



**XXI International Baldin Seminar  
on High Energy Physics Problems**  
***Relativistic Nuclear Physics &  
Quantum Chromodynamics***

September 10-15, 2012, Dubna, Russia

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# Self-similarity of high- $p_T$ hadron production in pA collisions

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JINR, Dubna, Russia



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XXI International Baldin Seminar on High Energy Physics Problems  
"Relativistic Nuclear Physics and Quantum Chromodynamics",  
JINR, Dubna, Russia, September 10-15, 2012

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# Contents

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- Introduction
- $z$ -Scaling (ideas, definitions, properties,...)
- Self-similarity of hadron production in  $pp$  &  $pD$
- Self-similarity of high- $p_T$  cumulative hadron production in  $pA$  ( $A=C, Al, Cu, W$ ) at U70
- Conclusions

# Motivation & Goals

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Development of  **$z$** -scaling approach for description of hadron production in inclusive reactions to search for signatures of new physics phenomena.

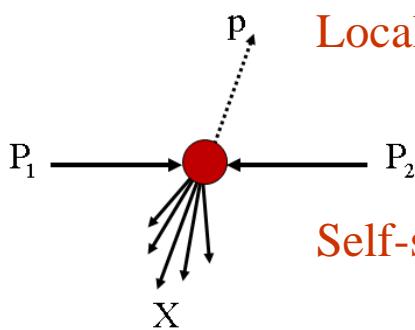
Analysis of new experimental data on inclusive spectra of hadron production in **pA** collisions obtained at **U70** to verify properties of  **$z$** -scaling in high- $p_T$  cumulative region

- **pA** is a reference frame for **pp** & **AA**
- High- $p_T$  process – hadron production at a constituent level
- Cumulative process:
  - enhancement of nuclear matter compression
  - particle formation is sensitive to state of matter
  - search for indications of phase transition & CP

Verification of fundamental principles ,  
(Lorenz invariance, self-similarity, scale relativity,... )  
properties of particles and hadronization process.

# **z**-Scaling

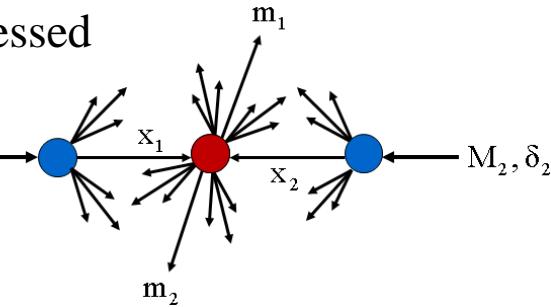
## Principles: locality, self-similarity, fractality



**Locality:** collisions of hadrons and nuclei are expressed via interactions of their constituents (partons, quarks and gluons,...).

**Self-similarity:** interactions of the constituents are mutually similar.

**Fractality:** the self-similarity over a wide scale range.



### Hypothesis of **z**-scaling :

$s^{1/2}, p_T, \theta_{cms}$  Inclusive particle distributions can be described in terms of constituent sub-processes and parameters characterizing bulk properties of the system.

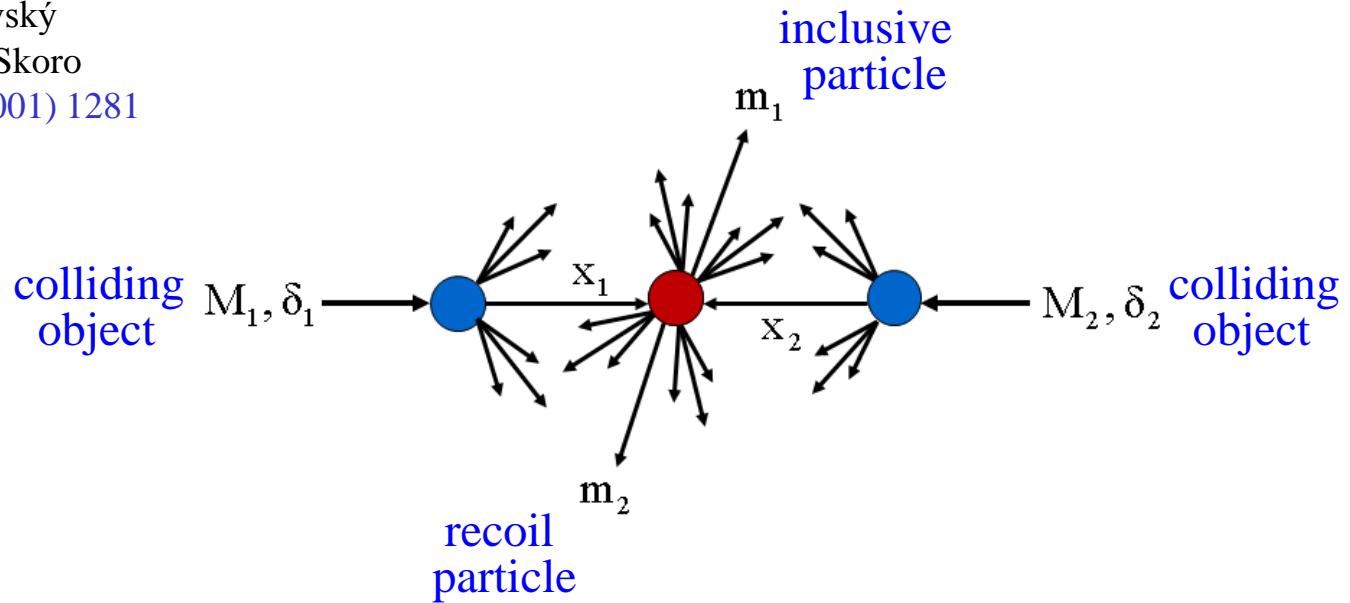
$E d^3\sigma/dp^3$  Scaled inclusive cross section of particles depends in a self-similar way on a single scaling variable **z**.

$x_1, x_2$   
 $\delta_1, \delta_2$

$\Psi(z)$

# Locality of hadron interactions

M.T. & I.Zborovský  
Yu.Panebratsev, G.Skoro  
Int. J. Mod. Phys. A16 (2001) 1281  
JINR E2-99-113



## Constituent subprocess

$$(\textcolor{red}{x}_1 M_1) + (\textcolor{red}{x}_2 M_2) \Rightarrow (m_1) + (\textcolor{red}{x}_1 M_1 + \textcolor{red}{x}_2 M_2 + m_2)$$

Kinematical condition (4-momentum conservation law):

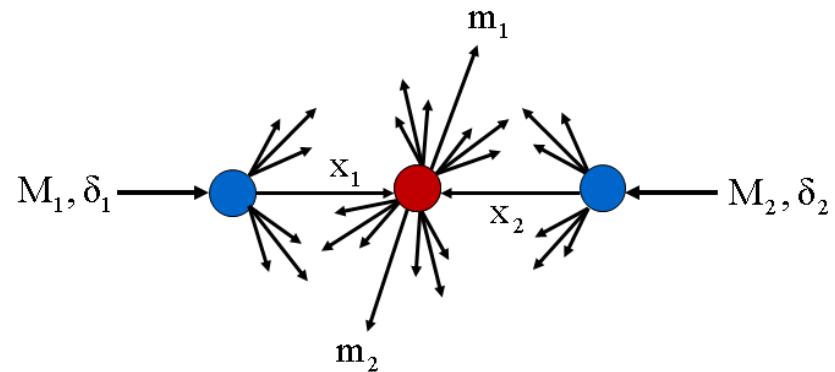
$$(\textcolor{red}{x}_1 P_1 + \textcolor{red}{x}_2 P_2 - p)^2 = M_X^2$$

Recoil mass:  $M_X = \textcolor{red}{x}_1 M_1 + \textcolor{red}{x}_2 M_2 + m_2$

# Self-similar parameter z

$$z = z_0 \Omega^{-1}$$

$$z_0 = \frac{s_{\perp}^{1/2}}{(dN_{ch}/d\eta|_0)m}$$



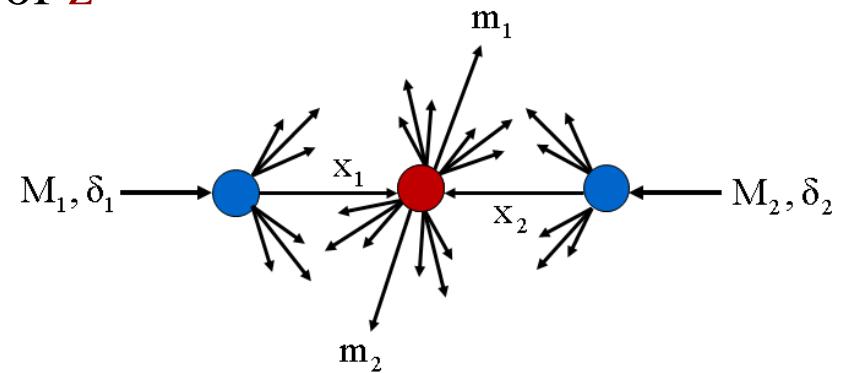
- $\Omega^{-1}$  is the minimal resolution at which a constituent subprocess can be singled out of the inclusive reaction
- $\sqrt{s}_{\perp}$  is the transverse kinetic energy of the subprocess consumed on production of  $m_1$  &  $m_2$
- $dN_{ch}/d\eta|_0$  is the multiplicity density of charged particles at  $\eta = 0$
- $m$  is an arbitrary constant (fixed at the value of nucleon mass)

# Fractal measure $z$

The fractality is reflected in definition of  $z$

$$z = z_0 \Omega^{-1}$$

$$\Omega = (1 - x_1)^{\delta_1} (1 - x_2)^{\delta_2}$$



$\Omega$  is relative number of configurations containing a sub-process with fractions  $x_1, x_2$  of the corresponding 4-momenta

$\delta_1, \delta_2$  are parameters characterizing structure of the colliding objects

$\Omega^{-1}(x_1, x_2)$  characterizes resolution at which a constituent sub-process can be singled out of the inclusive reaction

$$z(\Omega) \Big|_{\Omega^{-1} \rightarrow \infty} \rightarrow \infty$$

The fractal measure  $z$  diverges as the resolution  $\Omega^{-1}$  increases.

# Momentum fractions $x_1, x_2$

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**Principle of minimal resolution:** The momentum fractions  $x_1, x_2$  are determined in a way to minimize the resolution  $\Omega^{-1}$  of the fractal measure  $z$  with respect to all constituent sub-processes taking into account 4-momentum conservation:

$$\Omega = (1 - x_1)^{\delta_1} (1 - x_2)^{\delta_2}$$

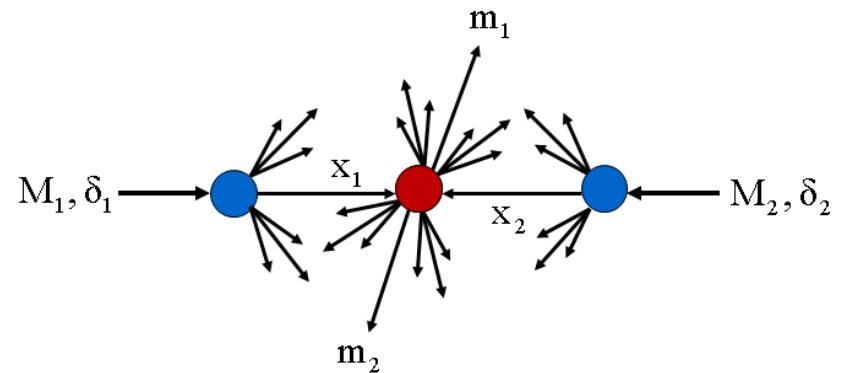
$$\partial \Omega / \partial x_1 \Big|_{x_2=x_2(x_1)} = 0$$

Momentum conservation law)

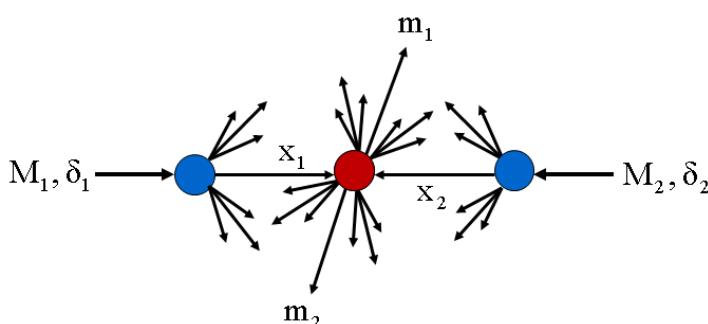
$$(x_1 P_1 + x_2 P_2 - p)^2 = M_X^2$$

Recoil mass

$$M_X = x_1 M_1 + x_2 M_2 + m_2$$

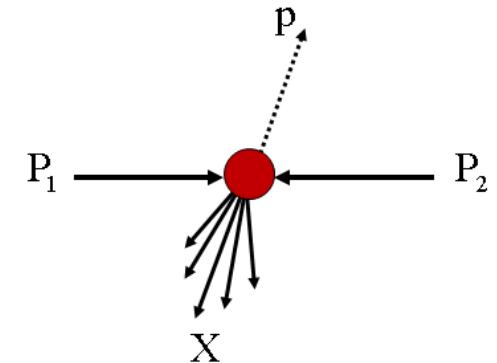


# Scaling function $\Psi(z)$



$$\int_0^\infty \Psi(z) dz = 1$$

$$z \rightarrow \alpha_F z, \quad \Psi \rightarrow \alpha_F^{-1} \Psi$$



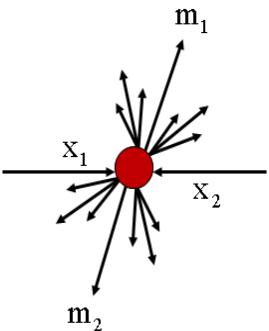
$$\Psi(z) = \frac{\pi}{(dN/d\eta) \cdot \sigma_{inel}} \cdot J^{-1} \cdot E \frac{d^3\sigma}{dp^3} \quad \longleftrightarrow \quad \int E \frac{d^3\sigma}{dp^3} dy d^2p_\perp = \sigma_{inel} \cdot N$$

- $\sigma_{in}$  - inelastic cross section
- $N$  - average multiplicity of the corresponding hadron species
- $dN/d\eta$  - pseudorapidity multiplicity density at angle  $\theta(\eta)$
- $J(z,\eta;p_T^2,y)$  - Jacobian
- $E d^3\sigma/dp^3$  - inclusive cross section

The scaling function  $\Psi(z)$  is probability density to produce an inclusive particle with the corresponding  $z$ .

# Transverse kinetic energy $\sqrt{s}_\perp$

$$s_\perp^{1/2} = \underbrace{(s_\lambda^{1/2} - M_1 \lambda_1 - M_2 \lambda_2) - m_1}_{\text{energy consumed for the inclusive particle } m_1} + \underbrace{(s_\chi^{1/2} - M_1 \chi_1 - M_2 \chi_2) - m_2}_{\text{energy consumed for the recoil particle } m_2}$$



Fraction decomposition:  $x_{1,2} = \lambda_{1,2} + \chi_{1,2}$

$$\lambda_{1,2} = \kappa_{1,2} + \nu_{1,2} \quad \kappa_{1,2} = \frac{(P_{2,1}p)}{(P_2 P_1) - M_1 M_2}, \quad \nu_{1,2} = \frac{M_{2,1}m_2}{(P_2 P_1) - M_1 M_2}$$

$$\chi_{1,2} = (\mu_{1,2}^2 + \omega_{1,2}^2)^{1/2} \mp \omega_{1,2}$$

$$\omega_{1,2} = \mu_{1,2} U, \quad U = \frac{\alpha - 1}{2\sqrt{\alpha}} \xi, \quad \alpha = \frac{\delta_2}{\delta_1}$$

$$\xi^2 = (\lambda_1 \lambda_2 + \lambda_0) / [(1 - \lambda_1)(1 - \lambda_2)]$$

$$\mu_{1,2}^2 = \alpha^{\pm 1} (\lambda_1 \lambda_2 + \lambda_0) \frac{1 - \lambda_{1,2}}{1 - \lambda_{2,1}}$$

$$\lambda_0 = \bar{\nu}_0 - \nu_0$$

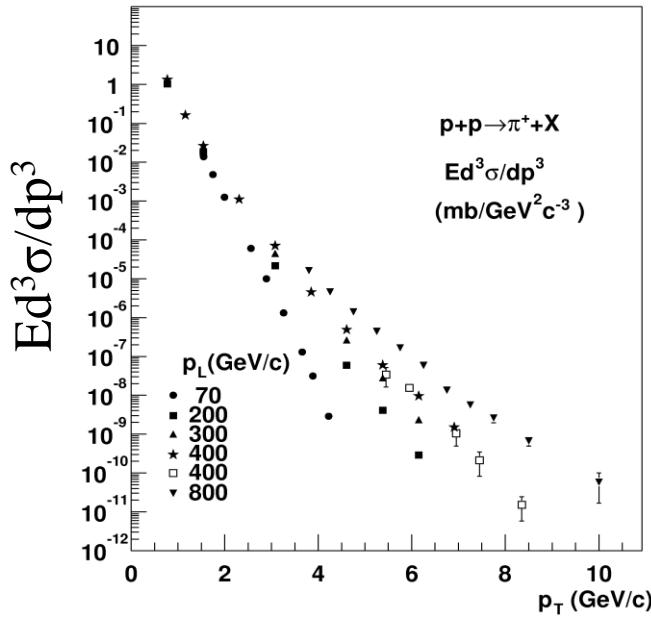
$$\bar{\nu}_0 = \frac{0.5m_2^2}{(P_1 P_2) - M_1 M_2}, \quad \nu_0 = \frac{0.5m_1^2}{(P_1 P_2) - M_1 M_2}$$

$$s_\lambda = (\lambda_1 P_1 + \lambda_2 P_2)^2$$

$$s_\chi = (\chi_1 P_1 + \chi_2 P_2)^2$$

The scaling variable  $z$  and scaling function  $\Psi(z)$   
are expressed via relativistic invariants.

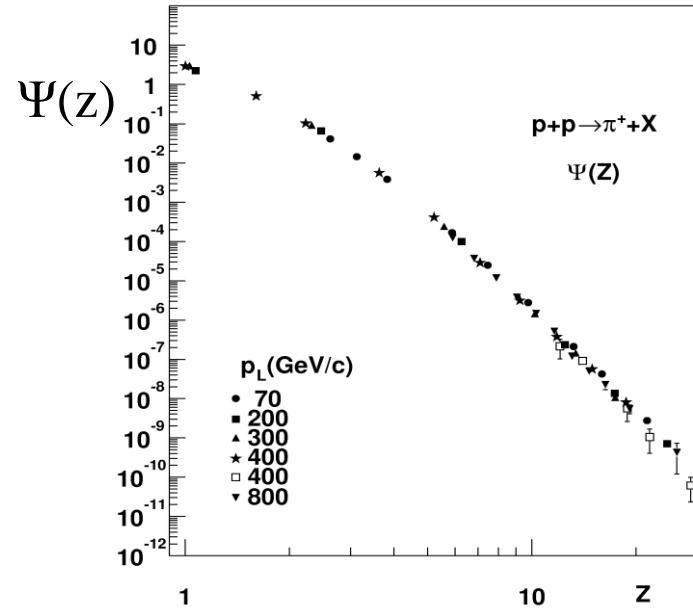
# Self-similarity of hadron production in pp



## Spectra

- 10 orders of magnitude
- Sensitive to energy  $\sqrt{s}$  at high  $p_T$
- Power law for high  $\sqrt{s}$  and  $p_T$

J.W. Cronin et.al., Phys. Rev. D11 (1975) 3105.  
D. Antreasyan et al., Phys. Rev. D19 (1979) 764.  
V.V. Abramov et al., Sov. J. Nucl. Phys. 41 (1985) 357.  
D.E. Jaffe et al., Phys. Rev. D40 (1989) 2777.

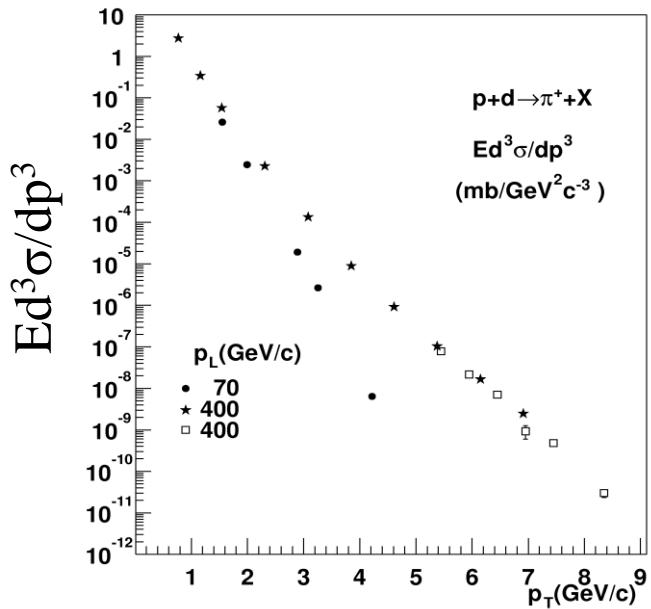


- Energy independence of  $\Psi(z)$
- Power law of  $\Psi(z)$  at high  $z$

## Scale invariance

Independence of the shape of the curve  
on  $\{z, \Psi\}$  plane on scale quantities  $\sqrt{s}, p_T, \theta$

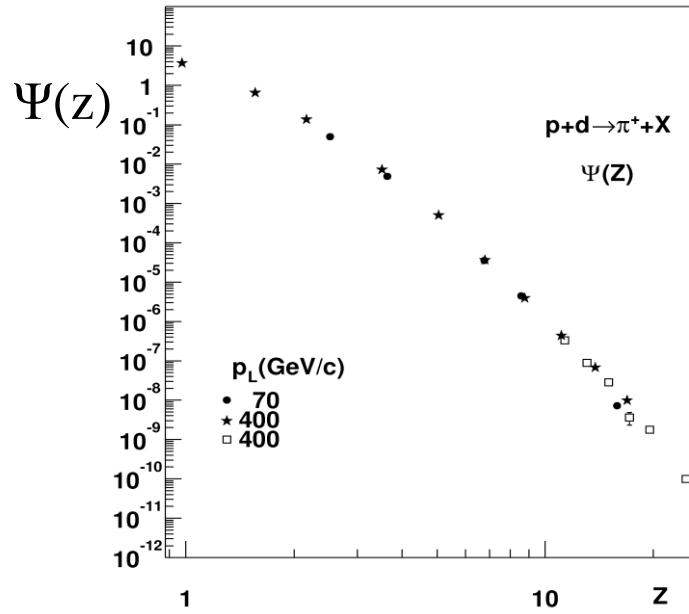
# Self-similarity of hadron production in pD



## Spectra

- 10 orders of magnitude
- Sensitive to energy  $\sqrt{s}$  at high  $p_T$
- Power law for high  $\sqrt{s}$  and  $p_T$

J.W. Cronin et.al., Phys. Rev. D11 (1975) 3105.  
D. Antreasyan et al., Phys. Rev. D19 (1979) 764.  
V.V. Abramov et al., Sov. J. Nucl. Phys. 41 (1985) 357.  
D.E. Jaffe et al., Phys. Rev. D40 (1989) 2777.



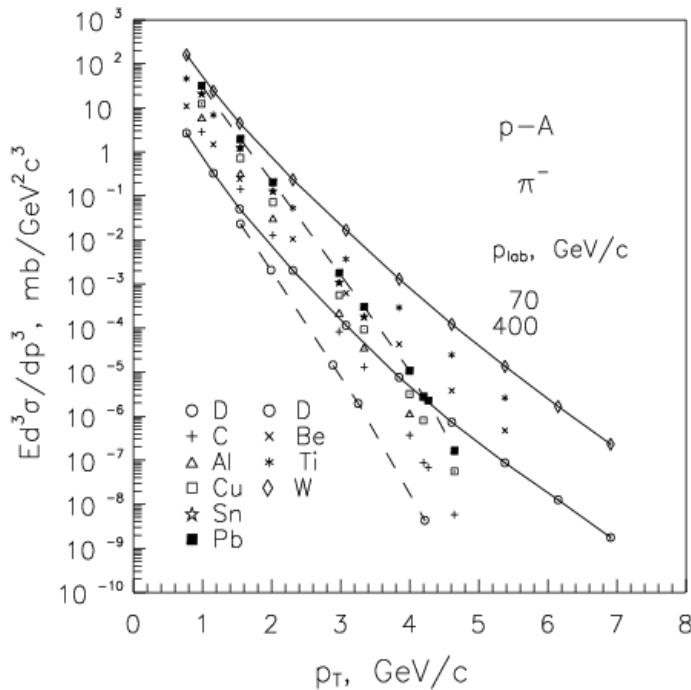
- Energy independence of  $\Psi(z)$
- Power law of  $\Psi(z)$  at high  $z$

## Fractal dimensions in pA & AA

$$\delta_1 = A_1 \delta, \quad \delta_2 = A_2 \delta$$

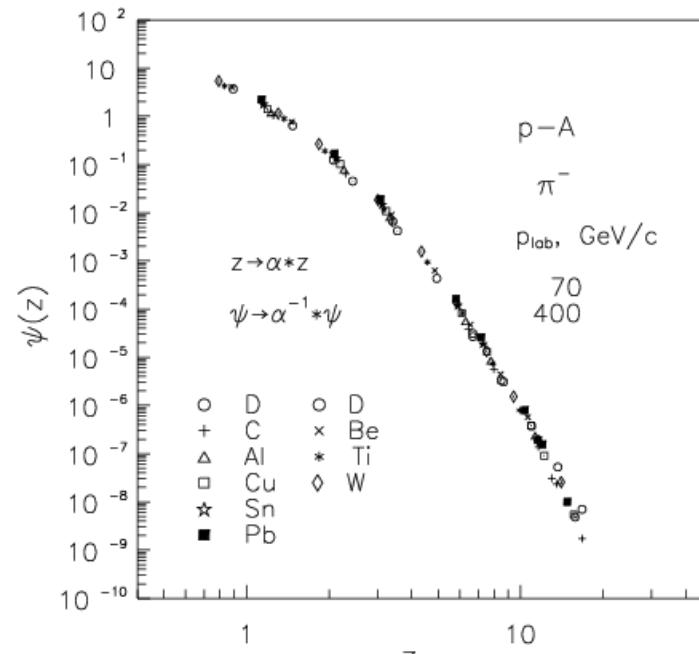


# Self-similarity of hadron production in pA



Strong dependence of spectra  
on  $\sqrt{s}$  at high  $p_T$

J.W. Cronin et al., Phys. Rev. D11 (1975) 3105.  
D. Antreasyan et al., Phys. Rev. D19 (1979) 764.  
V.V. Abramov et al., Sov. J. Nucl. Phys. 41 (1985) 357.  
M.T., Yu.Panebratsev, I.Zborovsky, G.Skoro  
JINR E2-99-113; Int. J. Mod. Phys. A16 (2001) 1281.



- Energy independence of  $\Psi(z)$
- Power law of  $\Psi(z)$  at high  $z$
- A-dependence of  $\Psi(z)$

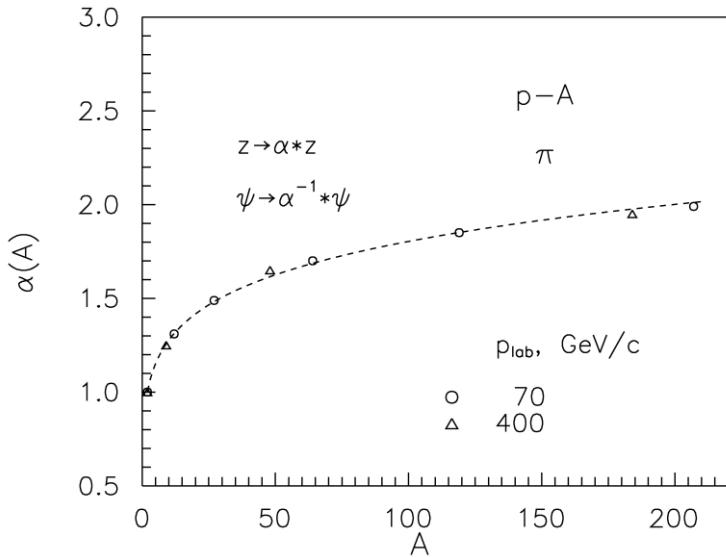
Scale invariance

Independence of the shape of the curve  
on  $\{z, \Psi\}$  plane on scale quantities  $\sqrt{s}, p_T, \theta$



# A-dependence of z-scaling

The scaling transformations of  $z$  and  $\Psi(z)$  allow us to compare scaling functions for different nuclei



$$z = z_0 \Omega^{-1}$$

$$\Omega = (1 - x_1)^{\delta_1} (1 - x_2)^{\delta_2}$$

$$\delta_1 = A_1 \delta, \quad \delta_2 = A_2 \delta$$

$$\begin{aligned} z &\rightarrow \alpha(A) \cdot z \\ \Psi(z) &\rightarrow \alpha^{-1}(A) \cdot \Psi(z) \\ &\downarrow \\ \alpha(A) &\approx 0.9 A^{0.15} \end{aligned}$$

Self-similarity of nuclear modification of constituent interactions and hadron formation.

Self-similar parameter  $z$   
“Critical exponents”  $\delta_1, \delta_2$   
Preservation of self-similarity and discontinuity of  $\delta_1, \delta_2$  is a signature of new physics

M.T., Yu.Panebratsev, I.Zborovsky, G.Skoro

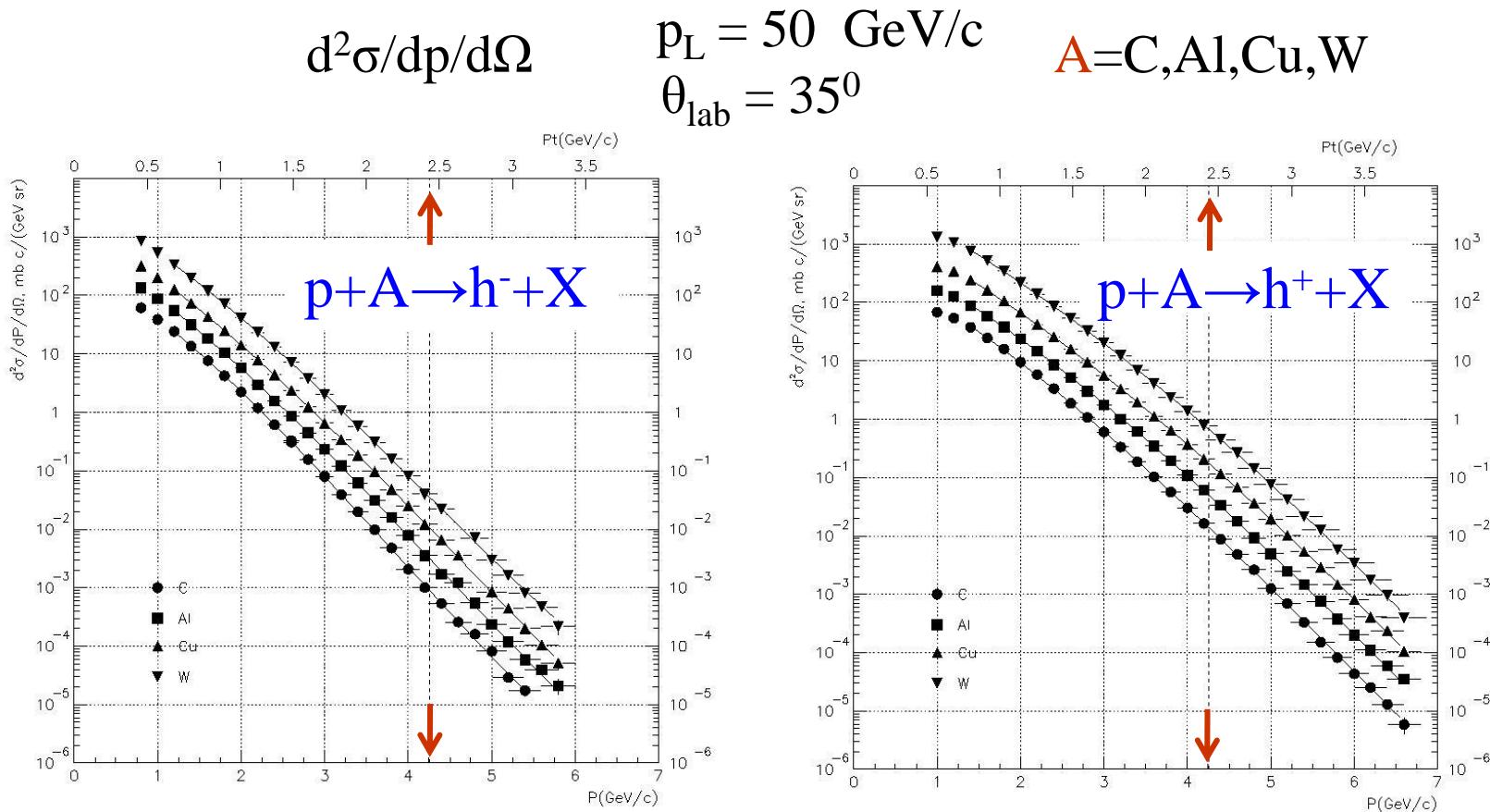
JINR E2-99-113; Int. J. Mod. Phys. A16 (2001) 1281.

A.Aparin

IBSHEPP XXI, Dubna, 2012



# Charged hadron spectra in pA at U70



- Spectra in cumulative region:  $p_T > 2.5 \text{ GeV}/c$
- Smooth behavior of spectra vs.  $p_T$

N.N. Antonov et al., "Physics of Fundamental Interactions"

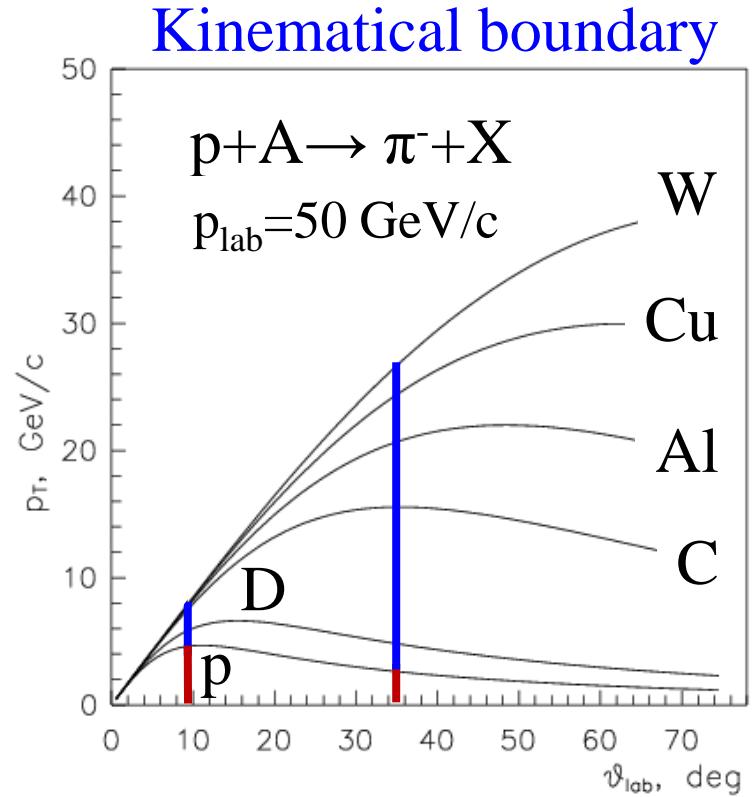
RAS, ITEP, Moscow, Russia, November 21 - 25, 2011

# Kinematics

| $p_T^{\pi} (\text{GeV}/c)$ |      |      |
|----------------------------|------|------|
| P                          | 2.6  | 5.6  |
| D                          | 4.8  | 7.4  |
| C                          | 15.6 | 10.0 |
| Al                         | 20.7 | 10.4 |
| Cu                         | 24.4 | 10.6 |
| W                          | 26.7 | 10.7 |

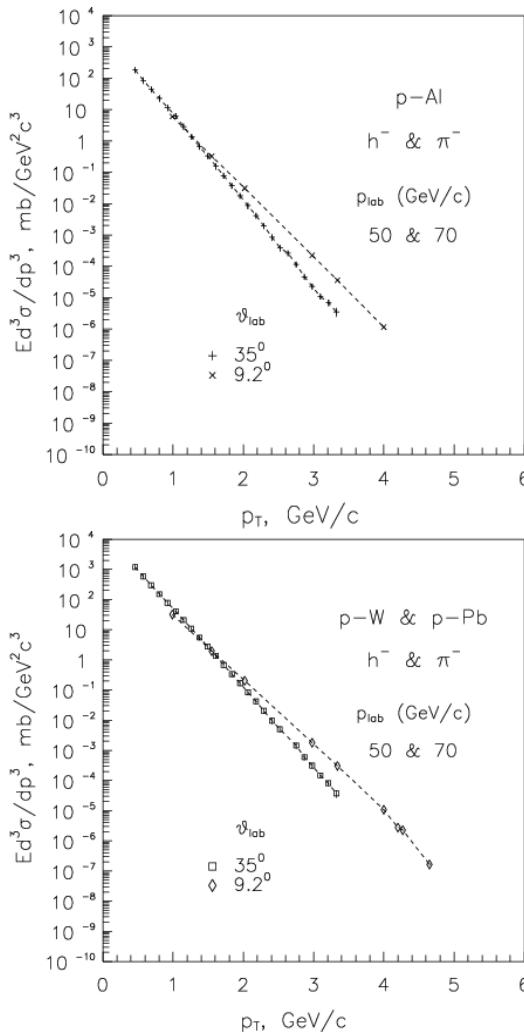
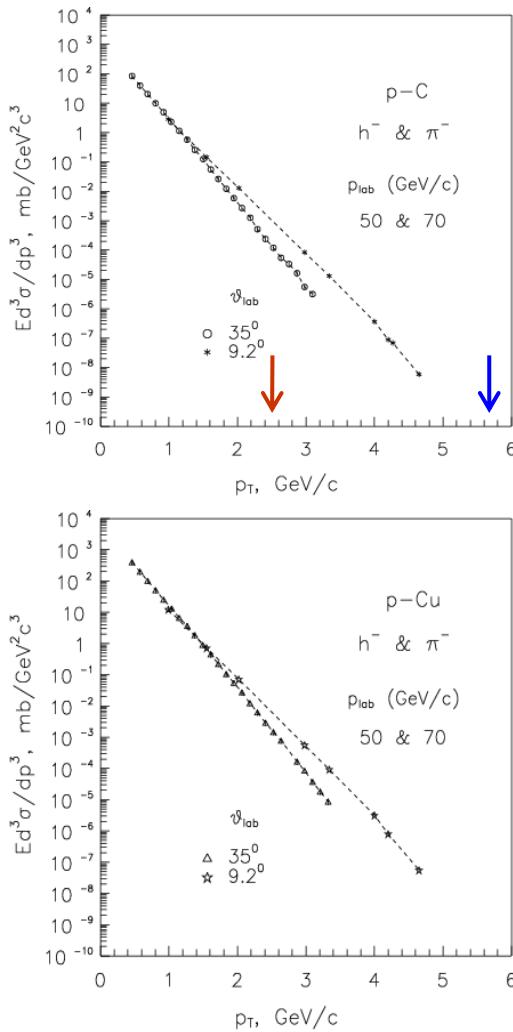
$p(\text{GeV}/c)$  50 70

$\vartheta_{\text{lab}}$   $35^0$   $9^0$



Cumulative region:  $p_{\text{max}}^{pA} > p_{\text{max}}^{pp}$

# High- $p_T$ hadron spectra in pA at U70



U70  
SPIN & FODS

SPIN, N.N. Antonov et al.,  
“Physics of Fundamental Interactions”  
RAS, ITEP, Moscow, Russia,  
November 21 - 25, 2011

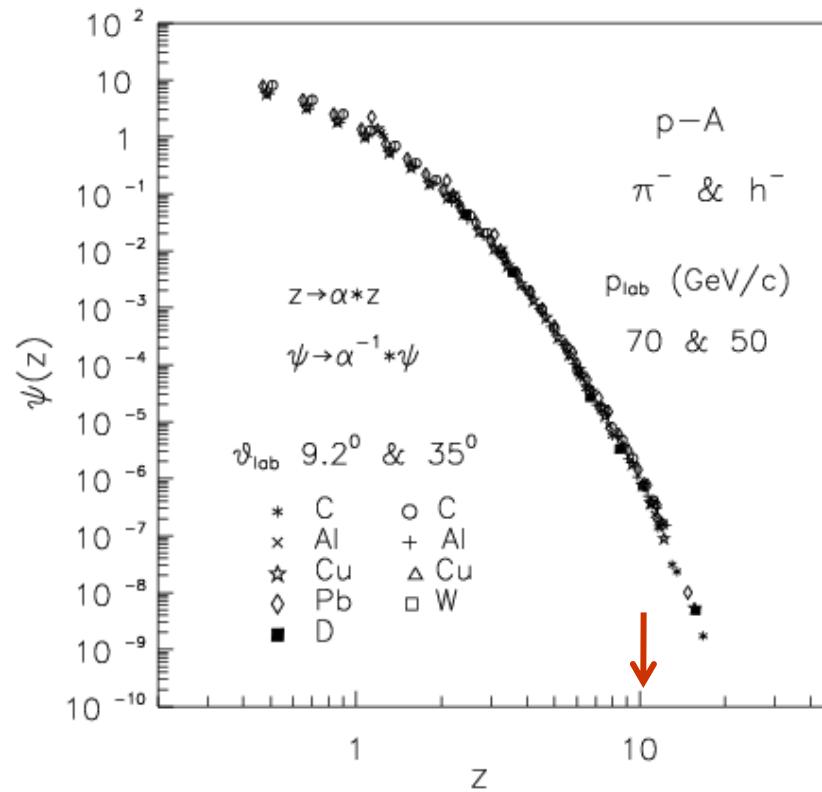
FODS, V.V.Abramov et al.,  
Sov. J. Nucl. Phys. 41 (1985) 357.

# Self-similarity High- $p_T$ and cumulative hadron production in pA

**U70**  
**SPIN & FODS**

$$z = z_0 \Omega^{-1}$$

$$\Omega = (1 - x_1)^{\delta_1} (1 - x_2)^{\delta_2}$$

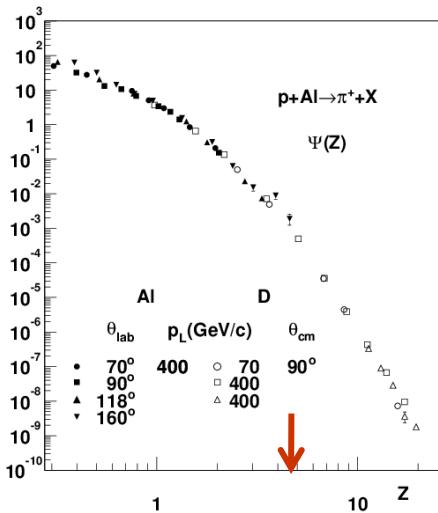
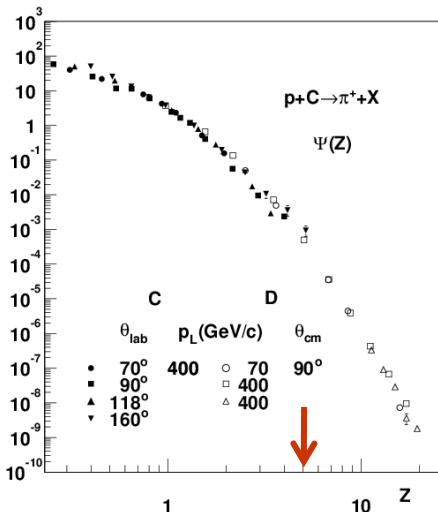
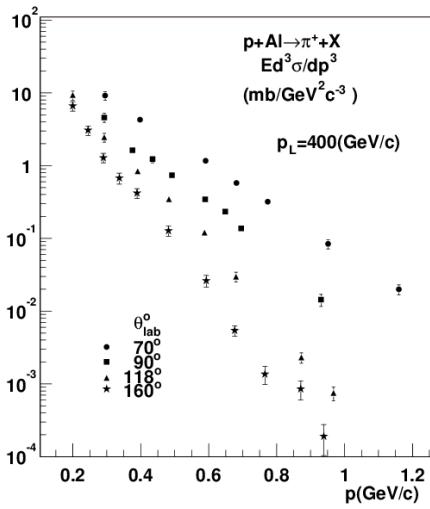
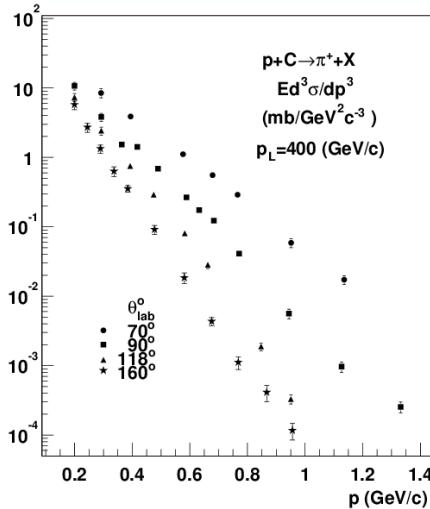


- Universal shape of  $\Psi(z)$
- Power law for  $z > 4$
- No discontinuity of  $\delta_2 = A_2 \delta$

Scale invariance

Independence of the shape of the curve  
on  $\{z, \Psi\}$  plane on scale quantities  $\sqrt{s}, p_T, \theta$

# High- $p_T$ and cumulative hardron production in pA

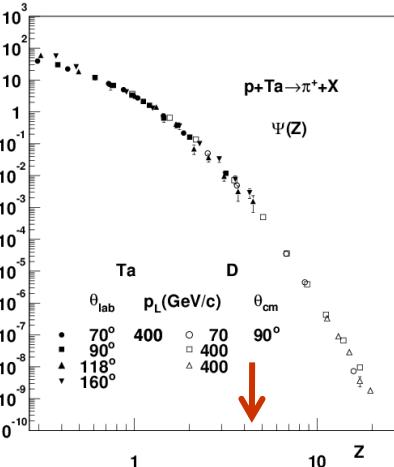
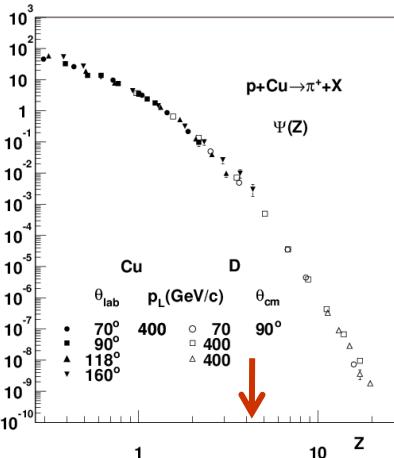
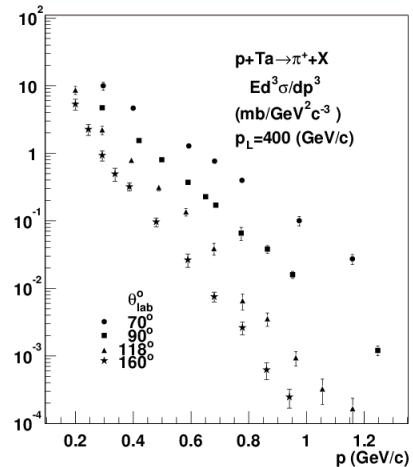
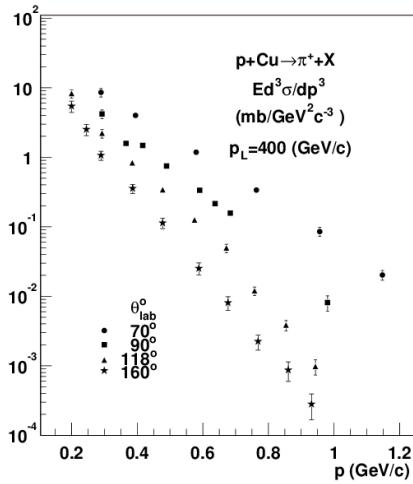


C, Al & D

- N.A. Nikiforov et al., Phys. Rev. C22 (1980) 700.  
J.W. Cronin et.al., Phys. Rev. D11 (1975) 3105.  
D. Antreasyan et al., Phys. Rev. D19 (1979) 764.  
V.V. Abramov et al., Sov. J. Nucl. Phys. 41 (1985) 357.  
D.E. Jaffe et al., Phys. Rev. D40 (1989) 2777.

| $\theta_{lab}^\pi = 180^0$ | $p_L$<br>(GeV/c) |      |      |
|----------------------------|------------------|------|------|
| $p$                        | D                | C    | Al   |
| 0.447                      | 0.905            | 5.13 | 10.6 |
| 0.456                      | 0.928            | 5.53 | 12.2 |
| 0.459                      | 0.933            | 5.63 | 12.7 |

# High- $p_T$ and cumulative hardron production in pA



## Cu, Ta & D

- Collapse of data point
- Universal shape of  $\Psi(z)$
- Self-similarity of hadron production over a wide range of energy  $\sqrt{s}$ , angle  $\theta$  and atomic number  $A$

$\theta_{lab}^\pi = 180^\circ$

|                        | p     | D     | Cu   | Ta   |
|------------------------|-------|-------|------|------|
| $p_\pi^{\max}$ (GeV/c) | 0.447 | 0.905 | 20.7 | 37.8 |
| $p_L$ (GeV/c)          | 0.456 | 0.928 | 27.9 | 69.8 |
|                        | 0.459 | 0.933 | 30.0 | 84.7 |

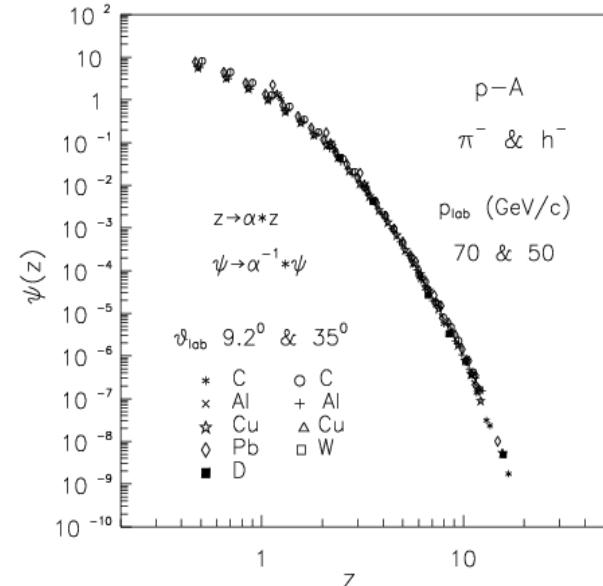
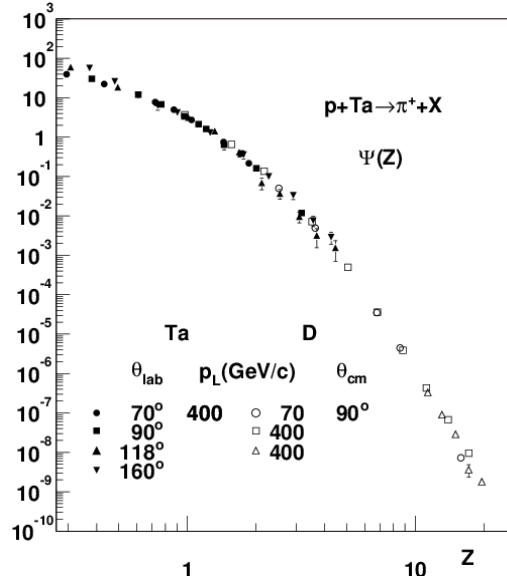
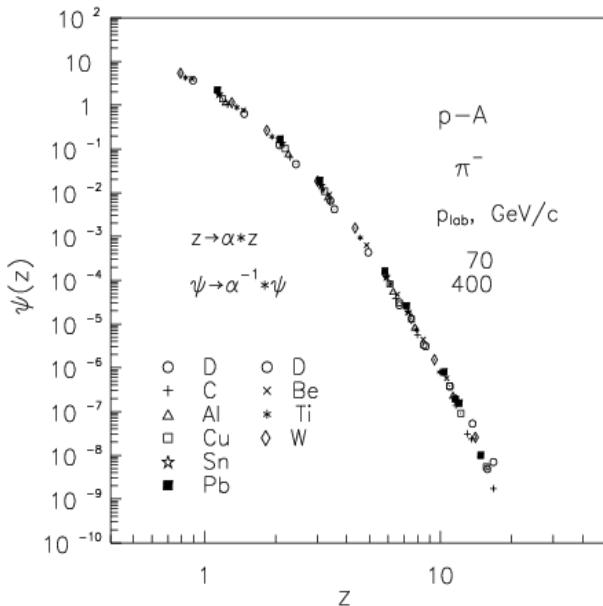
# Self-similarity High- $p_T$ and cumulative hadron production in pA

FNAL

J.Cronin & D.Jaffe

U70

SPIN & FODS



Open questions:

- Spectra of cumulative identified particles
- Multiplicity density  $dN_{ch}/d\eta$  vs.  $\sqrt{s}$  and  $\eta$
- Centrality dependence of the spectra
- Power law of  $\Psi(z)$  in cumulative region

Goal: Search for violation of  $z$ -scaling → search for phase transition & CP

# Conclusions

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- New **U70** data on charged hadron production in **pA** collisions were analyzed in the **z**-scaling approach.
- Results of new analysis were compared with previous one from data taken by J. Cronin, R. Sulyaev, G. Leksin and D. Jaffe groups.
- Confirmation of self-similarity of hadron production in **pA** collisions in high- $p_T$  cumulative region were obtained.
- **z**-Scaling of charged hadron production in **pA** collisions at high energies manifests self-similarity, locality and fractality of hadron interactions at a constituent level.

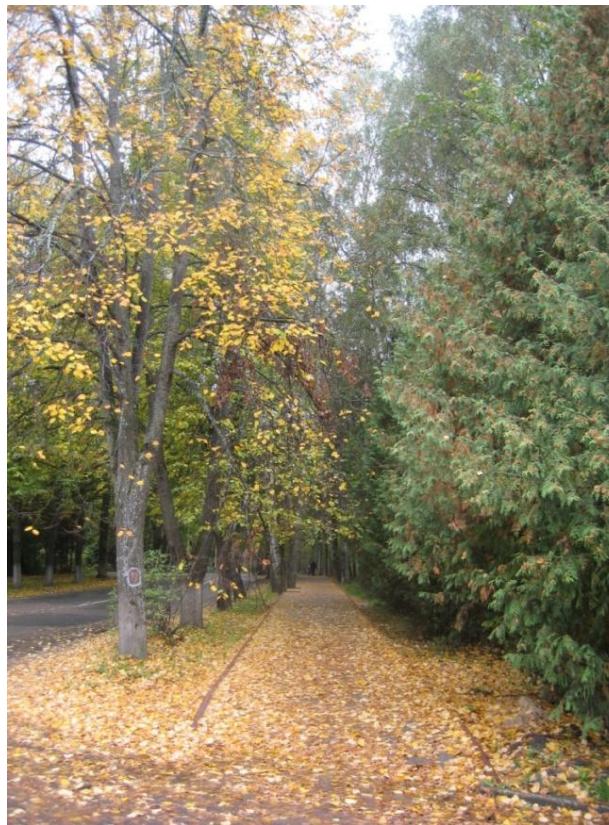
New kinematical region is available for search of new physics phenomena in hadron production at **U70**.



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***Thank you for your attention !***



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# *Back-up slides*

# Angular Dependence of Multiplicity Density

