Long-Range Rapidity Correlations in the Model with Independent Emitters

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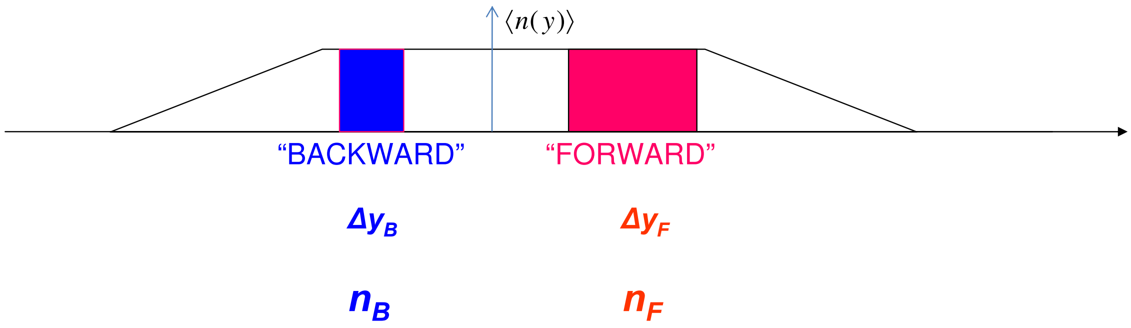
Long Range Rapidity Correlations

two rapidity intervals separated by a gap

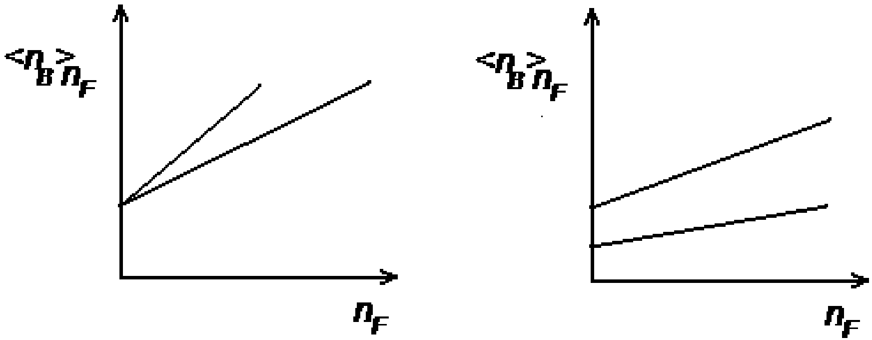


- the event multiplicity in the BACKWARD or FORWARD rapidity windows.

$$\langle n_B \rangle_{n_F} \equiv f(n_F) \ - \ \text{the correlation function (regression)}$$

The **linear** correlation function (linear regression):

$$\langle n_B \rangle_{n_F} = a^{abs} + b^{abs} n_F$$



$$\frac{\langle n_B \rangle_{n_F}}{\langle n_B \rangle} = a^{rel} + b^{rel}\,\frac{n_F - \langle n_F \rangle}{\langle n_F \rangle} = a^{rel} + b^{rel}\left(\frac{n_F}{\langle n_F \rangle} - 1\right)$$

$$b^{rel} = \frac{\langle n_F \rangle}{\langle n_B \rangle}\, b^{abs}\ , \qquad a^{rel} = \frac{\langle n_B \rangle_{n_F = \langle n_F \rangle}}{\langle n_B \rangle}$$

For a **nonlinear** correlation function $\langle n_B \rangle_{n_F} = f(n_F)$ (nonlinear regression), expanding in powers of $[n_F - \langle n_F \rangle]$ we have

$$\langle n_B \rangle_{n_F} \equiv f(n_F) = f_0 + f_1 [n_F - \langle n_F \rangle] + f_2 [n_F - \langle n_F \rangle]^2 + f_3 [n_F - \langle n_F \rangle]^3 + \ldots$$

$$b^{abs} \equiv \left. \frac{d \langle n_B \rangle_{n_F}}{d n_F} \right|_{n_F = \langle n_F \rangle} = f_1 \ , \qquad b^{rel} = \left. \frac{d \langle n_B \rangle_{n_F} / \langle n_B \rangle}{d n_F / \langle n_F \rangle} \right|_{n_F = \langle n_F \rangle} = \frac{\langle n_F \rangle}{\langle n_B \rangle} \, b^{abs}$$

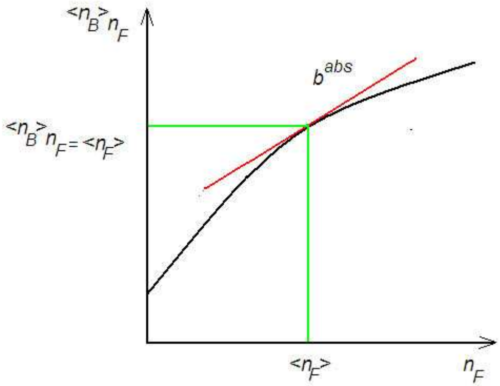
$$\langle n_B \rangle_{n_F = \langle n_F \rangle} = f(\langle n_F \rangle) = f_0 \ , \qquad a^{rel} = \frac{\langle n_B \rangle_{n_F = \langle n_F \rangle}}{\langle n_B \rangle}$$

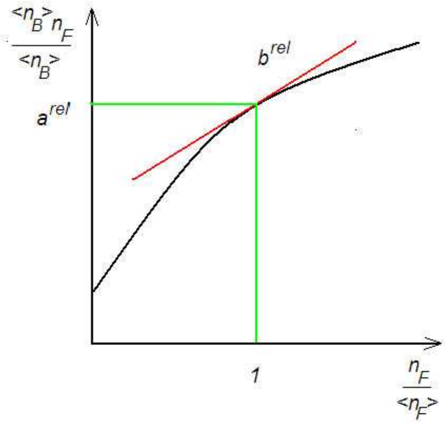
To exclude the trivial dependence on the lengths of the forward Δy_F and backward Δy_B rapidity windows we define the correlation coefficient b^{rel} using the scaled variables:

Definition 1 :

$$b^{rel} \equiv \left. \frac{d\langle n_B\rangle_{n_F}/\langle n_B\rangle}{dn_F/\langle n_F\rangle} \right|_{n_F=\langle n_F\rangle} = \frac{\langle n_F\rangle}{\langle n_B\rangle} \left. \frac{d\langle n_B\rangle_{n_F}}{dn_F} \right|_{n_F=\langle n_F\rangle} = \frac{\langle n_F\rangle}{\langle n_B\rangle} b^{abs}$$

where $\langle n_F\rangle$ and $\langle n_B\rangle$ are the mean multiplicities in the forward and backward rapidity windows. The $\langle n_B\rangle_{n_F}$ is the correlation function (regression) - the mean multiplicity in the backward window Δy_B as a function of the multiplicity in the forward window Δy_F.





In the framework of the model with independent emitters in paper [1] using methods developed in [2] under some very specific assumptions the following formula for the defined correlation coefficient b^{rel} was obtained:

$$b^{rel} = \frac{\kappa\,\overline{\mu}_F}{\kappa\,\overline{\mu}_F + 1} \; .$$

Here the κ is the ratio of two scaled variances:

$$\kappa = \frac{V_N}{V_{\mu_F}} \;, \qquad V_N = \frac{D_N}{\langle N \rangle} \;, \qquad V_{\mu_F} = \frac{D_{\mu_F}}{\overline{\mu}_F} \;,$$

$\langle N \rangle$ and $D_N = \langle N^2 \rangle - \langle N \rangle^2$ - the mean number of emitters and the event-by-event variance of the number of emitters.
$\overline{\mu}_F$ and $D_{\mu_F} = \overline{\mu_F^2} - \overline{\mu}_F^2$ - the mean multiplicity produced by *one* emitter in the *forward* window and the corresponding variance.

1. V.V. Vechernin, R.S. Kolevatov, hep-ph/0304295 (2003); Vestnik SPbU, ser.4, no.2, 12 (2004).

2. M.A. Braun, C. Pajares and V.V. Vechernin, Phys. Lett. B493, 54 (2000).

For Poisson distributions $V_N = V_{\mu_F} = 1$ and $\kappa = 1$. Clear that the $\overline{\mu}_F$ is depends on the length of the forward rapidity window. In a first approximation we can assume

$$\overline{\mu}_F = \mu_{0F} \Delta y_F$$

where μ_{0F} is the average multiplicity produced by *one* emitter in the *forward* window per a *unit* of rapidity.

$$b^{rel} = \frac{\kappa \mu_{0F} \, \Delta y_F}{\kappa \mu_{0F} \, \Delta y_F + 1} \; .$$

So the multiplicity correlation coefficient b^{rel} even *defined for scaled variables* nevertheless depends through μ_F on the length of the *forward* rapidity window Δy_F and does not depend on the length of the backward one Δy_B.

This is because the regression procedure is being made by the forward window. One can find the physical discussion of this phenomenon in ref.:

V.V. Vechernin, R.S. Kolevatov, hep-ph/0304295 (2003); Vestnik SPbU, ser.4, no.2, 12 (2004).

For a **linear** correlation function:

$$\langle n_B\rangle_{n_F} = a^{abs} + b^{abs} n_F \ , \qquad \frac{\langle n_B\rangle_{n_F}}{\langle n_B\rangle} = a^{rel} + b^{rel}\left(\frac{n_F}{\langle n_F\rangle} - 1\right)$$

we have *exactly*:

$$b^{abs} = \frac{\langle n_B n_F\rangle - \langle n_B\rangle\langle n_F\rangle}{\langle {n_F}^2\rangle - \langle n_F\rangle^2} = \frac{\langle n_B n_F\rangle - \langle n_B\rangle\langle n_F\rangle}{D_{n_F}} \ , \qquad b^{rel} = \frac{\langle n_F\rangle}{\langle n_B\rangle}\, b^{abs} \ ,$$

$$a^{rel} = \frac{\langle n_B\rangle_{n_F=\langle n_F\rangle}}{\langle n_B\rangle} = 1$$

.

So we can take as

Definition 2 :

$$b^{abs} = \frac{\langle n_B n_F\rangle - \langle n_B\rangle\langle n_F\rangle}{\langle {n_F}^2\rangle - \langle n_F\rangle^2} \ , \qquad b^{rel} = \frac{\langle n_F\rangle}{\langle n_B\rangle}\, b^{abs} \ ,$$

Note that for a **nonlinear** correlation function $\langle n_B\rangle_{n_F} = f(n_F)$

Definition 1 $\neq$ Definition 2

If we use the Definition 2

we can obtain the above formula for b^{rel} at very general assumptions.

Because to calculate such defined correlation coefficient

we *need not to calculate the correlation function* $\langle B\rangle_F = f(F)$,

but only four averages: $\langle F\rangle$, $\langle B\rangle$, $\langle BF\rangle$ and $\langle F^2\rangle$.

Calculation of the correlation coefficient

Simplified notations:

$$\langle B\rangle_F = a + bF \ , \qquad F \equiv n_F \ , \qquad B \equiv n_B$$

$$P(B,F) \text{ - basic} \ , \qquad \sum_{B,F} P(B,F) = 1 \ , \qquad \langle BF\rangle \equiv \sum_{B,F} BF\ P(B,F)$$

$$P(F) = \sum_B P(B,F) \ , \qquad \sum_F P(F) = 1 \ , \qquad \langle F\rangle \equiv \sum_F FP(F) = \sum_{B,F} FP(B,F)$$

$$P(B) = \sum_F P(B,F) \ , \qquad \sum_B P(B) = 1 \ , \qquad \langle B\rangle \equiv \sum_B BP(B) = \sum_{B,F} BP(B,F)$$

$$P(B,F) = P(F)P_F(B) \ \Rightarrow \ P_F(B) = P(B,F)/P(F) \qquad \langle B\rangle_F \equiv \sum_B BP_F(B)$$

For independent identical emitters:

$$P(B,F)=\sum_N w(N)\sum_{B_1,\dots,B_N}\sum_{F_1,\dots,F_N}\delta_{B\ B_1+\dots+B_N}\delta_{F\ F_1+\dots+F_N}\prod_{i=1}^{N}p(B_i,F_i)$$

For LRC:

$$p(B_i,F_i)=p_B(B_i)\ p_F(F_i)$$

Clear that for identical emitters:

$$\sum_{F_i}p_F(F_i)=1\ ,\qquad \sum_{F_i}F_i\ p_F(F_i)=\overline{\mu}_F\ ,\qquad \sum_{F_i}F_i^2\ p_F(F_i)=\overline{\mu_F^2}$$

$$\sum_{B_i}p_B(B_i)=1\ ,\qquad \sum_{B_i}B_i\ p_B(B_i)=\overline{\mu}_B\ ,\qquad \sum_{B_i}B_i^2\ p_B(B_i)=\overline{\mu_B^2}$$

We denote also

$$\sum_N w(N)=1\ ,\qquad \sum_N Nw(N)=\langle N\rangle\ ,\qquad \sum_N N^2\ w(N)=\langle N^2\rangle$$

The variances:

$$D_N=\langle N^2\rangle-\langle N\rangle^2\qquad ,\qquad D_{\mu_F}=\overline{\mu_F^2}-\overline{\mu}_F^2$$

and the scaled variances:

$$V_N=D_N/\langle N\rangle\qquad ,\qquad V_{n_F}=D_{\mu_F}/\overline{\mu}_F$$

Calculation of $\langle n_F^2 \rangle \equiv \langle F^2 \rangle$ as an example

$$\langle n_F^2 \rangle \equiv \langle F^2 \rangle \equiv \sum_F F^2 P(F) = \sum_F F^2 \ \sum_N w(N) \sum_{F_1,\dots,F_N} \delta_{F \ F_1+\dots+F_N} \prod_{i=1}^{N} p_F(F_i) =$$

$$= \sum_N w(N) \sum_{F_1,\dots,F_N} (F_1 + \dots + F_N)^2 \prod_{i=1}^{N} p_F(F_i) =$$

$$= \sum_N w(N) \sum_{F_1,\dots,F_N} [\sum_{i=1}^{N} F_i^2 + \sum_{i \neq j=1}^{N} F_i F_j] \prod_{i=1}^{N} p_F(F_i) =$$

$$= \sum_N w(N)[N\overline{\mu_F^2} + (N^2 - N)\overline{\mu}_F^2] = \langle N \rangle \overline{\mu_F^2} + (\langle N^2 \rangle - \langle N \rangle)\overline{\mu}_F^2 =$$

$$= \langle N \rangle (\overline{\mu_F^2} - \overline{\mu}_F^2) + \langle N^2 \rangle \overline{\mu}_F^2$$

So we find

$$\langle n_F^2 \rangle = \langle N \rangle D_{\mu_F} + \langle N^2 \rangle \overline{\mu}_F^2$$

and

$$D_{n_F} \equiv \langle n_F^2 \rangle - \langle n_F \rangle^2 = \langle N \rangle D_{\mu_F} + \langle N^2 \rangle \overline{\mu}_F^2 - \langle N \rangle^2 \overline{\mu}_F{}^2 = \langle N \rangle D_{\mu_F} + D_N \overline{\mu}_F^2$$

Gathering we find

$$b^{abs} \equiv \frac{\langle BF\rangle - \langle B\rangle\langle F\rangle}{\langle F^2\rangle - \langle F\rangle^2} = \frac{\langle BF\rangle - \langle B\rangle\langle F\rangle}{D_{n_F}} = \frac{D_N\overline{\mu}_B\overline{\mu}_F}{\langle N\rangle D_{\mu_F} + D_N\overline{\mu}_F^2}$$

and

$$b^{rel} = \frac{\langle n_F\rangle}{\langle n_B\rangle}\, b^{abs} = \frac{\langle N\rangle\overline{\mu}_F}{\langle N\rangle\overline{\mu}_B}\, b^{abs} = \frac{\overline{\mu}_F}{\overline{\mu}_B}\, b^{abs} = \frac{D_N\overline{\mu}_F^2}{\langle N\rangle D_{\mu_F} + D_N\overline{\mu}_F^2} = \frac{\kappa\,\overline{\mu}_F}{\kappa\,\overline{\mu}_F + 1}\ ,$$

where the κ is the ratio of two scaled variances:

$$\kappa = \frac{V_N}{V_{\mu_F}}\ , \qquad V_N = \frac{D_N}{\langle N\rangle}\ , \qquad V_{\mu_F} = \frac{D_{\mu_F}}{\overline{\mu}_F}\ ,$$

$$D_N = \langle N^2\rangle - \langle N\rangle^2\ , \qquad D_{\mu_F} = \overline{\mu_F^2} - \overline{\mu}_F^2$$

Comparing the definitions

In the case of **nonlinear** regression:

$$\langle n_B\rangle_{n_F} \equiv f(n_F) = f_0 + f_1[n_F - \langle n_F\rangle] + f_2[n_F - \langle n_F\rangle]^2 + f_3[n_F - \langle n_F\rangle]^3 + \ldots$$

Definition 1 $\neq$ Definition 2

By **Definition 1 :**

$$\overline{b}^{abs} \equiv \left.\frac{d\langle n_B\rangle_{n_F}}{dn_F}\right|_{n_F=\langle n_F\rangle} = f_1 \ , \qquad \overline{b}^{rel} = \frac{\langle n_F\rangle}{\langle n_B\rangle}\ \overline{b}^{abs} \ ,$$

By **Definition 2 :**

$$b^{abs} = \frac{\langle n_B n_F\rangle - \langle n_B\rangle\langle n_F\rangle}{\langle {n_F}^2\rangle - \langle n_F\rangle^2} = \frac{\langle n_B n_F\rangle - \langle n_B\rangle\langle n_F\rangle}{D_{n_F}} \ , \qquad b^{rel} = \frac{\langle n_F\rangle}{\langle n_B\rangle}\ b^{abs} \ ,$$

$$b^{abs} - \overline{b}^{abs} = \frac{f_2\langle[n_F - \langle n_F\rangle]^3\rangle + f_3\langle[n_F - \langle n_F\rangle]^4\rangle + \ldots}{D_{n_F}}$$

Coefficient a^{rel}

$$a^{rel} = \frac{\langle n_B \rangle_{n_F = \langle n_F \rangle}}{\langle n_B \rangle} = \frac{f(\langle n_F \rangle)}{\langle f(n_F) \rangle}$$

$$\langle n_B \rangle_{n_F} \equiv f(n_F) = f_0 + f_1[n_F - \langle n_F \rangle] + f_2[n_F - \langle n_F \rangle]^2 + f_3[n_F - \langle n_F \rangle]^3 + \ldots$$

$$\langle n_B \rangle_{n_F = \langle n_F \rangle} = f(\langle n_F \rangle) = f_0$$

$$\langle n_B \rangle = \langle f(n_F) \rangle = f_0 + f_2 \langle [n_F - \langle n_F \rangle]^2 \rangle + f_3 \langle [n_F - \langle n_F \rangle]^3 \rangle + \ldots$$

$$\boxed{\langle n_B \rangle - \langle n_B \rangle_{n_F = \langle n_F \rangle} = f_2 D_{n_F} + f_3 \langle [n_F - \langle n_F \rangle]^3 \rangle + \ldots\ .}$$

So $a^{rel} = 1$ for linear correlation function.

In the next (quadratic) approximation:

If $a^{rel} < 1$ - the correlation function is convex downwards: $f_2 > 0$.

If $a^{rel} > 1$ - the correlation function is convex upwards: $f_2 < 0$.

Conclusions

◇ The formula for the long-range multiplicity correlation coefficient in the model with independent emitters is obtained at very general assumptions:

$$b^{rel} = \frac{\kappa\,\overline{\mu}_F}{\kappa\,\overline{\mu}_F + 1}\ ,$$

where the κ is the ratio of two scaled variances: $\kappa = \frac{V_N}{V_{\mu_F}}$, $V_N = \frac{D_N}{\langle N\rangle}$, $V_{\mu_F} = \frac{D_{\mu_F}}{\overline{\mu}_F}$
and $\overline{\mu}_F$ - the mean multiplicity produced by *one* emitter in the *forward* window.

◇ The multiplicity correlation coefficient *defined for scaled variables* nevertheless depends through μ_F on the length of the *forward* rapidity window Δy_F and does not depend on the length of the backward one Δy_B:

$$b^{rel} = \frac{\kappa\mu_{0F}\,\Delta y_F}{\kappa\mu_{0F}\,\Delta y_F + 1}\ , \qquad \overline{\mu}_F = \mu_{0F}\Delta y_F$$

where μ_{0F} is the average multiplicity produced by *one* emitter in the *forward* window per a *unit* of rapidity.

◇ This is due to the regression procedure is being made by the forward window.
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