XX INTERNATIONAL BALDIN SEMINAR ON HIGH ENERGY PHYSICS PROBLEMS
"RELATIVISTIC NUCLEAR PHYSICS & QUANTUM CHROMODYNAMICS"

Saturation of Hadron Production in pp/p$\bar{p}$ collisions at low p_T

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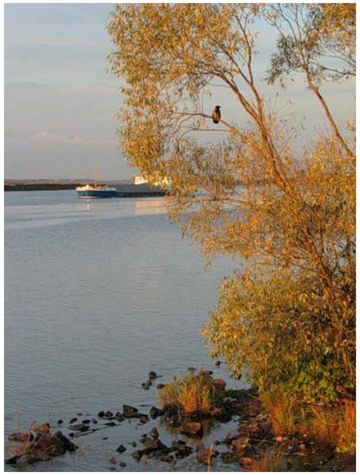

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J. Phys. G: Nucl. Part. Phys. 37 (2010) 085008

Contents

- Self-similarity in physics
- z-Scaling in pp/p$\bar{p}$ collisions
  (manifestation of self-similarity in inclusive reactions)
  - properties, soft & hard p_T region
  - entropy & constituent sub-processes
- Flavor independence of $\psi(z)$
- Saturation of $\psi(z)$ at low z (low p_T)
  - kinematics of constituent sub-processes
  - estimation of energy losses in fragmentation processes
  - coherence in processes with low p_T
- First LHC data on charged hadron spectra

  confirm the saturation observed at lower energies
- Conclusions

Self-similarity principle in physics

- Self-similarity means that a pattern is similar to a part of itself.
- Universal description using self-similarity parameters constructed as suitable combinations of physical quantities.

Examples of self-similarity parameters Π (Re, π, M,…)

Hydrodynamics

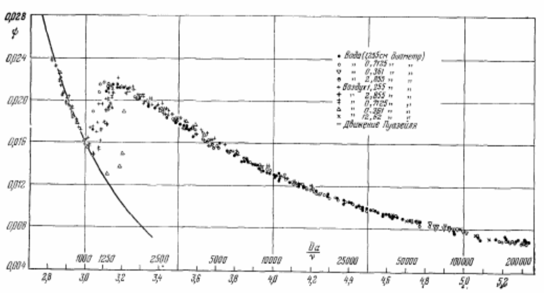
$Re = dU\rho/\mu$

d-diameter

U-velocity of the fluid

ρ-density of the fluid

μ-viscosity of the fluid



Point explosion

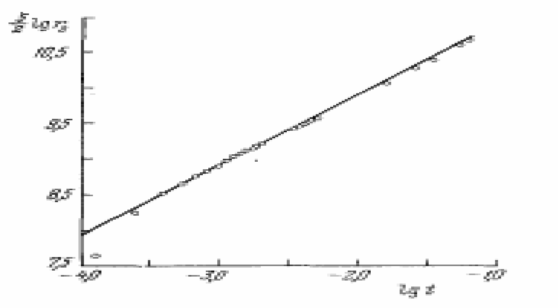
$\pi = r(Et^2/\rho)^{-1/5}$

r-radius of the front wave

E-energy of the explosion

t-elapsed time

ρ-density of the environment

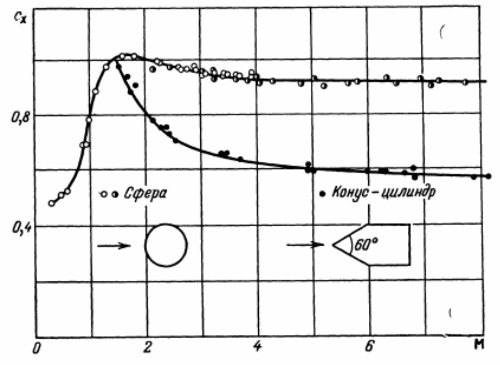


Aerodynamics

$M = v/c$

v - velocity of medium

c – velocity of sound



Self-similarity in inclusive particle production

$M_1+M_2 \rightarrow m+X$

Assumptions:

Self-similarity of hadron interactions at constituent level (partons, q, g, …) is reflected in similarity of inclusive spectra.

There exists unified description of spectra via a self-similarity variable z (adequate, physically meaningful, but still simple…)

Variable z includes suitable physical quantities:

1. reaction characteristics (A_1, A_2, P_1, P_2)
2. particle characteristics (m, p, θ)
3. structural and dynamical characteristics of the interaction (δ, ε,… $dN/d\eta$,...)

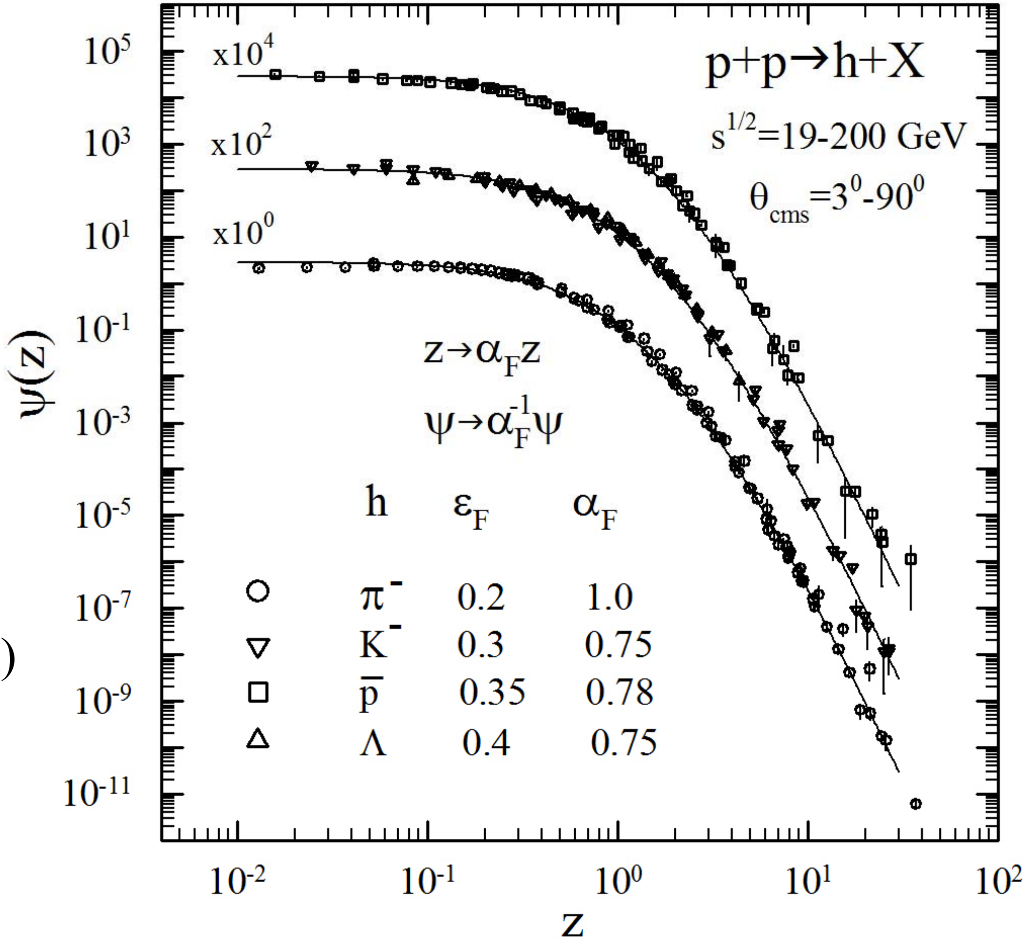
Search for a universal (scaling) function $\psi(z) \sim Ed^3\sigma/dp^3$ reflecting self-similarity of hadron interactions as revealed by data on inclusive distributions at high energies.

Scaling function ψ(z) in pp collisions

FNAL:
PRD 75 (1979) 764

ISR:
NPB 100 (1975) 237
PLB 64 (1976) 111
NPB 116 (1976) 77
NPB 56 (1973) 333

STAR:
PLB 616 (2005) 8
PLB 637 (2006) 161
PRC 75 (2007) 064901



- Energy & angular independence
- Flavor independence (π, K, p, Λ)
- Saturation for $z<0.1$
- power law $\psi(z)\sim z^{-\beta}$ at large z

ε_F, α_F independent of p_T, $s^{1/2}$, θ

Self-similarity of I & II type

G.I. Barenblatt (1978)

Self-similarity of I type:

Self-similar solutions $F_\sigma(\alpha,\beta,\gamma,..)$ expressed by single scaling function $\Phi(\Pi_1, \Pi_2, ...)$ via self-similarity parameters $\Pi_1(\alpha,\beta,\gamma,..)$, $\Pi_2(\alpha,\beta,\gamma,..)$ ....
(F_σ, α, β, γ – dimensional quantities; Φ, Π_1, Π_2 – dimensionless functions)

V.S. Stavinsky (1972): cumulative particle production
$F_\sigma = E d^3\sigma/dp^3$; $\alpha,\beta,\gamma = p,\theta,s$
$\Phi(\Pi_i) = \exp(\Pi_i/c)$; $\Pi_i = 1 - x_i$; x_1, x_2 - cumulative numbers
$\Phi(\Pi_0) = \exp(-\Pi_0/c_0)$; $\Pi_0 = \sqrt{}(x_1P_1 + x_2P_2)^2/m_N$
…but universality is broken by power asymptotic at high p_T !!!

Self-similarity of II type (intermediate asymptotics):

If Φ does not converge but has power asymptotic for extreme values of Π_1, Π_2,..
the self-similar solutions F_σ can be expressed by

$$\psi(\pi,...), \quad \pi = \Pi_0/\Pi_i^{\Delta_i}$$

A.M. Baldin (1998):
Hypothesis of self-similarity in Relativistic nuclear physics:
... search for $\Phi(\Pi_1, \Pi_2, ...)$ or $\psi(\pi,…)$ otherwise.
… parameters Δ_i have to be found from experiment…

$$\psi(\pi,...) = ? \quad \pi = ?$$

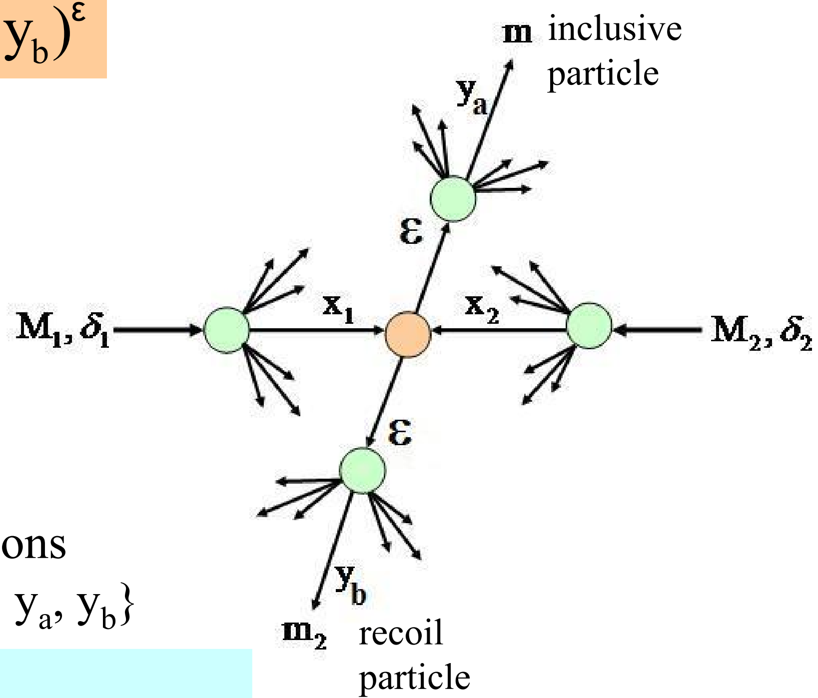
(Functional) self-similarity of II type & variable z

$$z \cong \frac{s_{\perp}^{1/2}}{\Omega} \leftarrow \Omega=(1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon}(1-y_b)^{\varepsilon}$$

$\Pi_0 \approx$ „$\sqrt{(x_1P_1+x_2P_2)^2}-\Sigma m_i$“

Δ_i

Momentum fractions $\{x_1, x_2, y_a, y_b\}$ define constituent subprocess



z - is self-similarity parameter of II type
- is expressed via momentum fractions x_i, y_i
- is a fractal measure

parameters:

δ_1,δ_2 - structure of the colliding objects M_1 M_2

ε - fragmentation process

Ω ~ relative number of all constituent configurations containing the subprocess defined by $\{x_1, x_2, y_a, y_b\}$

Ω^{-1} ~ resolution at which the constituent subprocess can be singled out of the inclusive reaction.

fractal property of z: $z(\Omega) \rightarrow \infty \quad \text{if} \quad \Omega^{-1} \rightarrow \infty \, (x,y \rightarrow 1)$

Variable z, constituent subprocess & entropy S

$$\boxed{z \equiv \frac{1}{(dN_{ch}/d\eta|_0)^c} \times \frac{s_\perp^{1/2}}{\Omega}} \Rightarrow \boxed{z \equiv \frac{s_\perp^{1/2}}{W}}$$

$$\Omega = (1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon}(1-y_b)^{\varepsilon}$$

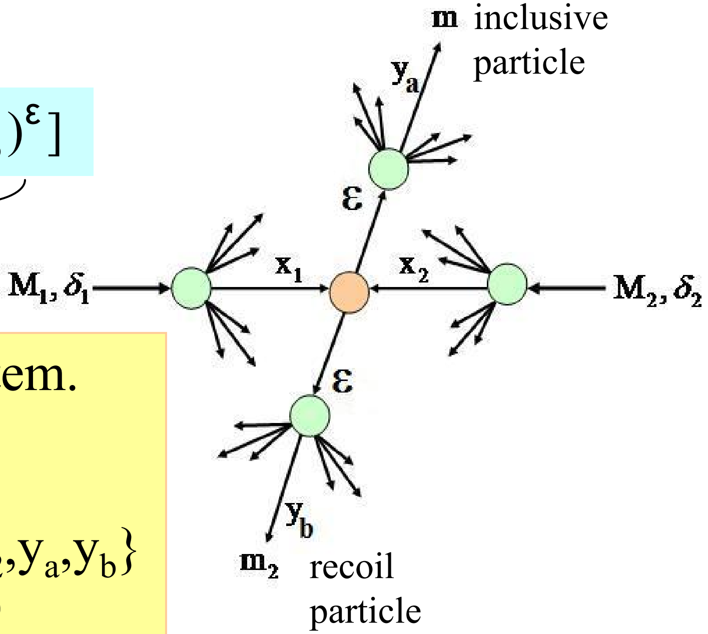
$$W = (dN_{ch}/d\eta|_0)^c \cdot \Omega$$

$$S = \ln W$$

W = relative number of all configurations
which include the configuration $\{x_1, x_2, y_a, y_b\}$.

$$S = \underbrace{c \cdot \ln(dN_{ch}/d\eta|_0)}_{} + \underbrace{\ln[(1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon}(1-y_b)^{\varepsilon}]}_{}$$

$$S = c_V \ln T + R \ln V$$



- $dN_{ch}/d\eta|_0$ characterizes "temperature" of the system.
- $dN_{ch}/d\eta|_0 \sim T^3$ for high temperatures and small μ.
- c - "specific heat" of the produced medium.
- $\delta_1, \delta_2, \varepsilon$ - fractal dimensions in the space of $\{x_1, x_2, y_a, y_b\}$
- $\varepsilon \equiv \varepsilon_F$ - depends on the type (F) of the hadron (*m*)

$S(x_1,x_2,y_a,y_b)$ = entropy of the rest of the system

Maximal entropy => Constituent subprocess

(calculation of z)

$$S_{\max} = \ln W_{\max}$$

$$W_{\max} = (dN_{ch}/d\eta|_0)^c \cdot \Omega_{\max}$$

$$\Omega_{\max} = (1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon}(1-y_b)^{\varepsilon}$$

with $(x_1P_1+x_2P_2-p/y_a)^2 = M_X^2$,

$M_X = x_1M_1+x_2M_2+m_2/y_b$

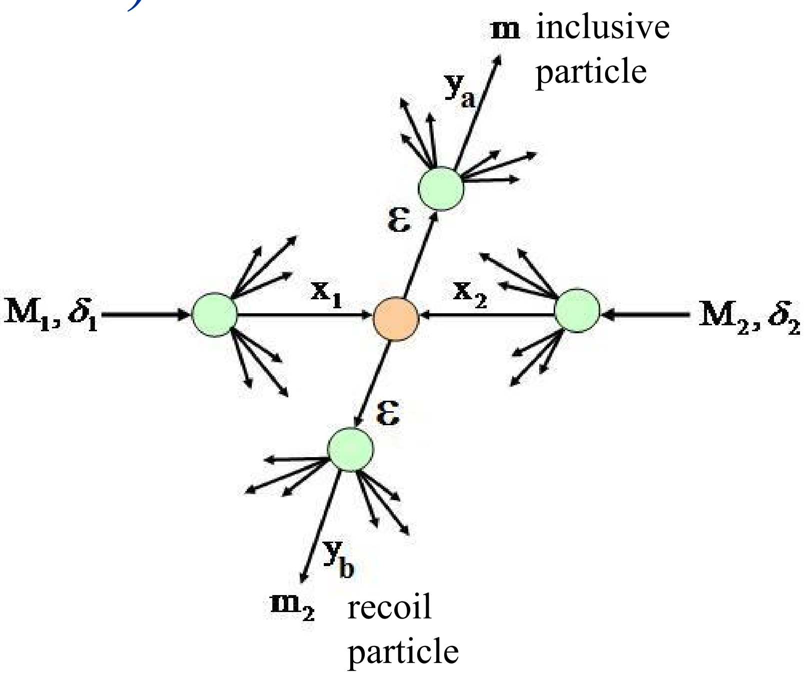
$$y_a, y_b; \quad x_i = \lambda_i + \chi_i$$

$$s_\lambda = (\lambda_1 P_1 + \lambda_2 P_2)^2 \qquad s_\chi = (\chi_1 P_1 + \chi_2 P_2)^2$$

$$T_a = y_a(\sqrt{s_\lambda} - M_1\lambda_1 - M_2\lambda_2) - m \qquad T_b = y_b(\sqrt{s_\chi} - M_1\chi_1 - M_2\chi_2) - m_2$$

$$s_\perp^{1/2} = T_a + T_b$$

$$z \equiv \frac{s_\perp^{1/2}}{W_{\max}}$$



$$p_T/y_a = \bar{p}_T/y_b$$

$$T_a \cong \sqrt{p_T^2 + m^2} - m$$

$$T_b \cong \sqrt{\bar{p}_T^2 + m_2^2} - m_2$$

Data analysis => $m_2 = m$

Scaling function $\Psi(z)$

$$\Psi(z) = \frac{1}{N\sigma_{in}}\frac{d\sigma}{dz} \Rightarrow \Psi(z) = \frac{\pi}{(dN/d\eta)\sigma_{in}} J^{-1} E \frac{d^3\sigma}{dp^3}$$

- σ_{in} - inelastic cross section
- N – average multiplicity of the corresponding hadron species
- $dN/d\eta$ - pseudorapidity multiplicity density of particles (*m*) at θ (η)
- $J(z,\eta;p_T^2,y)$ – Jacobian
- $Ed^3\sigma/dp^3$ - inclusive cross section

Normalization of $\psi(z)$:

$$\int_0^\infty \Psi(z)dz = 1$$

Scale transformation of z

$$z = \frac{s_{\perp}^{1/2}}{W_{\max}}$$

$$z' = z/W_0 \quad \Psi'(z') = W_0 \Psi(z)$$

Scale transformation of z is connected with absolute value of entropy.

$$S_{\max} = \ln W_{\max} + \ln W_0$$

W_0 - absolute number of the constituent configurations
(drops out of the z-scaling).
$W_0 = W_0(F)$ - depends on the type (F) of the inclusive particle (*m*).

Scaling functions for different hadrons collapse
to a single curve using the transformation

$$z \rightarrow \alpha_F z \quad \Psi \rightarrow \alpha_F^{-1} \Psi$$

$\alpha_F = W_0(F)/W_0(\pi)$ for the corresponding particle type (F)

The scale transformation of z preserves the normalization $\int_0^{\infty} \Psi(z) dz = 1$

Properties of the scaling function ψ(z) in pp/p$\bar{p}$ collisions

- Energy independence ($s^{1/2} > 20$ GeV)
- Angular independence ($\theta_{cms}=3^0-90^0$)
- Multiplicity independence ($dN_{ch}/d\eta=1.5-26$)
- Power law, $\psi(z) \sim z^{-\beta}$, at high z (z>4)
- Flavor independence (π,K,φ,Λ,..,D,J/ψ,B,ϒ,…)
- Saturation at low z (z<0.1)

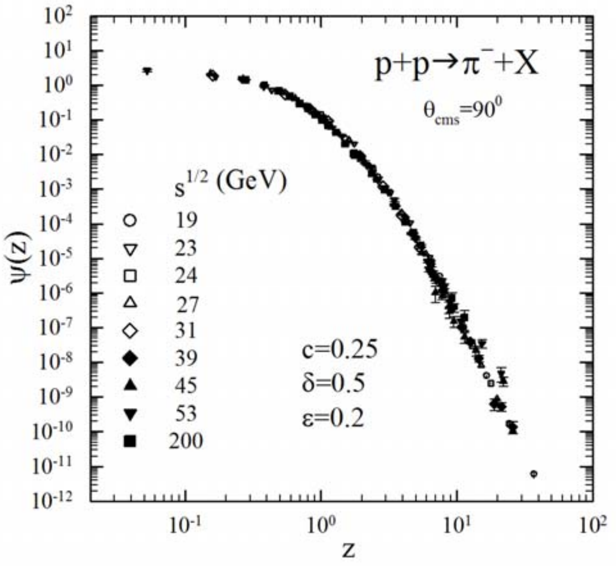
Scaling function at very high z is for pp and p$\bar{p}$ different

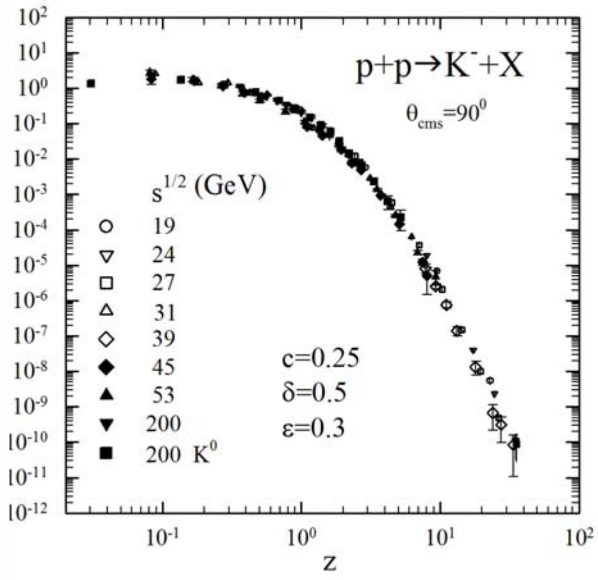
Energy independence of ψ(z)

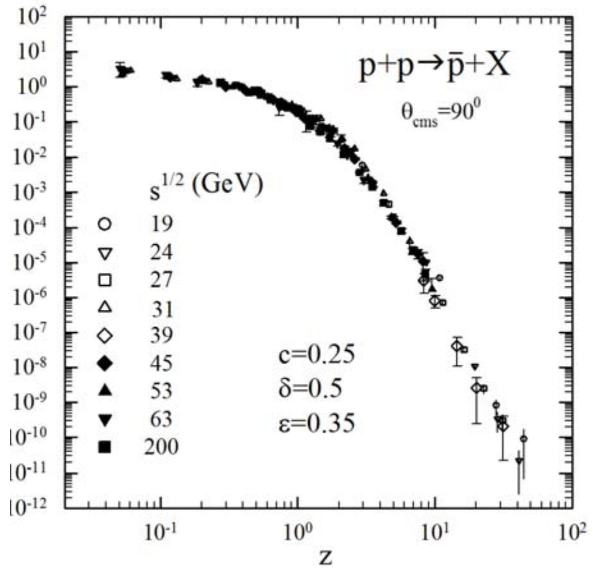
Identified hadrons - π^-, K^-, $\bar{p}$ in pp collisions

FNAL: PRD 19 (1979) 764 ; PRD 40 (1989) 2777
ISR: NPB 100 (1975) 237

STAR: PLB 637 (2006) 161; PLB 616 (2005) 8.
J. Adams, M. Heinz, nucl-ex/0403020





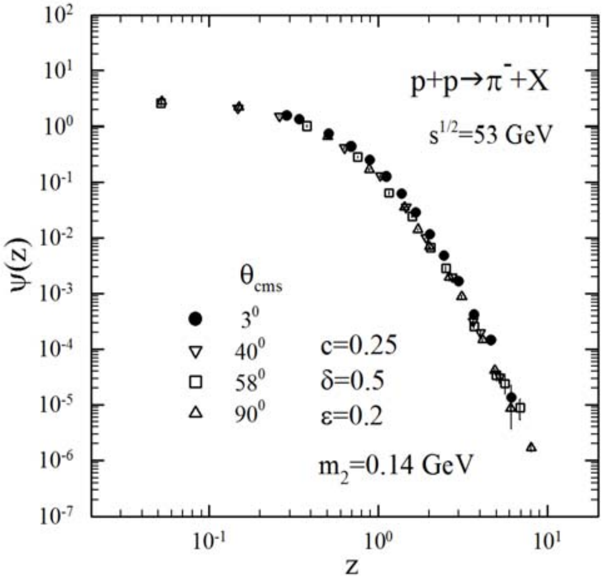


- The shape of ψ(z) is the same for different hadrons
- The power law, $\psi(z)\sim z^{-\beta}$, at large z
- ψ(z) is sensitive to δ and ε at large z
- ε increases with the particle mass ($\varepsilon\equiv\varepsilon_F$)

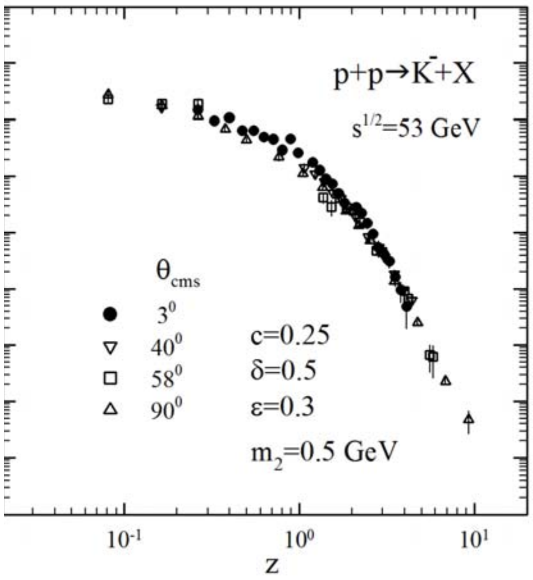
Angular independence of $\psi(z)$

Identified hadrons - π^-, K^-, $\bar{p}$ in pp collisions

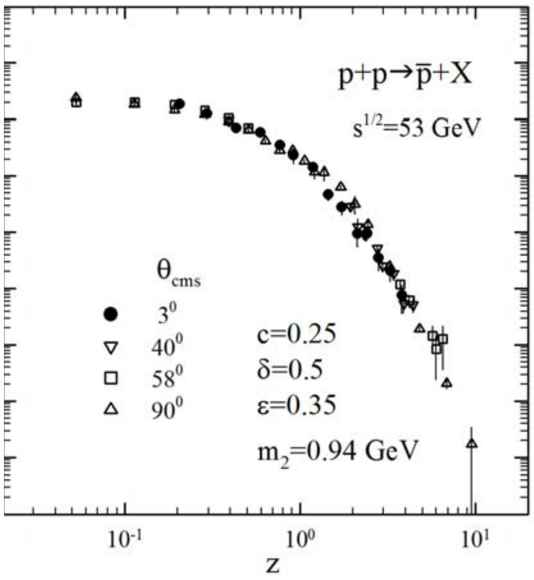
ISR: NPB 56 (1973) 333; NPB 100 (1975) 237



$m_2=m(\pi)$



$m_2=m(K)$



$m_2=m(p)$

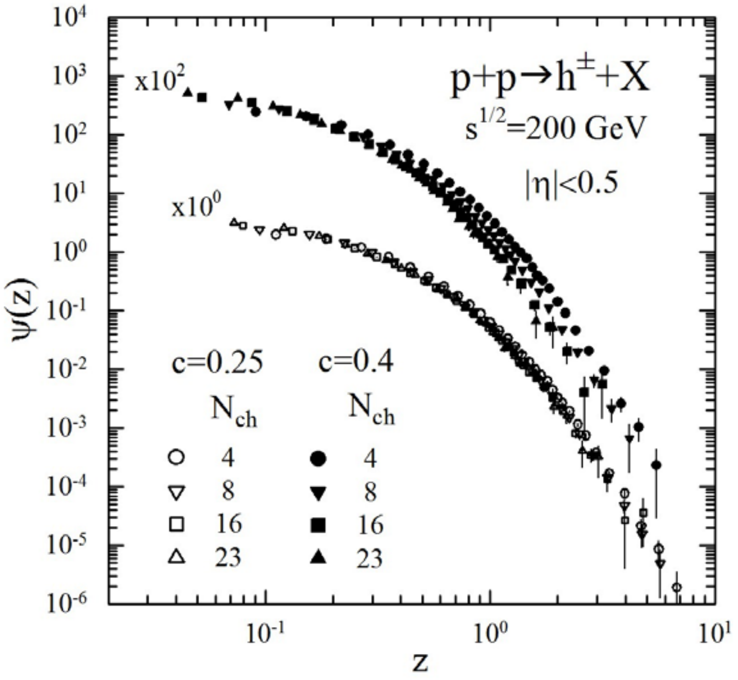
- Sensitivity of $\psi(z)$ to m_2 in the fragmentation region ($\theta_{cms}=3^0$)
- ε increases with the particle mass ($\varepsilon\equiv\varepsilon_F$)

$$(x_1P_1+x_2P_2-p/y_a)^2 = (x_1M_1+x_2M_2+m_2/y_b)^2$$

Sensitivity of data z-presentation to parameters

Sensitivity to “specific heat c”

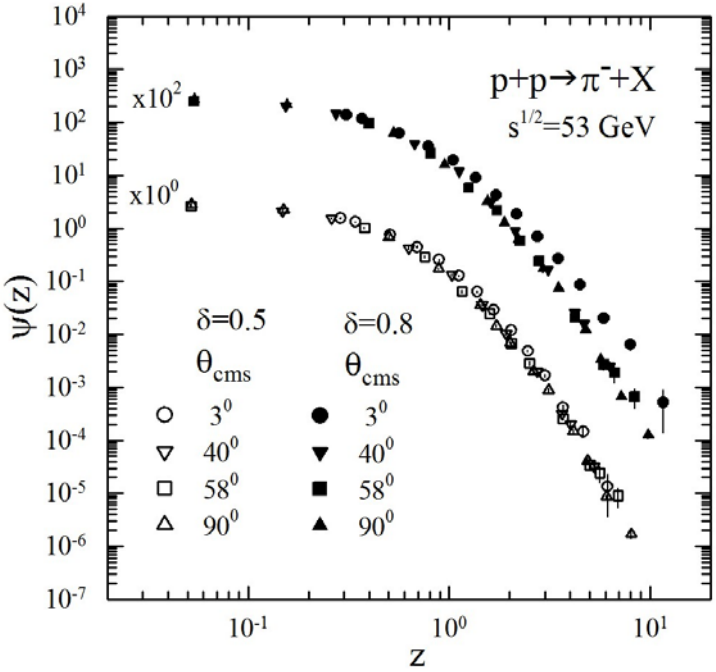
STAR: PRD 74 (2006) 032006



Sensitivity to fractal dimension δ

BS: NPB 100 (1975) 237

CHLM: NPB 56 (1973) 333



$$z = \frac{s_\perp^{1/2}}{W}$$

$$W=(dN_{ch}/d\eta|_0)^c(1-x_1)^\delta(1-x_2)^\delta(1-y_a)^\varepsilon(1-y_b)^\varepsilon$$

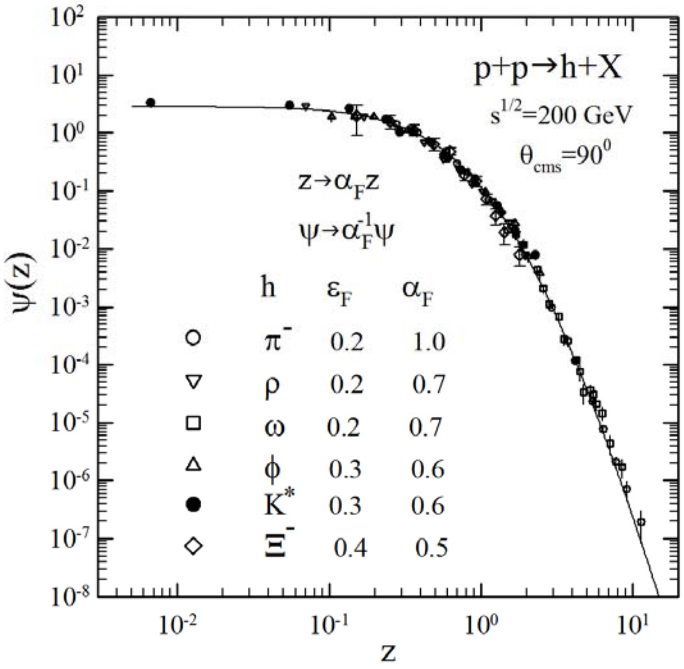
F-independence of $\psi(z)$ and saturation at low z

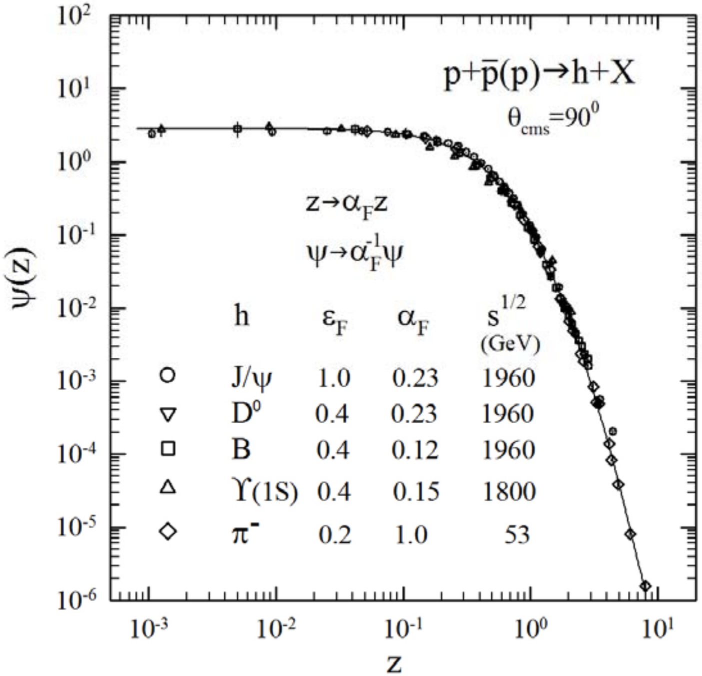
π^-, ρ, ω, ϕ, K*, Ξ, J/ψ, D, B, Υ in pp/p$\bar{\text{p}}$ collisions

STAR:
PRL 92 (2004) 092301
PLB 612 (2005) 181
PRC 71 (2005) 064902
PRC 75 (2007) 064901

PHENIX:
PRC 75 (2007) 051902

CDF:
PRL 88 (2002) 161802
PRL 91 (2003) 241804
PRD 71 (2005) 032001





- Energy independence
- Flavor independence (ρ, ω, ϕ, Ξ, J/ψ, D, B, Υ)
- Saturation for z = 0.001-0.1

- Power law $\psi(z) \sim z^{-\beta}$ at large z
- ε_F, α_F independent of p_T, $s^{1/2}$

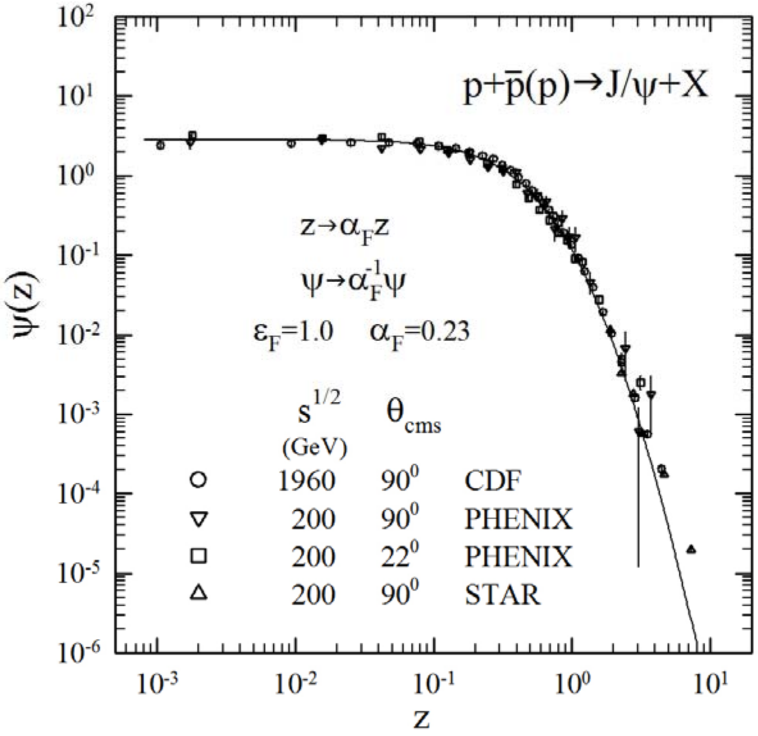
J/ψ: scaling function & saturation at low z

J/ψ in pp/p$\bar{p}$ collisions

CDF:
PRD 71 (2005) 032001

PHENIX:
PRL 98 (2007) 232002

STAR:
QM2008,Jaipur,India
arXiv:0804.4846



- $\psi(z) \sim z^{-\beta}$ at large z
- ε_F, α_F independent of p_T, $s^{1/2}$, θ_{cms}

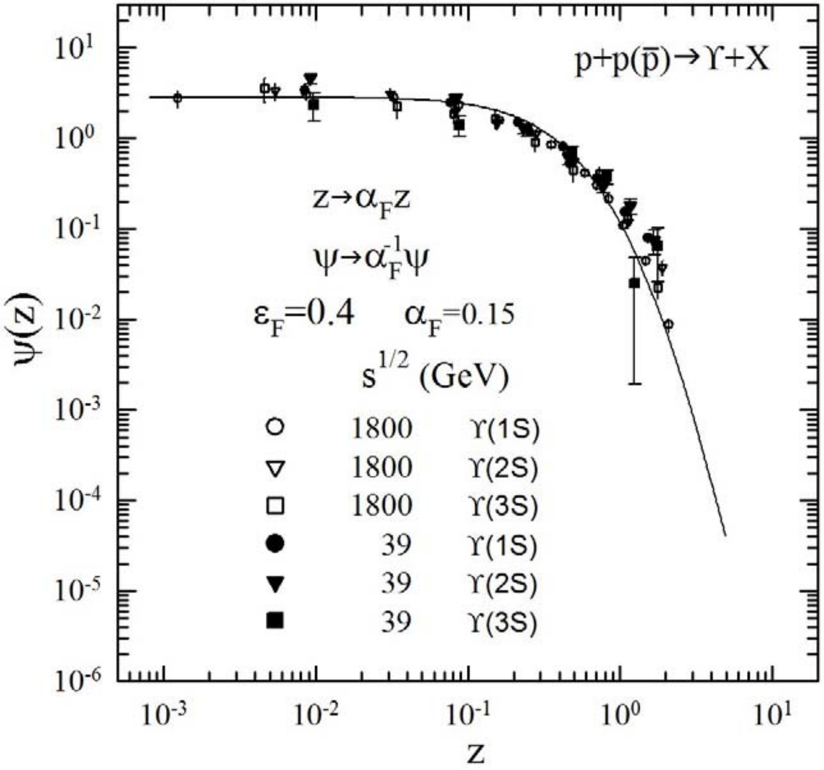
- Energy & angular independence
- Saturation for z=0.001-0.1
- Extra large ε_F=1 for J/ψ

Υ: scaling function & saturation at low z

Υ in pp/p$\bar{p}$ collisions

CDF Coll.:
PRL 88 (2002) 161802

E866/NuSea Coll.:
PRL 100 (2008) 062301

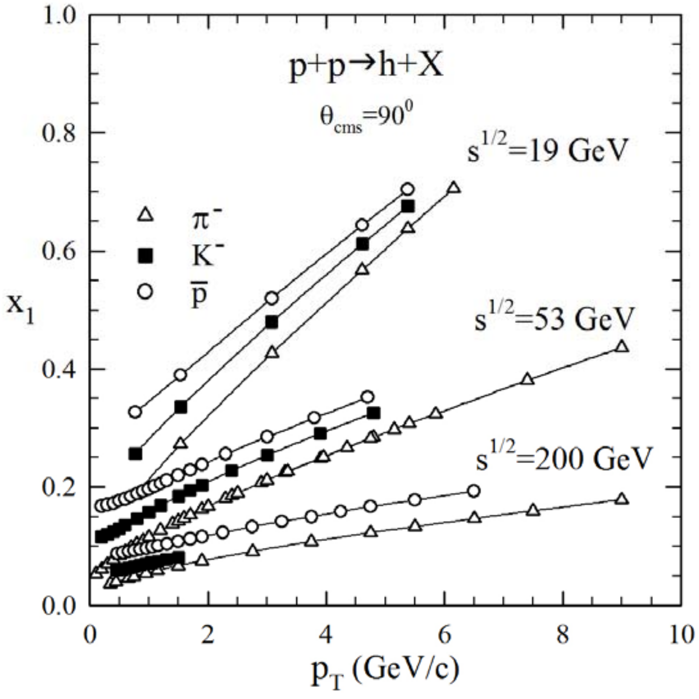


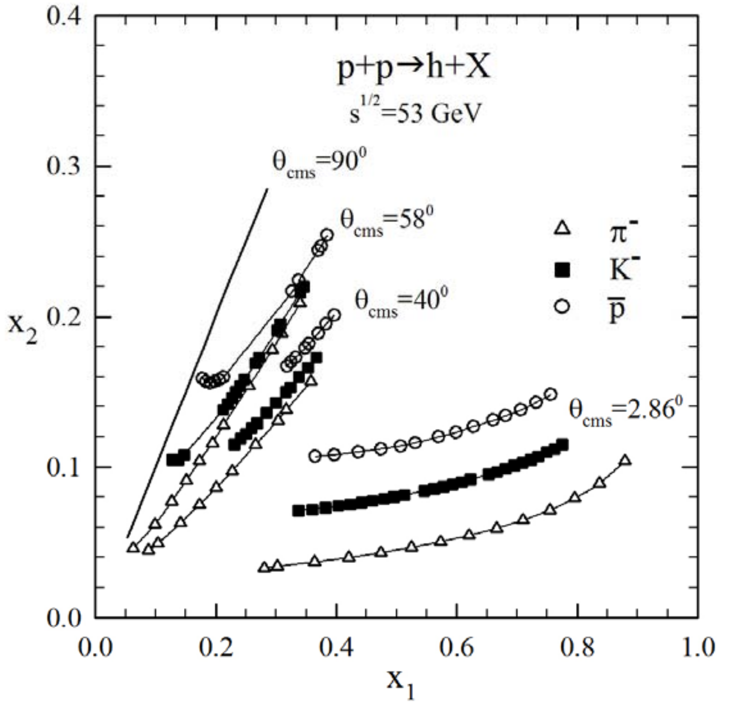
ε_F, α_F
independent of p_T, $s^{1/2}$

- Energy independence
- Mass independence: ψ(1S)≈ ψ(2S)≈ ψ(3S)
- Saturation for z=0.001-0.1

Momentum fractions x_1 & x_2

FNAL, ISR, RHIC





- x_1, x_2 increase with p_T and decrease with $s^{1/2}$
- x_1, x_2 increase slightly with the particle mass
- $x_1=x_2$ at $\theta_{cms}=90^0$; $x_1>>x_2$ at $\theta_{cms}=2.86^0$
- Considerable increase of the small fraction x_2 with the particle mass at $\theta_{cms}=2.86^0$

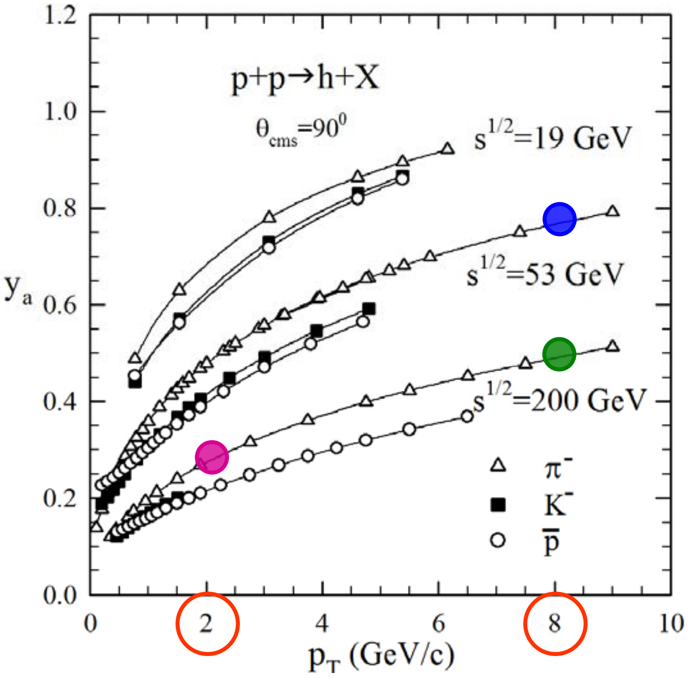
$$(x_1P_1+x_2P_2-p/y_a)^2 = (x_1M_1+x_2M_2+m/y_b)^2$$

Energy loss ($\Delta E/E \sim 1-y_a$) in the final state

(momentum fraction y_a)

$$(x_1P_1+x_2P_2-\underbrace{p/y_a}_{q})^2 = (x_1M_1+x_2M_2+m/y_b)^2$$

$y_a q = p$



FNAL (fixed target)
CERN ISR
BNL RHIC

20% energy loss
$q \approx 10$ GeV/c

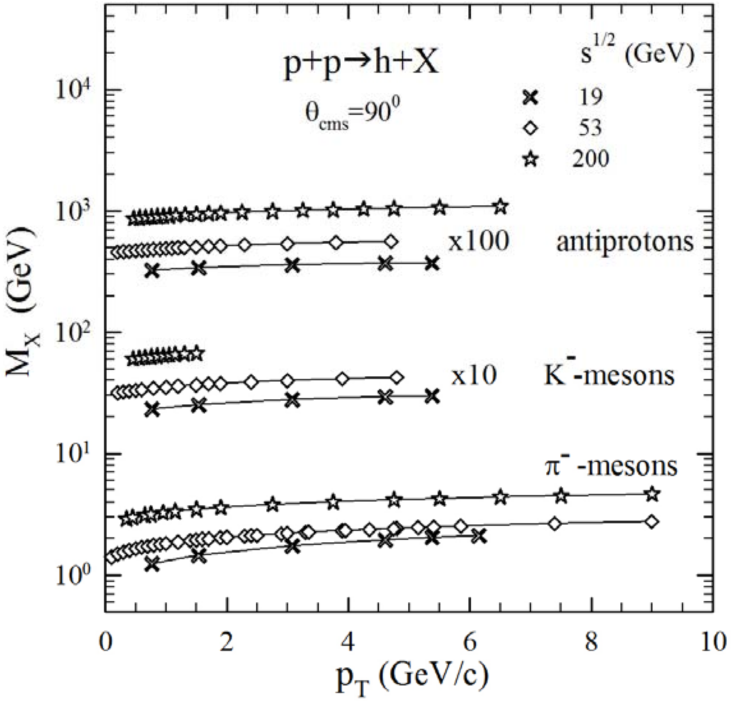
50% energy loss
$q \approx 16$ GeV/c

70% energy loss
$q \approx 7$ GeV/c

- y_a increases with p_T ⇒ energy losses decrease with p_T
- y_a decreases with $s^{1/2}$ ⇒ energy losses increase with $s^{1/2}$
- y_a decreases with *m* ⇒ energy losses increase with particle mass *m*.

Recoil mass $M_X = x_1 M_1 + x_2 M_2 + m/y_b$

$$(x_1 P_1 + x_2 P_2 - p/y_a)^2 = (x_1 M_1 + x_2 M_2 + m/y_b)^2$$

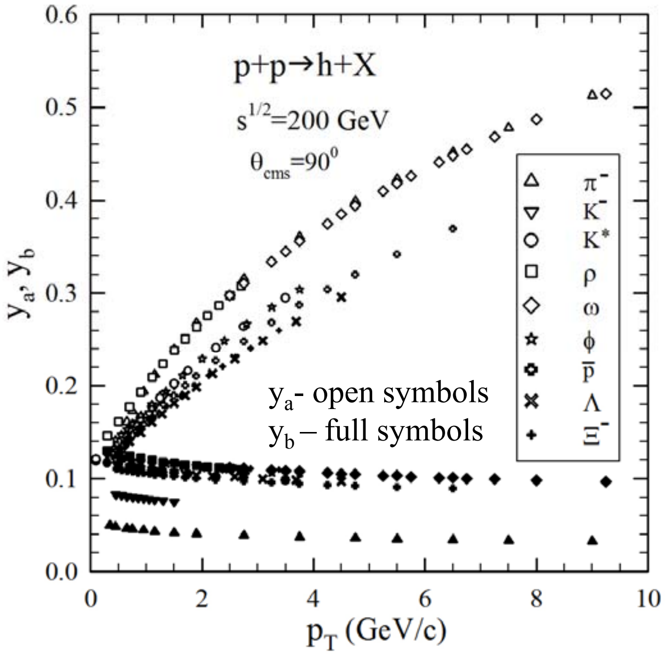


FNAL (fixed target),
CERN ISR,
BNL RHIC

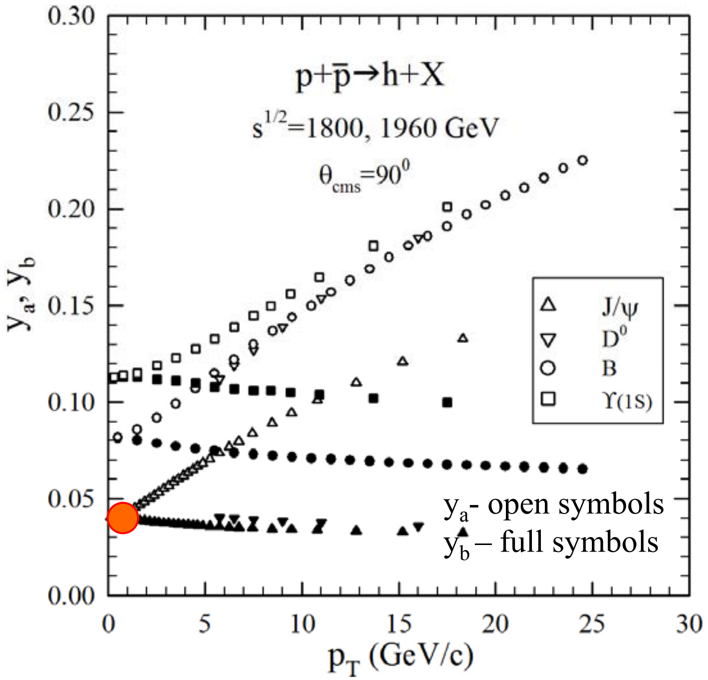
- $y_b \ll y_a$ for $p_T > 1$ GeV/c $\Rightarrow$ high multiplicity recoil M_X
- y_b is flat with p_T $\Rightarrow$ week p_T-dependence of M_X
- y_b decreases with $s^{1/2}$ $\Rightarrow$ M_X increases with $s^{1/2}$
- y_b decreases with m $\Rightarrow$ M_X increases with m

Momentum fractions y_a & y_b (various particles)

RHIC



Tevatron

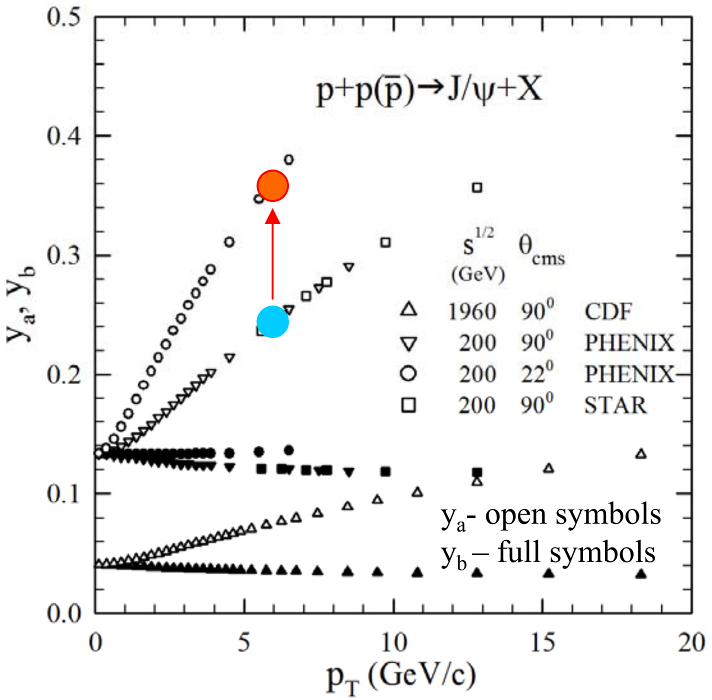


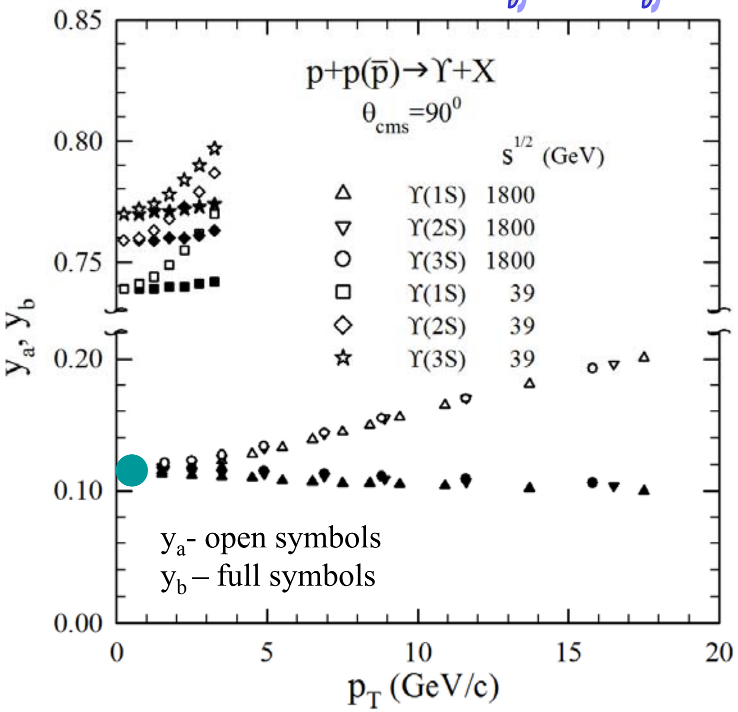
- y_a increases with p_T ⇒ energy loss decreases with p_T
- y_b is flat with p_T ⇒ weak dependence of M_X on p_T
- $y_b \approx y_a$ at low p_T ⇒ $M_X \approx m/y_a$ (for heavy particles)
- Extra small y_a for J/ψ ⇒ extra large energy loss in J/ψ production
- Extra small y_b for J/ψ ⇒ extra soft (high multiplicity) recoil M_X in J/ψ production

$(x_1P_1+x_2P_2-p/y_a)^2 = (x_1M_1+x_2M_2+m/y_b)^2$

Momentum fractions y_a & y_b (heavy flavors)

FNAL, RHIC, Tevatron





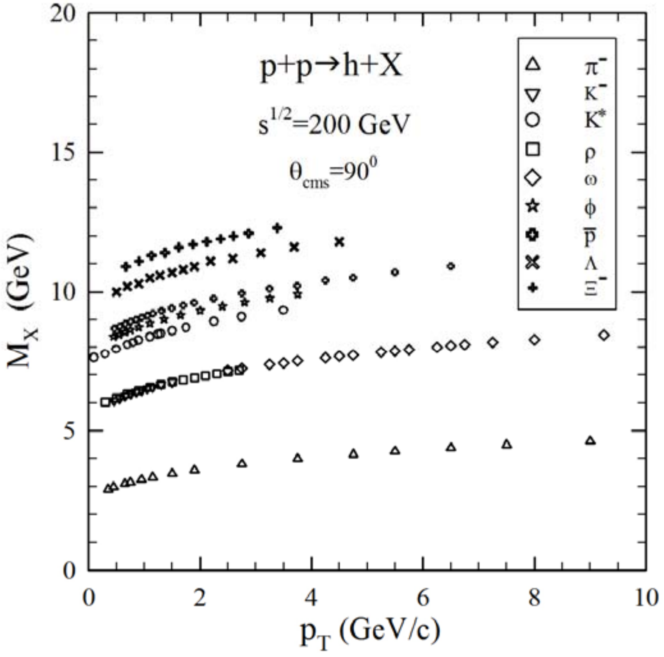
- y_a decreases with $s^{1/2}$ ⇒ increase of energy loss with $s^{1/2}$
- y_b decreases with $s^{1/2}$ ⇒ increase of the recoil multiplicity with $s^{1/2}$
- y_a large for small θ ⇒ small energy loss ($\Delta E/E \sim 1-y_a$) in the fragmentation region
- $y_a \approx y_b$ at small p_T ⇒ $M_X \approx m/y_a$ for heavy quarkonia (independent on $s^{1/2}$)
- y_a, y_b depend on ϒ state at lower energy (y_a, y_b do not depend on ϒ state at higher energy)

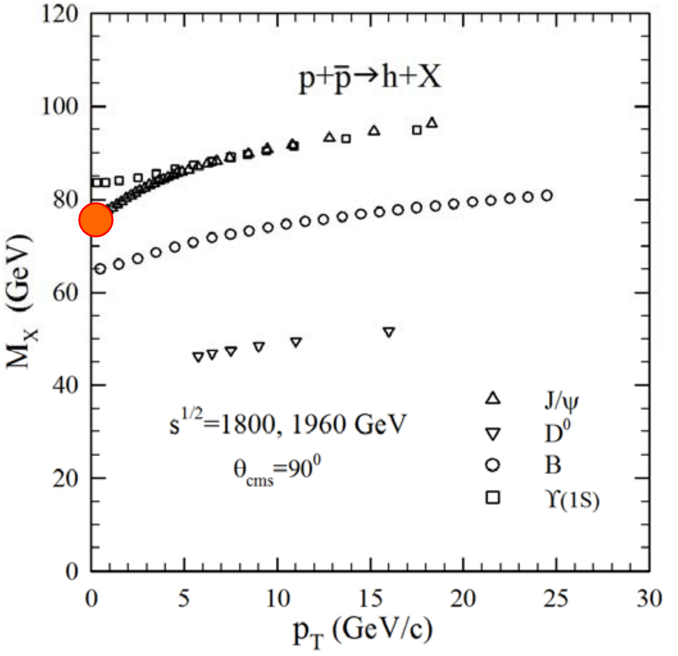
$(x_1P_1+x_2P_2-p/y_a)^2 = (x_1M_1+x_2M_2+m/y_b)^2$

Recoil mass M_X (various particles)

$M_X = x_1M_1 + x_2M_2 + m/y_b$

FNAL, ISR, RHIC, Tevatron





- M_X slightly increases with p_T for all particles
- M_X increases with the particle mass *m*
- M_X increases with $s^{1/2}$ for all particles

Extra large M_X for J/ψ ⇒ extra soft (high multiplicity) recoil M_X in J/ψ production

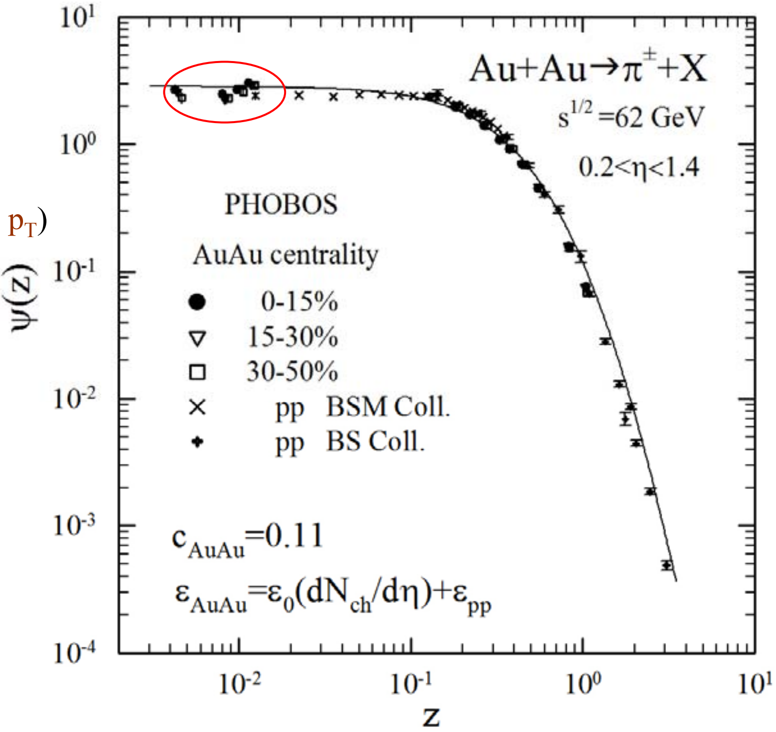
$(x_1P_1+x_2P_2-p/y_a)^2 = (x_1M_1+x_2M_2+m/y_b)^2$

Saturation of ψ(z) at low z in AuAu collisions

π in pp & AuAu collisions

PHOBOS:
PRC 75 (2007) 024910

ISR:
NPB 100 (1975) 237
PLB 64 (1976) 111 (low p_T)



At low z (low p_T) x,y are small,

$$\Omega=(1-x_1)^{\delta}(1-x_2)^{\delta}(1-y_a)^{\varepsilon}(1-y_b)^{\varepsilon}\cong 1$$

$$\boxed{z \cong \frac{s_{\perp}^{1/2}}{(dN_{ch}/d\eta|_0)^c}}$$

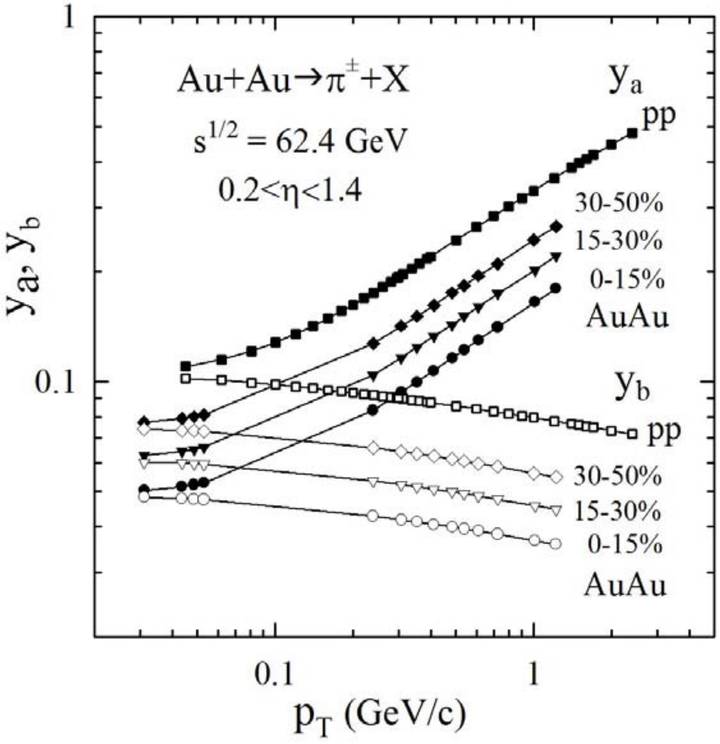
…but subprocess kinematics still depends on δ and ε

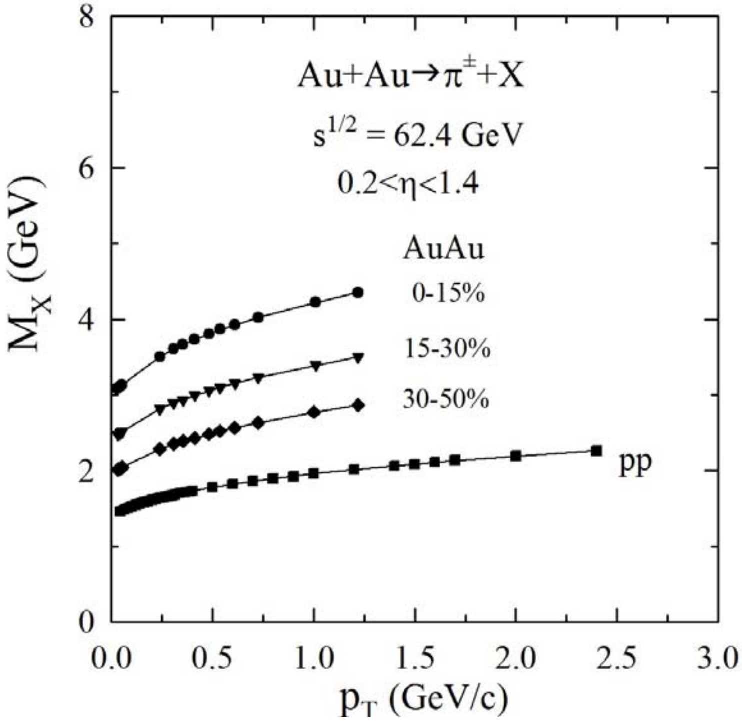
- Saturation of ψ(z) in AuAu for z<0.1
- Saturation in AuAu extrapolates the saturation in pp down to z=0.004
- Centrality (multiplicity) independence of ψ(z) in AuAu

Fractions y_a, y_b & Recoil mass M_X

(low p_T pions in AuAu)

$M_X = x_1 M_1 + x_2 M_2 + m/y_b$





- $y_b \ll y_a$ for $p_T > 1$ GeV/c ⇒ high multiplicity M_X
- y_a, y_b decrease with centrality ⇒ increase of energy loss ($\Delta E/E \sim 1-y_a$) with centrality
- $y_a \approx y_b$ at $p_T < 0.1$ GeV/c ⇒ $M_X \approx m/y_a$

- M_X slightly increases with p_T in pp
- M_X increases with p_T in AuAu
- M_X increases with AuAu centrality

$(x_1 P_1 + x_2 P_2 - p/y_a)^2 = (x_1 M_1 + x_2 M_2 + m/y_b)^2$

First LHC data on charged hadron production

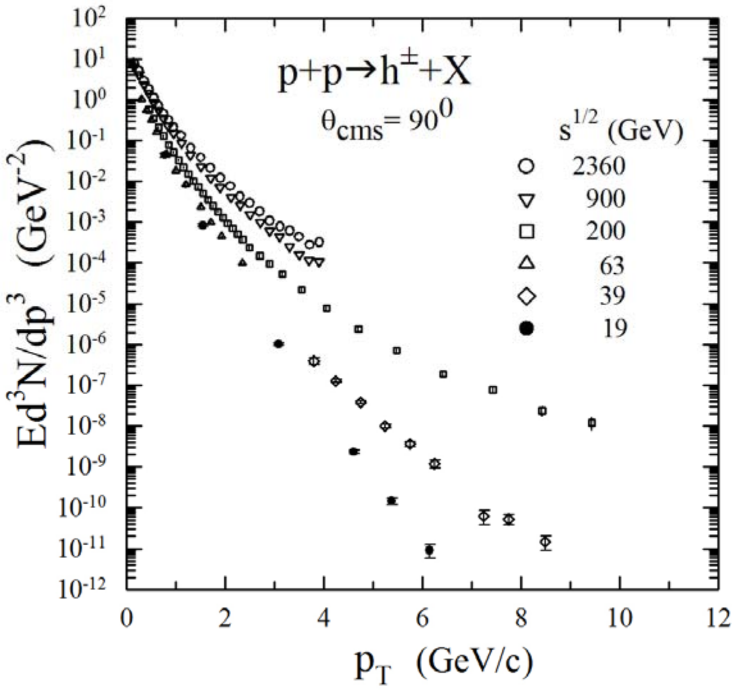
charged hadrons in pp collisions at low p_T

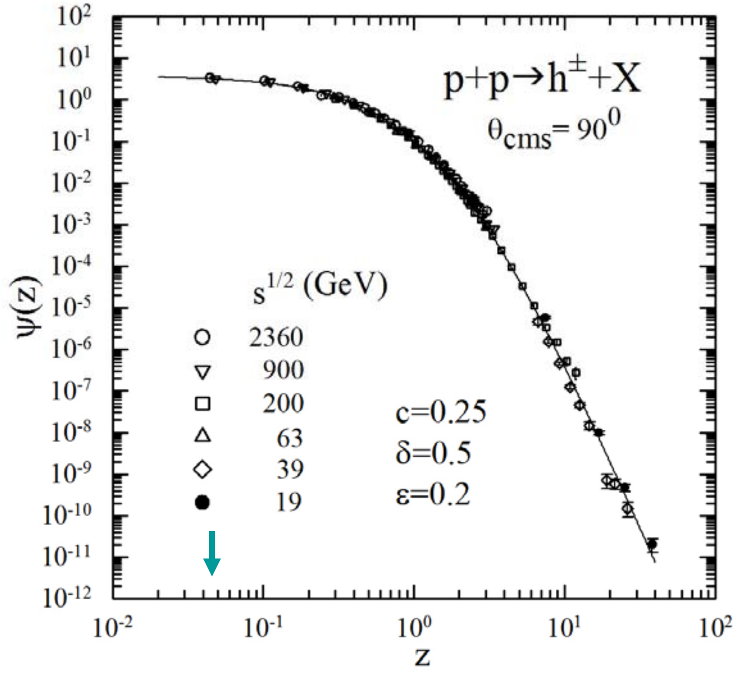
FNAL (fixed target):
PRD 19 (1979) 764
PRD 40 (1989) 2777

ISR: BS Coll.:
Nucl.Phys. B 100 (2007) 237

RHIC: STAR Coll.:
PRL 91 (2003) 172302

LHC: CMS Coll.:
JHEP02(2010) 041





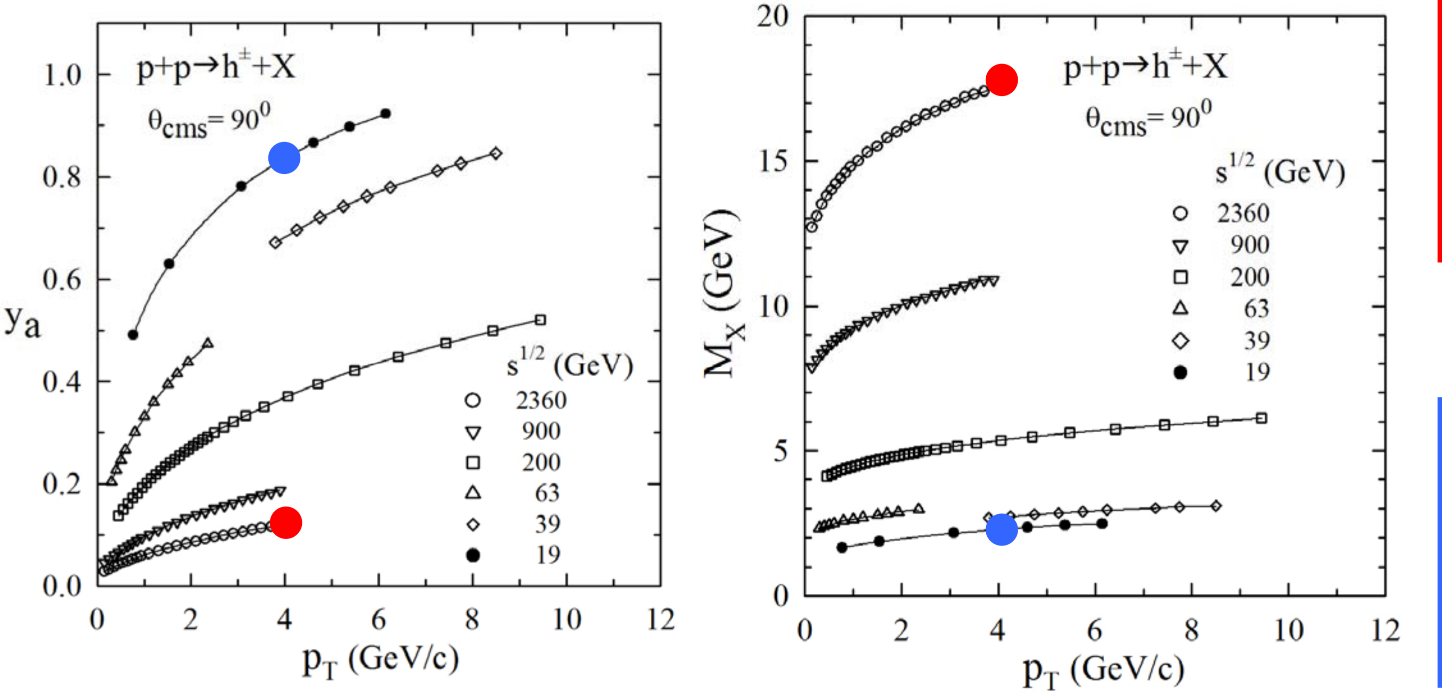
M.Tokarev & I.Z.
J.Phys.G: Nucl.Part.Phys.
37 (2010) 085008

- Energy independence of ψ(z) at low p_T
- Saturation for $z < 0.1$

Momentum fraction y_a & Recoil mass M_X

$$(x_1P_1+x_2P_2-\underbrace{p/y_a}_{q})^2=(M_X)^2 \qquad y_a\,q=\bigcirc\!\!\!\!p$$

$$M_X=x_1M_1+x_2M_2+m/y_b$$



LHC

$s^{1/2}$=2360 GeV
p_T=4 GeV/c:

90% energy loss
q ≈ 40 GeV/c
M_X ≈ 18 GeV

FNAL

$s^{1/2}$=19 GeV
p_T=4 GeV/c:

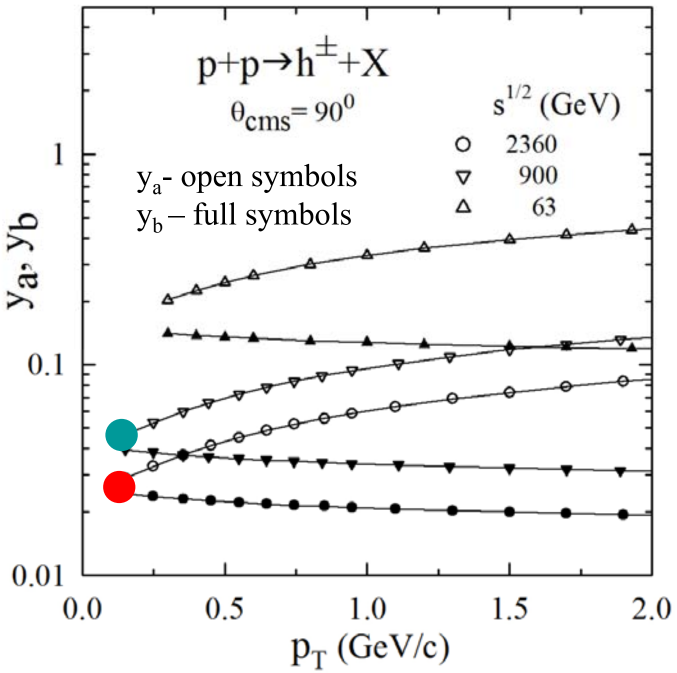
20% energy loss
q ≈ 5 GeV/c
M_X ≈ 2 GeV

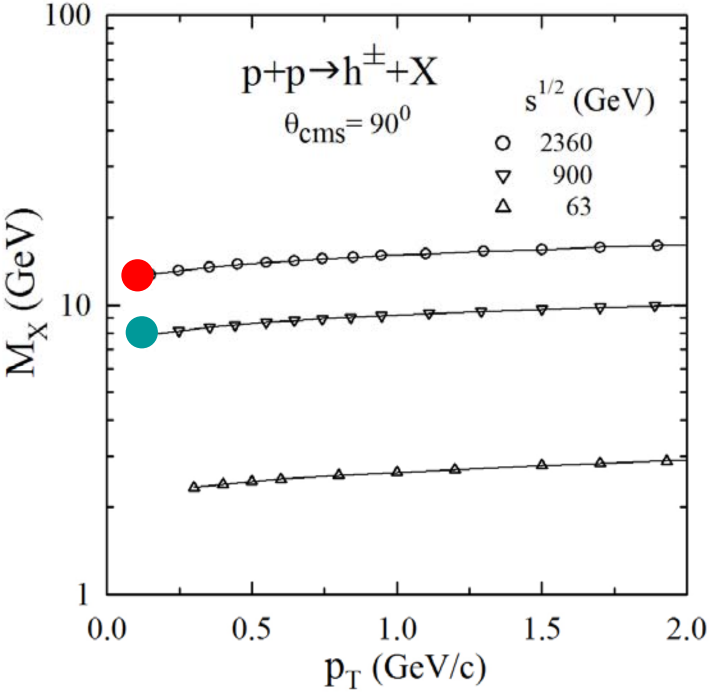
- y_a increases with p_T => energy loss ($\Delta E/E \sim 1-y_a$) decreases with p_T
- y_a decreases with $s^{1/2}$ => energy loss increases with $s^{1/2}$
- M_X increases with $s^{1/2}$ => multiplicity increase in the away-side

Low p_T limit - saturation of $\psi(z)$

$$(x_1P_1+x_2P_2-\underbrace{p/y_a}_{q})^2=(M_X)^2 \qquad y_a\, q = \boxed{p}$$

$$M_X = x_1M_1+x_2M_2+m/y_b$$





LHC

$s^{1/2} = 2360$ GeV
p_T =150 MeV/c:

97% energy loss
$q \approx 5$ GeV/c
$M_X \approx 13$ GeV

$s^{1/2} = 900$ GeV
p_T=150 MeV/c:

95% energy loss
$q \approx 3$ GeV/c
$M_X \approx 8$ GeV

- $y_a \approx y_b$ at p_T<0.5 GeV/c => $M_q \approx M_X$
- $q < M_X$ at low p_T => slow moving secondary objects (fireballs, resonances…) which fragment to low p_T hadrons
- y_a is small at low p_T @ LHC => large energy loss ($\Delta E/E \sim 1-y_a$) in the soft region

LHC data on K^0_s at low p_T & $\psi(z)$ saturation

K^0_s in pp collisions

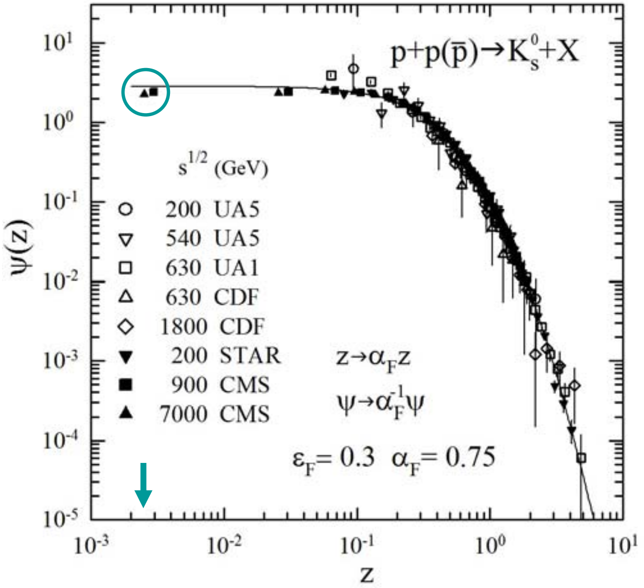
CERN: UA5:
PLB 199 (1987) 311
NPB 258 (1985) 505

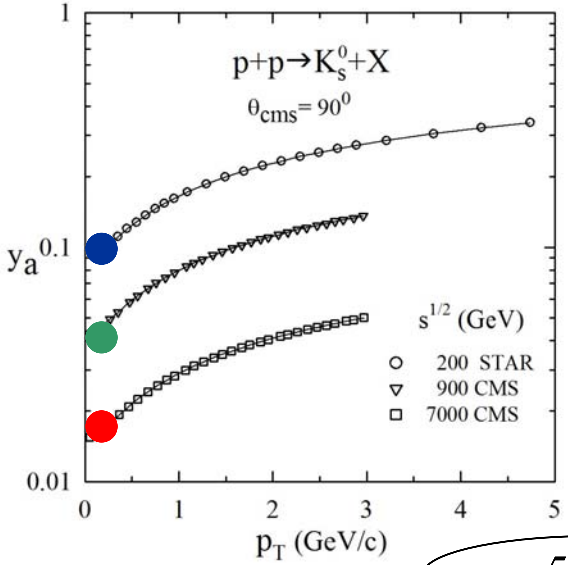
CERN: UA1:
PLB 366 (1996) 441

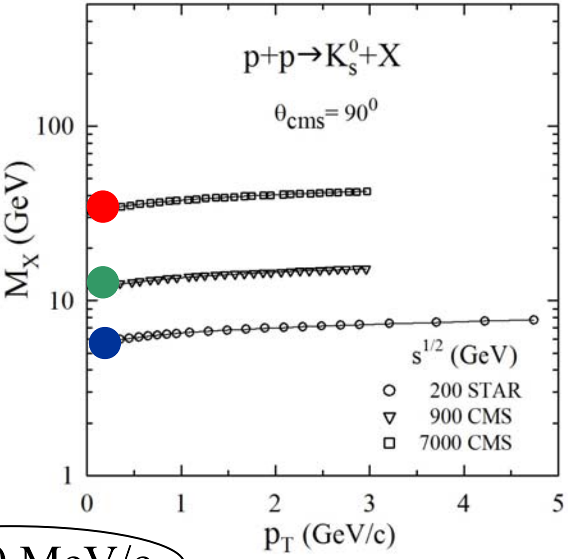
FNAL: CDF Coll.:
PRD 40 (1989) 3791

RHIC: STAR Coll.:
PRC 75 (2007) 064901

LHC: CMS Coll.:
CERN CMS PAS QCD-10-007







- Energy independence at low p_T
- Saturation down to $z \sim 0.0025$
- K^0_s suitable to study low z region
- $\underbrace{y_a \approx y_b \Rightarrow q/M_X \approx p_T/m}$

Coherent processes at low p_T

p_T=50 MeV/c

$s^{1/2}$ = 200 GeV
90% energy loss
$q \approx 0.5$ GeV/c
$M_X \approx 5.5$ GeV

$s^{1/2}$ = 900 GeV
96% energy loss
$q \approx 1.2$ GeV/c
$M_X \approx 12$ GeV

$s^{1/2}$ = 7000 GeV
98.5% energy loss
$q \approx 3$ GeV/c
$M_X \approx 30$ GeV

$$(x_1P_1+x_2P_2-\underbrace{p/y_a}_{q})^2 = (M_X)^2$$

$$y_a q = \boxed{p}$$

Conclusions

- ❑ The main features of z-presentation of inclusive spectra at high energies were summarized.
- ❑ New properties of the z-scaling in $pp/p\bar{p}$ collisions
  - flavor independence and saturation at low z - were established.
- ❑ Kinematic properties of the constituent sub-processes were discussed.
- ❑ Energy losses in pp collisions were estimated.
- ❑ New LHC pp data on charged hadrons and K_s^0 confirm saturation of z-scaling at low p_T
- ❑ Saturation of $\psi(z)$ at low z indicates on coherence in processes with low p_T
- ❑ The results may be of interest in searching for new physics in soft p_T region of particle production at RHIC, Tevatron, and LHC.

Forthcoming data from LHC should resolve some opened questions

- Saturation of heavy flavor hadrons in the soft region.
- Difference of z-presentation in pp & $p\bar{p}$ in the hard region.
- Constraints on the parameters used in the z-scaling approach.
- Hard regime of particle production in data z-presentation.
- Relation between jets and identified particles.
- Clarification of some theoretical issues concerning z-scaling (not discussed here)…

Thank You for Attention