Effects of the final-state interaction for the exclusive deuteron electrodisintegration

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Motivation

The electrodisintegration of the deuteron is a useful tool to investigate the electromagnetic structure of the deuteron and unbound neutron-proton (np) system.
The separable kernels proposed in previous talk make possible to avoid non-integrable singularities appeared in the calculations. Using these kernels final-state interaction (FSI) can be taken into account for the deuteron electrodisintegration in a wide range of energy.

Unpolarized exclusive cross-section

The 5-fold differential cross section in one-photon approximation (in laboratory system - LS)

$$\frac{d^3\sigma}{dE'_e d\Omega'_e d\Omega_p} = \frac{\sigma_{\text{Mott}}}{8M_d(2\pi)^3} \frac{p_p^2\sqrt{s}}{\sqrt{1+\eta}|p_p| - E_p\sqrt{\eta}\cos\theta_p}$$

$$\times \left[l^0_{00}W_{00} + l^0_{++}(W_{++} + W_{--}) + 2l^0_{+-}\cos 2\phi \ \text{Re}W_{+-} - 2l^0_{+-}\sin 2\phi \ \text{Im}W_{+-} \right.$$

$$\left. -2l^0_{0+}\cos\phi \ \text{Re}(W_{0+} - W_{0-}) - 2l^0_{0+}\sin\phi \ \text{Im}(W_{0+} + W_{0-}) \right]$$

$\sigma_{\text{Mott}} = (\alpha\cos\frac{\theta}{2}/2E_e\sin^2\frac{\theta}{2})^2$ - Mott cross section
$\alpha = e^2/(4\pi)$ - fine structure constant
M_d - mass of the deuteron, m - mass of the nucleon
$q = p_e - p'_e = (\omega, \boldsymbol{q})$ - momentum transfer
$p_e = (E_e, \boldsymbol{l})$ and $p'_e = (E'_e, \boldsymbol{l}')$ - initial and final electron momenta
Ω'_e - outgoing electron solid angle
p_p - momentum of outgoing proton
$\Omega_p = (\theta_p, \phi)$ - outgoing proton solid angle
$\eta = \boldsymbol{q}^2/s$ - Lorentz boost factor

Density matrices

The virtual photon density matrix

$$l^0_{00} = \frac{Q^2}{\boldsymbol{q}^2}, \quad l^0_{0+} = \frac{Q}{|\boldsymbol{q}|\sqrt{2}}\sqrt{\frac{Q^2}{\boldsymbol{q}^2} + \tan^2\frac{\theta}{2}},$$

$$l^0_{++} = \tan^2\frac{\theta}{2} + \frac{Q^2}{2\boldsymbol{q}^2}, \quad l^0_{+-} = -\frac{Q^2}{2\boldsymbol{q}^2}$$

here $Q^2 = -q^2$

The hadron density matrix

$$W_{\lambda\lambda'} = W_{\mu\nu}\varepsilon^{\mu}_{\lambda}\varepsilon^{\nu}_{\lambda'}$$

λ, λ' - photon helicity components

Cartesian components

$$W_{\mu\nu} = \frac{1}{3}\sum_{s_d s_n s_p} |< np : SM_S|j_\mu|d : 1M >|^2$$

ε - photon polarization vectors

S - spin of the np pair, M_S - projection

s_d, s_n and s_p deuteron, neutron and proton momentum projections

EM current matrix element $< np : SM_S|j_\mu|d : 1M >$
Matrix element within the relativistic impulse approximation (in LS)

$$< np : SM_S|j_\mu|d : 1M >= i \sum_{n=1,2} \int \frac{d^4p^{\mathsf{CM}}}{(2\pi)^4} \times$$

$$\mathrm{Sp}\left\{\Lambda(\mathcal{L}^{-1})\bar{\psi}_{SM_S}(p^{\mathsf{CM}}, p^{*\mathsf{CM}}; P^{\mathsf{CM}})\Lambda(\mathcal{L})\Gamma_\mu^{(n)}(q) \times\right.$$

$$\left. S^{(n)}\left(\frac{K_{(0)}}{2} - (-1)^n p - \frac{q}{2}\right)\Gamma^M\left(p + (-1)^n \frac{q}{2}; K_{(0)}\right)\right\}$$

P^{CM} - total, $p^{*\mathsf{CM}}$ - relative momenta of the outgoing nucleons
p^{CM} - relative momenta in the center-of-mass system (CM)
p - relative np pair momentum in LS (p, q)
$K_{(0)} = (M_d, \mathbf{0})$ - deuteron total momentum in LS
$S^{(n)}$ - propagator of the nth nucleon
$\mathcal{L}$ - Lorenz-boost transformation along the $\boldsymbol{q}$ direction

Λ - boost operator from CM to LS:

$$\Lambda(\mathcal{L}) = \left(\frac{1+\sqrt{1+\eta}}{2}\right)^{\frac{1}{2}} \left(1 + \frac{\sqrt{\eta}\gamma_0\gamma_3}{1+\sqrt{1+\eta}}\right),$$

$\Gamma_\mu^{(n)}$ - photon-nucleon interaction vertex (on-mass-shell):

$$\Gamma_\mu(q) = \gamma_\mu F_1(q^2) - \frac{1}{4m}\left(\gamma_\mu \not{q} - \not{q}\gamma_\mu\right) F_2(q^2), \quad (1)$$

ψ_{SM_S} - np pair wave function Γ^M - deuteron vertex function
are solutions of the Bethe-Salpeter equation with separable kernel

Complex separable kernel

$$V_{l'l}(p_0', |\mathbf{p}'|; p_0, |\mathbf{p}|; s) = \sum_{m,n=1}^{N} \left[\underline{\lambda_{mn}^r(s)} + i\lambda_{mn}^i(s)\right] \underline{g_i^{[l']}(p_0', |\mathbf{p}'|) g_j^{[l]}(p_0, |\mathbf{p}|)}$$

underlined part $\equiv$ MYN kernels,

$$\lambda_{mn}^i(s) = \theta(s - s_{th}) \left(1 - \frac{s_{th}}{s}\right) \bar{\lambda}_{mn}^i$$

s_{th} - the inelasticity threshold

Deuteron vertex function

$$\Gamma^{JM}(k;K_{(0)}) = \sum_a \mathcal{Y}_{aM}(-\mathbf{k})\ g_a(k_0,|\mathbf{k}|)$$

$\mathcal{Y}_{aM}$ - spin-angular parts
g_a - radial parts
a - ${}^3S_1, {}^3D_1$-states
radial parts

$$g_a(p_0,|\mathbf{p}|) = \sum_{i,j=1}^{N} \lambda_{ij}(s) g_i^{[a]}(p_0,|\mathbf{p}|) c_j(s)$$

coefficients

$$c_i(s) - \sum_{k,j=1}^{N} h_{ik}(s)\lambda_{kj}(s)c_j(s) = 0$$

with functions

$$h_{ij}(s) = -\frac{i}{4\pi^3}\sum_a \int dk_0 \int \mathbf{k}^2 d|\mathbf{k}| \frac{g_i^{[a]}(k_0,|\mathbf{k}|)g_j^{[a]}(k_0,|\mathbf{k}|)}{(\sqrt{s}/2 - E_{\mathbf{k}} + i\epsilon)^2 - k_0^2}$$

np pair wave function

BS amplitude of the np pair

$$\psi_{SM_S}(p,p^*;P) = \psi^{(0)}_{SM_S}(p,p^*;P) + \frac{i}{4\pi^3} S_2(p;P) \int d^4k\, V(p,k;P)\psi_{SM_S}(k,p^*;P)$$

Plane-wave approximation

$$\psi_{SM_S}(p,p^*;P) = \psi^{(0)}_{SM_S}(p,p^*;P) = (2\pi)^4 \chi_{SM_S}(p;P)\delta(p-p^*)$$

Interacting term

$$\psi^{(t)}_{SM_S}(p,p^*;P) = 4\pi i\, S_2(p;P) T(p,p^*;P) \chi_{SM_S}(p^*;P)$$

The partial-wave decomposition of the np-pair BS amplitude

$$\psi^{(t)}_{SM_S}(p, p^*; P) = 4\pi i \times \sum_{LmJMa} C^{JM}_{LmSM_S} Y^*_{Lm}(\hat{p}^*) \mathcal{Y}_{aM}(p) \phi_{a,J:LS+}(p_0, |p|; s), \tag{2}$$

where $p^* = (0, p^*)$ with $|p^*| = \sqrt{s/4 - m^2}$ is the relative momentum of on-mass-shell nucleons in CM, $\hat{p}^*$ denotes the azimuthal angle $\theta_{\boldsymbol{p}^*}$ between the p^* and $\boldsymbol{q}$ vectors and zenithal angle ϕ. Since only positive-energy partial-wave states are considered here the radial part is:

$$\phi_{a,J:LS+}(p_0, |p|; s) = \frac{t_{a,J:LS+}(p_0, |p|; 0, |p^*|; s)}{(\sqrt{s}/2 - E_{\boldsymbol{p}} + i\epsilon)^2 - p_0^2}. \tag{3}$$

Spin-angular parts

$$\mathcal{Y}_{aM}(p) = \frac{1}{\sqrt{8\pi}} \frac{1}{4E_{\boldsymbol{p}}(E_{\boldsymbol{p}}+m)} (m + \not{p}_1)(1+\gamma_0)\mathcal{G}_{aM}(m - \not{p}_2)$$

where matrices $\mathcal{G}_{aM}$

$a = \{^{2S+1}L_J^{\rho}\}$	$\mathcal{G}_{aM}$
$^1S_0^+$	$-\gamma_5$
$^3S_1^+$	$\not{\xi}_M$
$^1P_1^+$	$\frac{\sqrt{3}}{\lvert p \rvert}(p_1 \cdot \xi_M)\gamma_5$
$^3P_0^+$	$-\frac{1}{2\lvert p \rvert}(\not{p}_1 - \not{p}_2)$
$^3P_1^+$	$-\sqrt{\frac{3}{2}}\frac{1}{\lvert p \rvert}\left[(p_1 \cdot \xi_M) - \frac{1}{2}\not{\xi}_M(\not{p}_1 - \not{p}_2)\right]\gamma_5$
$^3D_1^+$	$\frac{1}{\sqrt{2}}\left[\not{\xi}_M + \frac{3}{2}\frac{1}{p^2}(p_1 \cdot \xi_M)(\not{p}_1 - \not{p}_2)\right]$

Spin-angular parts $\mathcal{G}_{aM}$ (11) for the np pair; $p_1 = (E_{\boldsymbol{p}}, p)$, $p_2 = (E_{\boldsymbol{p}}, -p)$ are on-mass-shell momenta, $E_{\boldsymbol{p}} = \sqrt{p^2 + m^2}$

Matrix element

Plane-wave approximation

$$< np : SM_S | j_\mu | d : 1M >^{(0)} = i \sum_{n=1,2} \{ \Lambda(\mathcal{L}^{-1}) \bar{\chi}_{SM_S}(p^{*\,\mathsf{CM}}; P^{\mathsf{CM}}) \Lambda(\mathcal{L}) \Gamma_\mu^{(n)}(q)$$

$$\times S^{(n)}(\frac{K_{(0)}}{2} - (-1)^n p^* - \frac{q}{2}) \Gamma^M(p^* + (-1)^n \frac{q}{2}; K_{(0)}) \}$$

$\Lambda(\mathcal{L})$ - Lorentz-boost matrix
$\Gamma_\mu^{(n)}(q) = \gamma_\mu F_1(q) + (\gamma_\mu \hat{q} - \hat{q}\gamma_\mu) F_2(q)$ - photon-NN vertex
$F_{1,2}(q)$ - electromagnetic nucleon form-factors

Matrix element

Final-state interaction

$$< np : SM_S|j_\mu|d : 1M >^{(t)} = \frac{i}{4\pi^3} \sum_{n=1,2} \sum_{LmJM_JL'lm'} C^{JM_J}_{LmJM_J} Y_{Lm}(\hat{p}^*)$$

$$\int_{-\infty}^{\infty} dp_0^{\mathbf{CM}} \int_0^{\infty} (p^{\mathbf{CM}})^2 d|p^{\mathbf{CM}}| \int_{-1}^{1} d\cos\theta_{\boldsymbol{p}}^{\mathbf{CM}} \int_0^{2\pi} d\phi$$

$$\mathrm{Sp}\left\{\Lambda(\mathcal{L}^{-1})\bar{\mathcal{Y}}_{JL'SM_J}(p^{\mathbf{CM}})\Lambda(\mathcal{L})\Gamma_\mu^{(n)}(q)S^{(n)}\left(\frac{K_{(0)}}{2} - (-1)^n p - \frac{q}{2}\right)\right.$$

$$\left.\mathcal{Y}_{JlSm'}\left(p + (-1)^n \frac{\boldsymbol{q}}{2}\right)\right\}$$

$$\frac{t^*_{L'L}(p_0^{\mathbf{CM}}, |p^{\mathbf{CM}}|; 0, |p^*|; s)}{(\sqrt{s}/2 - E_{\boldsymbol{p}} + i\epsilon)^2 - p_0^2} g_l\left(p_0 + (-1)^n \frac{\omega}{2}, p + (-1)^n \frac{\boldsymbol{q}}{2}; K_{(0)}\right)$$

Calculations

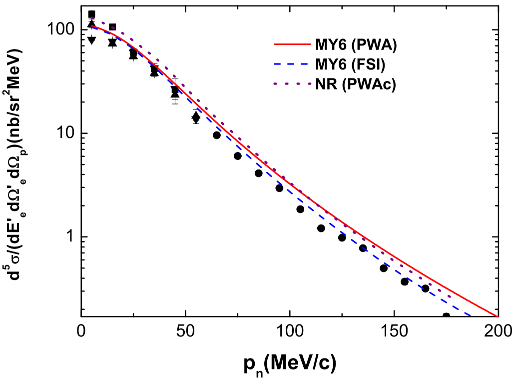
- calculate trace in MAPLE, perform analytic integration over ϕ and convert expressions to FORTRAN
- analyze the poles in complex p_0 plane (poles from propagators, radial parts of deuteron vertex function and np-pair amplitude can cross the Wick rotation contour and give additional contribution)
- perform (3,2,1)-fold numerical integrations in FORTRAN

Sacley data

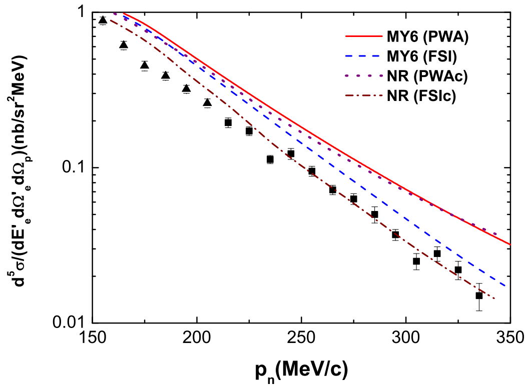
S_I, S_{II}– M. Bernheim et al., Nucl. Phys. **A365**, 349 (1981).
S_{III}– S. Turck-Chieze et al., Phys. Lett. **B142**, 145 (1984).

S.G. Bondarenko, V.V. Burov, E.P. Rogochaya. "Final-state interaction effects in electrodisintegration of the deuteron within the Bethe-Salpeter approach". JETP Lett. 94 (2012) 3.

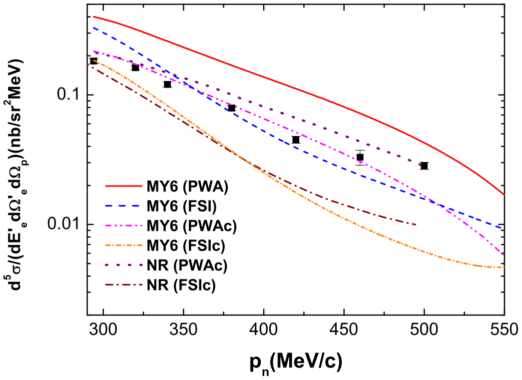
		S_I	S_{II}	S_{III}
E_e, GeV		0.500	0.500	0.560
E'_e, GeV		0.395	0.352	0.360
θ, $^\circ$		59	44.4	25
p_n, GeV/c	min	0.005	0.165	0.294
	max	0.350	0.350	0.550
θ_n, $^\circ$	min	101.81	172.07	153.01
	max	37.78	70.23	20.81
θ_{qe}, $^\circ$	min	48.79	44.74	33.06
p_p, GeV/c	min	0.451	0.514	0.557
	max	0.276	0.403	0.306
θ_p, $^\circ$	min	0.622	2.54	13.86
	max	51.03	54.90	140.28
θ_{pe}, $^\circ$	min	49.41	47.28	46.92
	max	99.81	99.64	173.35
$\sqrt{s}$, GeV		1.929	1.993	2.057
$\sqrt{s}-2m$, GeV		0.051	0.115	0.176
Q^2, (GeV/c)^2		0.192	0.101	0.038
ω, GeV		0.105	0.148	0.200
$\lvert\boldsymbol{q}\rvert$, GeV/$c$		0.450	0.350	0.279



Cross section depending on recoil neutron momentum $|p_n|$ calculated under kinematic conditions set I of the Sacley experiment (S_I). MY6 (PWA) (red solid line) - relativistic calculation in the plane-wave approximation with the MY6 potential; MY6 (FSI) (blue dashed line) - relativistic calculation including FSI effects; NR (PWAc) (violet dotted line) - nonrelativistic calculation (Shebeko *et al.*)



The same as in previous figure but under kinematic conditions set II of the Scaly experiment (S_{II}). The nonrelativistic calculation NR (FSIc) (brown dashed-dotted line) includes FSI effects



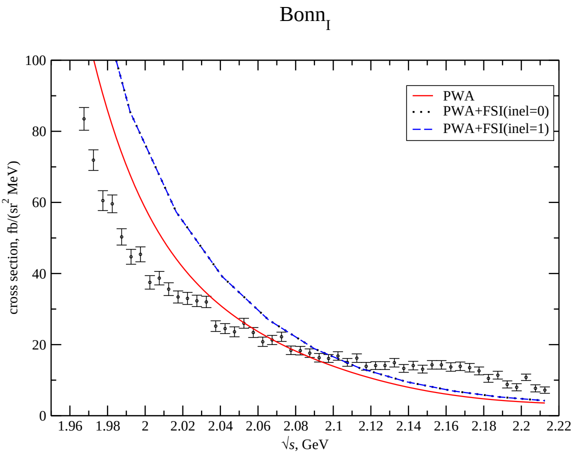
The same as in previous figures but under kinematic conditions set III of the Scale experiment (S_{III}). Two additional results are presented for comparison: MY6 (PWAc) (pink dashed-dotted-dotted line) - relativistic PWA calculation; MY6 (FSIc) (orange dashed-dotted line) - relativistic calculation with FSI effects; both obtained under current conservation condition $\omega J_0 = q_z J_z$

Bonn data

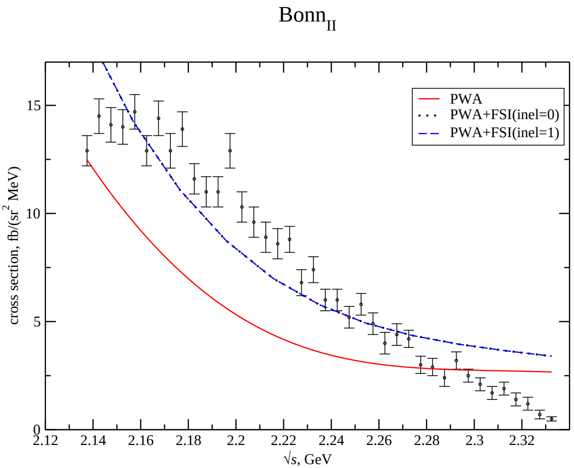
B_I, B_{II}– H. Breuker *et al.*, Nucl. Phys. **A455**, 641 (1986).
B_{III}, B_{IV}, B_V– B. Boden *et al.*, Nucl. Phys. **A549**, 471 (1992).

S.G. Bondarenko, V.V. Burov, E.P. Rogochaya. "Inelasticity of the NN-kernel for the final-state interaction in the deuteron breakup". In preparation

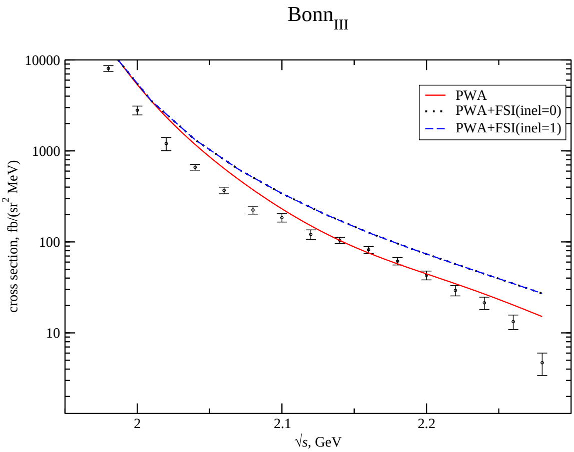
		B_I	B_{II}	B_{III}	B_{IV}	B_V
E, GeV		1.464	1.569	1.2	1.2	1.2
E', GeV	min	1.175	1.118	0.895	0.895	0.895
	max			0.800	0.800	0.800
θ, $^\circ$		21	21	20.15	20.15	20.15
p_n, GeV/c	min	0.314	0.500	0.126	0.197	0.197
	max	0.660	0.773	0.564	0.423	0.488
p_p, GeV/c	min	0.466	0.681	0.525	0.620	0.622
	max	0.664	0.791	0.834	0.929	0.889
$\sqrt{s}$, GeV	min	1.9675	2.1375	1.98	2.04	2.04
	max	2.2125	2.3325	2.28	2.28	2.28
$\sqrt{s} - 2m$, GeV	min	0.090	0.260	0.101	0.161	0.161
	max	0.335	0.455	0.401	0.401	0.401
Q^2, GeV$^2/c^2$	min	0.257	0.255	0.154	0.145	0.145
	max	0.206	0.209	0.106	0.106	0.106
ω, GeV	min	0.162	0.348	0.148	0.210	0.210
	max	0.422	0.568	0.476	0.476	0.476
q_z, GeV/c	min	0.532	0.613	0.420	0.435	0.435
	max	0.620	0.729	0.577	0.577	0.577



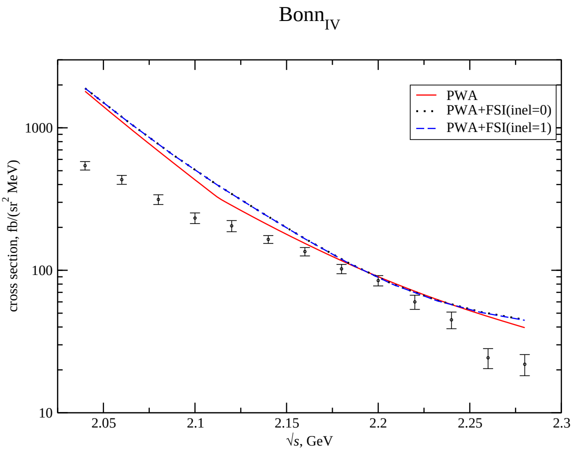
Cross section depending on $\sqrt{s}$ - invariant mass of the np-pair - calculated under kinematic conditions set I of the Bonn experiment (B$_I$). Solid red line - plane-way calculations, dotted black curve - final-state interaction calculations without taking into account inelasticities in NN kernel, dashed blue curve -



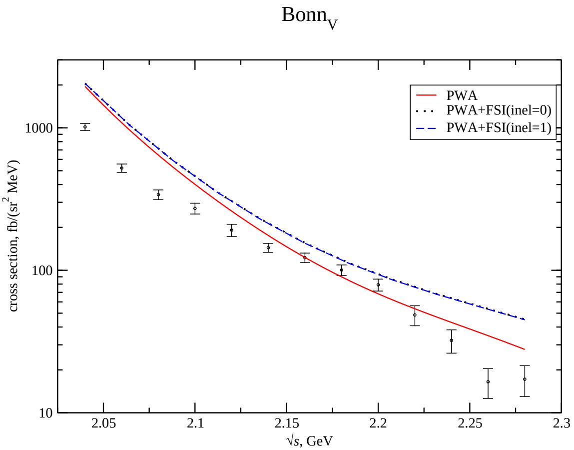
The same as in previous figure but under kinematic conditions set II of the Bonn experiment (B_{II}). Solid red line - plane-way calculations, dotted black curve - final-state interaction calculations without taking into account inelasticities in NN kernel, dashed blue curve - with inelasticities



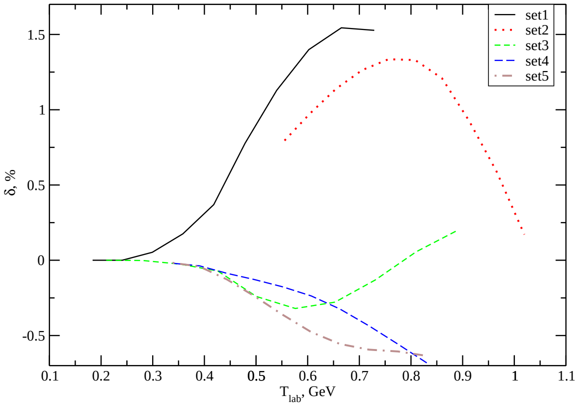
The same as in previous figure but under kinematic conditions set III of the Bonn experiment (B_{III}). Solid red line - plane-way calculations, dotted black curve - final-state interaction calculations without taking into account inelasticities in NN kernel, dashed blue curve - with inelasticities



The same as in previous figure but under kinematic conditions set IV of the Bonn experiment (B$_{IV}$). Solid red line - plane-way calculations, dotted black curve - final-state interaction calculations without taking into account inelasticities in NN kernel, dashed blue curve - with inelasticities



The same as in previous figure but under kinematic conditions set V of the Bonn experiment (B_V). Solid red line - plane-way calculations, dotted black curve - final-state interaction calculations without taking into account inelasticities in NN kernel, dashed blue curve - with inelasticities



δ = FSI(inel=1)/FSI(inel=0)-1, in % for all Bonn experiments kinematic conditions

Conclusion

- The multirank separable kernels of the neutron-proton interaction for states with the total angular momentum J=0,1 are used to calculate final-state interaction effects for the deuteron electrodisintegration.
- The effects of the FSI are small at low momentum-transfer squared and energy of np-pair but become sizable at higher values of them (tenth of per cents).
- The effects of the inelasticities are small (not exceed 1.5 %) in the region of the laboratory kinetic energy of the np-pair from 0.2 till 1.1 GeV for unpolarized cross-section.

Outlook

We have a powerful tool for investigation of the reactions with the deuteron (as well as reactions with the few-body systems).

Nearest plans

- To investigate the FSI effects for the JLab data.
- To investigate the influence of the FSI and complex part of the interaction kernel. to the polarization characteristics of the deuteron electrodisintegration