Phys.Rev. C**84**(1951) 1232
2. S. Mandelstam, Proc.Roy.Soc. **233**A (1955) 248.
3. S.Bondarenko et.al, Prog.Part.Nucl.Phys. **48**(2002)449;
4. S.Bondarenko et.al, NP, **A832**(2010)233; NP, **A848** (2010) 75; NP, **B219-220c** (2011) 216; FBS, **49** (2011) 121; PLB, **705**(2011)264; JETP Letters, **94**(2011)800.

Solution of BS Equation

- Separable Kernel of Interaction
  - BSE for T-matrix after partial expansion can be written as:

$$T_{\alpha\beta}\left(p_0',|\vec{p}'|,p_0,|\vec{p}|;s\right)=V_{\alpha\beta}\left(p_0',|\vec{p}'|,p_0,|\vec{p}|;s\right)+\frac{i}{2\pi^2}\int dq_0\vec{q}^2d|\vec{q}|\times$$

$$\sum_{\gamma\delta}V_{\alpha\gamma}\left(p_0',|\vec{p}'|,q_0,|\vec{q}|;s\right)S_{\gamma\delta}\left(q_0,|\vec{q}|;s\right)T_{\delta\beta}\left(q_0,|\vec{q}|,p_0,|\vec{p}|;s\right).$$

  - Separable *anzats*:

$$V_{\alpha\beta}\left(p_0',|\vec{p}'|,p_0,|\vec{p}|;s\right)=\sum_{ij=1}^{N}\lambda_{ij}g_i^{(\alpha)}\left(p_0',|\vec{p}'|\right)g_j^{(\beta)}\left(p_0,|\vec{p}|\right),\ \lambda_{ij}=\lambda_{ji}$$

Then for $\boldsymbol{T}$ – matrix we can write:

$$T_{\alpha\beta}\left(p_0',|\vec{p}'|,p_0,|\vec{p}|;s\right)=\sum_{ij=1}^{N}\tau_{ij}(s)\,g_i^{(\alpha)}\left(p_0',|\vec{p}'|\right)g_j^{(\beta)}\left(p_0,|\vec{p}|\right).$$

Substitution $\boldsymbol{V}$, $\boldsymbol{T}$ in BSE for $\boldsymbol{T}$–matrix we can find $\tau_{ij}(s)$:

$$\tau_{ij}^{-1}(s)=\lambda_{ij}^{-1}-H_{ij}(s),$$

where the $H_{ij}(s)$ can be written as:

$$H_{ij}(s)=\frac{i}{2\pi^2}\sum_{\alpha\beta}\int dq_0\,\vec{q}^{\,2}d|\vec{q}|\,S_{\alpha\beta}\left(q_0,|\vec{q}|;s\right)g_i^{(\alpha)}\left(q_0,|\vec{q}|\right)g_j^{(\beta)}\left(q_0,|\vec{q}|\right).$$

Then radial part of BSA has following form:

$$\phi_\alpha\left(p_0,|\vec{p}|\right)=\sum_{\beta}\sum_{i,j=1}^{N}S_{\alpha\beta}\left(p_0,|\vec{p}|;s\right)\lambda_{ij}\,g_i^{(\beta)}\left(p_0,|\vec{p}|\right)c_j(s),$$

where coefficients $c_j(s)$ satisfy the equation:

$$c_i(s)-\sum_{k,j=1}^{N}H_{ik}(s)\lambda_{kj}c_j(s)=0.$$

NN-scattering

- Let us consider *NN*-scattering in ${}^3S_1 - {}^3D_1$ -channel (${}^{2S+1}L_J^{\rho}$ - notation). In this case nucleons are on mass shell:

$$p_0 = p_0' = 0, \quad |\vec{p}| = |\vec{p}\,'| = |\vec{p}^{\,*}| = \sqrt{s/4 - m^2} = \sqrt{mE_{lab}/2},$$

and ***T***-matrix can be parameterized as:

$$T^{(0s)} = -\frac{2i}{|\vec{p}^{\,*}|\sqrt{s}} \begin{pmatrix} \cos 2\varepsilon\, e^{2i\delta_S} - 1 & i\sin 2\varepsilon\, e^{i(\delta_S+\delta_D)} \\ i\sin 2\varepsilon\, e^{i(\delta_S+\delta_D)} & \cos 2\varepsilon\, e^{2i\delta_D} - 1 \end{pmatrix}.$$

Here $\delta_S\,(\delta_D)$ are phase shifts of ${}^3S_1\,({}^3D_1)$ waves, ε - is mixing parameter. For low energy ***NN*** – scattering we can express phase shift through scattering length ***a***, effective radius of interaction r_0:

$$|\vec{p}^{\,*}|\cot\delta_S(s) = -\frac{1}{a} + \frac{r_0}{2}|\vec{p}^{\,*}|^2 + O\left(|\vec{p}^{\,*}|^3\right).$$

Covariant Graz-II kernel of interaction

- As a starting point we will use for $^3S_1^+\left(^3D_1^+\right)$ channels:

$$g_1^{(S)}\left(p_0,|\vec{p}|\right)=\frac{1-\gamma_1\left(p_0^2-|\vec{p}|^2\right)}{\left(p_0^2-|\vec{p}|^2-\beta_{11}^2\right)^2},$$

$$g_2^{(S)}\left(p_0,|\vec{p}|\right)=-\frac{\left(p_0^2-|\vec{p}|^2\right)}{\left(p_0^2-|\vec{p}|^2-\beta_{12}^2\right)^2},$$

$$g_3^{(D)}\left(p_0,|\vec{p}|\right)=\frac{\left(p_0^2-|\vec{p}|^2\right)\left[1-\gamma_2\left(p_0^2-|\vec{p}|^2\right)\right]}{\left(p_0^2-|\vec{p}|^2-\beta_{21}^2\right)\left(p_0^2-|\vec{p}|^2-\beta_{22}^2\right)^2},$$

$$g_1^{(D)}\left(p_0,|\vec{p}|\right)=g_2^{(D)}\left(p_0,|\vec{p}|\right)=g_3^{(S)}\left(p_0,|\vec{p}|\right)\equiv 0$$

Solution of BSE

- The solution of BSE with separable potential can be written as:

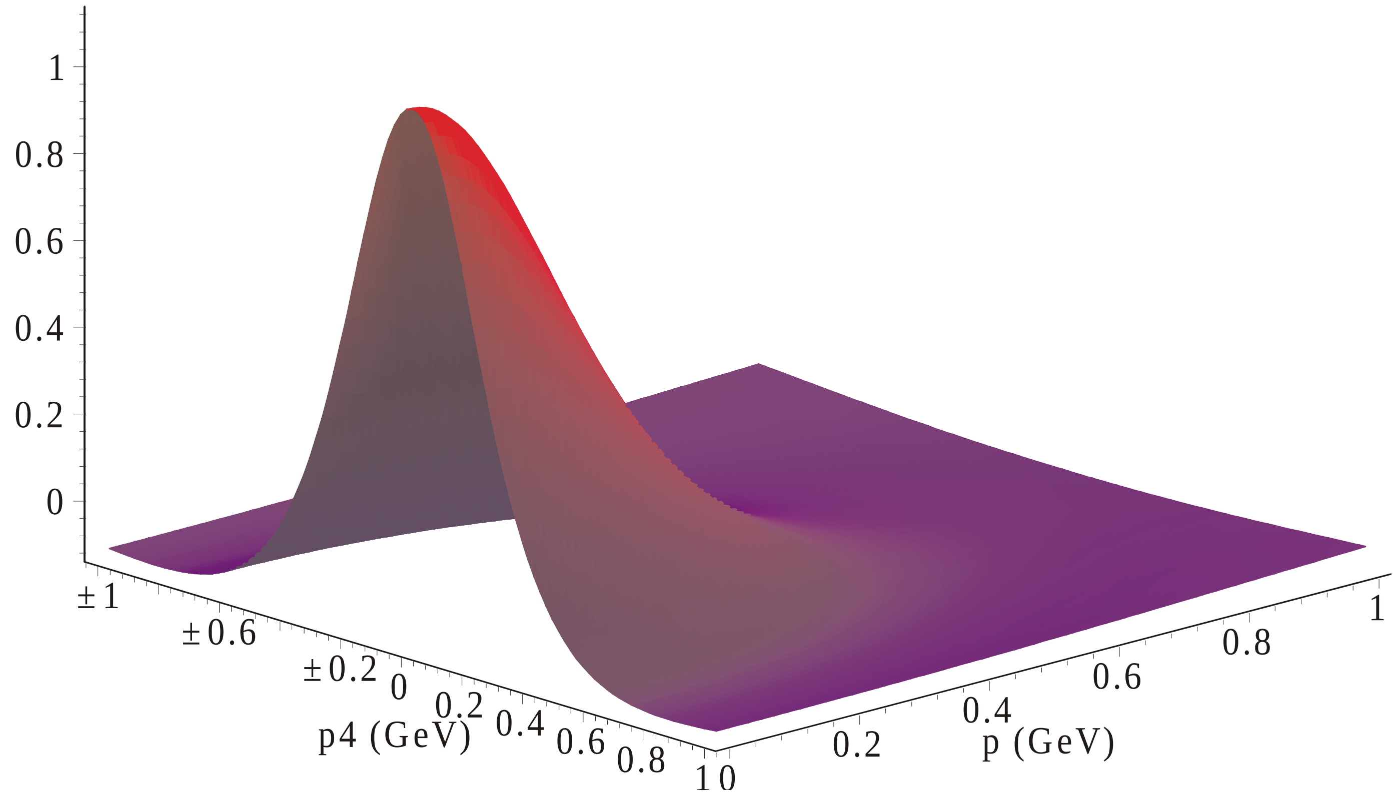
$$g_{{}^3S_1^+}\left(p_0,|\vec{p}|\right)=\sum_{i=1}^{2}\sum_{j=1}^{3}c_j(s)\lambda_{ij}g_i^{(S)}\left(p_0,|\vec{p}|\right);$$

$$g_{{}^3D_1^+}\left(p_0,|\vec{p}|\right)=\sum_{j=1}^{3}c_j(s)\lambda_{3j}g_3^{(D)}\left(p_0,|\vec{p}|\right);$$

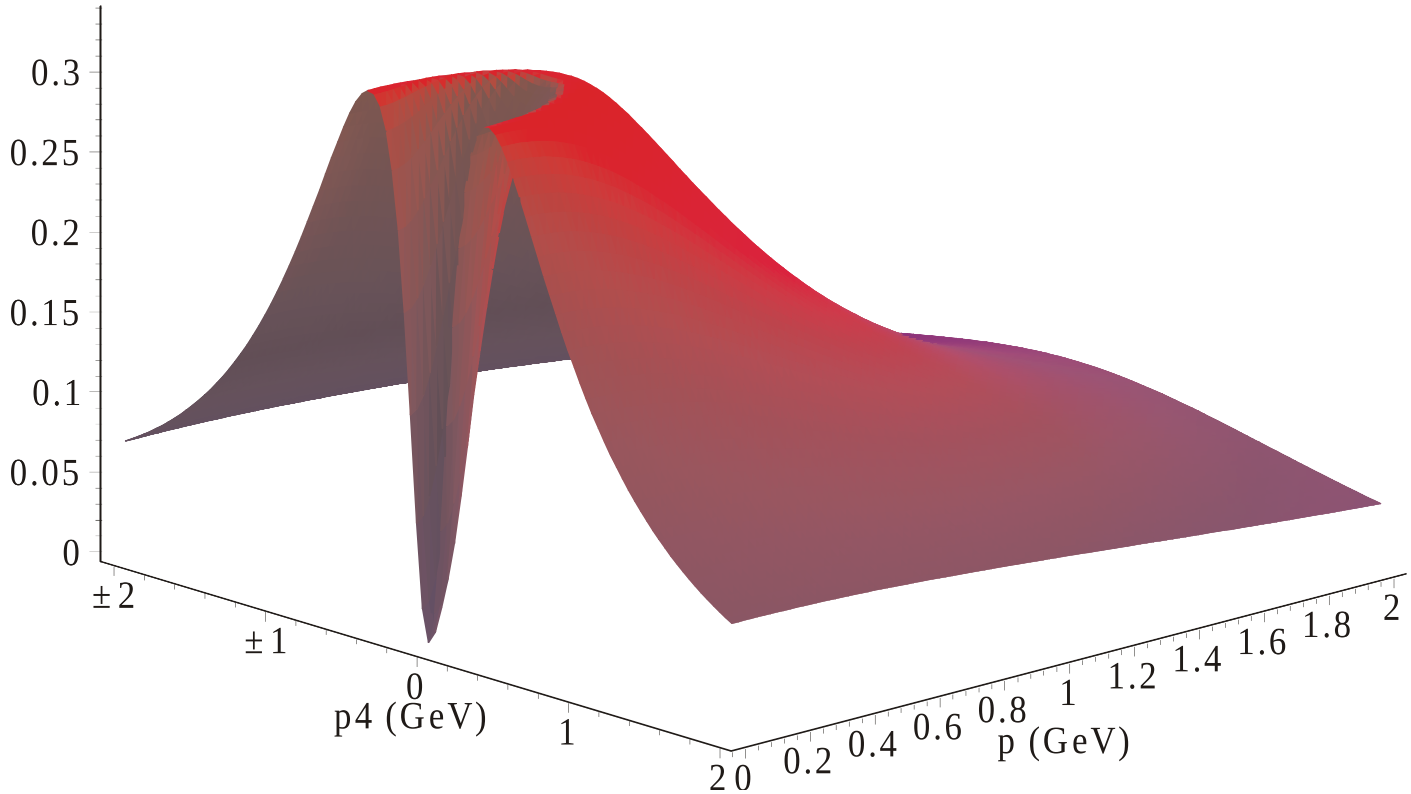
Properties of the deuteron and low energy *NN*-scattering ${}^3S_1$.

${}^2D$	p_D	ε_D	Q_D	μ_D	$\rho_{D/S}$	r_0	a
	%	MeV	fm^{-2}	$e/2m$		fm	fm
NR	4	2.225	0.2499	0.8565	0.0241	1.786	5.419
RIA	4.82	2.225	0.2812	0.8522	0.0274	1.78	5.420
Exp.		2.2246	0.286	0.8574	0.0263	1.759	5.424

Vertex function $g_{^3S_1^+}\left(p_0 \equiv p_4, |\vec{p}| \equiv p\right).$



Vertex function $g_{^3D_1^+}\left(p_0 \equiv p_4, |\vec{p}| \equiv p\right).$



Relativistic description

Yamaguchi

$$g(|\boldsymbol{p}|) = \frac{1}{\boldsymbol{p}^2 + \beta^2}$$

$$\boldsymbol{p}^2 \to -p^2 = -p_0^2 + \boldsymbol{p}^2 \Rightarrow g_p(p) = \frac{1}{-p^2+\beta^2} \xrightarrow{center-of-mass} \frac{1}{-p_0^2 + \boldsymbol{p}^2 + \beta^2 + i0}$$

Y. Avishai, T. Mizutani, Nucl. Phys. A 338 (1980) 377-412

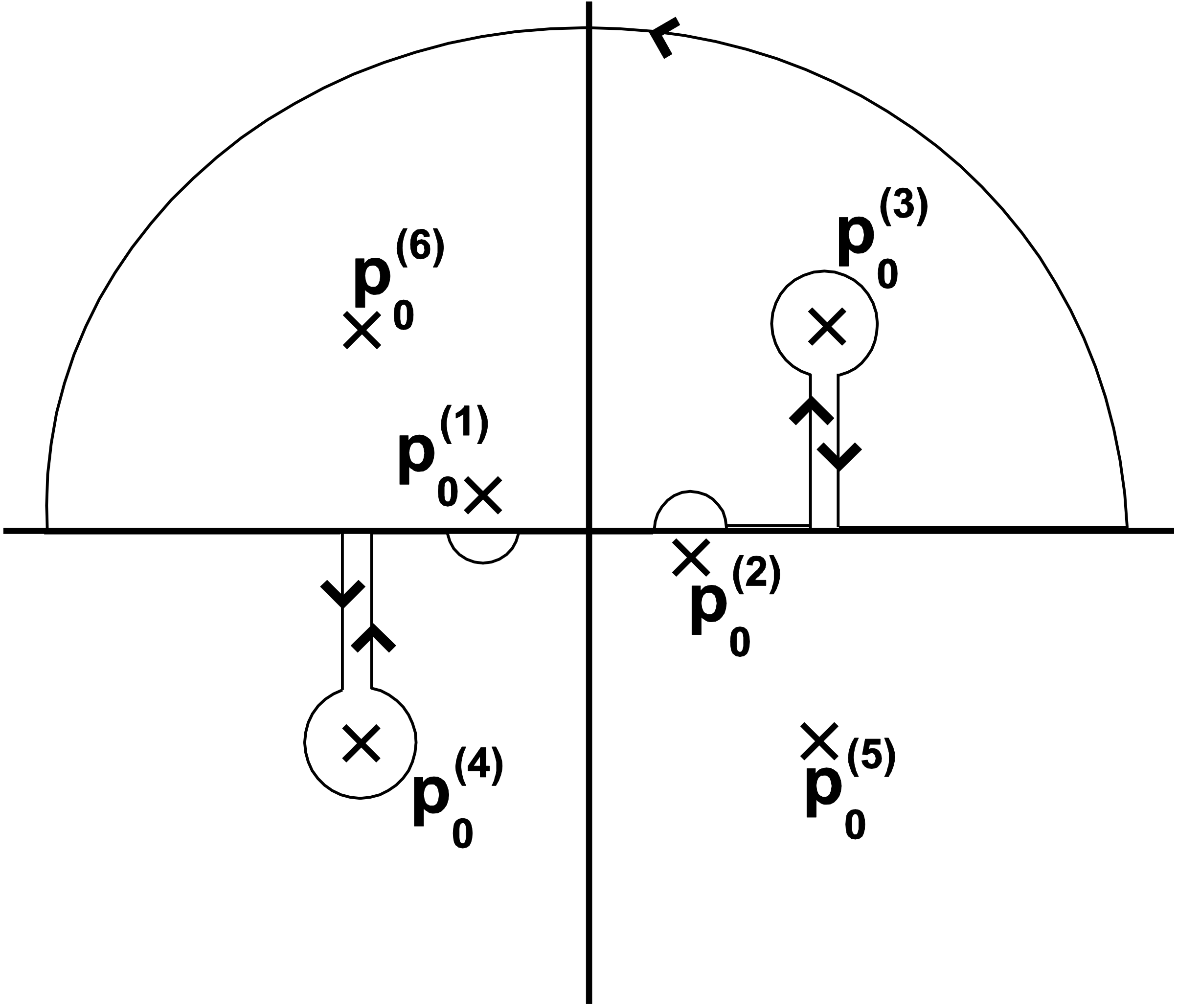
$$Q = p - \frac{P \cdot p}{s} P \Rightarrow g_Q(p) = \frac{1}{-Q^2+\beta^2} \xrightarrow{center-of-mass} \frac{1}{\boldsymbol{p}^2 + \beta^2}$$

$$g_p \to p_0 = \pm\sqrt{\boldsymbol{p}^2 + \beta^2} \mp i0$$

$g_Q \to$ No poles

K. Schwarz, J. Frohlich, H.F.K. Zingl, L. Streit, Acta Phys. Austr. 53 (1981) 191-202

$$g_p(p) = \frac{1}{(p_0^2 - \boldsymbol{p}^2 - \beta^2)^2 + \alpha^4}$$



$$p_0^{(1,2)} = \pm\sqrt{s}/2 \mp E_{\boldsymbol{p}} \pm \boldsymbol{i}\varepsilon$$

$$p_0^{(3,4)} = \pm\sqrt{\boldsymbol{p}^2 + \beta^2 + \boldsymbol{i}\alpha^2}$$

$$p_0^{(5,6)} = \pm\sqrt{\boldsymbol{p}^2 + \beta^2 - \boldsymbol{i}\alpha^2}$$

At $\alpha \to 0$:

1) Poles do not cross the counter
2) Good limit

R.E. Cutkosky, P.V. Landshoff, D.I. Olive, J.C. Polkinghorne, Nucl. Phys. B 12 (1969) 281-300

Form factors of the separable kernel

The uncoupled channels

$^3P_0, {}^1P_1, {}^3P_1$:

$$g_1^{[P]}(p) = \frac{\sqrt{-p_0^2 + \boldsymbol{p}^2}}{(p_0^2 - \boldsymbol{p}^2 - \beta_1^2)^2 + \alpha_1^4}$$

$$g_2^{[P]}(p) = \frac{\sqrt{(-p_0^2 + \boldsymbol{p}^2)^3}\,(p_{c2} - p_0^2 + \boldsymbol{p}^2)}{((p_0^2 - \boldsymbol{p}^2 - \beta_2^2)^2 + \alpha_2^4)^2}$$

$^1S_0$:

$$g_1^{[S]}(p) = \frac{(p_{c1} - p_0^2 + \boldsymbol{p}^2)}{(p_0^2 - \boldsymbol{p}^2 - \beta_1^2)^2 + \alpha_1^4}$$

$$g_2^{[S]}(p) = \frac{(p_0^2 - \boldsymbol{p}^2)(p_{c2} - p_0^2 + \boldsymbol{p}^2)^2}{((p_0^2 - \boldsymbol{p}^2 - \beta_2^2)^2 + \alpha_2^4)^2}$$

$$g_3^{[S]}(p) = \frac{(p_0^2 - \boldsymbol{p}^2)}{(p_0^2 - \boldsymbol{p}^2 - \beta_3^2)^2 + \alpha_3^4}$$

The coupled channel $^3S_1$-$^3D_1$

$$g_1^{[S]}(p) = -\frac{(p_{c1} - p_0^2 + \boldsymbol{p}^2)}{(p_0^2 - \boldsymbol{p}^2 - \beta_1^2)^2 + \alpha_1^4}$$

$$g_3^{[S]}(p) = g_4^{[S]}(p) = g_1^{[D]}(p) = g_2^{[D]}(p) = 0$$

$$g_2^{[S]}(p) = -\frac{(p_0^2 - \boldsymbol{p}^2)(p_{c2} - p_0^2 + \boldsymbol{p}^2)^2}{((p_0^2 - \boldsymbol{p}^2 - \beta_2^2)^2 + \alpha_2^4)^2}$$

$$g_3^{[D]}(p) = \frac{(p_0^2 - \boldsymbol{p}^2)(p_{c3} - p_0^2 + \boldsymbol{p}^2)^2}{((p_0^2 - \boldsymbol{p}^2 - \beta_{31}^2)^2 + \alpha_{31}^4)((p_0^2 - \boldsymbol{p}^2 - \beta_{32}^2)^2 + \alpha_{32}^4)}$$

$$g_4^{[D]}(p) = \frac{(p_0^2 - \boldsymbol{p}^2)}{(p_0^2 - \boldsymbol{p}^2 - \beta_4^2)^2 + \alpha_4^4}$$

The vertex functions of the deuteron

$$g_{^3S_1^+}(p) = (c_1\lambda_{11} + c_2\lambda_{12} + c_3\lambda_{13} + c_4\lambda_{14})g_1^{[S]}(p) + (c_1\lambda_{11} + c_2\lambda_{22} + c_3\lambda_{23} + c_4\lambda_{24})g_2^{[S]}(p)$$

$$g_{^3D_1^+}(p) = (c_1\lambda_{13} + c_2\lambda_{23} + c_3\lambda_{33} + c_4\lambda_{34})g_3^{[D]}(p) + (c_1\lambda_{14} + c_2\lambda_{24} + c_3\lambda_{34} + c_4\lambda_{44})g_4^{[D]}(p)$$

The normalization

$$p_l = \frac{\boldsymbol{i}}{2M_d(2\pi)^4}\int dk_0 \int \boldsymbol{k}^2 d|\boldsymbol{k}| \frac{(E_k - M_d/2)[g_l(k_0,|\boldsymbol{k}|)]^2}{((M_d/2 - E_k + \boldsymbol{i}0)^2 - k_0^2)^2}$$

The calculation scheme

$^1P_1^+, {}^3P_1^+ \quad \rightarrow \quad \lambda_{ij}(s) = \bar{\lambda}_{ij} = const$

$^1S_0^+, {}^3P_0^+ \quad \rightarrow \quad \lambda_{ij}(s) = (s_0 - s)\bar{\lambda}_{ij}$

$^3S_1^+ \text{-} {}^3D_1^+$: $\quad \det|\tau_{ij}^{-1}(s = M_d)| = 0 \quad \rightarrow \lambda_{ij}(s) = \dfrac{\bar{\lambda}_{ij}}{s - m_0^2}$

The Minimization procedure

P-waves:

$$\chi^2 = \sum_{i=1}^{n} (\delta^{\exp}(s_i) - \delta(s_i))^2 / (\Delta\delta^{\exp}(s_i))^2$$

$^1S_0^+$:

$$\chi^2 = \sum_{i=1}^{n} (\delta^{\exp}(s_i) - \delta(s_i))^2 / (\Delta\delta^{\exp}(s_i))^2 + (a^{\exp} - a)^2 / (\Delta a^{\exp})^2$$

$^3S_1^+$-$^3D_1^+$:

$$\chi^2 = \sum_{i=1}^{n} (\delta_S^{\exp}(s_i) - \delta_S(s_i))^2 / (\Delta\delta_S^{\exp}(s_i))^2 + (\delta_D^{\exp}(s_i) - \delta_D(s_i))^2 / (\Delta\delta_D^{\exp}(s_i))^2$$

$$+ (\varepsilon^{\exp}(s_i) - \varepsilon(s_i)) / (\Delta\varepsilon^{\exp}(s_i))^2 + (a^{\exp} - a)^2 / (\Delta a^{\exp})^2$$

$^1S_0^+$:

	a_s (fm)	r_{0s} (fm)
MY3	-23.750	2.70
MYQ3	-23.754	2.78
Experiment	-23.748(10)	2.75(5)

$^3S_1^+$-$^3D_1^+$:

	p_d(%)	a_t(fm)	r_{0t}(fm)	E_d(MeV)
MY4	6	5.417	1.75	2.2246
MYQ4	6	5.417	1.75	2.2246
CD-Bonn	4.85	5.4196	1.751	2.224575
Graz II	4.82	5.42	1.78	2.225
Experiment	-	5.424(4)	1.759(5)	2.224644(46)

O. Dumbrajs et al., Nucl Phys. B 216 (1983) 277

To describe the influence of the inelastic channels into the elastic NN scattering the inelasticity parameter is introduced.

S matrix (Arndt-Roper parametrization)

$$S = \frac{1 - K_i + iK_r}{1 + K_i - iK_r} = \eta \exp(2i\delta)$$

$$K = K_r + iK_i$$

$$K_r = \tan\delta, \quad K_i = \tan^2\rho$$

δ - the phase shift, ρ - the inelasticity parameter.

$$\eta^2 = \frac{1 + K^2 - 2K_i}{1 + K^2 + 2K_i}$$

Inelasticity!

$$K^2 = K_r^2 + K_i^2$$

$$\delta = \frac{1}{2}\{\tan^{-1}[K_r/(1 - K_i)] + \tan^{-1}[K_r/(1 + K_i)]\}$$

If there are no inelastic channels: $(\rho = 0)$, $\delta = \delta_e$, $\eta = 1$ and $S = S_e = \exp(2i\delta_e)$.

Complex separable kernel

$$V_r \to V = V_r + iV_i.$$

$$V_{l'l}(p_0', |\mathbf{p}'|; p_0, |\mathbf{p}|; s) = \sum_{m,n=1}^{N} \left[\underline{\lambda_{mn}^r(s)} + i\lambda_{mn}^i(s) \right] \underline{g_i^{[l']}(p_0', |\mathbf{p}'|) g_j^{[l]}(p_0, |\mathbf{p}|)}$$

underlined part $\equiv$ MYN kernels,

$$\lambda_{mn}^i(s) = \theta(s - s_{th}) \left(1 - \frac{s_{th}}{s}\right) \bar{\lambda}_{mn}^i$$

s_{th} - the inelasticity threshold.

Procedure

Fix the parameters of the real part (λ^r, p_c, β, α) and calculate the parameters λ^i to describe the inelasticity to minimaze the function:

$$\chi^2 = \sum_{m=m_{th}}^{n} (\delta^{\exp}(s_m) - \delta(s_m))^2 / (\Delta\delta^{\exp}(s_m))^2$$

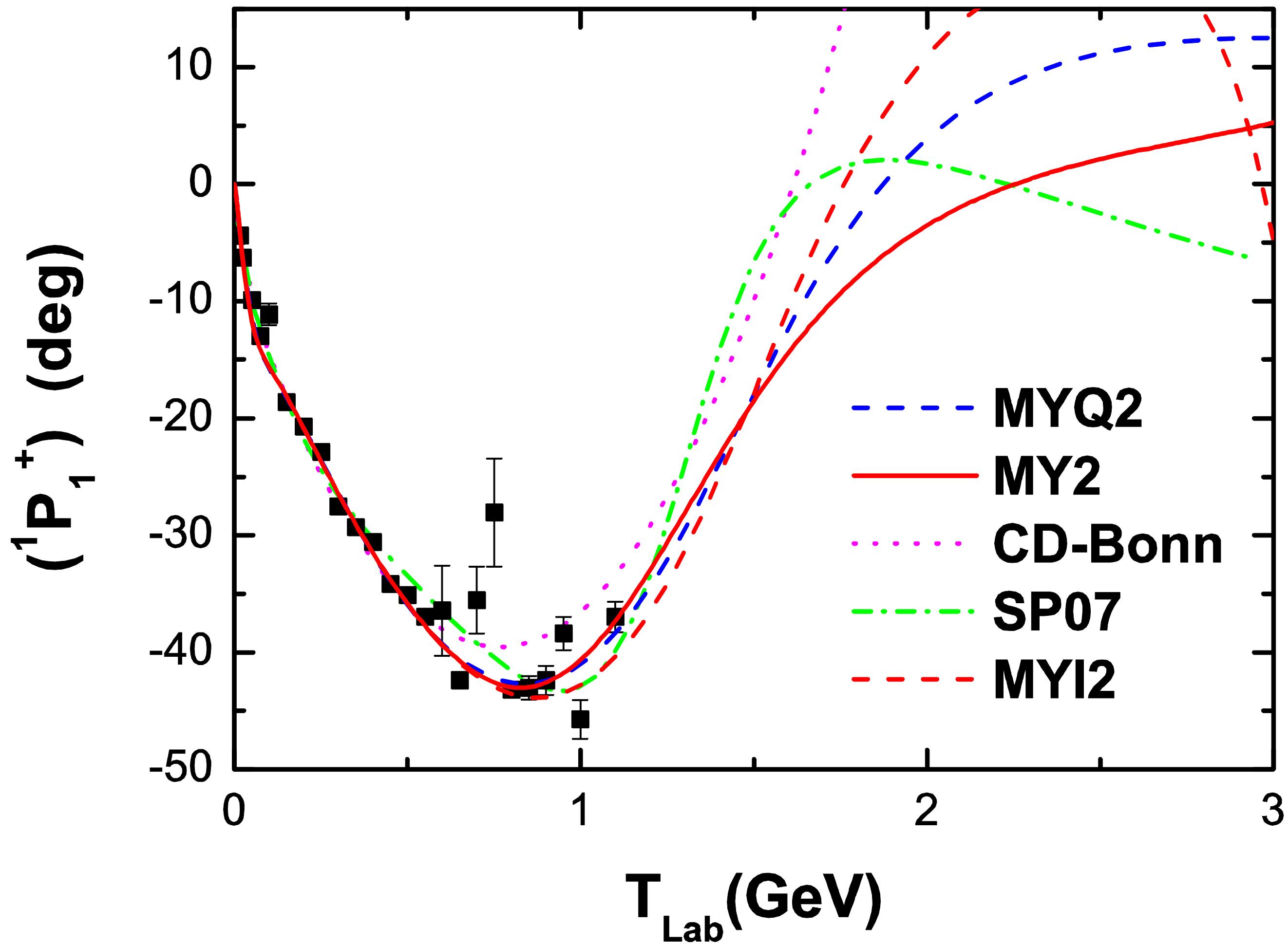
$$+ \sum_{m=m_{th}}^{n} (\rho^{\exp}(s_m) - \rho(s_m))^2 / (\Delta\rho^{\exp}(s_m))^2$$

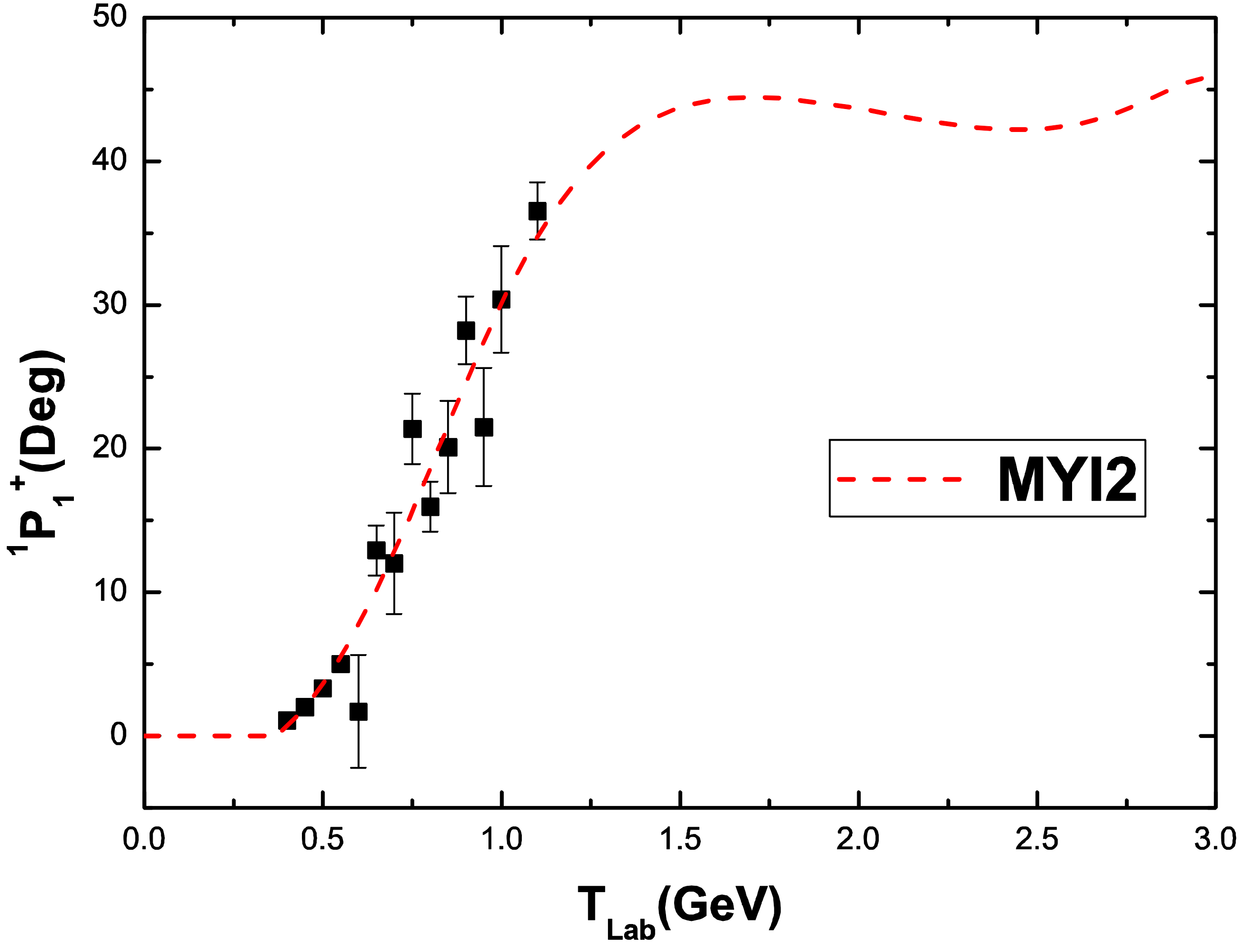
MYN (λ^r, p_c, β, α) $\rightarrow$ MYIN (λ^r, λ^i, p_c, β, α)

SAID (http://gwdac.phys.gwu.edu)

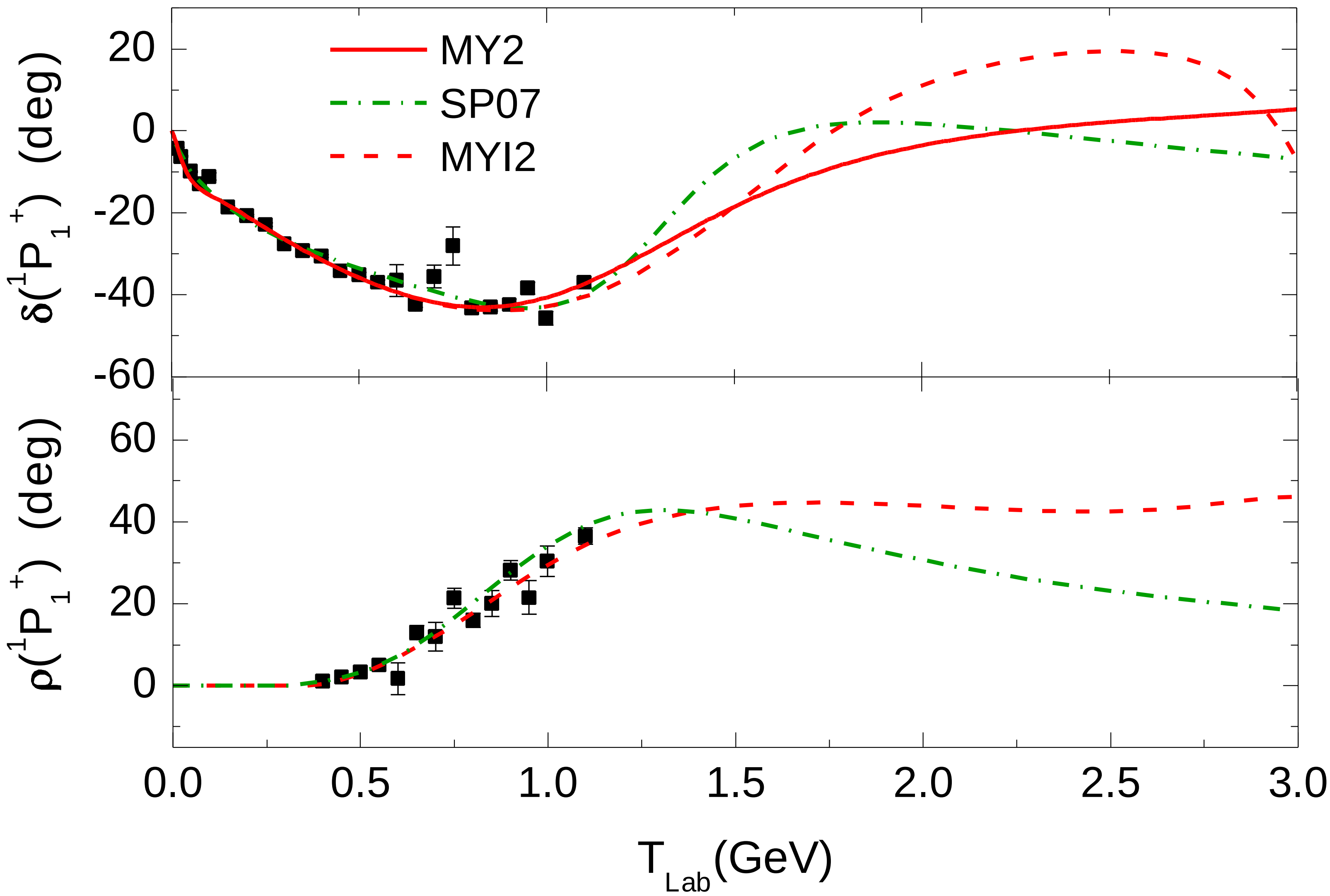
CD-Bonn: R. Machleidt, Phys. Rev. C 63 (2001) 024001

SP07: R.A. Arndt et al., Phys. Rev. C 76 (2007) 025209

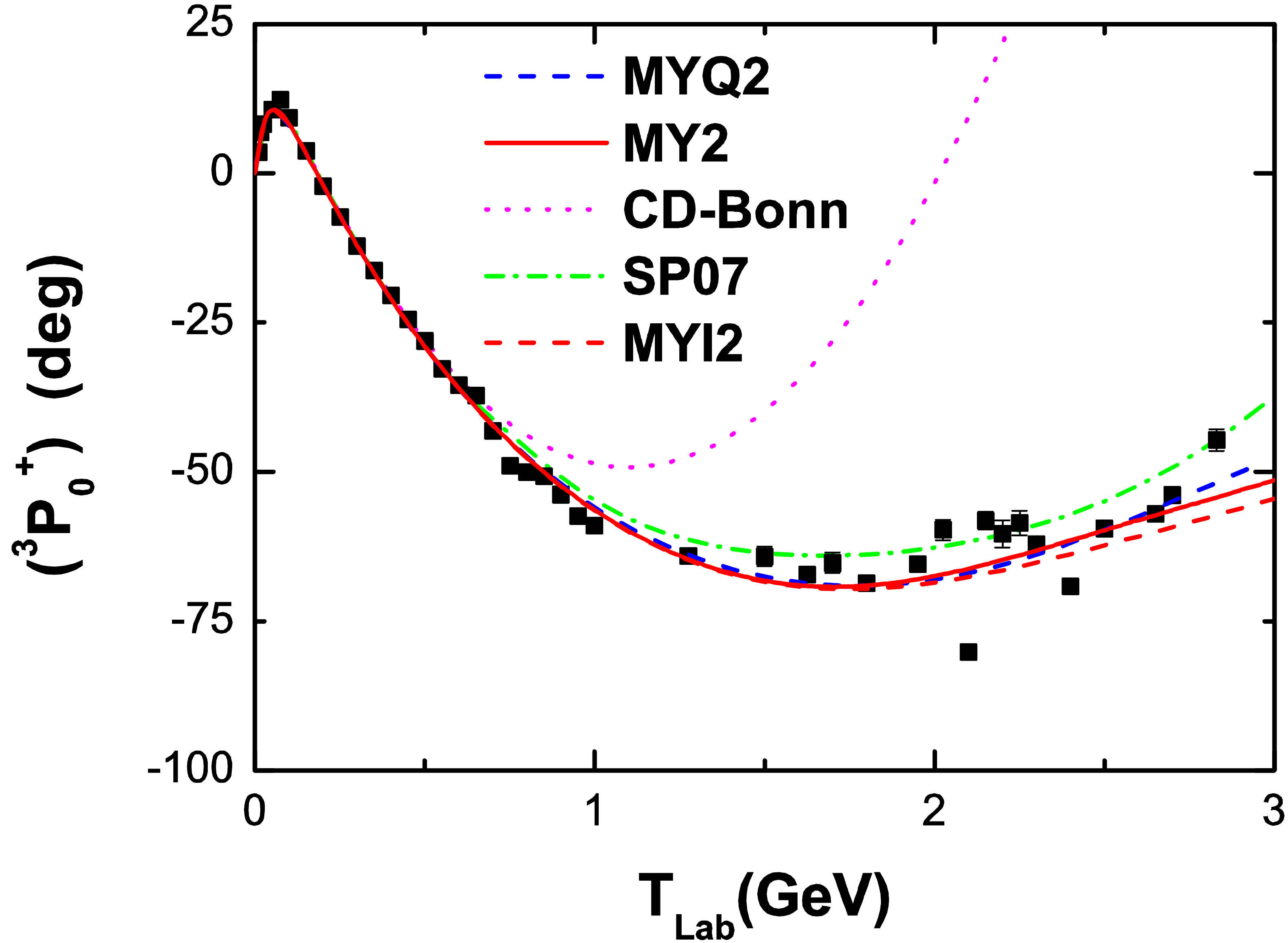




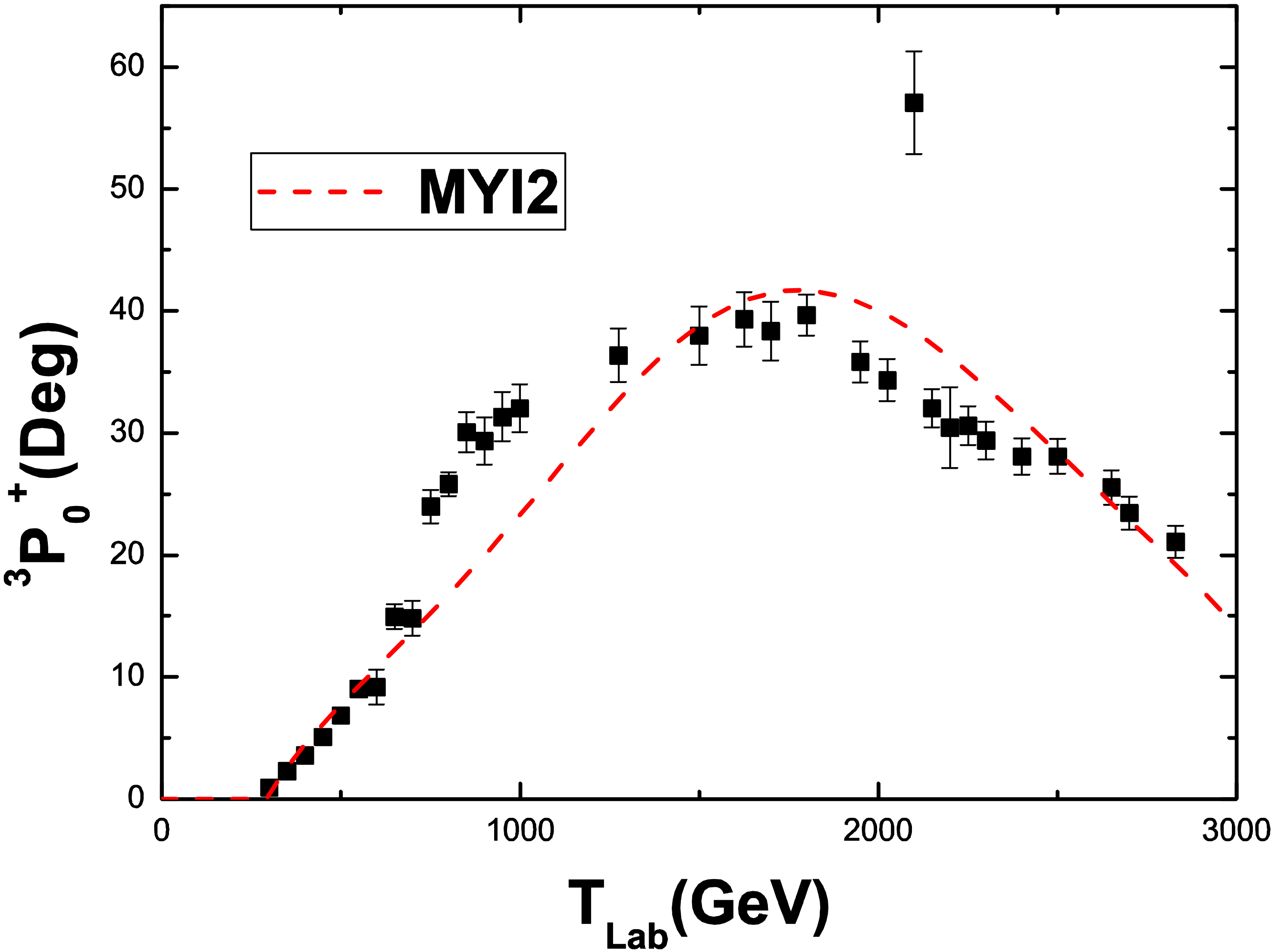
50
40
30
20
10
0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$^1P_1^+$(Deg)
T_{Lab}(GeV)
MYI2

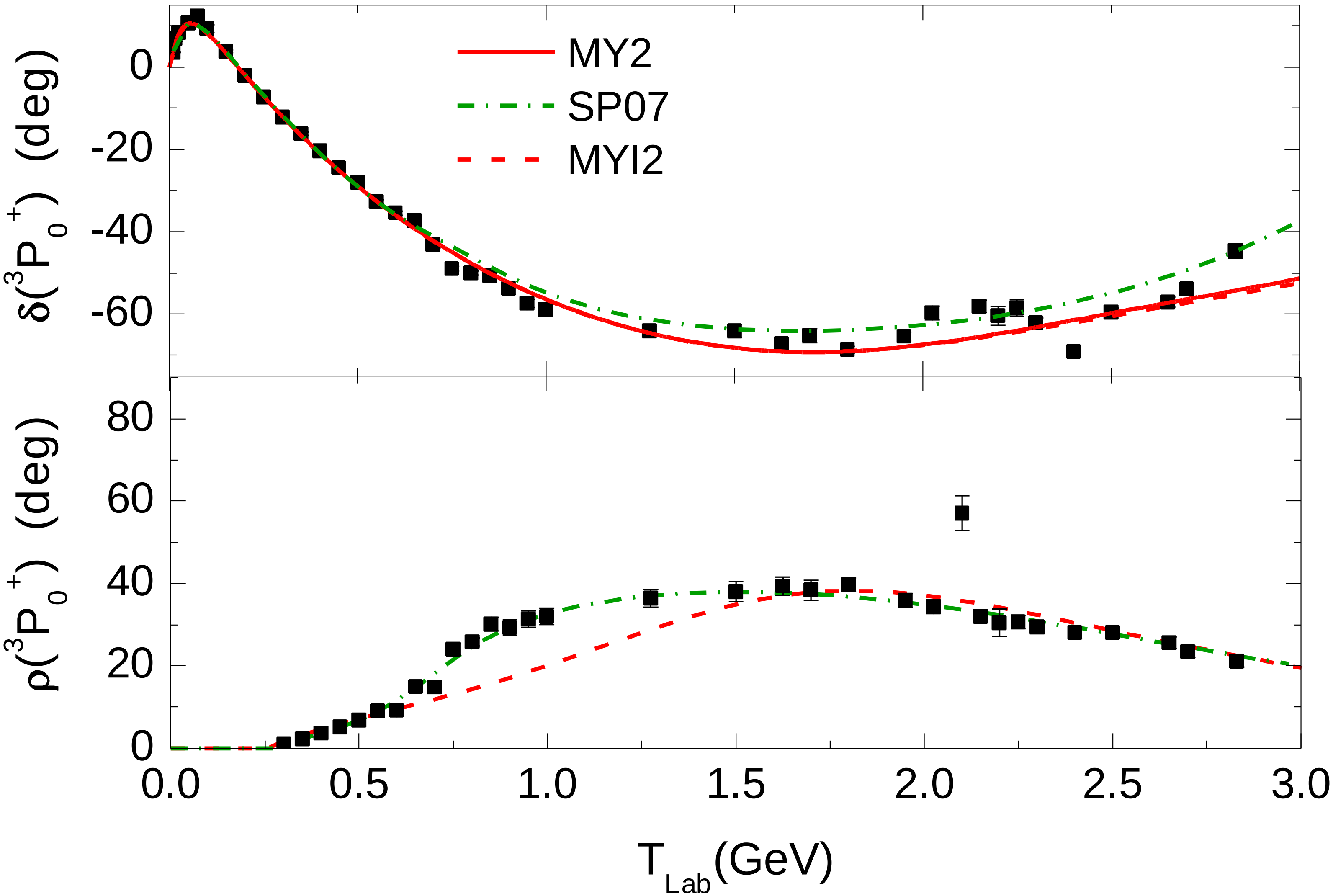


MY2
SP07
MYI2
$\delta(^1P_1^+)$ (deg)
$\rho(^1P_1^+)$ (deg)
20
0
-20
-40
-60
60
40
20
0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
T_{Lab}(GeV)

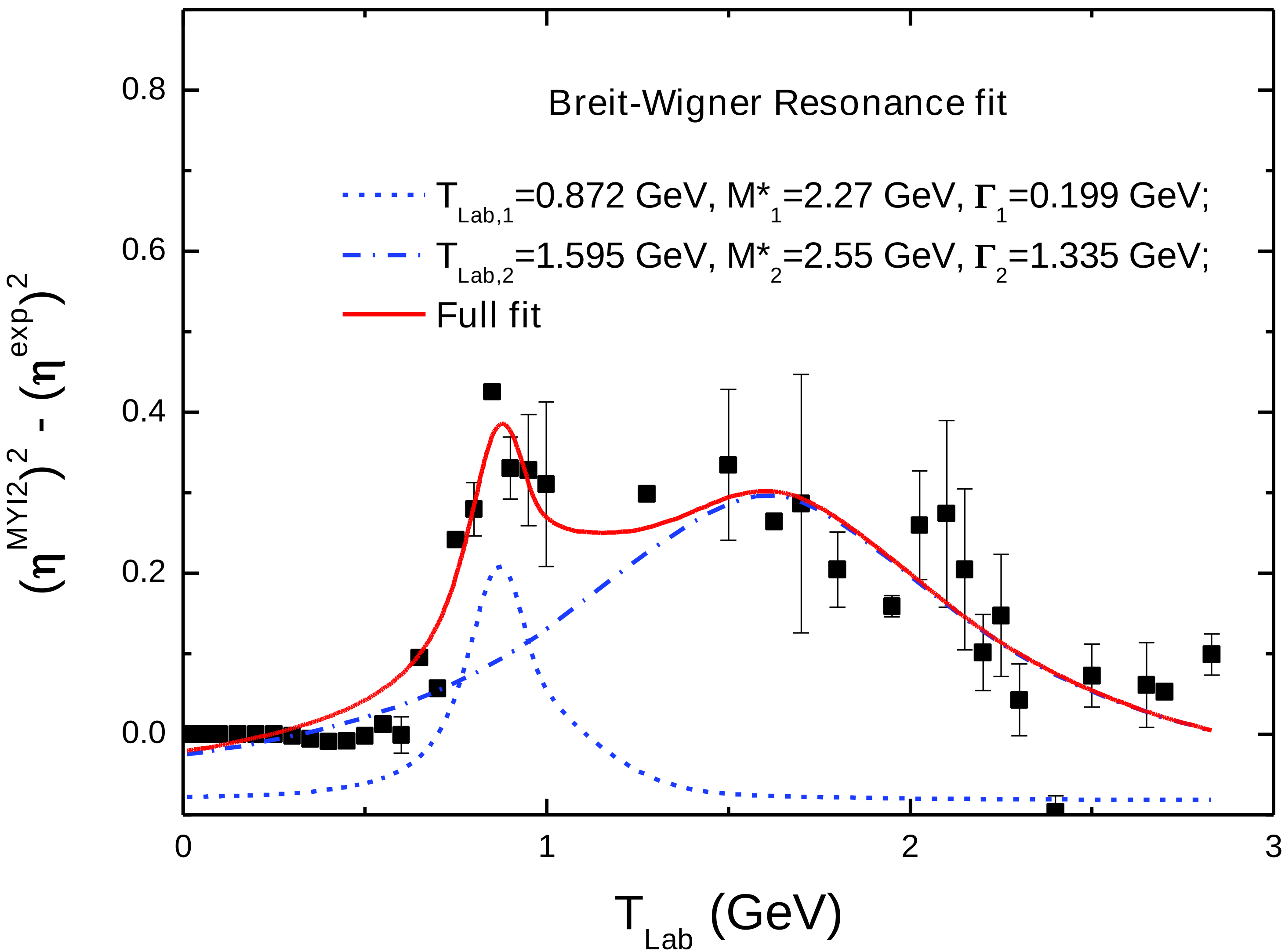


25
0
-25
-50
-75
-100
0
1
2
3
MYQ2
MY2
CD-Bonn
SP07
MYI2
$(^3P_0^+)$ (deg)
T_{Lab}(GeV)

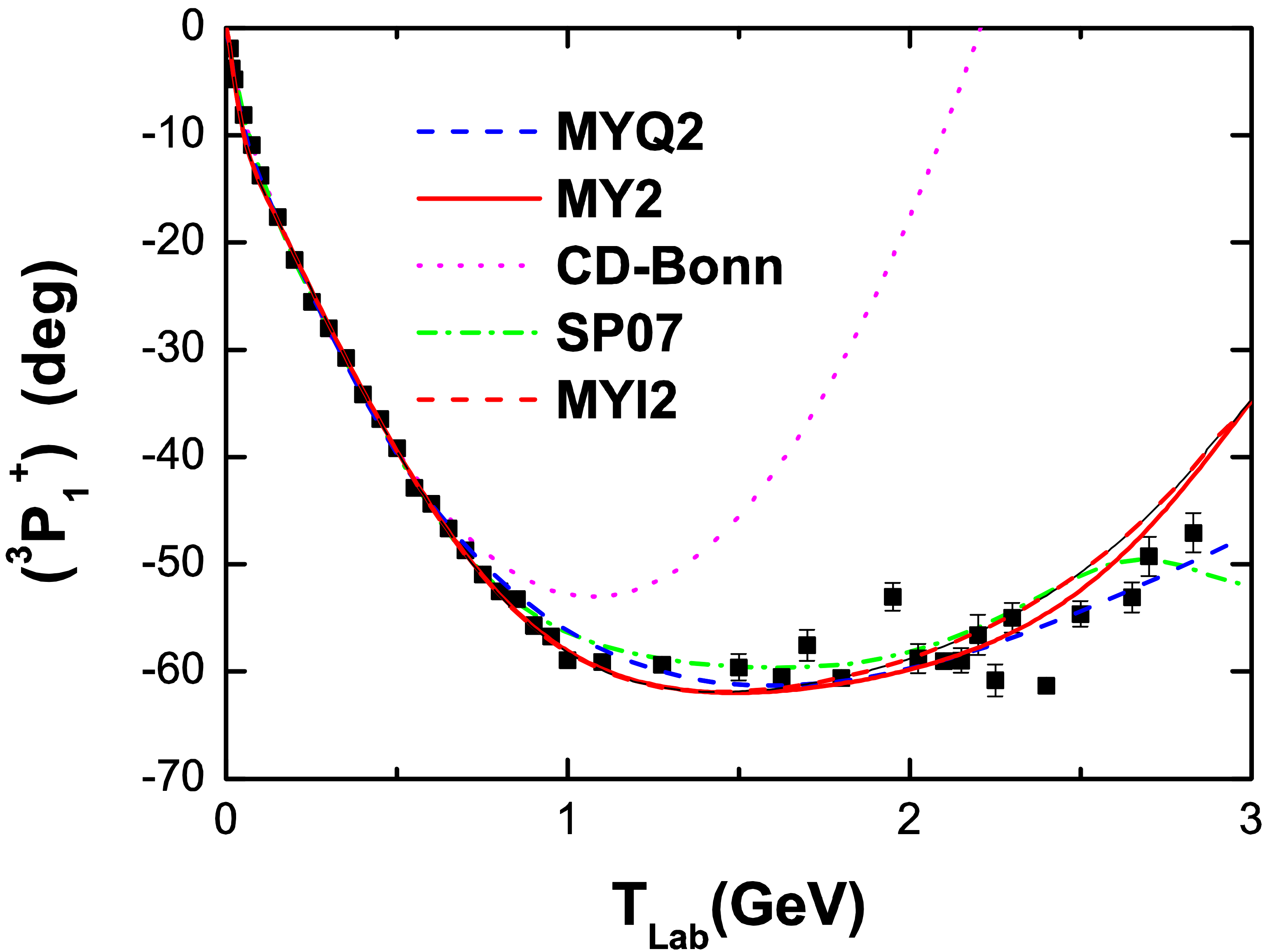

MYI2
60
50
40
30
20
10
0
0
1000
2000
3000
$^3P_0^+$(Deg)
T_{Lab}(GeV)



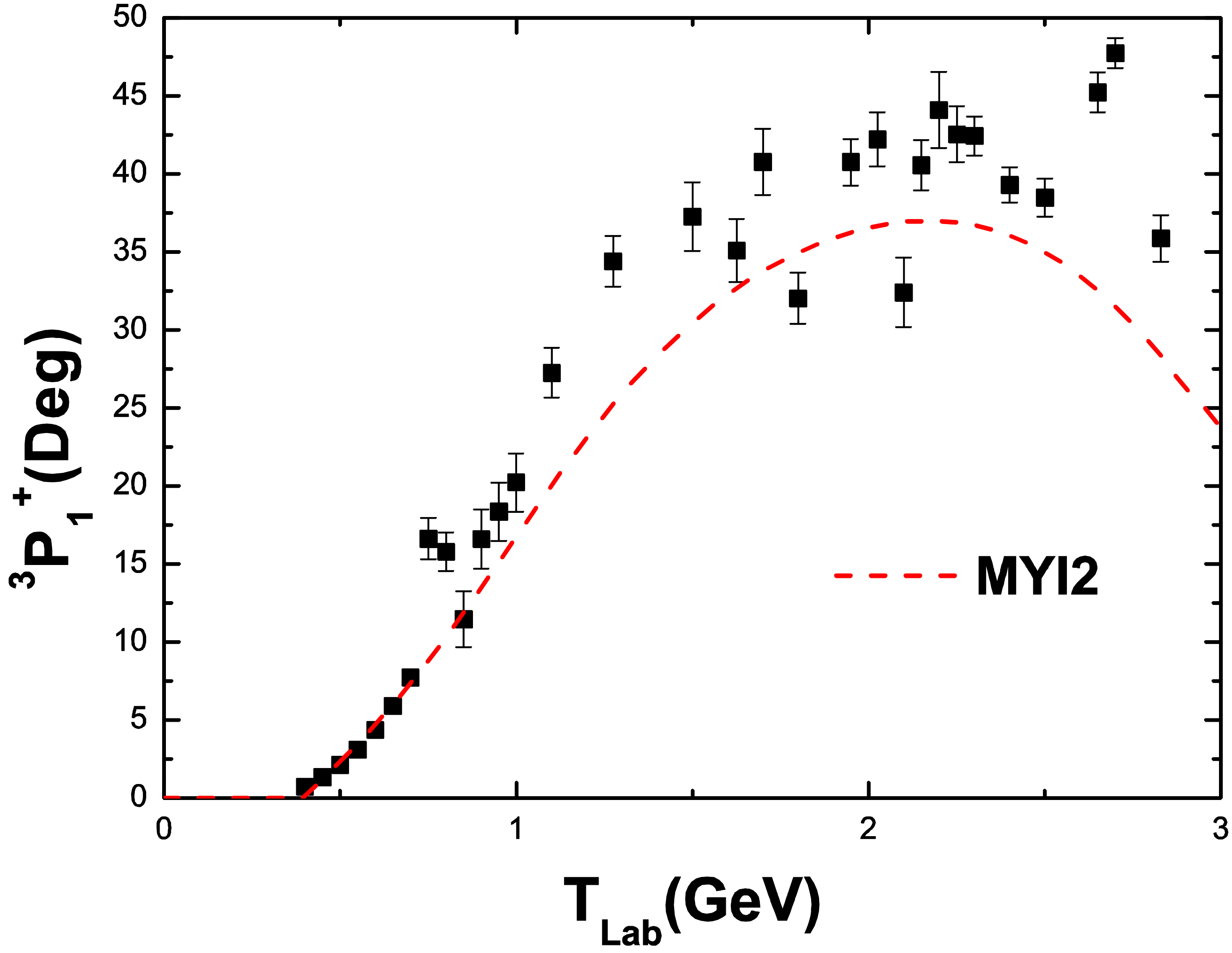
MY2
SP07
MYI2
0
-20
-40
-60
δ(³P₀⁺) (deg)
80
60
40
20
0
ρ(³P₀⁺) (deg)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
T Lab (GeV)



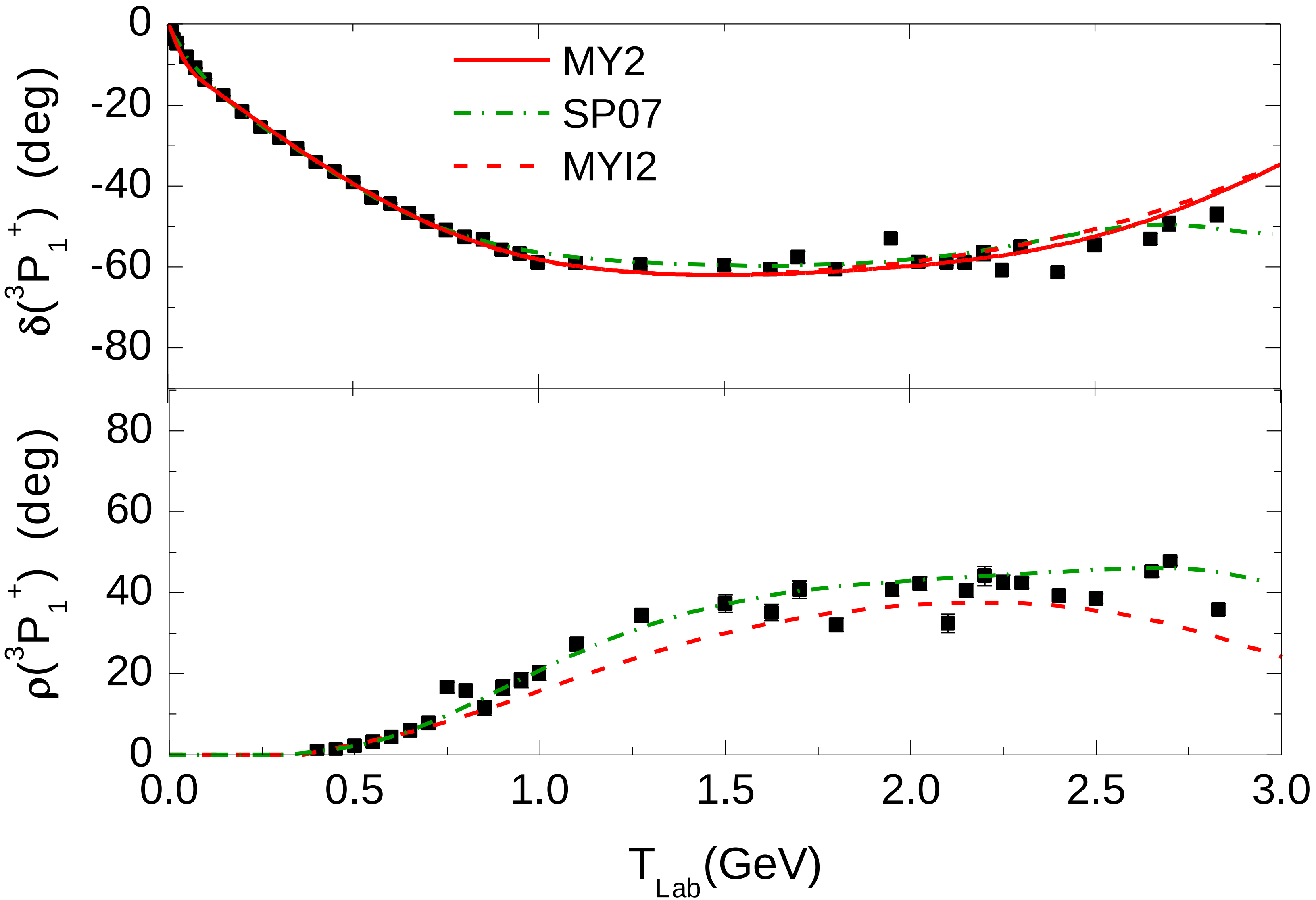
Breit-Wigner Resonance fit
T_Lab,1=0.872 GeV, M*_1=2.27 GeV, Γ_1=0.199 GeV;
T_Lab,2=1.595 GeV, M*_2=2.55 GeV, Γ_2=1.335 GeV;
Full fit
0.8
0.6
0.4
0.2
0.0
0
1
2
3
(η^MYI2)^2 - (η^exp)^2
T_Lab (GeV)



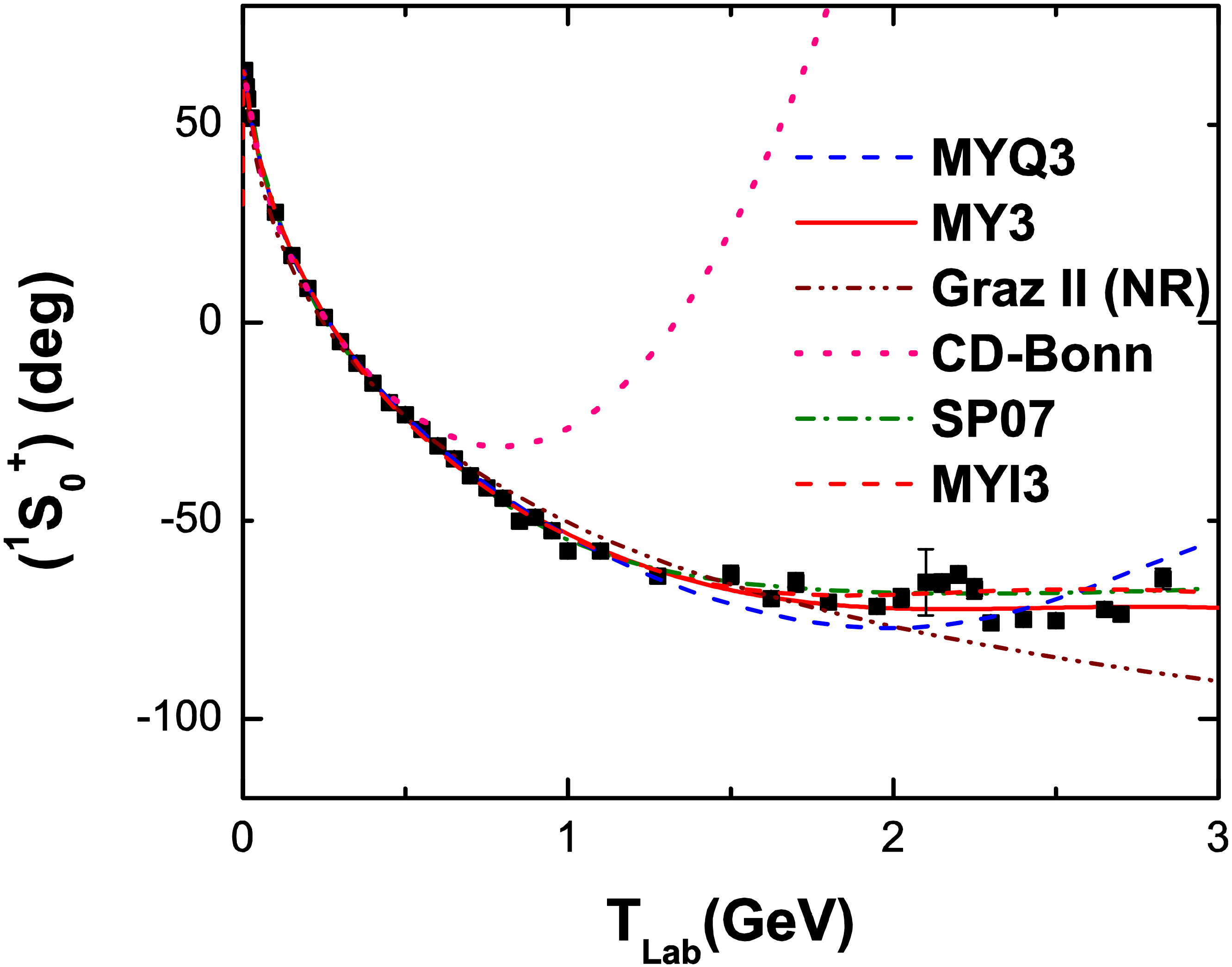
MYQ2
MY2
CD-Bonn
SP07
MYI2
0
-10
-20
-30
-40
-50
-60
-70
0
1
2
3
$(^3P_1^+)$ (deg)
T_{Lab}(GeV)

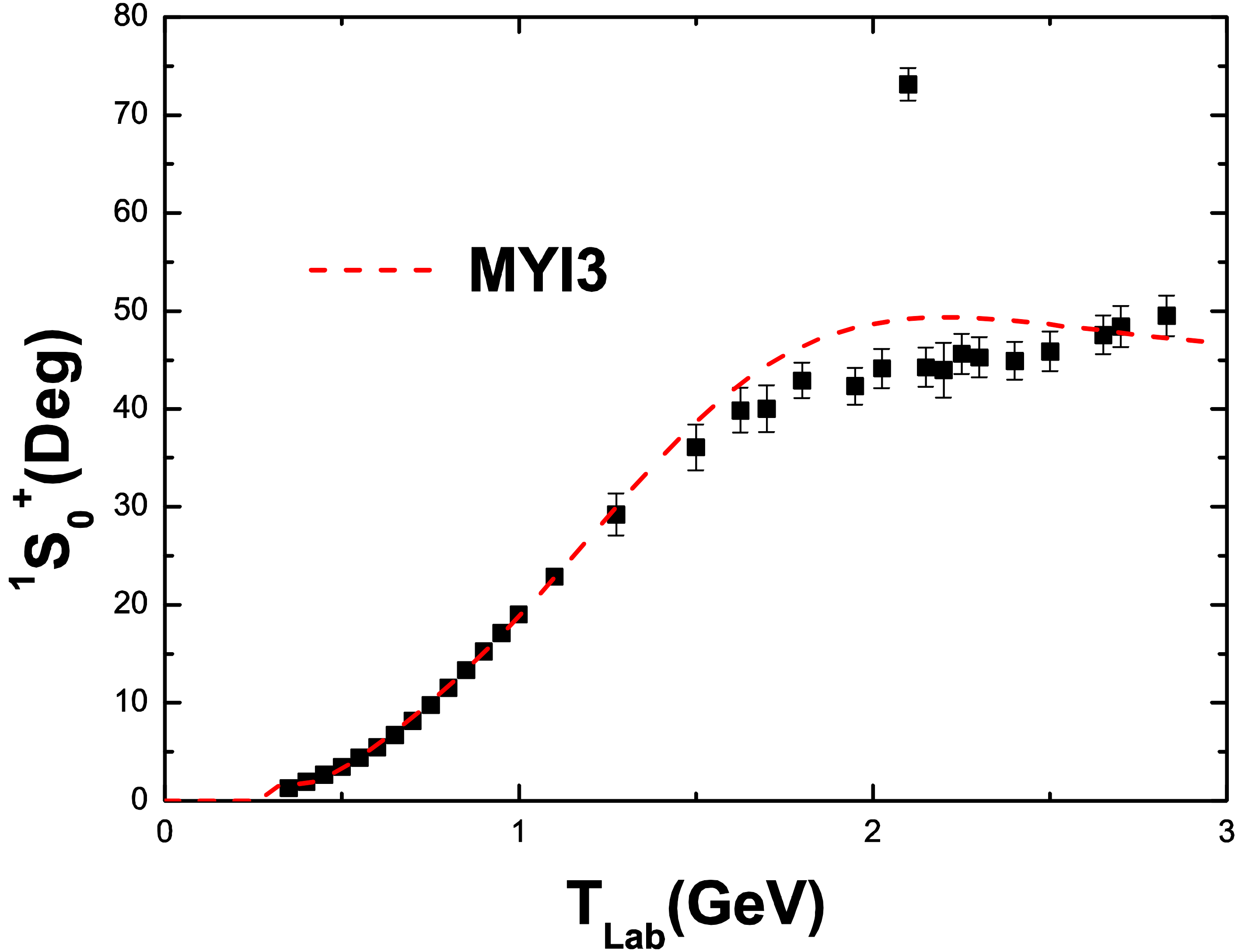


50
45
40
35
30
25
20
15
10
5
0
0
1
2
3
$^{3}P_{1}^{+}$(Deg)
T_{Lab}(GeV)
MYI2

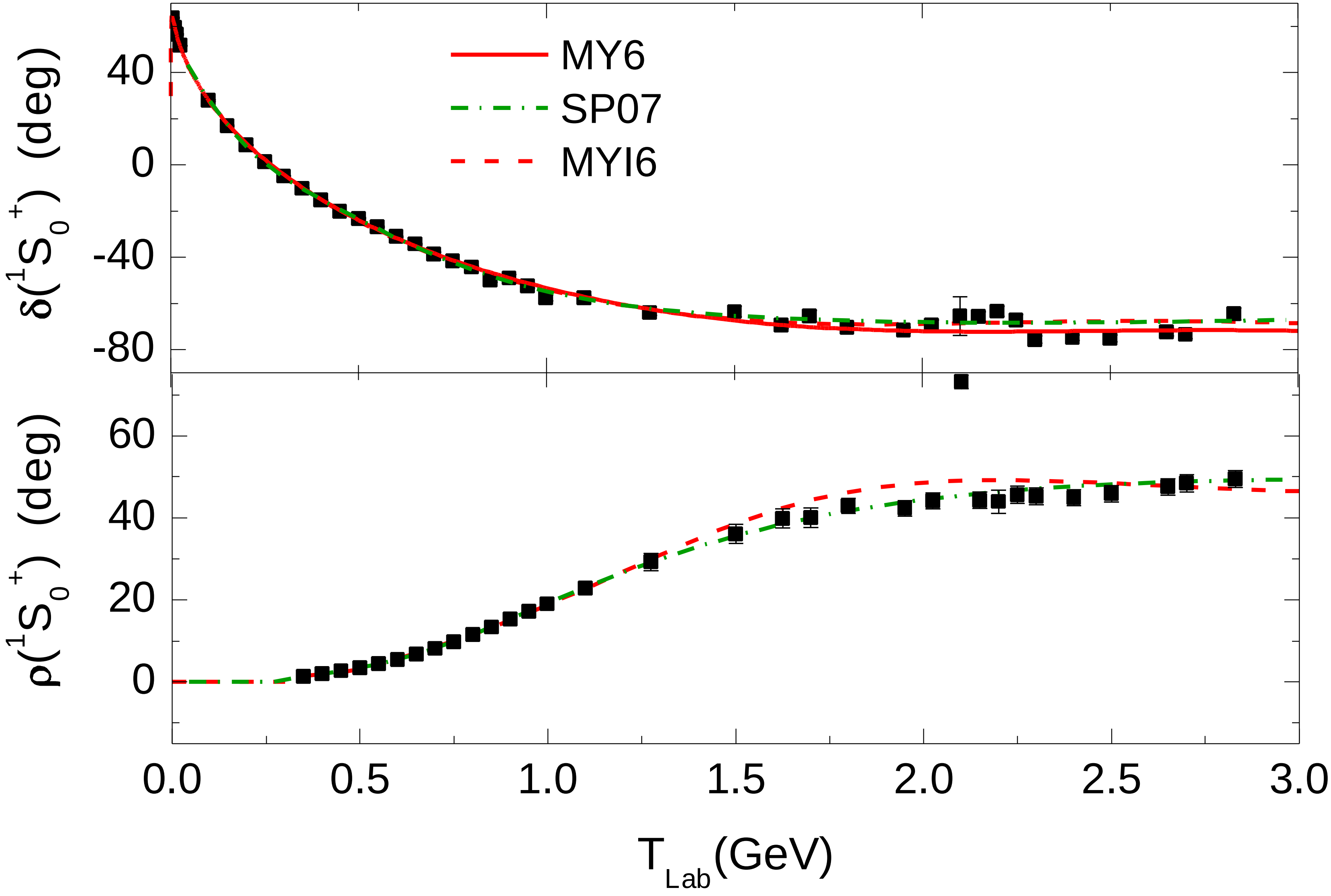

MY2
SP07
MYI2
$\delta(^3P_1^+)$ (deg)
$\rho(^3P_1^+)$ (deg)
T_{Lab}(GeV)
0
-20
-40
-60
-80
80
60
40
20
0
0.0
0.5
1.0
1.5
2.0
2.5
3.0

Graz II: L. Mathelitsch, W. Plessas, M. Schweiger, Phys. Rev. C 26 (1982) 65

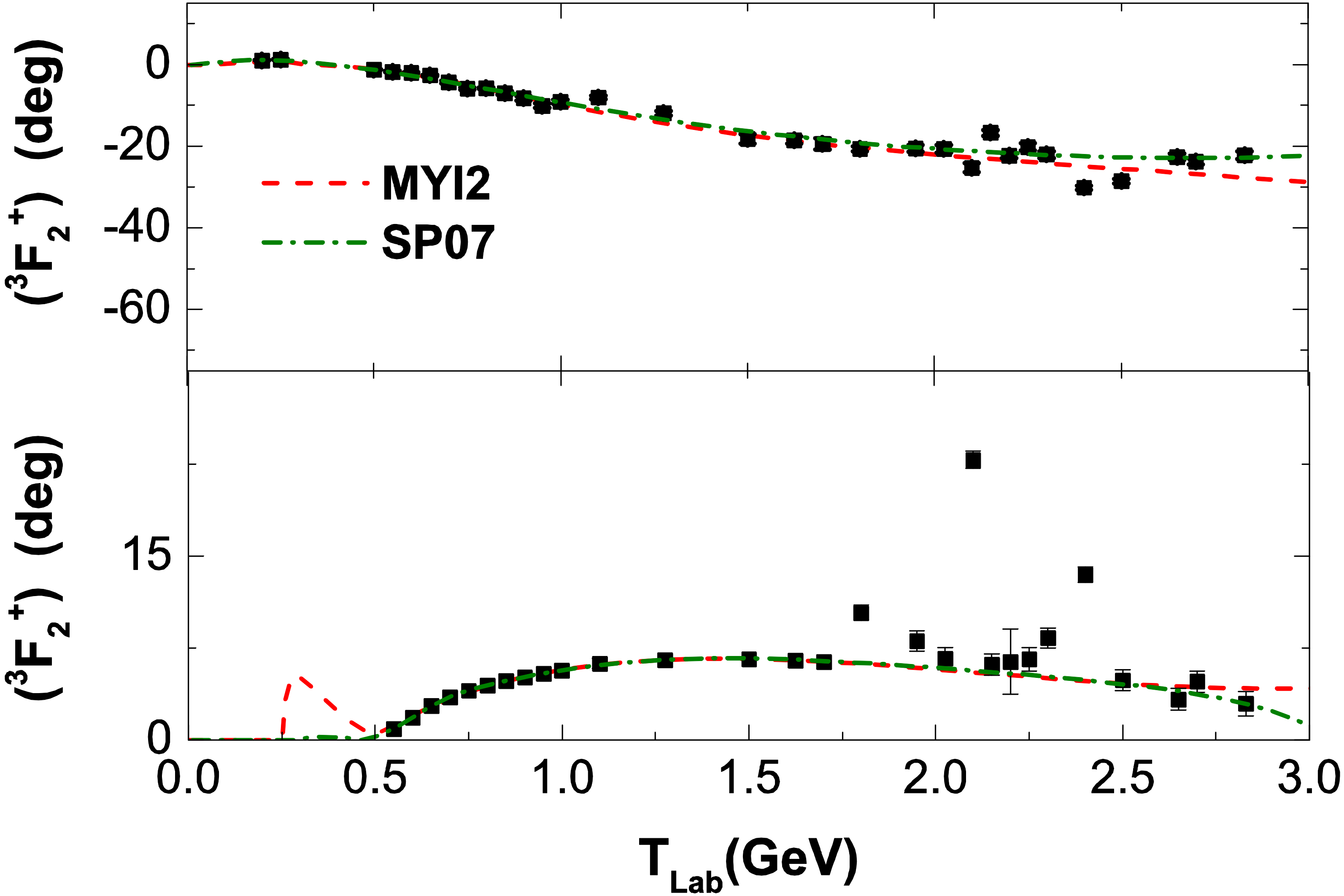




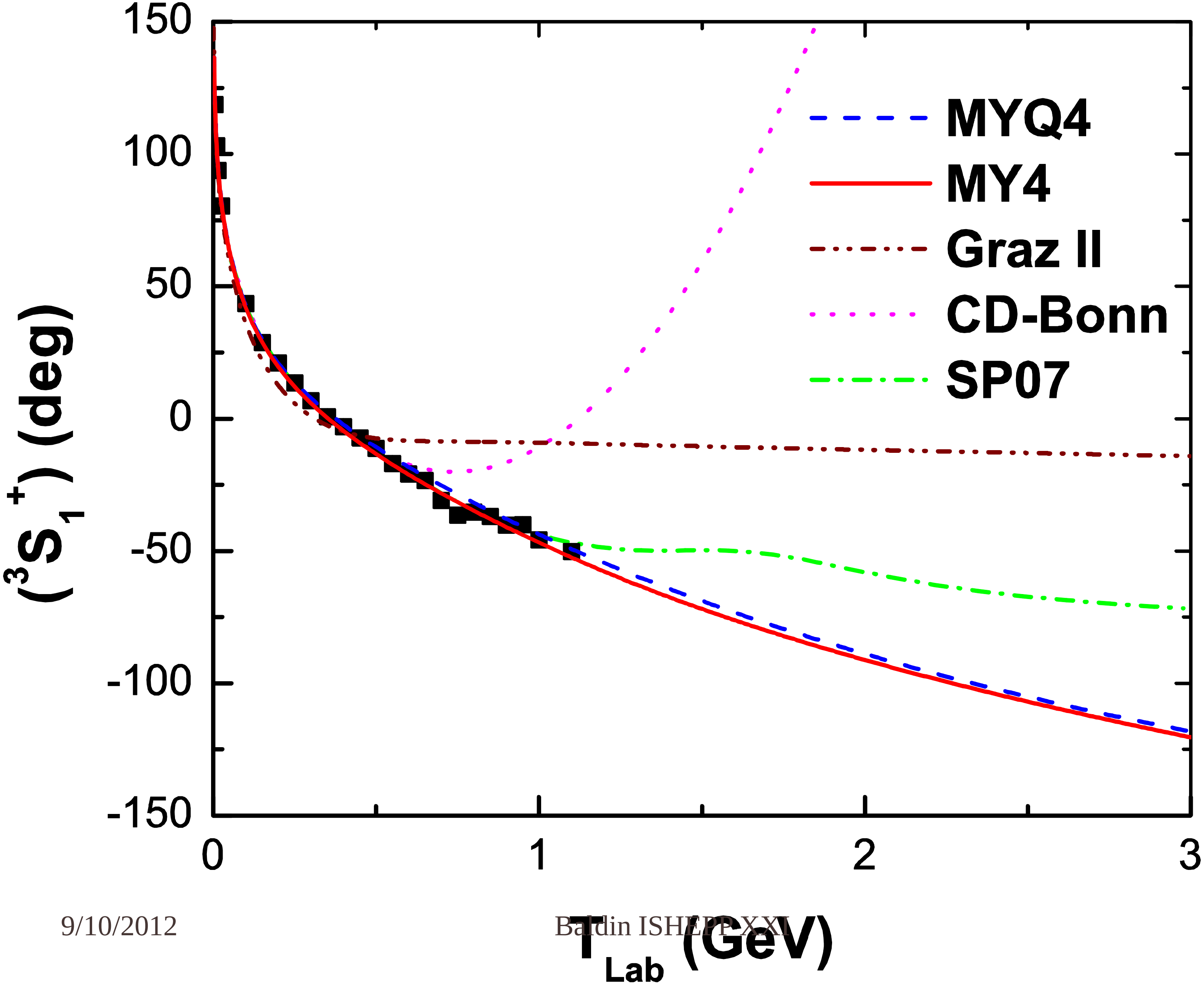
80
70
60
50
40
30
20
10
0
0
1
2
3
MYI3
$^1S_0^+$(Deg)
T_{Lab}(GeV)

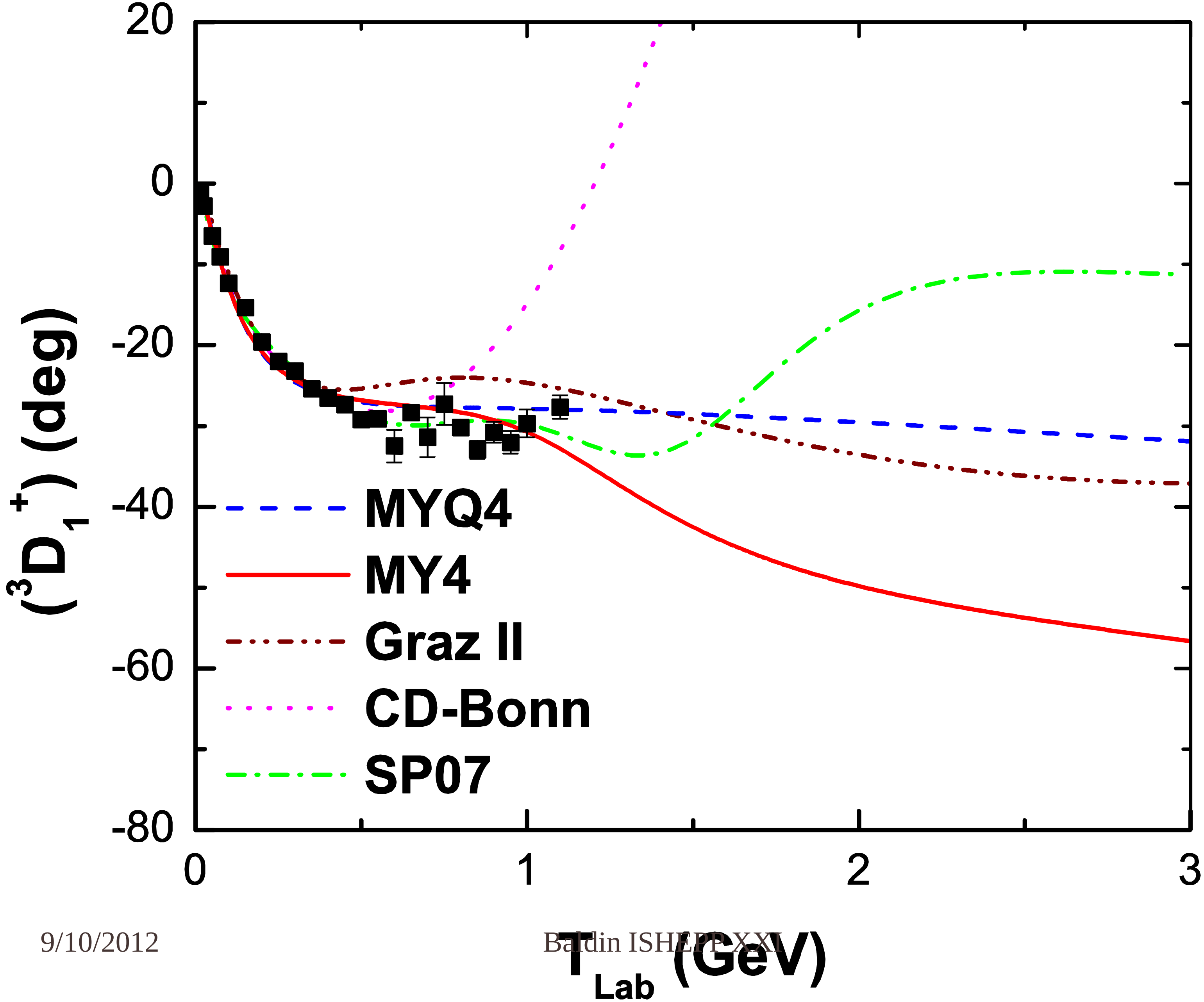


MY6
SP07
MYI6
$\delta(^1S_0^+)$ (deg)
$\rho(^1S_0^+)$ (deg)
40
0
-40
-80
60
40
20
0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
T_{Lab}(GeV)

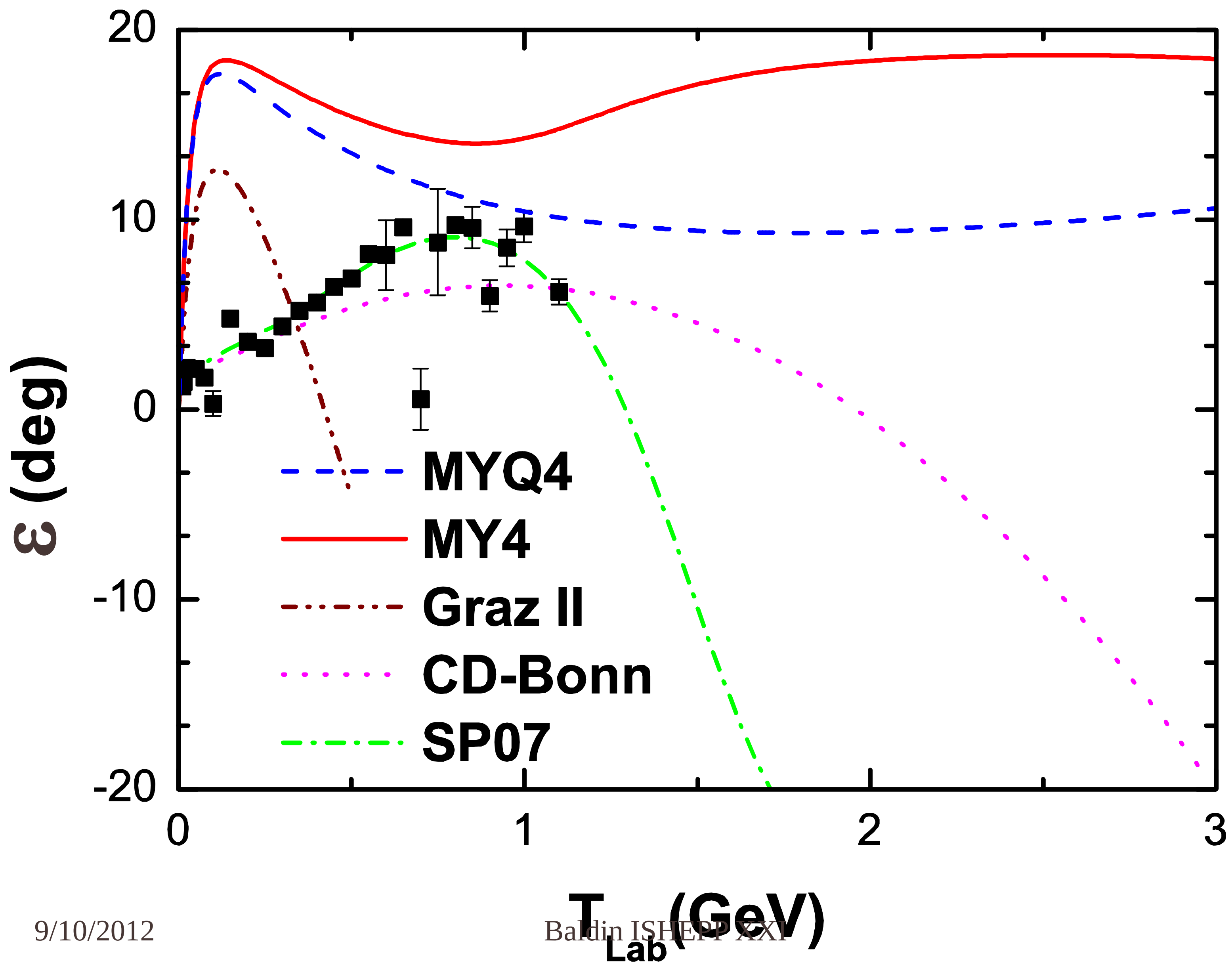


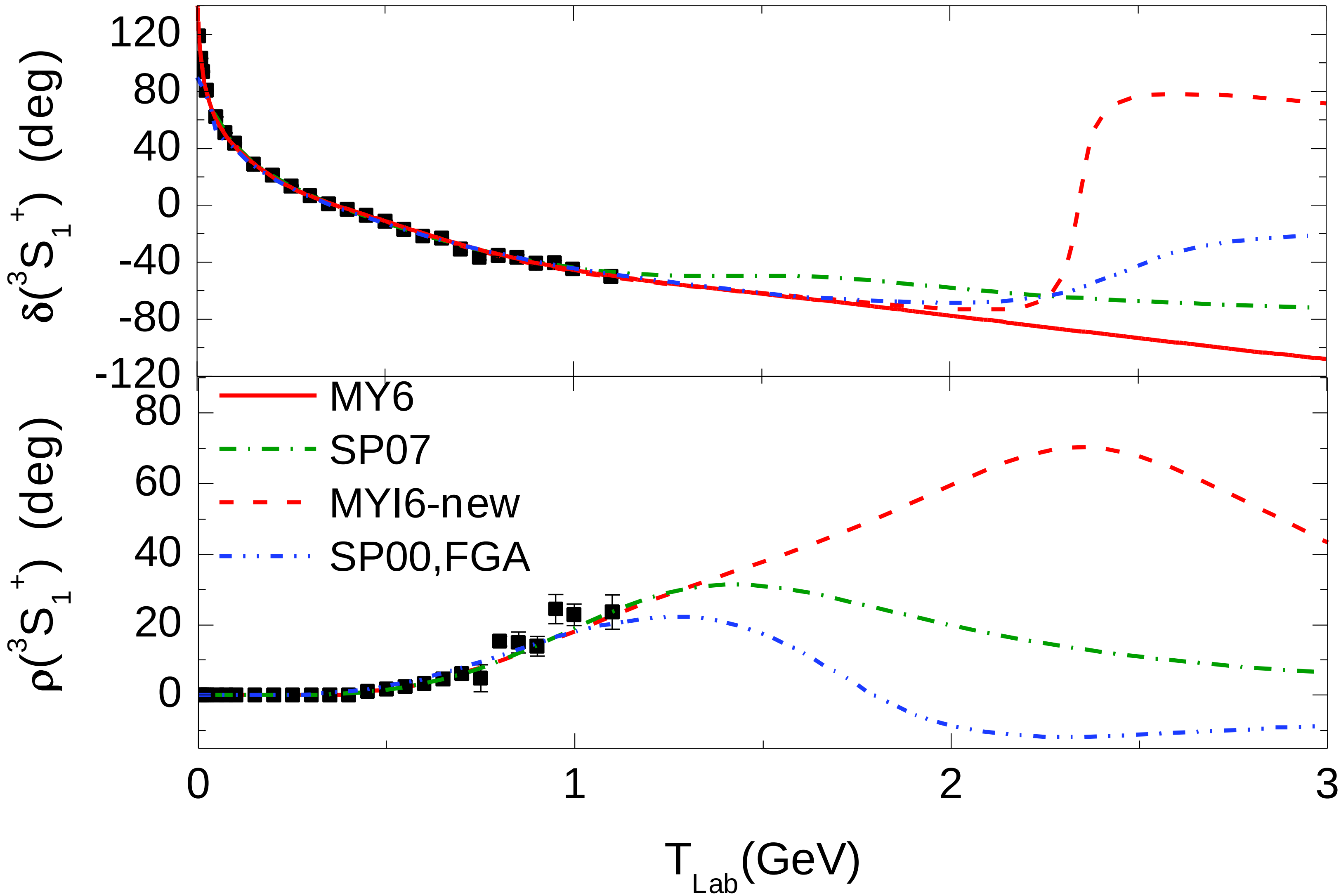
0
-20
-40
-60
MYI2
SP07
$(^3F_2^+)$ (deg)
15
0
$(^3F_2^+)$ (deg)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
T_{Lab}(GeV)


150
100
50
0
-50
-100
-150
0
1
2
3
($^3S_1^+$) (deg)
T_{Lab} (GeV)
MYQ4
MY4
Graz II
CD-Bonn
SP07

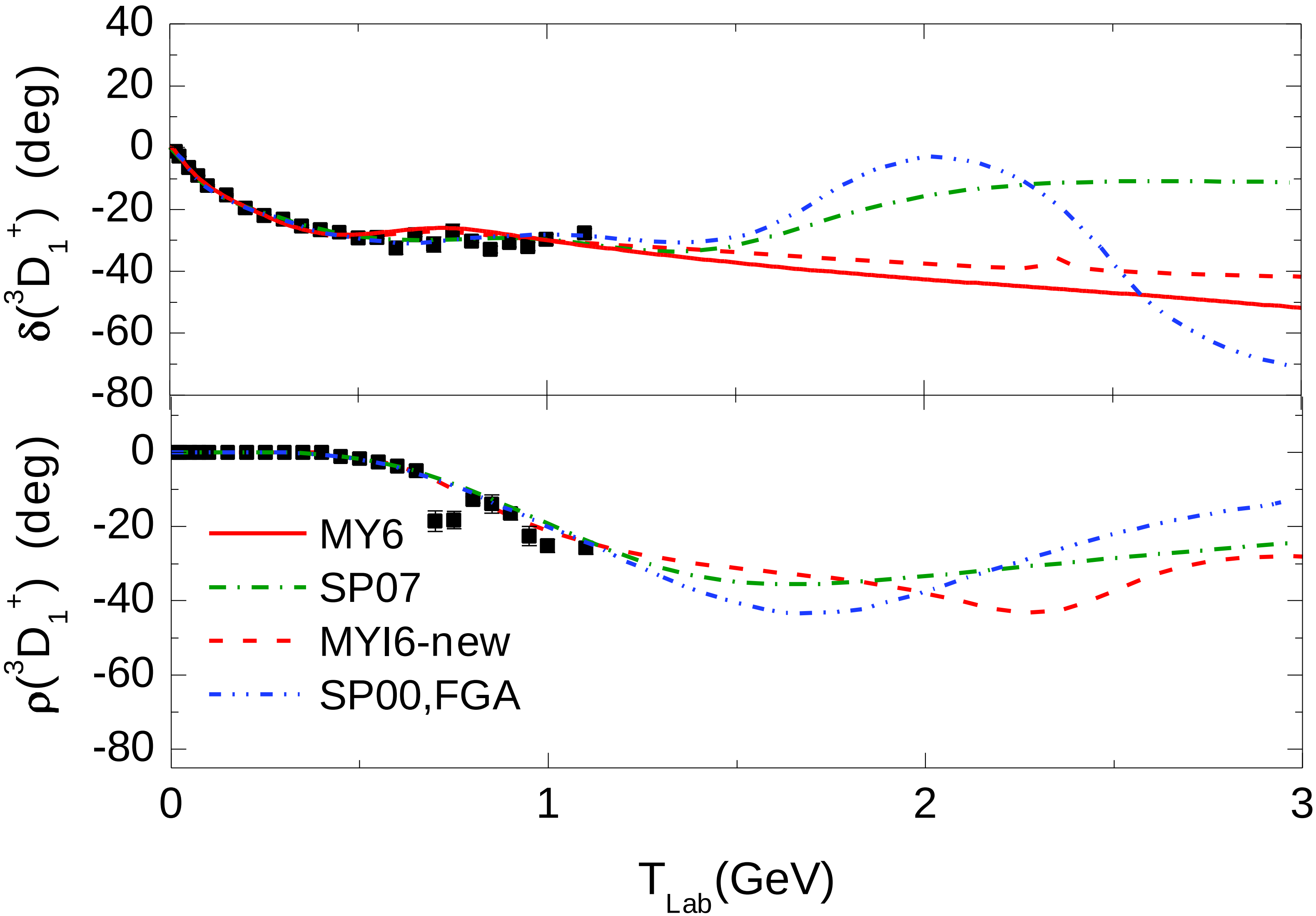


20
0
-20
-40
-60
-80
0
1
2
3
($^3D_1^+$) (deg)
T_{Lab} (GeV)
MYQ4
MY4
Graz II
CD-Bonn
SP07


20
10
0
-10
-20
0
1
2
3
ε (deg)
T_Lab (GeV)
MYQ4
MY4
Graz II
CD-Bonn
SP07

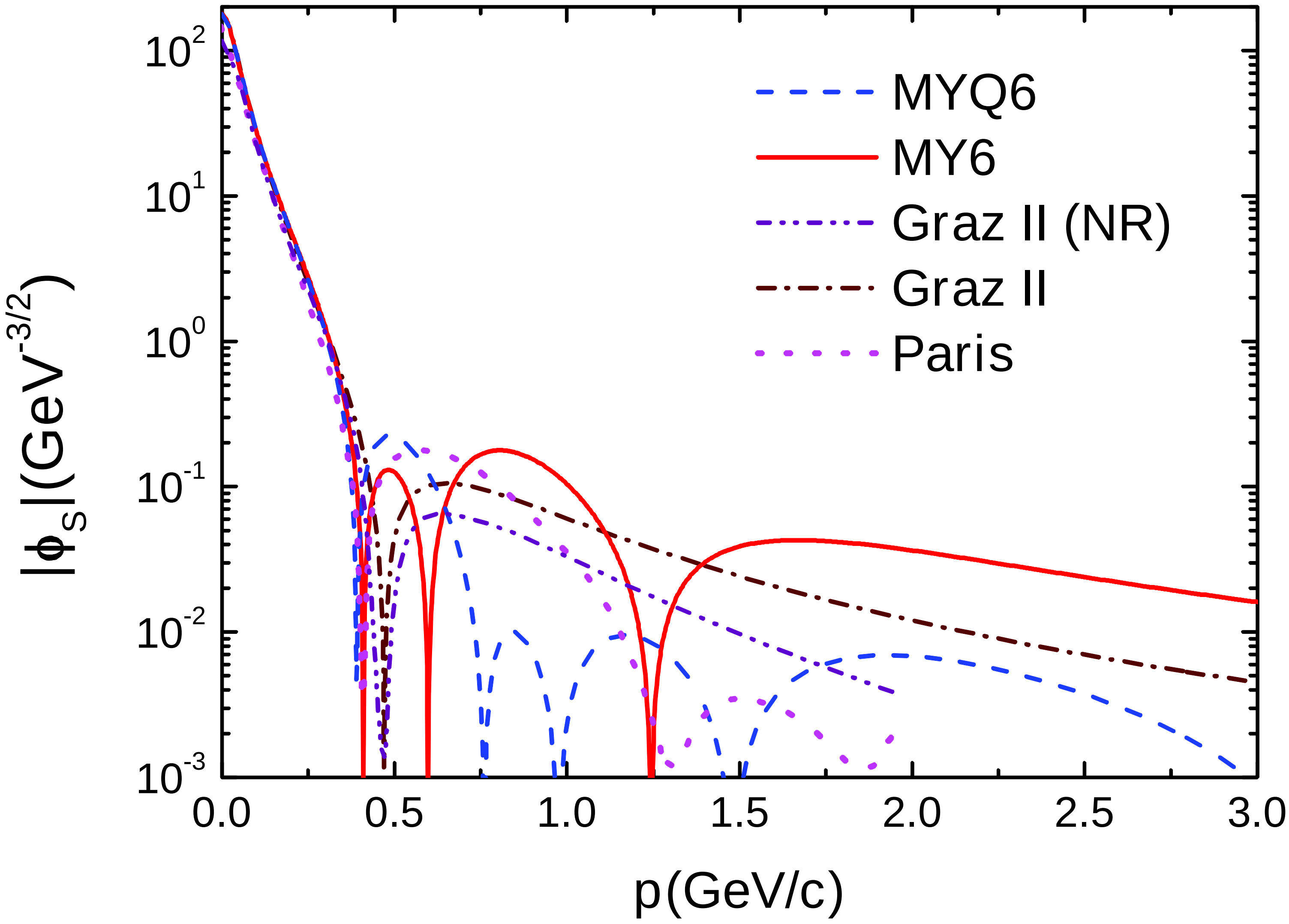


120
80
40
0
-40
-80
-120
δ($^3S_1^+$) (deg)
80
60
40
20
0
ρ($^3S_1^+$) (deg)
MY6
SP07
MYI6-new
SP00,FGA
0
1
2
3
T_{Lab}(GeV)

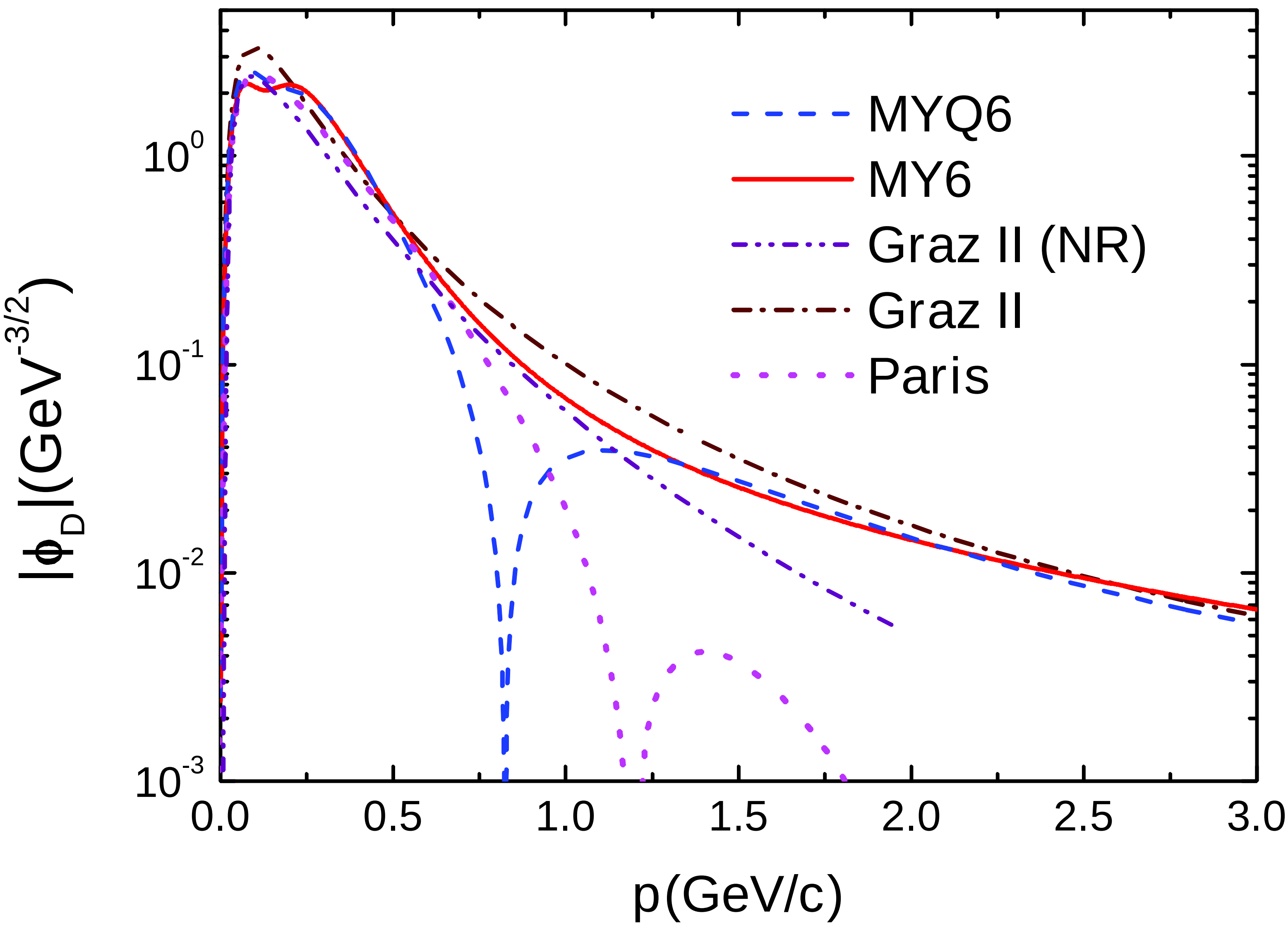


40
20
0
-20
-40
-60
-80
$\delta(^3D_1^+)$ (deg)
0
-20
-40
-60
-80
$\rho(^3D_1^+)$ (deg)
MY6
SP07
MYI6-new
SP00,FGA
0
1
2
3
T_{Lab}(GeV)

Deuteron "Wave function"



Deuteron "Wave function"



Summary

- BS approach
  - **is full covariant descriptions of two body system**;
  - allows to build the multirank covariant separable potential ***MYN*** and ***MYIN*** of the neutron-proton interaction for coupled and uncoupled partial-waves states with the total angular momentum *J=0,1,2* till the kinetic energy *3GeV*.
    - The description of the phases and inelasticity parameter with ***MYN*** and ***MYIN*** is very good.
    - Deuteron ***MYN wave functions are very close to nonrelativistic one at small momenta less then 0.7 GeV/c.***
    - ***Dibaryon resonances are proposed.***

Summary

- BS approach:
  - can give very reasonable explanation structure functions, form factors and tensor polarization of deuteron in elastic eD-scattering;
  - gives in one iteration approximation pair mesonic currents
  - gives foundations of light cone dynamics approaches;
  - gives good instrument to study polarization phenomena in elastic, inelastic, deep-inelastic lepton deuteron scattering;
  - ***is a powerful tool for investigation of the reactions with the deuteron (as well as reactions with the few-body systems).***

Plans

- *To investigate the influence of the complex part of the interaction kernel (namely, influence of the inelasticity parameter) to the exclusive cross section and polarization characteristics of the deuteron electrodisintegration for several kinematic conditions (Sacle, JLab).*
- *To calculate the observables in the photo- and hadron-deuteron reactions.*