

# Adiabatic Chemical Freeze-out and Wide Resonance Modification in a Thermal Media

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# Outline

- New results obtained within Statistical Model of Chemical FO
- Model for Adiabatic Chemical FO and surprises on Hagedorn spectrum
- Wide Resonance Width Modification in a Thermal Media
- Application to Quark Gluon Bags
- Conclusions

# Hadron Resonance Gas: a Multi-component Model

Traditional HRG model: one hard-core radius  $R=0.25-0.3$  fm

A. Andronic, P.Braun-Munzinger, J. Stachel, NPA (2006)777

Two hard-core radii:  $R_{\pi}=0.62$  fm,  $R_{\text{other}}=0.8$  fm

G. D.Yen. M. Gorenstein, W. Greiner, S.N. Yang, PRC (1997)56

Or:  $R_{\text{mesons}}=0.25$  fm,  $R_{\text{baryons}}=0.3$  fm

A. Andronic, P.Braun-Munzinger, J. Stachel, NPA (2006) 777,  
PLB (2009) 673

Overall description of data (mid-rapidity or  $4\pi$  multiplicities or ratios is good !

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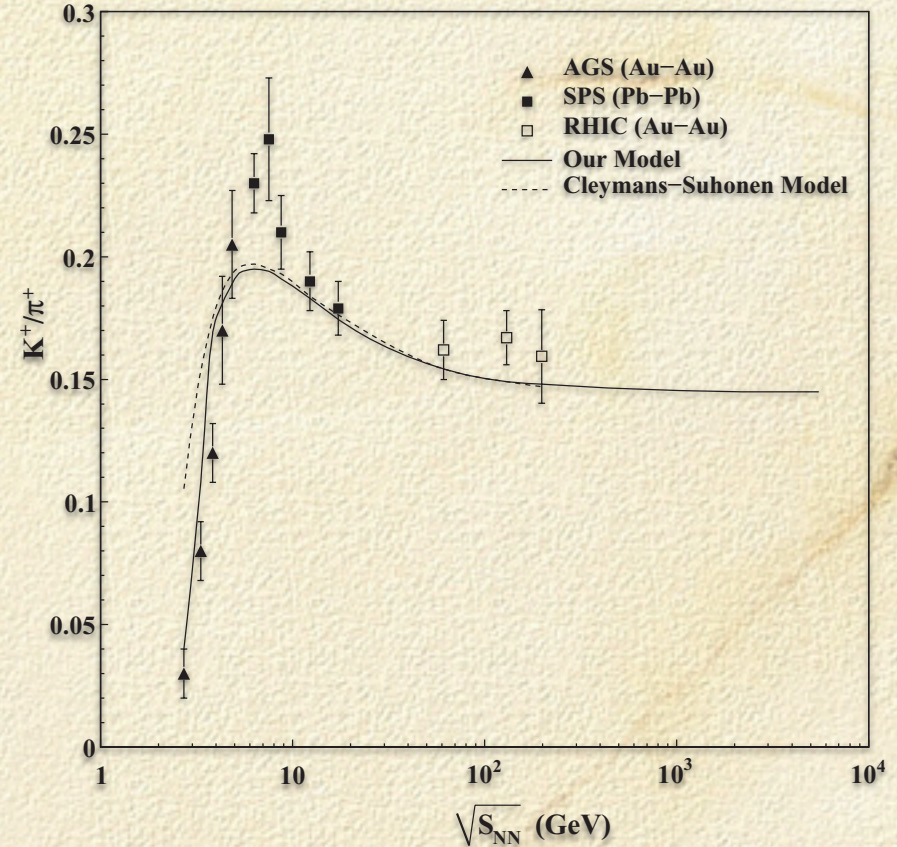
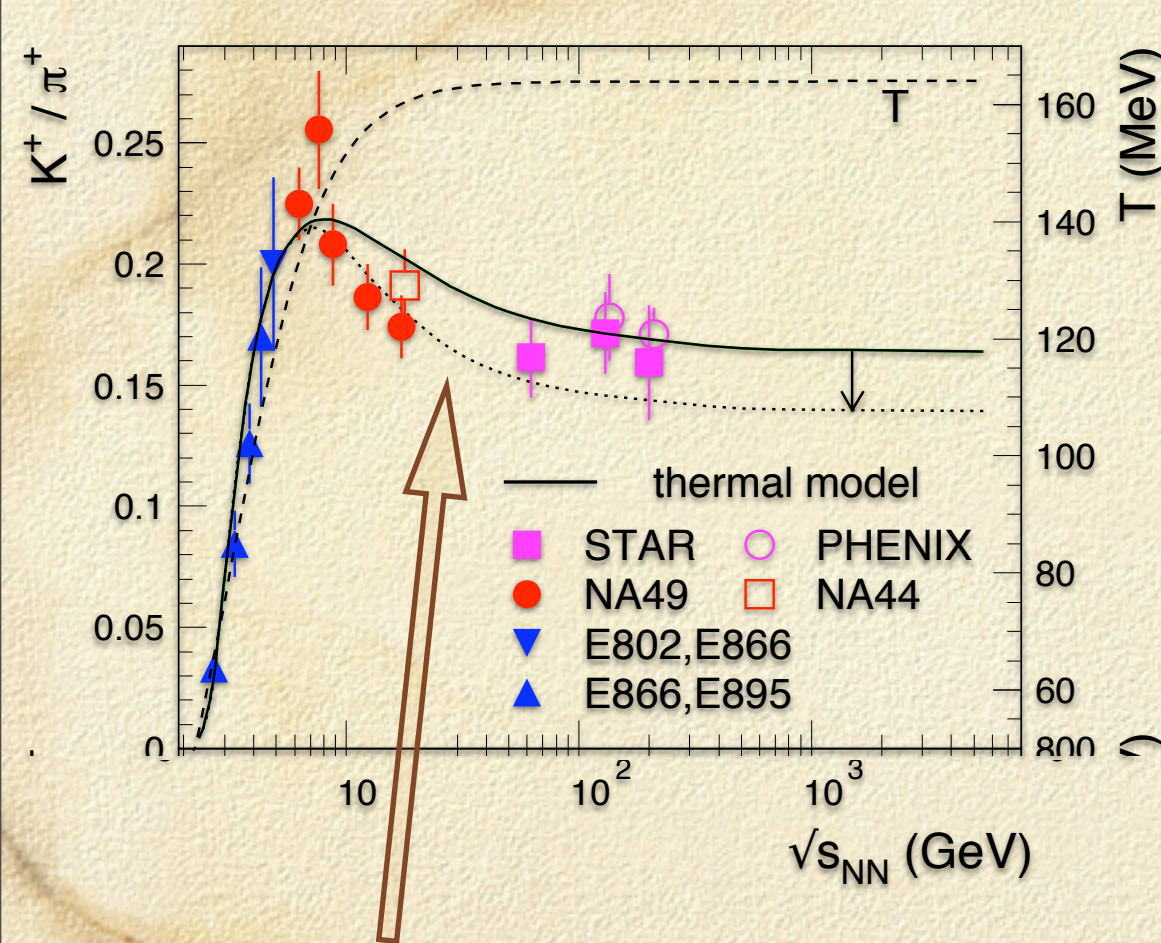
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A. Andronic, P.Braun-Munzinger, J. Stachel, NPA (2006) 777,  
PLB (2009) 673

**But there are problems with  $K^+/\pi^+$  and  $\Lambda/\pi^-$  ratios at  
SPS energies!!!**

# Strangeness Horn Description Puzzle

Too slow decrease after maximum!



S. K. Tiwari, P. K. Srivastava, and C. P. Singh,  
PRC(2012) 85

Short dashed line: a desired result

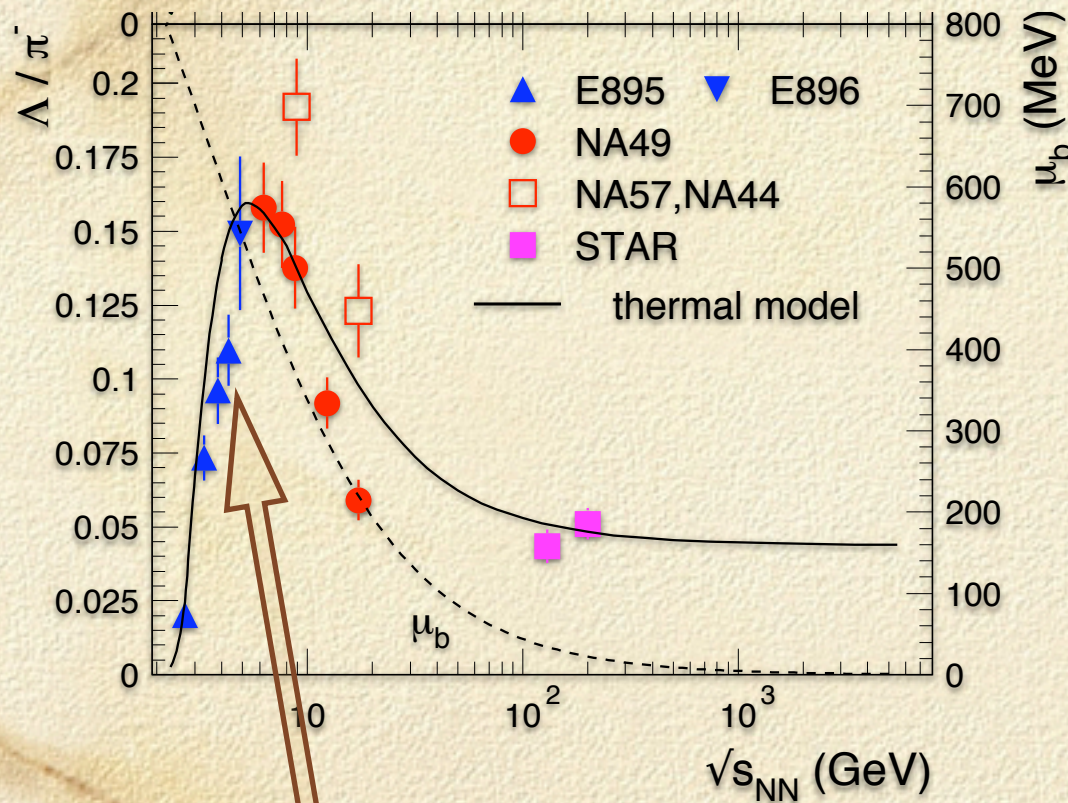
A. Andronic, P. Braun-Munzinger, J. Stachel,  
PLB (2009) 673

$R_{\pi} = 0. \text{ fm}$ ,  $R_{\text{other}} = 0.8 \text{ fm}$

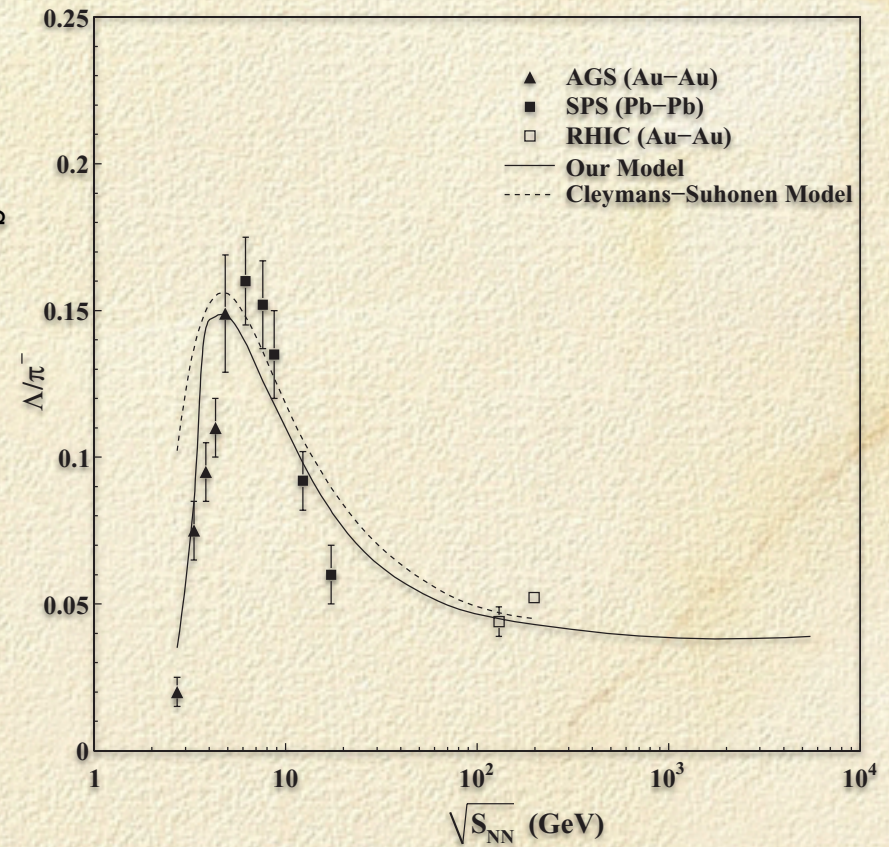
Very sophisticated excluded volume, but  
thermodynamically inconsistent model!

# Further Problems at SPS Energies

Too steep increase before maximum and too slow decrease after it!



Anti Lambda problem!



A. Andronic, P. Braun-Munzinger, J. Stachel,  
PLB (2009) 673

S. K. Tiwari, P. K. Srivastava, and C. P. Singh,  
PRC(2012) 85

# Simple Solution to Horn Puzzle

Use four hard-core radii:  $R_{pi}$ ,  $R_K$  are fitting parameters;  
 $R_{mesons} = 0.3$  fm,  $R_{baryons} = 0.5$  fm are fixed

G. Zeeb, K.A. Bugaev, P.T. Reuter and H. Stoecker, Ukr. J. Phys. 53, 279 (2008)

D.R. Oliinychenko, K.A. Bugaev and A.S. Sorin, arXiv:1204.0103 [hep-ph].

$p$  is pressure  $K$ -th charge density of  $i$ -th hadron sort is  $n_i^K$  ( $K \in \{B, S, I3\}$ )

$\mathcal{B}$  the second virial coefficients matrix  $b_{ij} \equiv \frac{2\pi}{3}(R_i + R_j)^3$

$$p = T \sum_{i=1}^N \xi_i, \quad n_i^K = Q_i^K \xi_i \left[ 1 + \frac{\xi^T \mathcal{B} \xi}{\sum_{j=1}^N \xi_j} \right]^{-1}, \quad \xi = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \dots \\ \xi_s \end{pmatrix},$$

**NO strangeness suppression is included!**

the variables  $\xi_i$  are the solution of the following system:

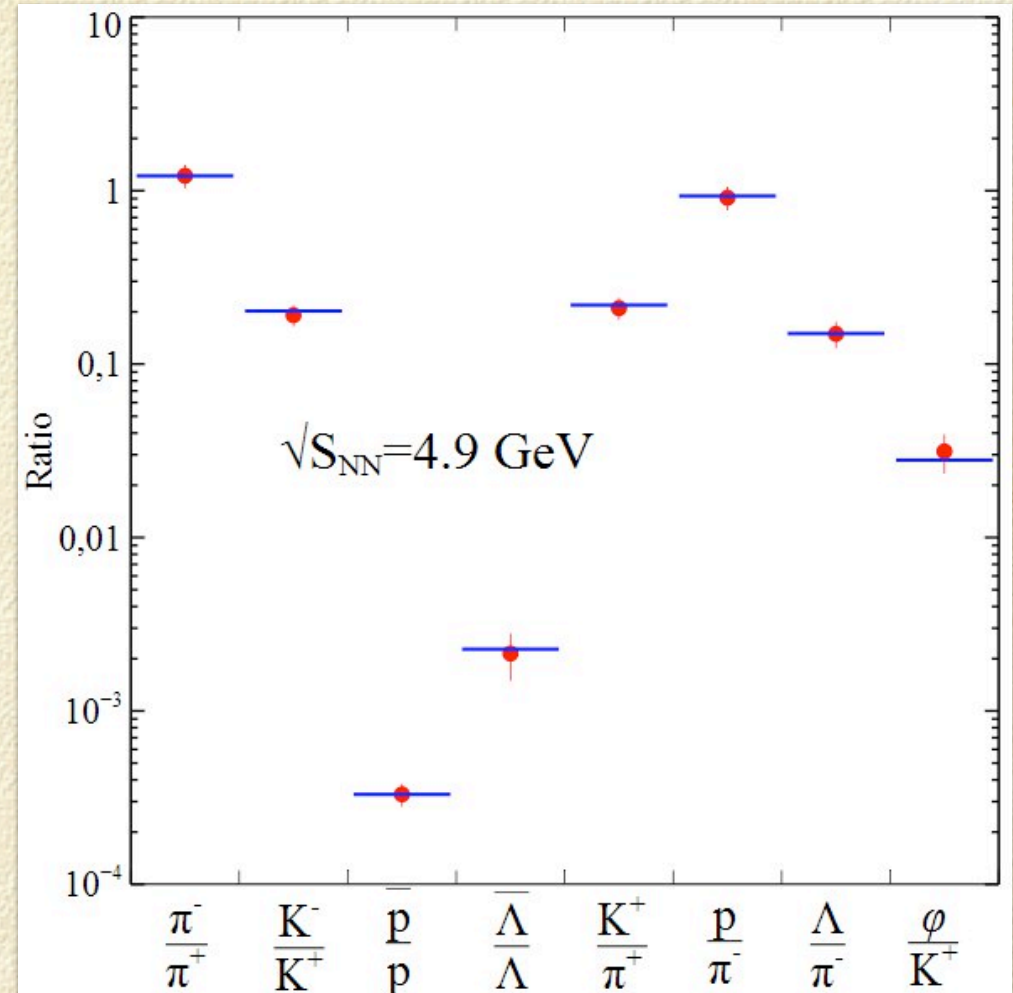
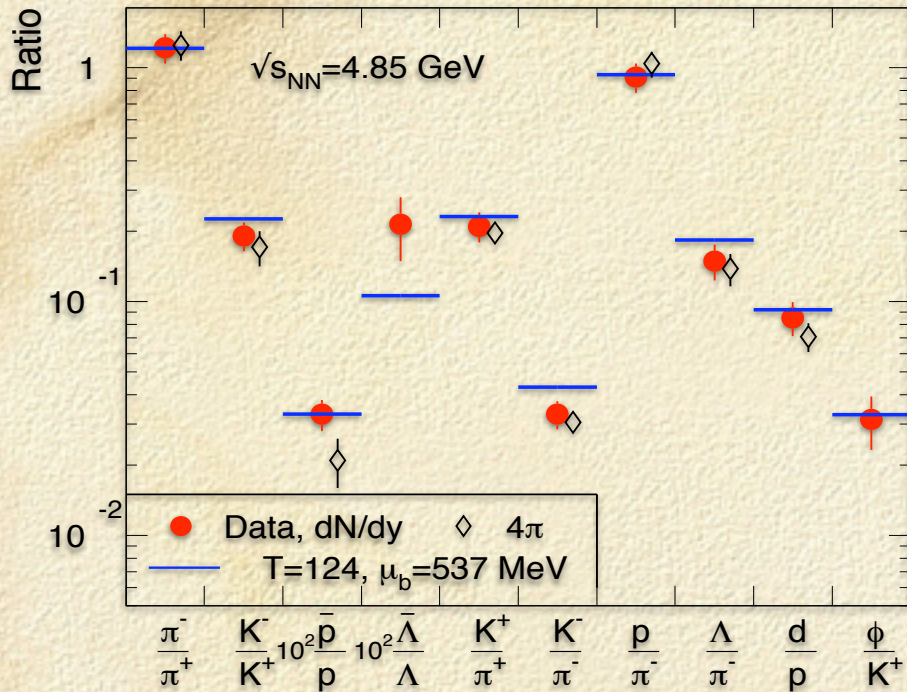
$$\xi_i = \phi_i(T) \exp \left( \frac{\mu_i}{T} - \sum_{j=1}^N 2\xi_j b_{ij} + \frac{\xi^T \mathcal{B} \xi}{\sum_{j=1}^N \xi_j} \right), \quad \phi_i(T) = \underbrace{\frac{g_i}{(2\pi)^3} \int \exp \left( -\frac{\sqrt{k^2 + m_i^2}}{T} \right) d^3k}_{\text{THERMAL DENSITY}}$$

Chemical potential of  $i$ -th hadron sort:  $\mu_i \equiv Q_i^B \mu_B + Q_i^S \mu_S + Q_i^{I3} \mu_{I3}$

$Q_i^K$  are charges,  $m_i$  is mass and  $g_i$  is degeneracy of the  $i$ -th hadron sort

# Results for Ratios (AGS)

There is NO anti Lambda problem here  
and all ratios are well described!



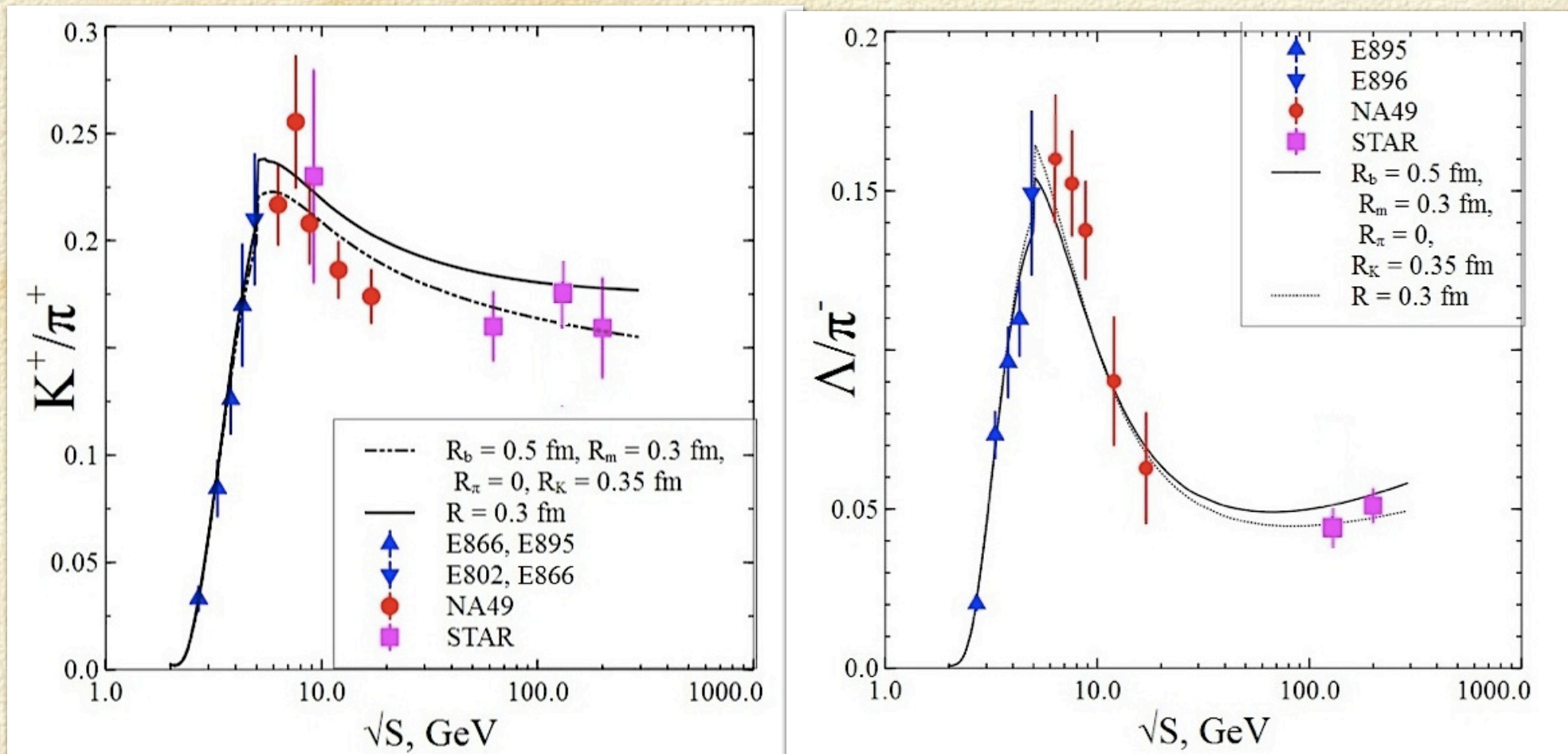
**There is an anti Lambda problem!**  
**Also K-/K+ and K/pi and Lambda/pi-**  
**are not well described!**

$T \simeq 131$  MeV,  $\mu_B \simeq 539$  MeV,  $\mu_{I3} \simeq -16$  MeV

A. Andronic, P. Braun-Munzinger, J. Stachel, K.A.B., D.R. Oliinychenko, A.S. Sorin, G.M. Zinovjev,  
NPA (2006)777 arXiv:1208.5968 [hep-ph].



# Description of Horns at SPS



K.A.B., D.R. Oliinychenko, A.S. Sorin, G.M. Zinovjev, arXiv:1208.5968 [hep-ph].

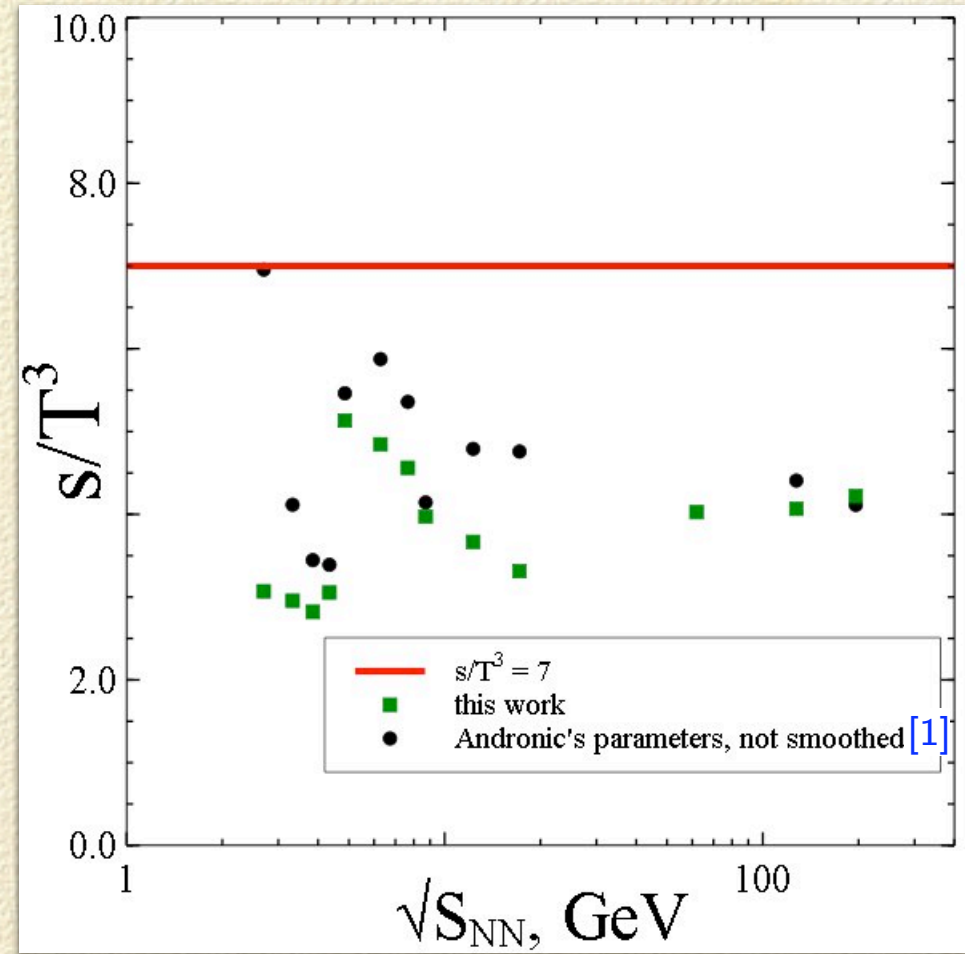
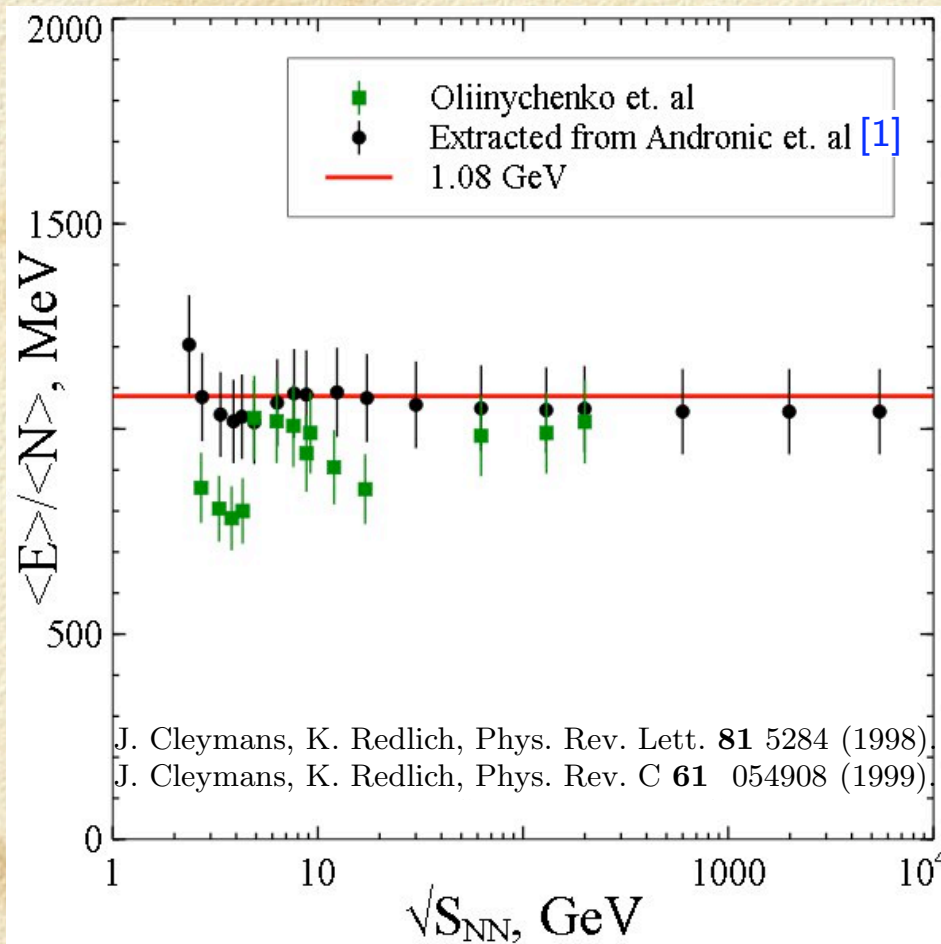
Best global fit of all ratios gives  $R_\pi=0$ . fm,  $R_K=0.35$  fm,

$\chi^2/\text{dof}=1.018$  for fixed:  $R_{\text{baryons}}=0.5$  fm,  $R_{\text{mesons}}=0.3$  fm

Note that Lambda and other hyperons can be described better!

# Chemical Freeze-out Criteria

All these (and other) FO criteria were suggested with too few hadron states!  
Particle tables with hadron masses up to 1.8-1.9 GeV



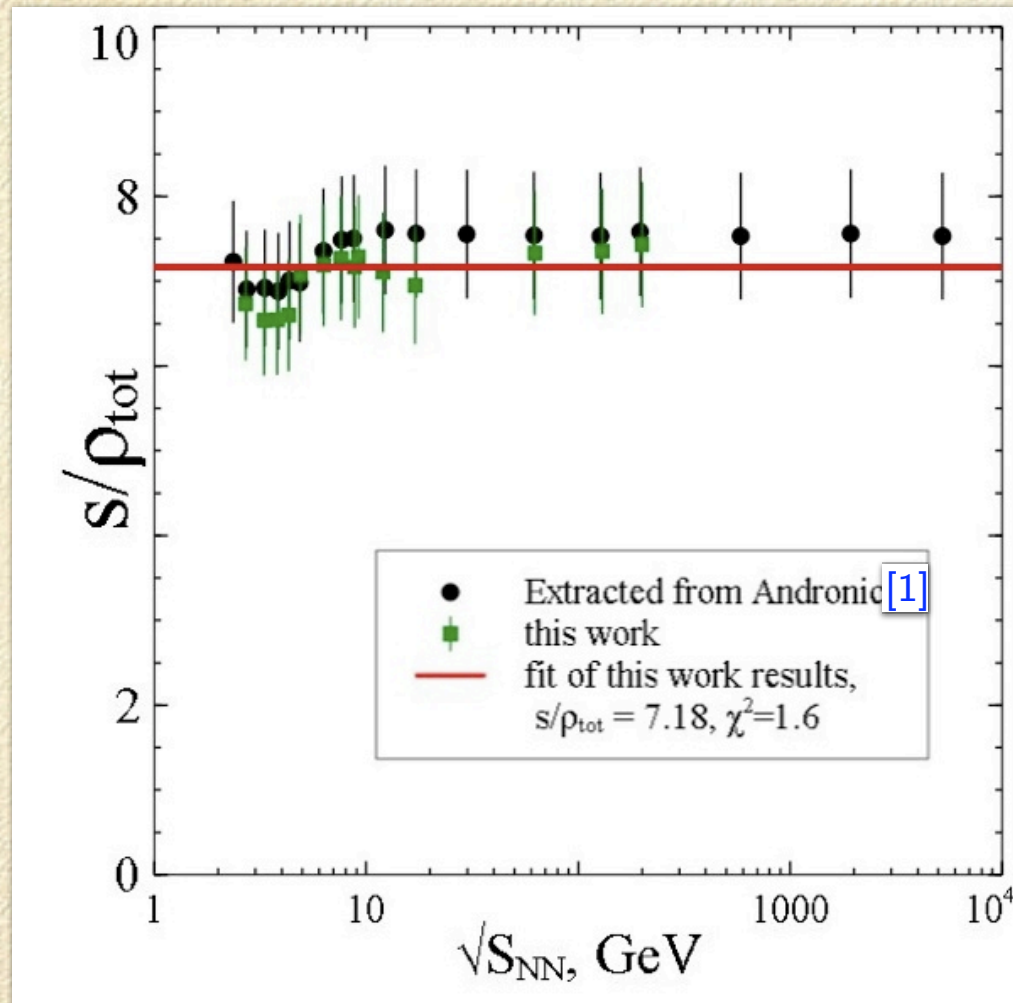
M. Stankiewicz, B. Sc. Honours Thesis, University of Cape Town (unpublished),  
J. Cleymans, H. Oeschler, K. Redlich, S. Wheaton, Phys. Lett. B **615** 50 (2005)  
A. Tawfik, J. Phys. G Nucl. Part. Phys. G **31** S1105 (2005)

[1] A. Andronic, P. Braun-Munzinger, J. Stachel, M. Winn, arXiv:1201.0693 [nucl-th].

# Adiabatic Chemical Freeze-out

Entropy per particle 7.18 is a really robust chemical freeze-out criterion!

But what is the reason for it?



D.R. Oliinychenko, K.A. Bugaev and A.S. Sorin, arXiv:1204.0103 [hep-ph].

[1] A. Andronic, P. Braun-Munzinger, J. Stachel, M. Winn, arXiv:1201.0693 [nucl-th].

# Model for Adiabatic Chemical Freeze-out

The simplest model pressures (no hard-core repulsion) for low densities

$$\text{mesons} \quad p = C_M T^{A_M} \exp \left[ \frac{\mu_M - m_M}{T} \right]$$

$$\text{baryons} \quad p = C_B T^{A_B} \exp \left[ \frac{\mu_B - m_B}{T} \right]$$

Constants: powers  $A_k$ , normaliz.  $C_k$ ,  
masses  $m_k$ , mesonic chem.potential  $\mu_M$

For  $\sqrt{S_{NN}} \leq 10$  GeV

Entropy density

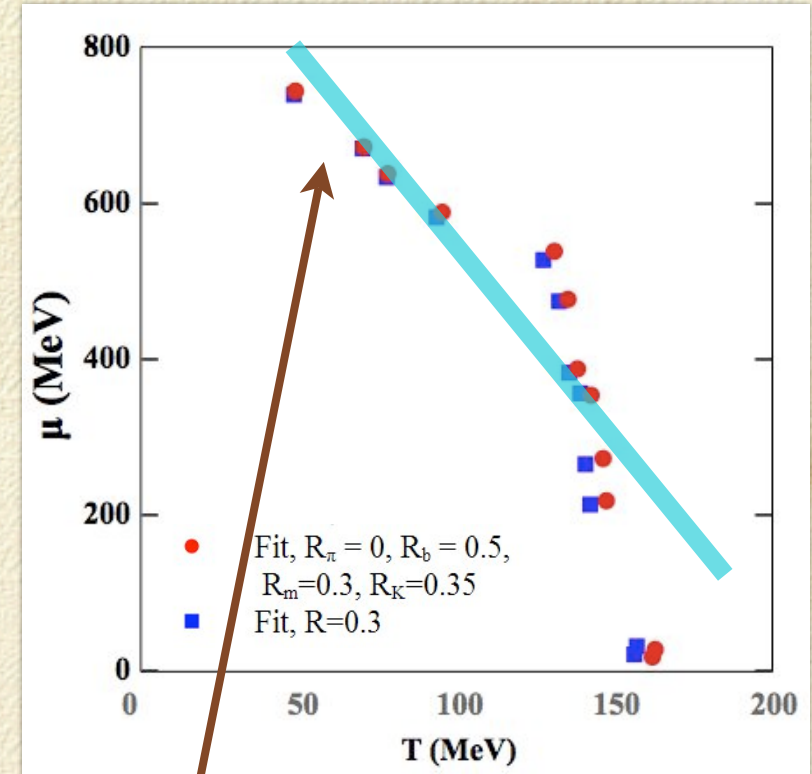
$$s = A_M \rho_M + \frac{\mu_M - m_M}{T} \rho_M + A_B \rho_B + \frac{\mu_B - m_B}{T} \rho_M$$

particle density

$$\rho_P = \rho_M + \rho_B$$

baryochem potential

$$\mu_B(T) \simeq 910 + \mu'_B T \simeq 910 - 3.6 T$$



# Explanation of Adiabatic Chemical FO

Entropy per particle

$$\frac{s}{\rho_P} = \underbrace{A_B - \mu'_B \frac{\rho_B}{\rho_P} + (A_M - A_B) \frac{\rho_M}{\rho_P}}_{\text{constant}} + \frac{(m_B - 910)\rho_B + (m_M - \mu_M)\rho_M}{T\rho_P}$$

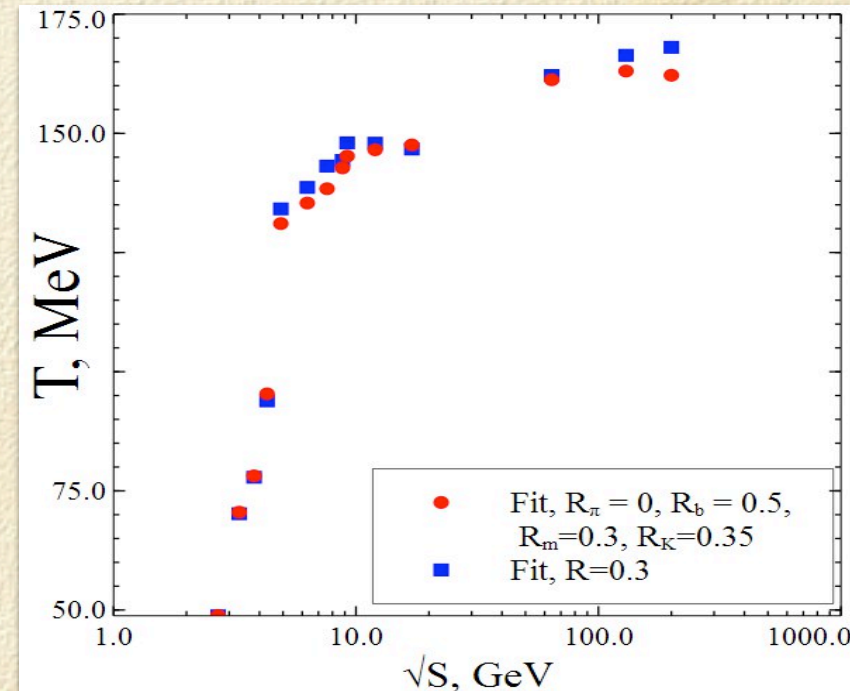
For  $\sqrt{S_{NN}} \leq 10$  GeV  $\Rightarrow$   $\frac{\rho_M}{\rho_P} \simeq 0.4$  but  $T$  grows fast!

To have  $\frac{s}{\rho_P} \simeq 7$   $\Rightarrow$   $m_B \simeq 910$  MeV,  $\frac{m_M - \mu_M}{T} \simeq 0$

Additionally  $0.4A_M + 0.6A_B \simeq 4.84$

For  $\sqrt{S_{NN}} > 10$  GeV same model works,  
but add anti-baryons!

It seems that const. entropy per particle  
is a combination of mean power of  $T$  and  
and nearly  $T$ -linear decrease of  $\mu$  at FO!



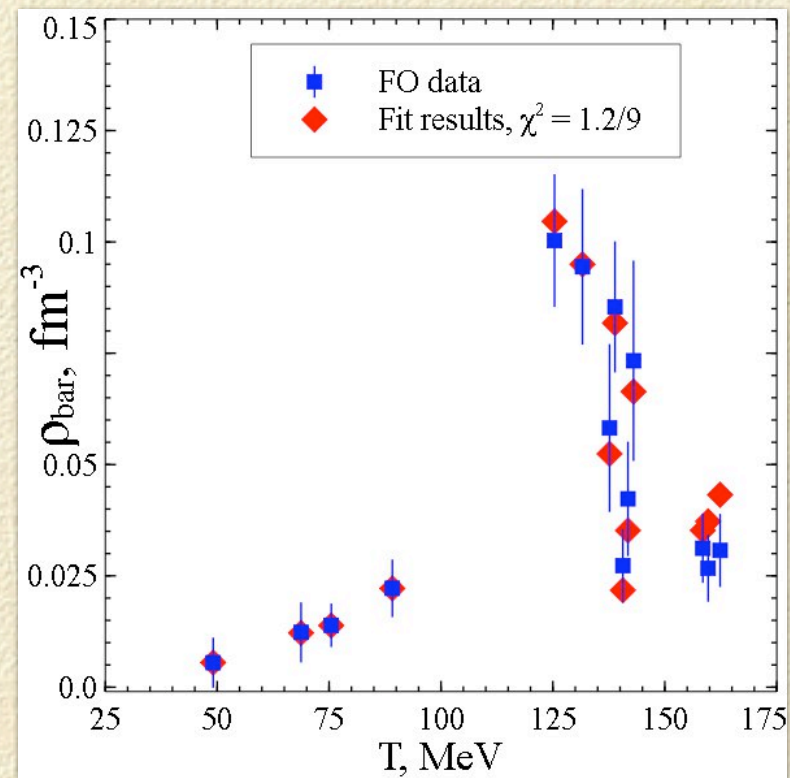
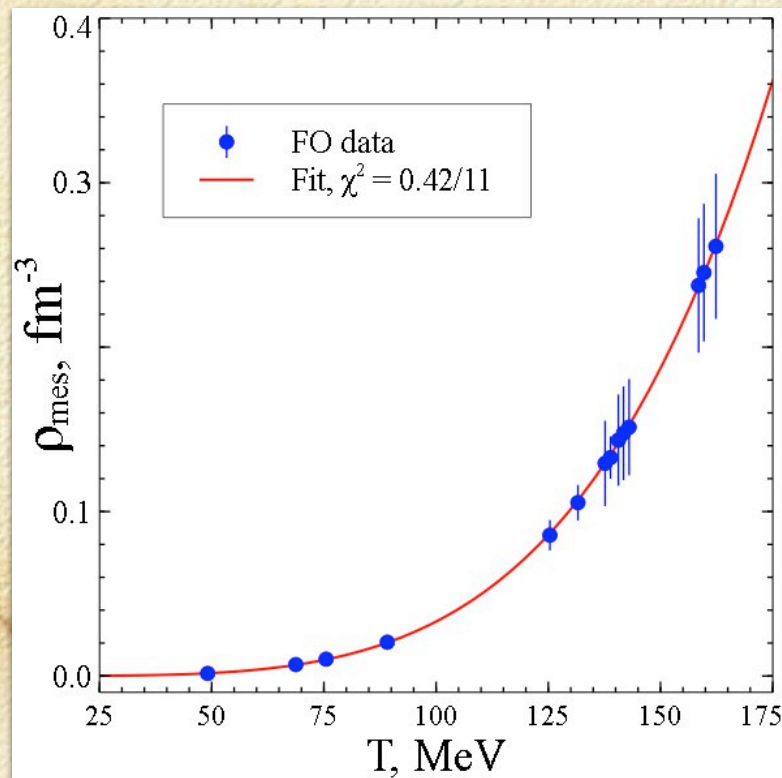
# Actual fit of Chemical FO data

Model works very well everywhere!

What does it mean ?

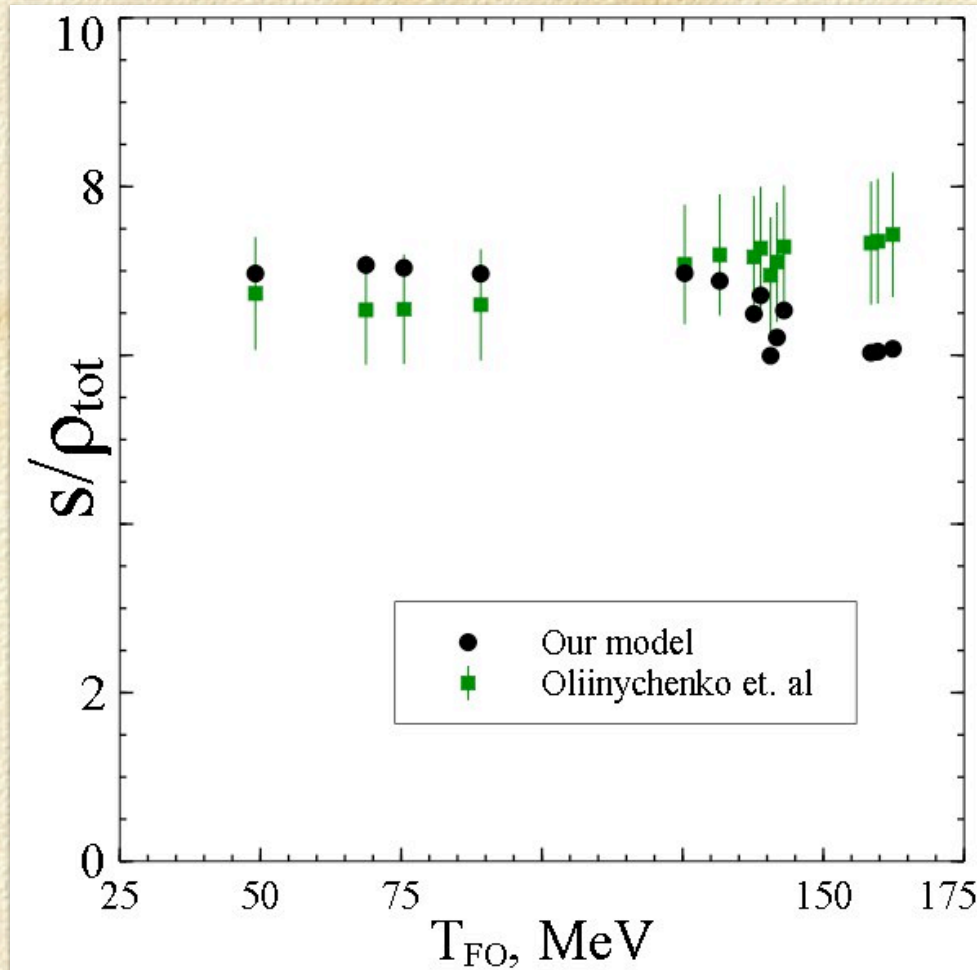
$$\begin{aligned} \text{mesons} \quad p &= C_M T^{A_M} \exp \left[ \frac{\mu_M - m_M}{T} \right], \quad A_M \simeq 5.31, \quad \mu_M - m_M \simeq 5 \text{ MeV} \quad ?? \\ \text{baryons} \quad p &= C_B T^{A_B} \exp \left[ \frac{\mu_B - m_B}{T} \right], \quad A_B \simeq 4.62, \quad m_B \simeq 930 \text{ MeV} \end{aligned}$$

Meson data give perfect fit, baryon data are bit worse at small  $\mu_B$



Check out our estimates  $0.4A_M + 0.6A_B \simeq 4.9!$

# Results



1. No additional fitting!

2. last 3 points are out because  $\mu$  dependence differs!

3. can be improved by additional fitting!

**Surprise No I: a constant entropy per particle is a combination of mean power of T and nearly T-linear decrease of  $\mu$  at FO!**

# What is Baryonic Mass Spectrum?

Consider baryon density and write it as integral of mass spectrum

$$\begin{aligned}
 \rho_B &= C_B T^{A_B-1} \exp\left[\frac{\mu_B - m_B}{T}\right] = C_B T^{A_B-2.5} \exp\left[\frac{\mu_B - m_B}{T}\right] \underbrace{T^{\frac{3}{2}}}_{d^3k \text{ integr.}} \\
 &= C_B \exp\left[\frac{\mu_B}{T}\right] \int_{m_B}^{\infty} dm \frac{(m - m_B)^{A_B-3.5}}{\Gamma(A_B - 2.5)} \underbrace{T^{\frac{3}{2}}}_{d^3k \text{ integr.}} \exp\left[-\frac{m}{T}\right] \\
 &= (2\pi)^{\frac{3}{2}} C_B \exp\left[\frac{\mu_B}{T}\right] \int_{m_B}^{\infty} dm \frac{(m - m_B)^{A_B-3.5}}{m^{\frac{3}{2}} \Gamma(A_B - 2.5)} \int \frac{d^3k}{(2\pi)^3} \exp\left[-\frac{\sqrt{k^2 + m^2}}{T}\right]
 \end{aligned}$$

Since Laplace representation is unique  $\Rightarrow$  Effective baryonic mass spectrum

$$W_B(m) \simeq (m - m_B)^{A_B-3.5} m^{-\frac{3}{2}} = (m - m_B)^{1.12} m^{-\frac{3}{2}}$$

Is uniquely defined by chem. FO data!



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**Surprise No II: effective baryonic mass spectrum is power-like!  
 $\Rightarrow$  chemical FO data for rHIC evidence for NO Hagedorn  
 spectrum for baryons!**

# What is Mesonic Mass Spectrum?

$\mu_M - m_M \simeq 5 \text{ MeV}$  deliberately means that  $m_M \ll T$

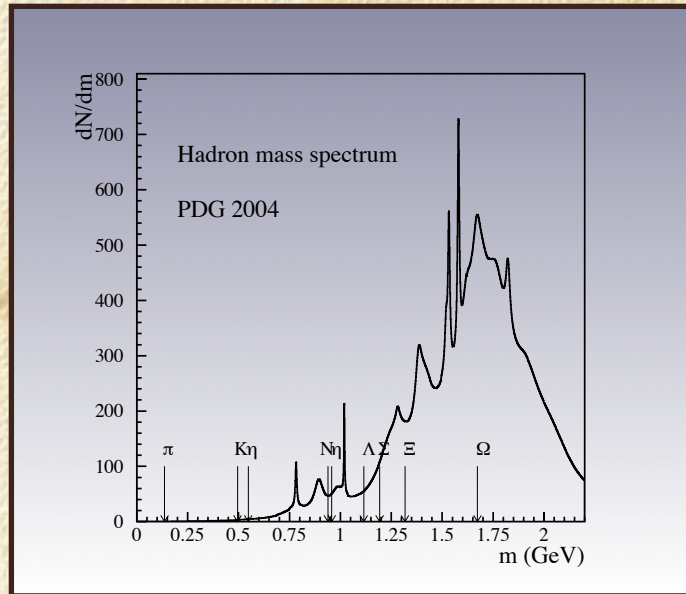
⇒ Effective mesons are ultrarelativistic

⇒ one finds effective mesonic mass spectrum as

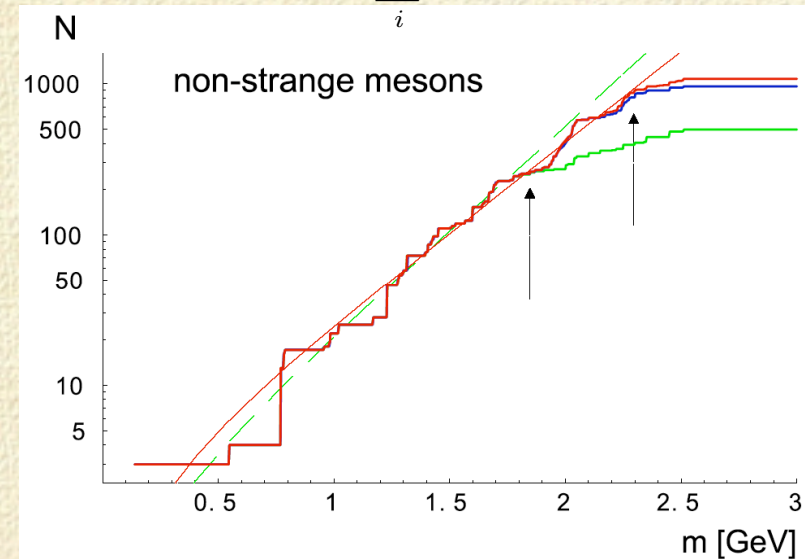
$$W_M(m) \simeq (m - m_M)^{A_M - 4} \simeq (m - m_M)^{1.31}$$

**Surprise No III: effective mesonic mass spectrum is NOT a Hagedorn mass spectrum!  
But what is the reason for this?**

# Problems with Empirical Mass Spectrum



$$N_{\text{exp}}(m) = \sum_i g_i \Theta(m - m_i),$$



1. In fact, at low masses it can be a non-exponential one!
2. For  $m > 2.5 \text{ GeV}$  there is a huge deficit of heavy hadrons compared to Hagedorn mass spectrum!
3. If it exists, then it is observed not where it was predicted, i.e. for  $1.3 \text{ GeV} < m < 2.5 \text{ GeV}$ !

# But Hagedorn Spectrum Follows from

Stat.Bootstrap Model,  
S.Frautschi, 1971

Hadrons are built from hadrons

Veneziano Model,  
K.Huang,S.Weinberg,  
1970

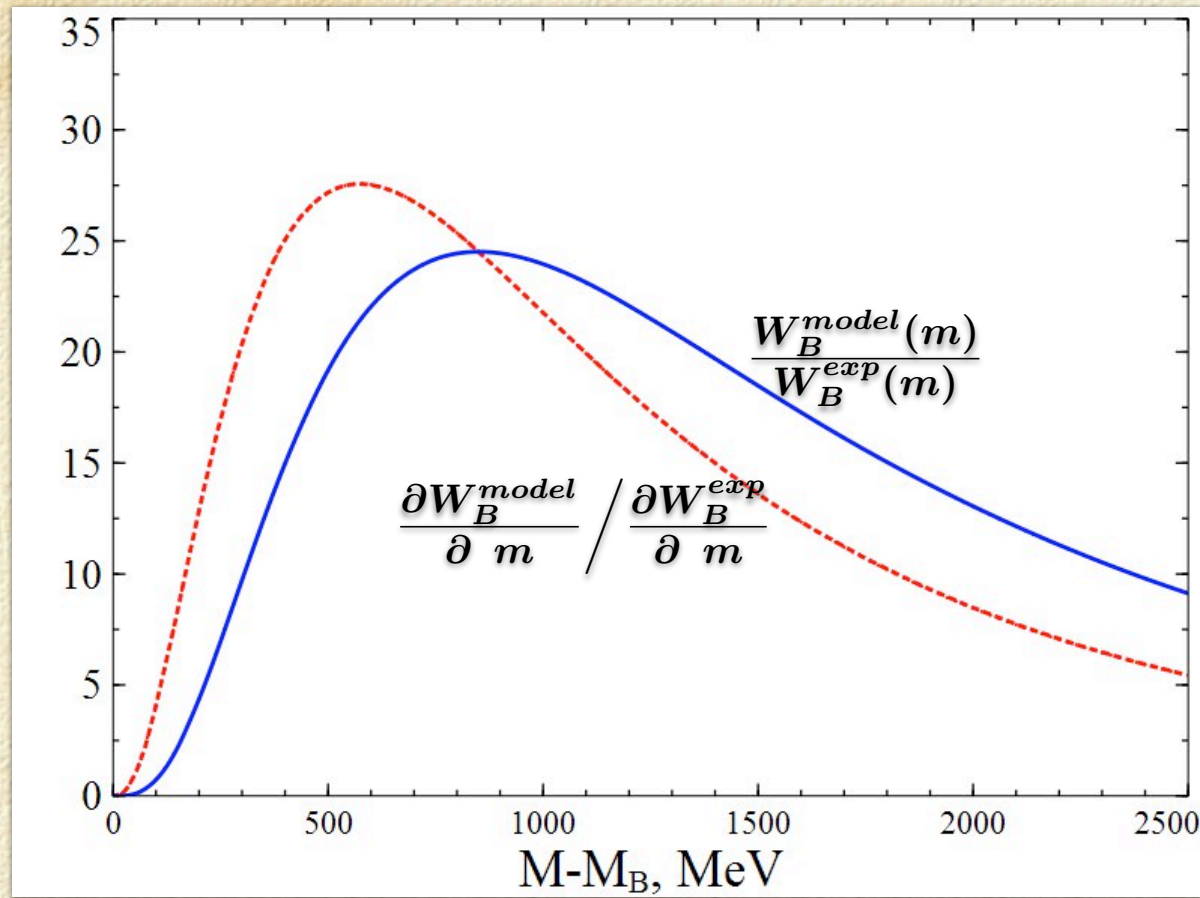
Used in string models

M.I.T. Bag Model,  
J.Kapusta, 1981

Hadrons are quark-gluon bags

Large  $N_c$  limit of 3+1  
QCD  
T. Cohen, 2009

# Comparison of Baryonic Mass Spectra



**Surprise No IV: a HUGE disagreement!**  
**What is the reason? Bad model or what?**

# Wide Resonances in Thermal Media

Wide resonances are VERY important in a thermal model.  
For instance, description of pions cannot be achieved without

$\sigma$  meson:  $m_\sigma = 484 \pm 24$  MeV, width  $\Gamma_\sigma = 510 \pm 20$  MeV

R. Garcia-Martin, J. R. Pelaez and F. J. Yndurain, PRD (2007) 76

$$n_X^{tot} = n_X^{thermal} + n_X^{decay} = n_X^{th} + \sum_Y n_Y^{th} Br(Y \rightarrow X)$$

$Br(Y \rightarrow X)$  is decay branching of Y-th hadron into hadron X

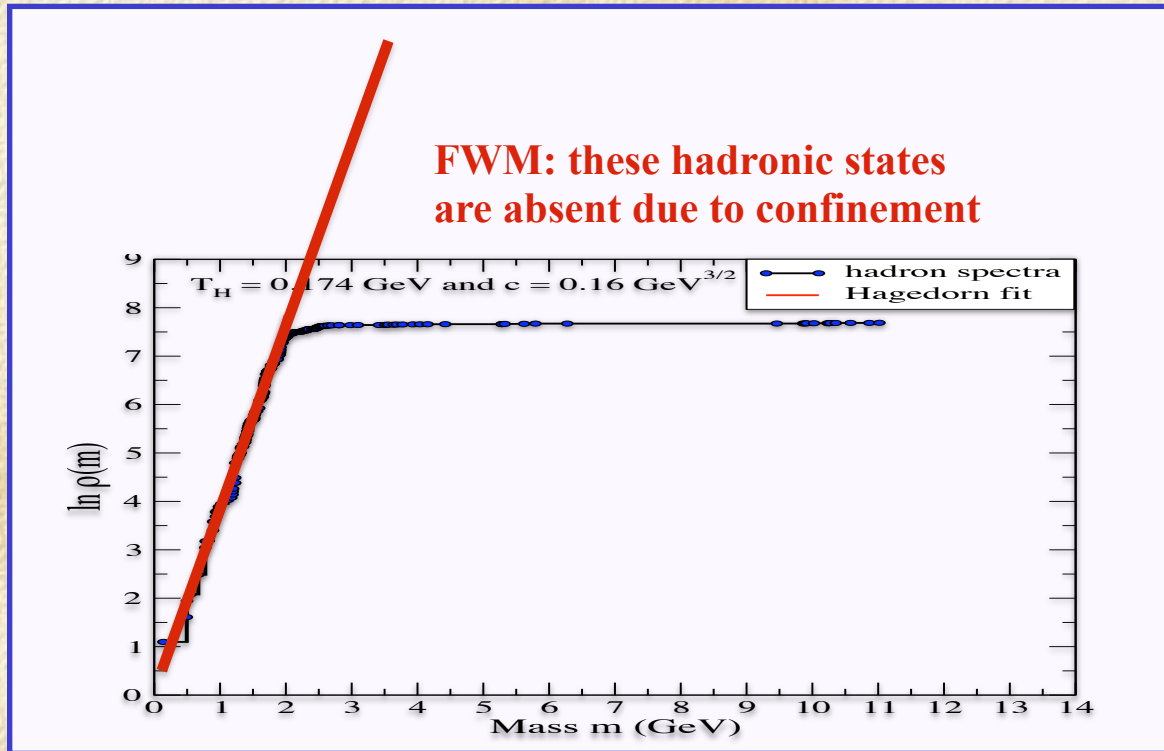
Large width of QG bags is absolutely necessary to explain the deficit in the number of observed heavy resonances compared to Hagedorn mass spectrum and to model

a color confinement phenomenon. See the finite width model of QG bags

K.A.B., V.K. Petrov, G.M. Zinovjev, Europhys. Lett. (2009) 85; PRC (2009) 79

Unfortunately, there is NO THEORY how to do this rigorously for an ensemble of about 150 different hadronic (mesonic & baryonic) species...

# Wide Resonances in Thermal Media



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R. Pelaez and F. J. Yndurain, PRD (2007) 76

$$h + \sum_Y n_Y^{th} Br(Y \rightarrow X)$$

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# Resonance Contribution in Thermal Media

Therefore, at chemical freeze-out we substitute

D. Hahn and H. Stoecker, Nucl. Phys. A (1986) 452  
K.G. Denisenko and St. Mrowczynski, PRC (1987) 35

$$\int \exp\left(-\frac{\sqrt{k^2 + m_j^2}}{T}\right) d^3k \rightarrow \frac{\int_{M_j^{Th}}^{\infty} dx \rho_j(x) \int \exp\left(-\frac{\sqrt{k^2 + x^2}}{T}\right) d^3k}{\int_{M_j^{Th}}^{\infty} dx \rho_j(x)},$$

$M_j^{Th}$  is a threshold of dominant decay channel

Mass distribution of j-th resonance is

$$\rho_j(x) = \begin{cases} \frac{1}{(x-m_j)^2 + \Gamma_j^2/4}, & \text{for hadrons with } m_j \leq 2.5 \text{ GeV} \\ \exp\left[-\frac{(m_j - x)^2}{2\sigma_j^2}\right] & \text{for hadrons \& QG bags with } m_j > 2.5 \text{ GeV} \end{cases}$$

**used in all advanced thermal models**

**used in finite width model of QG bags**

Gaussian width is  $\sigma_j = \Gamma_j/Q \simeq \Gamma_j/2.355$

K.A.B., V.K. Petrov, G.M. Zinovjev,  
Europhys. Lett. (2009) 85; PRC (2009) 79



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**This approximation is usually criticized, but**

- 1. within approaches that employ a few hadronic dof**
- 2. at least at low T the deviation from elaborate formulae is about experimental uncertainty for the Delta 1232 peak**

W. Weinhold, B. Friman, W. Noerenberg, PLB (1998) 433

However, since more elaborate approximations lead to huge complications without qualitative and quantitative improvements, the thermal models use this one at chemical freeze-out.

# Sigma Meson at Low T

In nonrelativistic approximation  $M_k^{Th} \gg T$

$$\Rightarrow \int \frac{d^3 p}{(2\pi)^3} \exp \left[ -\frac{\sqrt{p^2 + m^2}}{T} \right] = \left[ \frac{mT}{2\pi} \right]^{\frac{3}{2}} \exp \left[ -\frac{m}{T} \right]$$

For convenience let's use the Gaussian attenuation:

$$\underbrace{\Theta(m - M_k^{Th}) \exp \left[ -\frac{(m_k - m)^2}{2\sigma_k^2} \right]}_{\text{grows fast near threshold}} \overbrace{\exp \left[ -\frac{m}{T} \right]}^{\text{decreases fast}} \underbrace{\left[ \frac{mT}{2\pi} \right]^{\frac{3}{2}}}_{\simeq \text{const}}$$

After making a full square in exponential  $\Rightarrow$

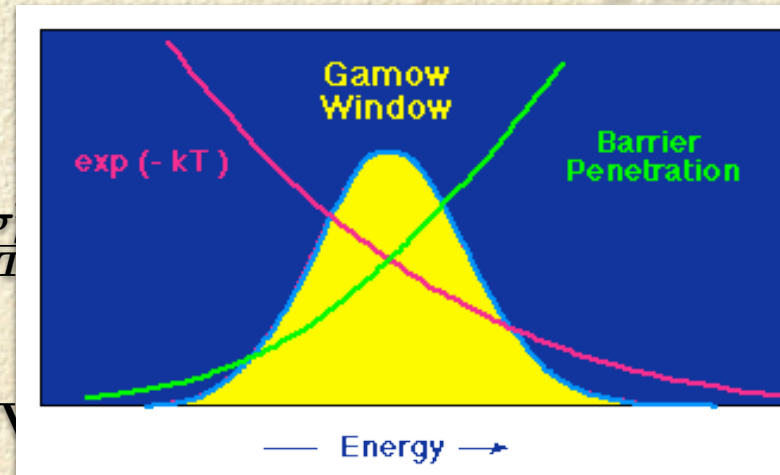
$$\Theta(m - M_k^{Th}) \underbrace{\exp \left[ -\frac{(m_k - \frac{\sigma_k^2}{T} - m)^2}{2\sigma_k^2} \right]}_{\text{peak is shifted to threshold}} \overbrace{\exp \left[ \frac{\sigma_k^2}{2T} \right]}^{\text{leads to enhancement}} \exp \left[ -\frac{m_k}{T} \right] \underbrace{\left[ \frac{mT}{2\pi} \right]^{\frac{3}{2}}}_{\simeq \text{const}}$$

The shifted peak goes UNDER threshold, if

$$M_k^{Th} \geq \tilde{m}_k \equiv m_k - \frac{\sigma_k^2}{T}$$

For  $\sigma$  meson  $\Rightarrow$

$$M_\sigma^{Th} = 2m_\pi \simeq 280 \text{ MeV}$$



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$$\underbrace{\Theta(m - M_k^{Th}) \exp \left[ -\frac{(m_k - m)^2}{2\sigma_k^2} \right]}_{\text{grows fast near threshold}} \overbrace{\exp \left[ -\frac{m}{T} \right]}^{\text{decreases fast}} \underbrace{\left[ \frac{mT}{2\pi} \right]^{\frac{3}{2}}}_{\simeq \text{const}}$$

After making a full square in exponential  $\Rightarrow$

$$\Theta(m - M_k^{Th}) \underbrace{\exp \left[ -\frac{(m_k - \frac{\sigma_k^2}{T} - m)^2}{2\sigma_k^2} \right]}_{\text{peak is shifted to threshold}} \overbrace{\exp \left[ \frac{\sigma_k^2}{2T} \right]}^{\text{leads to enhancement}} \exp \left[ -\frac{m_k}{T} \right] \underbrace{\left[ \frac{mT}{2\pi} \right]^{\frac{3}{2}}}_{\simeq \text{const}}$$

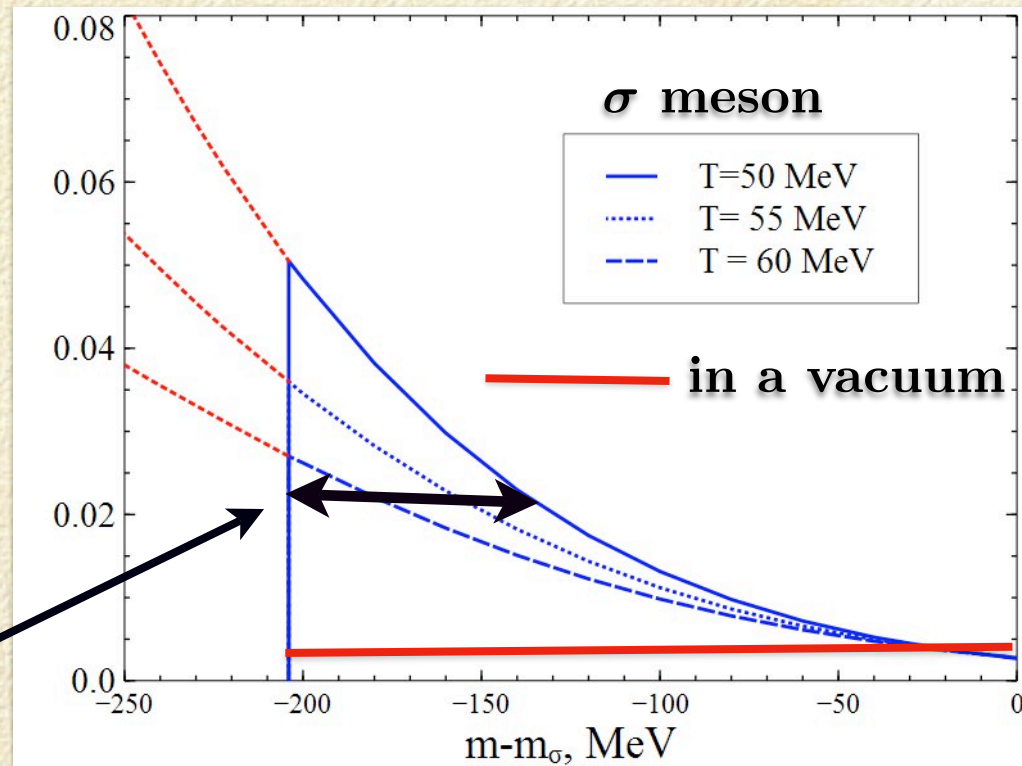
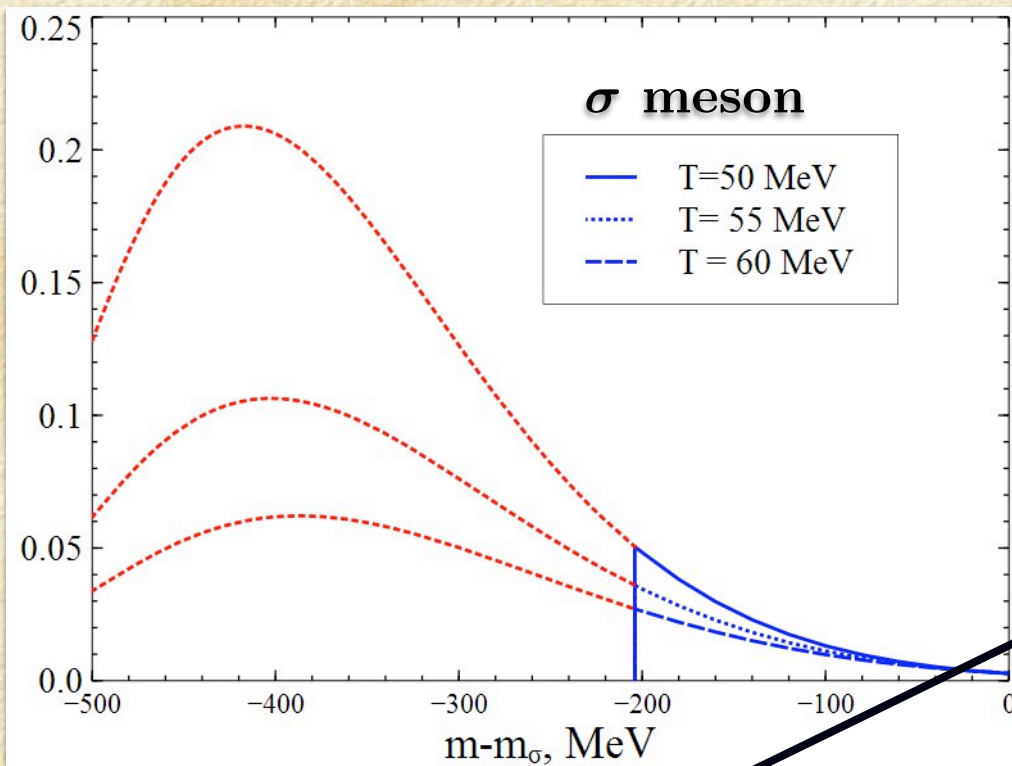
The shifted peak goes UNDER threshold, if

$$M_k^{Th} \geq \tilde{m}_k \equiv m_k - \frac{\sigma_k^2}{T} \Leftrightarrow \text{for } T < T_k^+ \equiv \frac{\sigma_k^2}{m_k - M_k^{Th}}$$

For  $\sigma$  meson  $\Rightarrow$

$$M_\sigma^{Th} = 2m_\pi \simeq 280 \text{ MeV} \quad \text{and} \quad T_\sigma^+ \simeq 92 \text{ MeV}$$

# Wide Resonance Sharpening and Enhancement Near Threshold



$\sigma$  meson effective width decreased in 7 times!

$$\Gamma_{\sigma}^{eff}(T = 50 \text{ MeV}) \simeq 62.5 \text{ MeV}, \quad \Gamma_{\sigma}^{eff}(T = 55 \text{ MeV}) \simeq 71.5 \text{ MeV},$$

$$\Gamma_{\sigma}^{eff}(T = 60 \text{ MeV}) \simeq 82.5 \text{ MeV}$$

Is well described by the formula

$$\Gamma_k^N(T) \simeq \frac{\ln(2)}{\frac{1}{T} - \frac{1}{T_k^+} - \frac{3}{2 M_k^{Th}}}$$

# Examples of Wide Resonance Sharpening Near Threshold at T=50 MeV

$$\Gamma_k^N(T) \simeq \frac{\ln(2)}{\frac{1}{T} - \frac{1}{T_k^+} - \frac{3}{2M_k^{Th}}}$$

Hadron	$m_k$ (MeV)	$\Gamma_k$ (MeV)	Decay channel	$M_k^{Th}$ (MeV)	$\beta_k$	$T_k^+$ (MeV)	$\Gamma_k^{eff}$ (MeV)	$\Gamma_k^N$ (MeV)
$\sigma$ -meson	484	510	$\sigma \rightarrow \pi\pi$	280	0.942	91.9	62.5	67.3
$P_{33}$	1232	120	$\Delta \rightarrow \pi N$	1080	2.98	11.6	43.5	N/A
$P_{11}$	1440	350	$N \rightarrow \pi N$	1080	2.42	38.74	129.5	N/A
$P_{33}$	1600	350	$\Delta \rightarrow \pi\Delta$	1372	1.53	50.4	68.7	80.8
$P_{33}$	1600	350	$\Delta \rightarrow \pi N$	1080	3.5	30.3	280.	N/A
$G_{17}$	2190	500	$\Delta \rightarrow \rho N$	1710	2.26	57.8	74.6	81.8

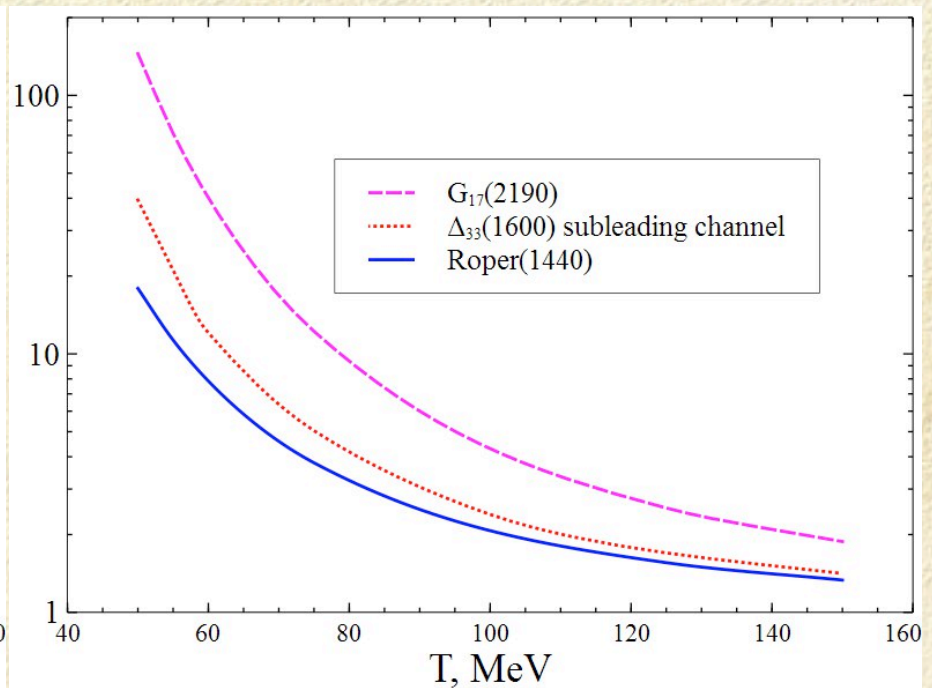
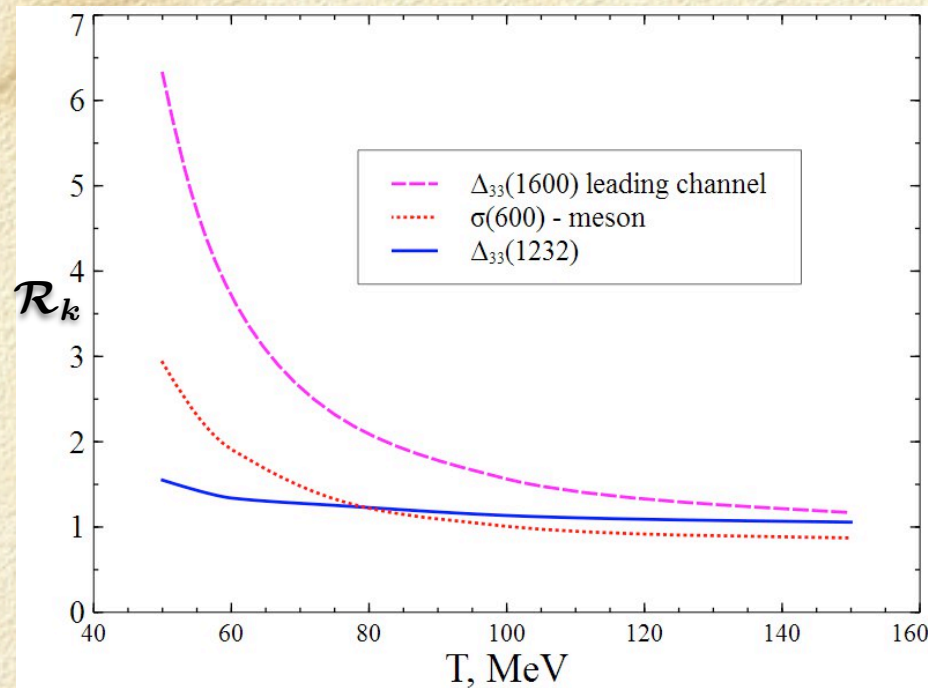
in vacuum

in media

**Conclusion: at low temperatures an effective width of wide resonances can be really small!**

# Examples of Wide Resonance Enhancement Near Threshold

k-th resonance enhancement factor  $\mathcal{R}_k = \frac{\rho_k(T, \Gamma_k)}{\rho_k(T, \Gamma_k \rightarrow 0)}$   $\rho_k(T, \Gamma_k)$  is thermal density



**Conclusion No I: The wide resonances can essentially modify the resulting mass spectrum at chemical FO!**

**Conclusion No II: There is no sense to discuss the hadron mass spectrum without accounting for resonance width!**

# A Few Remarks

1. Results are shown for extreme case, but formula

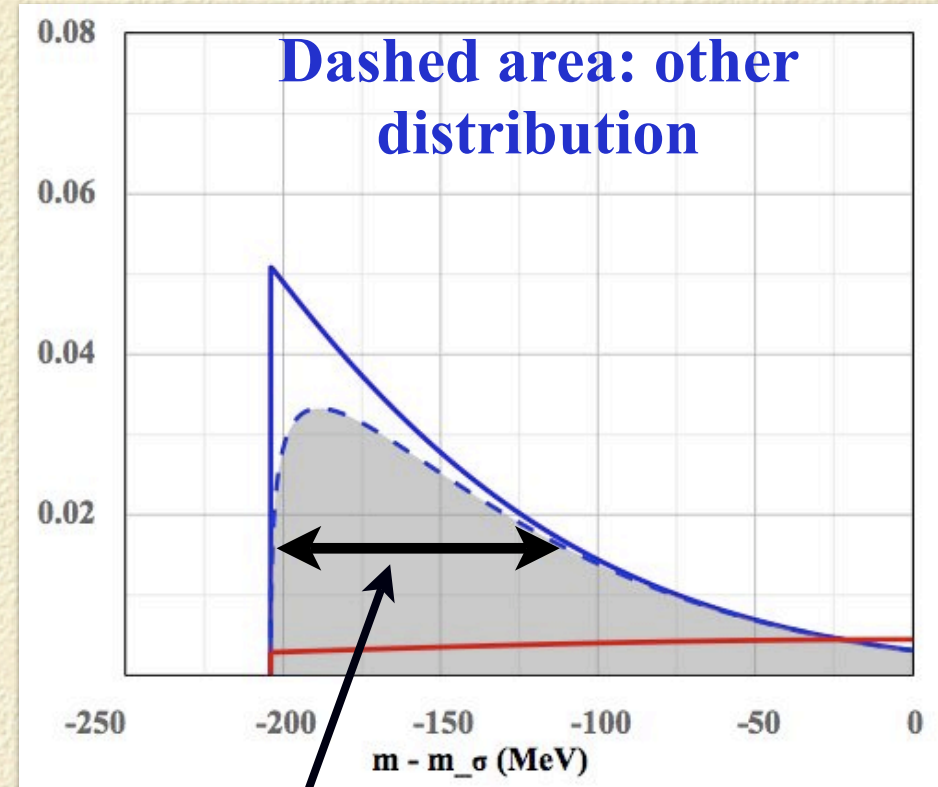
$$\Gamma_k^N(T) \simeq \frac{K}{\frac{1}{T} - \frac{1}{T_k^+} - \frac{3}{2 M_k^{Th}}}$$

with  $K \in [0.7; 1.5]$

works well for other parameterizations!

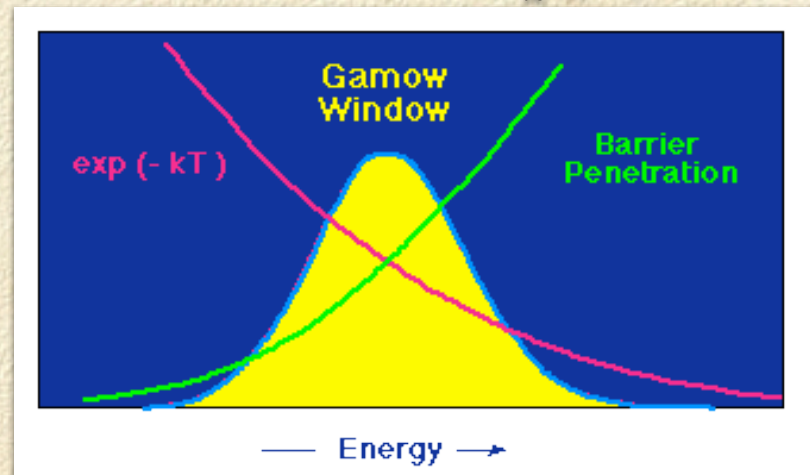
2. Source of resonance distribution has many similarities with famous Gamov window for thermonuclear reactions of charged particles!

The main difference is due threshold



Enhancement is 0.864 of blue line

Effective width is  $\Gamma_k^N(T) \simeq 90$  MeV



# Finite Width Model of QG Bags

Finite width model of QG bags predicts that

K.A.B., V.K. Petrov, G.M. Zinovjev, *Europhys. Lett.* (2009) 85

The bag width is  $\Gamma_B \simeq \Gamma_0(T) \left[ \frac{M_B}{M_0} \right]^{\frac{1}{2}}$

with  $M_0 \simeq 2.5 \text{ GeV}$  and

$$\Gamma_0(T) \simeq \begin{cases} 400 \text{ MeV}, & \text{if } T = 0 \text{ MeV}, \\ 800 \text{ MeV}, & \text{if } T \simeq 90 \text{ MeV}, \\ 1400 \text{ MeV}, & \text{if } T \simeq 170 \text{ MeV} \end{cases}$$

Finite width model was successfully verified on a variety of lattice QCD thermodynamics data

K.A.B., V.K. Petrov, G.M. Zinovjev, *PRC* (2009) 79

For  $T=0$  this width relation perfectly describes the imaginary part of leading Regge trajectories for  $\rho_{J--}$ ,  $\omega_{J--}$ ,  $a_{J++}$  and  $f_{J++}$  mesons with the spins  $J < 7$

K.A.B., E.G. Nikonov, A.S. Sorin, G.M. Zinovjev, *JHEP02* (2011) 059



# Application to Quark Gluon Bags

$$\Gamma_B^N(T) \simeq \frac{K}{\frac{1}{T} - \frac{1}{T_k^+}} \Big|_{T \ll T_B^+} \rightarrow T K, \quad \text{where } K \in [0.7; 1.5]$$

Thus, the lower is T, the sharper is bag!

However, the finite width model predicts a huge suppression for  $T < 80$  MeV  $\Rightarrow$  one can hope that for  $T = 90-120$  MeV the QG bag width can be between 60 and 160 MeV!

$\Rightarrow$  Perhaps the QG bags can be observed as sharp resonances with mass about 2.5 GeV which are absent in Particle Data Group

$\Rightarrow$  Such a hypothesis can be verified at NICA energies!

# Further Estimates

For a decay into  $L$  pions ( $L = 1-5$ ) one finds

$$T_B^+ \simeq \frac{[\Gamma_0(T)]^2}{Q^2 M_0 \left(1 - \frac{M_B^{Th}}{M_B}\right)} \simeq \left(1 + \frac{M_B^{Th}}{L m_\pi}\right) \cdot \begin{cases} 11.5 \text{ MeV, if } T = 0 \text{ MeV,} \\ 46.2 \text{ MeV, if } T \simeq 90 \text{ MeV,} \\ 141 \text{ MeV, if } T \simeq 170 \text{ MeV} \end{cases}$$

**Due to Boltzmann factor it is better to keep threshold mass about 2.5 GeV**

**$\Rightarrow$  For  $T = 90$  MeV the necessary condition for  $T_B^+$  is satisfied for  $L < 6$**

# Conclusions

**Hadron Resonance Model is able to describe Strangeness Horn.**

**Chemical FO criteria are analyzed with Hadron Resonance Model.**

**A new chemical FO criterion of constant entropy per particle is suggested. It is shown that such a criterion is robust.**

**Model for adiabatic chemical FO is developed. It evidences for absence of the Hagedorn mass spectrum for mesons and baryons!**

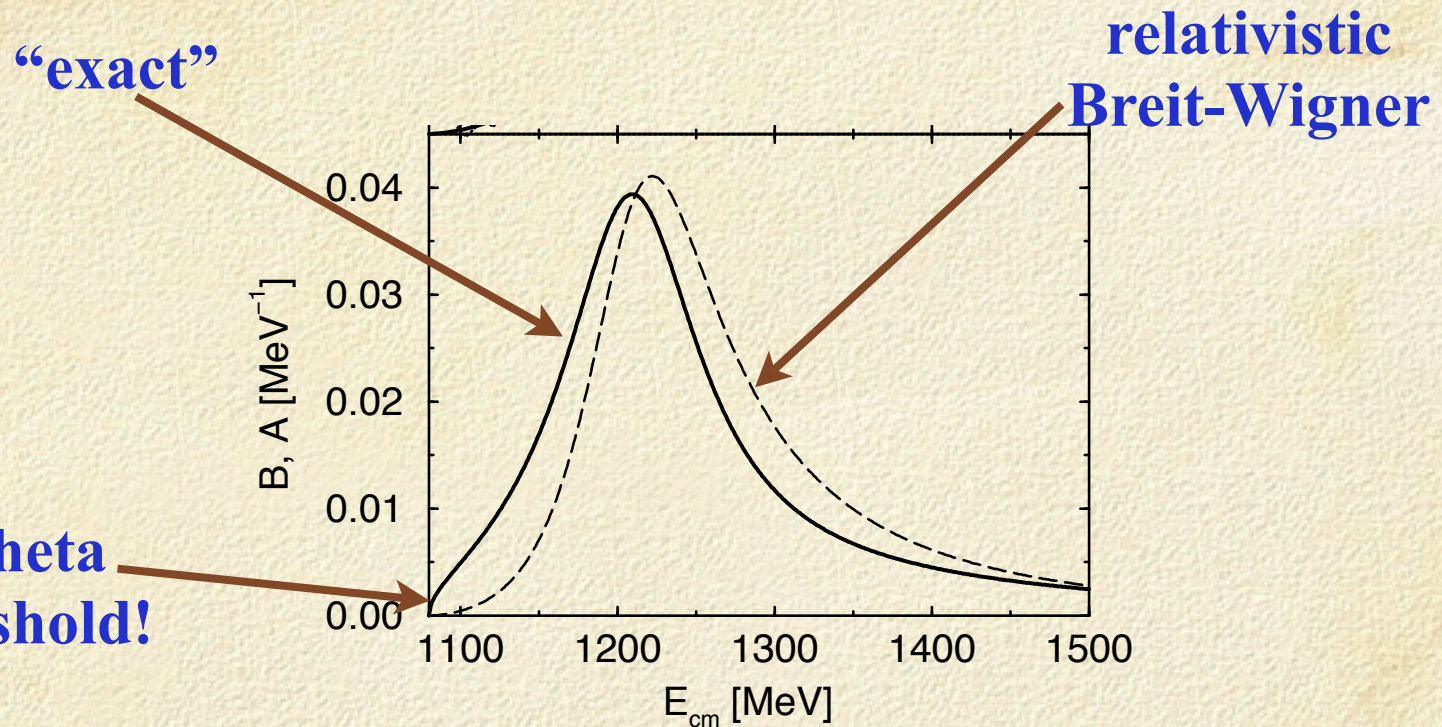
**The absence of the Hagedorn mass spectrum is explained by the wide resonance width modification and by resonance enhancement.**

**An effective width of QG bags is analyzed. QG bags presence in the experiments can be detected via appearance of narrow resonances of mass above 2.5 GeV that are absent in PDG.**

**Thanks for your attention!**

# Answer to Critique

## Weight function of Delta isobar



**W. Weinhold, B. Friman, W. Noerenberg, PLB (1998)433**

Fig. 2. Upper part: Fit of  $\delta_{33}$  from eq. (10) to the empirical  $P_{33}$  phase shifts [19]; Lower part: Weight function  $\mathcal{B}$  (solid line), see eqs. (6, 11), compared to the spectral function  $\mathcal{A}$  (dashed line).

**For Delta isobar the shift of profile is about 20 MeV, while the Delta width error is +/-10 MeV!**

**Since for most of heavier resonances the error in width and mass is between 100 and 200 MeV there is no sense to talk about such tiny defects of a simple approximations!**