On the continuum limit of Landau gauge gluon and ghost propagators in SU(2) lattice gauge gluodynamics

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Abstract We continue the systematic computation of gluon and ghost propagators in the $SU(2)$ gluodynamics within the Landau gauge using a sequence of increasing lattice sizes L^4 up to $L = 112$ with corresponding β-values chosen to keep the linear physical size $a(\beta)L \simeq 9.6$ fm fixed. To minimize the Landau gauge functional we employ the simulated annealing method combined with subsequent overrelaxation. Renormalizing the propagators at momentum $\mu = 2.2$ GeV we see quite strong lattice artifacts for the gluon propagator within the momentum region $q < 1.0$ Gev. At fixed momenta we provide extrapolations to the continuum limit. We also discuss the continuum limit for the running coupling.

Gauge fixing: standard approach

In order to fix the Landau gauge we apply a gauge transformation $g(x)$ to link variables $U_{x,\mu} \in SU(3)$ or $SU(2)$) such that the gauge functional is maximized

$$F_U[g] = \sum_{x,\mu} \frac{1}{N_c} \mathfrak{Re}\,\mathrm{Tr}\; {}^gU_{x,\mu} \, .$$

⇒ For $A_\mu(x+\hat{\mu}/2) := (1/2ig_0)\left(U_{x,\mu} - U^\dagger_{x,\mu}\right)_{\mathrm{traceless}}$
this is equivalent to $\Delta_\mu A_\mu = 0\,,$

⇒ but not unique: Gribov copies,

⇒ search for global maxima -
fundamental modular region (FMR).

Standard prescription:

i) $g(x)$ taken with periodic b.c.'s,

ii) maximize $F_U[g]$ with overrelaxation (OR) method.

Drawbacks of OR:

i) substantial slowing down of OR convergence with increasing lattice extension L,

ii) its possibilities to find global maximum of $F_U[g]$ are strongly limited.

Simulated annealing: the principle

- Simulated annealing (SA) is a "stochastic optimization method" – here with the statistical weight $W[g] \propto \exp\{F_U[g]/T\}$ – allowing quasi-equilibrium tunnelings through functional barriers, in the course of a "temperature" T decrease.

- In principle - with infinitely slow cooling down - it allows to reach global extrema (contrary to OR, "tied" to the (initially chosen) local maximum).

- Control parameters at hand:

  i) N_{iter}, T_{max} and T_{min},

  ii) schedule for temperature steps $T_i,\ i = 1, ..., N_{iter}$ can be optimized.

$\Rightarrow$ The larger N_{iter} the higher the local maxima, $N_{iter} \to \infty \implies$ global maximum.

$\Rightarrow$ Schedule in practice: $T_{max} = 0.45$ for $SU(3)$, $T_{max} = 1.1$ for $SU(2)$, $T_{min} = 0.01$, $N_{iter} = O(5 \cdot 10^3 - 15 \cdot 10^3)$ with smaller (larger) T-steps close to T_{max} (close to T_{min}).

Lattice Faddeev-Popov operator

Lattice Faddeev-Popov operator can be written in terms of the (gauge-fixed) link variables $U_{x,\mu}$ as

$$M^{ab}_{xy} = \sum_{\mu} A^{ab}_{x,\mu}\, \delta_{x,y} - B^{ab}_{x,\mu}\, \delta_{x+\hat{\mu},y} - C^{ab}_{x,\mu}\, \delta_{x-\hat{\mu},y}$$

with

$$\begin{aligned} A^{ab}_{x,\mu} &= \quad \mathfrak{Re}\,\mathrm{Tr}\left[\{T^a, T^b\}(U_{x,\mu} + U_{x-\hat{\mu},\mu})\right], \\ B^{ab}_{x,\mu} &= 2 \cdot \mathfrak{Re}\,\mathrm{Tr}\left[T^b T^a\, U_{x,\mu}\right], \\ C^{ab}_{x,\mu} &= 2 \cdot \mathfrak{Re}\,\mathrm{Tr}\left[T^a T^b\, U_{x-\hat{\mu},\mu}\right] \end{aligned}$$

and $T^a,\ a = 1, \ldots, N_c^2 - 1$ being the (hermitian) generators of the $\mathfrak{su}(N_c)$ Lie algebra satisfying $\mathrm{Tr}\left[T^a T^b\right] = \delta^{ab}/2$.

The ghost propagator is given by

$$G^{ab} = \sum_{x,y} \left\langle \mathrm{e}^{-ik\cdot(x-y)} [M^{-1}]^{ab}_{x,y} \right\rangle$$

M-inversion with conjugate gradient method and plane wave sources.

Finite-volume effects

Compute gluon propagator for $\beta = 2.3$ and various L: $L = 40, 56, 80, 112$

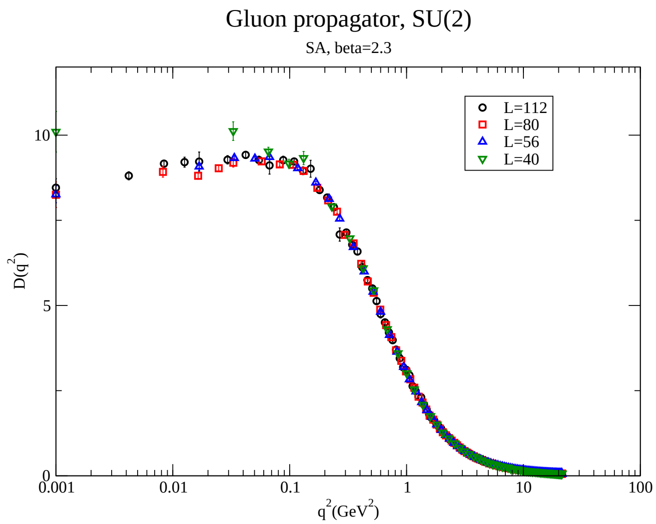

Figure 1: The gluon propagator for $\beta = 2.3$ and various lattice sizes . The data points drawn at $q^2 = 0.001$ represent the zero-momentum gluon propagator $D(0)$.

Fig.1 shows that finite volume effects (FVEs) for $\beta = 2.3$ are negligible if lattice size $L \geq 56$. For $L = 56$ and $\beta = 2.3$ physical lattice size $L_{FV} = L * a(\beta) = 9.6 fm$

One can assume that for all $\beta \in [2.3, 2.6]$ FVEs are small if $L * a(\beta) \geq 9.6 fm$.

Gluon propagator: fitting

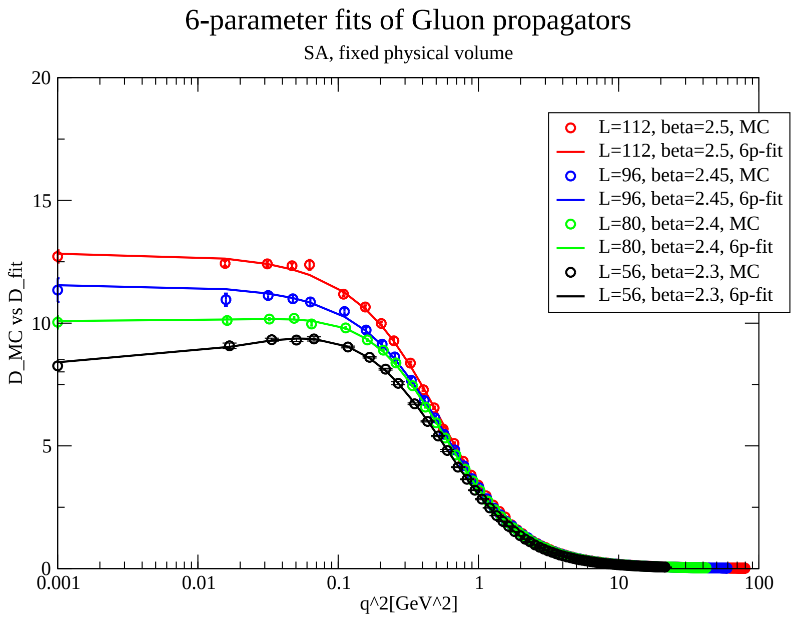

Figure 2: Fitting of non-normalized gluon propagators for various β at fixed physical volume
We use the 6 parameters fit (Cucchieri et al'11):

$$G(q) = C\frac{q^4 + Aq^2 + B}{q^6 + Dq^4 + Fq^2 + G}$$

⇒ Note universality of fitting formula for all β and q^2; 4-parameter fixed-gluon-mass fitting doesn't work well for gluon curves with turnover

⇒ Fitted gluon curves can be used further for renormalization and extrapolation to continuum limit.

Renormalized gluon and ghost

- Gluon propagators and ghost dressing functions have been renormalized requiring that values of dressing functions are equal to 1 at $\mu = 2.2$ GeV

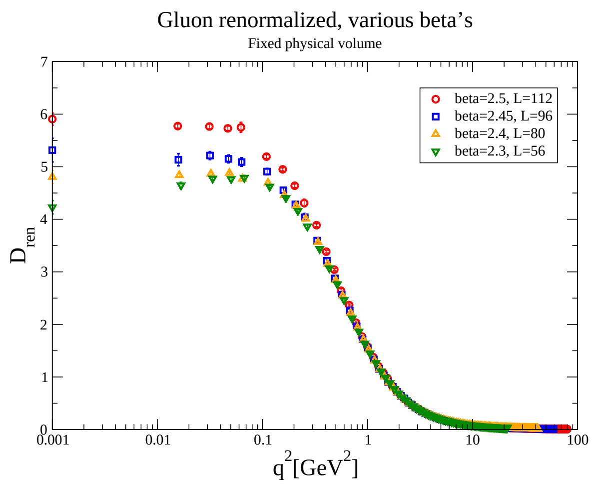

Figure 3: Renormalized gluon propagators computed at various β and fixed physical volume, $\mu = 2.2$GeV

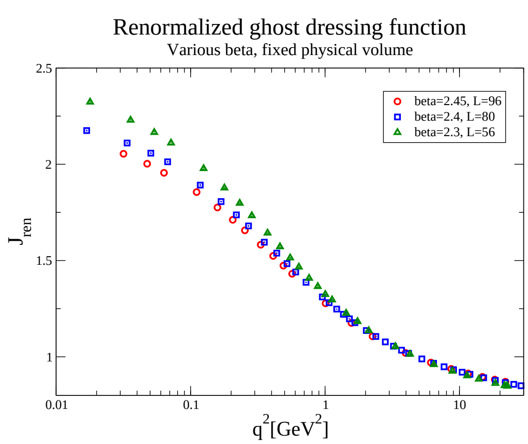

Figure 4: Renormalized ghost dressing functions computed at various β and fixed physical volume, $\mu = 2.2$GeV

Running coupling

Now compute gauge-invariant running coupling constant $\alpha_s(q^2) = \frac{g_0^2}{4\pi} J^2(q^2) Z(q^2)$.

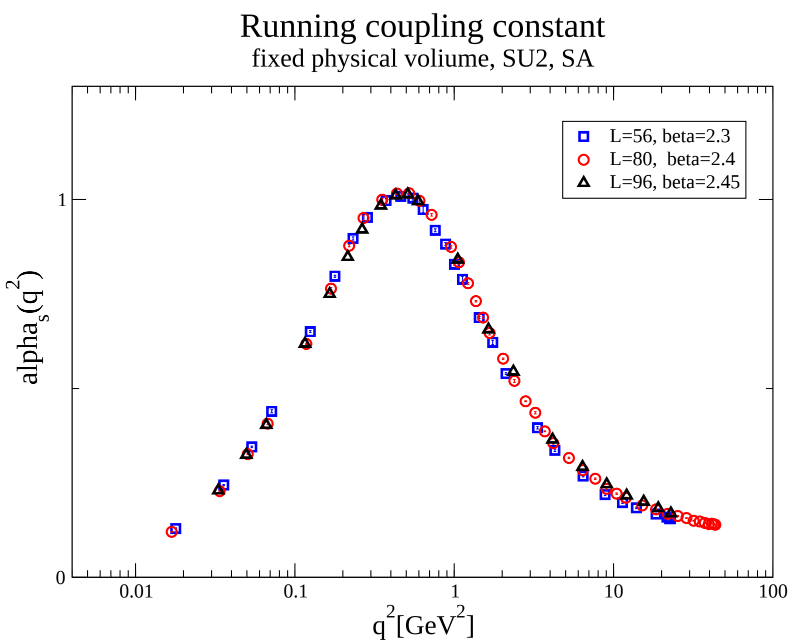

Figure 5: Running coupling $\alpha_s(q^2)$ computed for various β at fixed physical volume

$\Rightarrow$ No IR fixed point seen for $SU(2)$ $\alpha_s(q^2)$
The 3 curves are on top of each other!
$\Rightarrow$ Continuum limit seems to be reached already at $L = 56, \beta = 2.3$ for gauge-invariant $\alpha_s(q^2)$, which is not the case for gauge-variant renormalized gluon and ghost propagators.

Extrapolation to Continuum Limit

- We extrapolate renormalized gluon propagators (fitted) to continuum limit, i.e to $a(g) = 0$, using fit $D(a,q) = D_0 + C_2\ a^2 + C_4\ a^4$

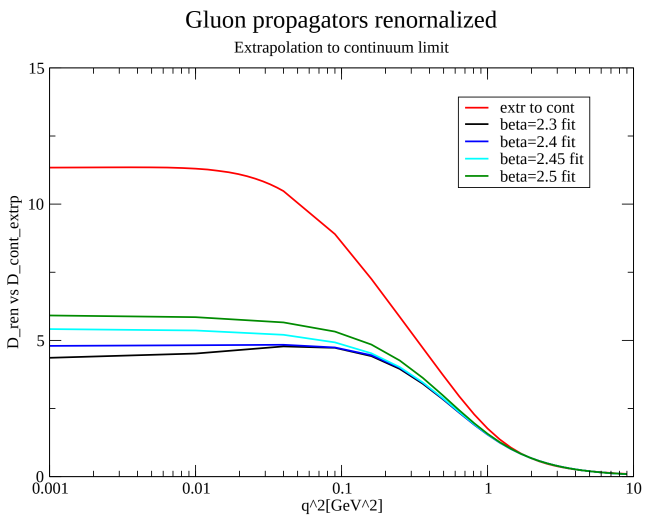

Figure 6: Reconstruction of gluon propagator in continuum limit using 3-parameter fit

$\Rightarrow$ Convergence of renormalized propagators to continuum limit is much slower than that of gauge-invariant running coupling $\alpha_s(q^2)$

$\Rightarrow$ Note plateau of $D_{cont}(q^2)$ for small q^2 !

$\Rightarrow$ Further efforts are needed (i) to check reliability of extrapolation to continuum and (ii) to study theoretical consequences

Conclusions and Questions

- Finite-volume effects for lattice gluon propagator at $\beta = 2.3$ are small when lattice extension $L \geq 56$ which corresponds to linear size $La(\beta) \geq 9.6$ fm .

- Systematic lattice simulations at various β and fixed physical volume $L^4 a(\beta)^4$ allow to study evolution of gluon and ghost propagators under decrease of lattice spacing $a(\beta)$. Such studies from first principles require intensive computations on modern parallel supercomputers

- Our studies confirmed so-called "decoupling" behaviour of gluon and ghost propagators for decreasing spacing a

- It was found that convergence to continuum limit of gauge-invariant observables (running coupling $\alpha_s(q^2)$ is much faster then that of gauge-variant propagators

- Difference between lattice gluon propagators and their extrapolation to continuum limit turned out to be surprisingly large! Further simulations and detailed study of extrapolation to continuum limit are needed

- Results of our simulations seems to be (of interest)/(a challenge) for the nonperturbative theory of gauge Yang-Mills fields. Possibly applicability of some widely used gauges (Landau, Coulomb,...) in IR region is to be critically reconsidered. Anyhow our results require theoretical explanation/interpretation.

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