

The nucleon structure function in a statistical quark model

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Outline:

- Resume and Introduction
- Statistical Quark Model
- Main results
- Conclusions

Related references:

- L.A. Trevisan et al, Eur. Phys. J. C 56 (2008) 221, "Quark sea structure functions of the nucleon in a statistical model" [EPJC2008]
- L.A.Trevisan et al, Eur. Phys. J. C 11 (1999) 351, "Srangeness content of the nucleon in a statistical quark model" [EPJC1999]
- C. Mirez et al., in preparation.



A statistical quark model for the sea-quark distribution in the nucleon is revisited, with improved parameterization for the experimental available data.

Within such model, we report recent results for the nucleon structure function, considering the ratio and difference between the corresponding proton and neutron structure functions.

The quark levels are given by a confining quark model, with the valence quark normalization inside the nucleon, for the up and down quarks, fixed by two chemical potentials. The flavor asymmetry in the nucleon sea, given by the Gottfried sum rule violation, is adjusted by a temperature parameter. Within such a model, reasonable results are obtained for the nucleon structure functions, in comparison with actual experimental data.

It is shown that the model results can be improved by considering some additional QCD effects, such as gluonic splitting processes.



QCD statistical models for the nucleon

The first confining model to obtain the hadron structure function in deep inelastic scattering is due to Jaffe in 1974 (without considering the Dirac sea).

Later on, several models have been proposed, by considering the confinement and also the Dirac sea (see, for example, the models of Cleymans-Thews [Z.Phys. C 37 (1988)315] and Mac-Ugaz [Z.Phys. C 43 (1989)655], which are close related to the model detailed in Trevisan et al. [Eur. Phys. J. C 56 (2008) 221]).

Field-Feynman's suggestion:

- □ It appears as an idea that was first presented by Field and Feynman [Phys. Rev. D 15 (1977) 2590.], based on the Pauli principle for the proton sea.
- ❑ As we have 5 empty states to be occupied by a quark "d" and 4 to be occupied by a quark "u" in the proton, within the 6 possibilities (spin times color), the pair creation from gluon splitting to quark-antiquark will favor d-dbar.
- □ Therefore, the Pauli Principle could be considered in statistical models in order to obtain different distributions for ubar and dbar in the sea of the nucleon.



Statistical quark model for the nucleon

In the statistical quark model we have considered, all individual quarks of the system, valence and sea quarks, are confined by a relativistic scalar+vector central effective interaction, with strength λ [Leal Ferreira, Helayel and Zagury, Nuovo Cim. A55 (1980) 215.], as given by

 $V(r) = (1+\beta)\frac{\lambda r}{2}.$

From the Dirac Equation:

$$[\alpha \mathbf{p} + \beta m + V(r)]\psi_i(\mathbf{r}) = \varepsilon_i \psi_i(\mathbf{r}) \,. \qquad \psi_i(\mathbf{r}) = \begin{pmatrix} 1 \\ \sigma \mathbf{p}/(m + \varepsilon_i) \end{pmatrix} \varphi_i(\mathbf{r}) \,,$$

The coupled equation will be reduced to a 2nd order equation:

$$\left[\mathbf{p}^{2}+(m+\varepsilon_{i})(m+\lambda r-\varepsilon_{i})\right]\varphi_{i}=0\,.$$

with solutions and energy levels given by

$$\varphi_i(r) = \sqrt{\frac{K_i}{4\pi}} \frac{\operatorname{Ai}(K_i r + a_i)}{r \left[\frac{\mathrm{dAi}(x)}{\mathrm{d}x}\right]}, \quad \varepsilon_i = m - \frac{\lambda}{K_i} a_i \cdot \varepsilon_i = \sqrt{\lambda} (-a_i)^{\frac{3}{4}}.$$



We use equal strengths for the confined light *u* and *d* quarks, as well as equal zero current masses.

By summing the first energy level of the quarks we should fit the Δ resonance mass, in a model where the instantons are responsible for the mass shift between Nucleon and Delta.

Fermi-Dirac distribution

In the present statistical model, we consider the Fermi-Dirac distribution.

The probability density of a quark system with temperature T is given by

$$\rho_q(\mathbf{r}) = \sum_i g_i \psi_i^{\dagger}(\mathbf{r}) \psi_i(\mathbf{r}) \frac{1}{1 + \exp\left(\frac{\varepsilon_i - \mu_q}{T}\right)}$$



The chemical potentials are used to adjust the quark flavor number inside the proton (neutron), and the temperature is a parameter that is obtained by the Gottfried sum rule violation.

The degeneracy of each level is given by the constant g.

$$\int \left[\rho_q(\mathbf{r}) - \bar{\rho}_q(\mathbf{r})\right] \mathrm{d}^3 r = \sum_i g_i \left[\frac{1}{1 + \exp\left(\frac{\varepsilon_i - \mu_q}{T}\right)} - \frac{1}{1 + \exp\left(\frac{\varepsilon_i + \mu_q}{T}\right)}\right] = \begin{cases} 1 & \text{for } q = d(u) \\ 2 & \text{for } q = u(d) \end{cases}$$



To obtain the structure function, we take the Fourier transform to the momentum space and use the null plan variables

$$p_z = p^+ - \varepsilon_i = M_N \left(x - \frac{\varepsilon_i}{M_N} \right), \quad p^+ = x P^+, \quad P^+ = M_N \quad (N \equiv n, p)$$

The structure function is given by:

$$q_T(x) = \sum_i \int \mathrm{d}^2 p_\perp \frac{\Phi_i^{\dagger} \Phi_i}{1 + \exp\left(\frac{\varepsilon_i - \mu_q}{T}\right)},$$

where

$$\Phi_{i} \equiv \Phi_{i} \left(M_{N} \left(x - \frac{\varepsilon_{i}}{M_{N}} \right), \mathbf{p}_{\perp} \right),$$

is the momentum-space Fourier transform of the quark-wave function.



 S_{GSR} (Gottfried sum rule) not equal to 1/3, is an evidence of the flavor symmetry breaking in the nucleon sea.

$$\int_0^1 [u(x) - \bar{u}(x)] \, dx = 2 \qquad \qquad \int_0^1 \left[d(x) - \bar{d}(x) \right] \, dx = 1$$

Therefore:

$$S_{GSR} = \int_{0}^{1} \frac{dx}{x} [F_{2}^{p}(x) - F_{2}^{n}(x)]$$

$$S_{GSR} = \frac{1}{3} \int_{0}^{1} dx [u(x) - d(x)] + \frac{1}{3} \int_{0}^{1} dx [\bar{u}(x) - \bar{d}(x)]$$

$$S_{GSR} = \frac{1}{3} \left[2 + \int_{0}^{1} dx \bar{u}(x) \right] - \frac{1}{3} \left[1 + \int_{0}^{1} dx \bar{d}(x) \right]$$

$$+ \frac{1}{3} \int_{0}^{1} dx [\bar{u}(x) - \bar{d}(x)] = \frac{1}{3} - \frac{2}{3} \int_{0}^{1} dx [\bar{d}(x) - \bar{u}(x)]$$

Where we define an integral for the violation of the GSR:

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$$I_{GSR} = \int_{0}^{1} dx \left[\overline{d}(x) - \overline{u}(x) \right] = \frac{1}{2} - \frac{3}{2} S_{GSR}$$



The violation of the GSR is used to fit the temperature parameter. Actually, this value is given by experimental analysis. In our recent readjustment of the model we have used the one obtained from the E866 Collab., PRD 64 (2002) 052002:

$$I_{\rm GSR} \equiv \frac{1}{2} - \frac{3}{2} S_{\rm G} = \int_0^1 \left(\bar{d}(x) - \bar{u}(x) \right) \, \mathrm{d}x = 0.118 \pm 0.012$$

I_{GSR}	=	0.148 ± 0.039 [NMC (94) [6K]]	
I_{GSR}	=	0.16 ± 0.03 [Hermes (98)	
I_{GSR}	=	0.118 ± 0.012 [E866/Nusea (02	



The chemical potentials are the other two constants to be adjusted (self-consistently). Their values will depend from other considerations of the statistical model, as it will be explained

In the statistical model considered in Eur. Phys. J C 56 (2008) 221, it was considered the value obtained in 1991 and 1994 by the NMC collaboration: 0.148 (+/- 0.039). With this value, the best values for the temperature and chemical potentials were: T=108, μ_u =135 MeV, and μ_d =78 MeV.

Next, from the analysis of the E866 experiments [Peng et al, PRD 58(1998) 092004; Hawker et al, PRL 80(1998) 3715], the new value for the violation of the GSR (0.100 ± 0.018) implied in a readjustment of the parameters of the istical model, such that T=103 MeV, μ_u =147 MeV, and μ_d =88 MeV.



The chemical potentials μ_u and μ_d adjusted to fix the normalization of the valence quarks inside the proton (μ_s =0), such that

$$\int_0^1 [u(x) - \overline{u}(x)] dx = 2 , \quad \int_0^1 [d(x) - \overline{d}(x)] dx = 1$$

$$\int_0^1 [s(x) - \overline{s}(x)] dx = 0$$



INSTANTONS, EFFECTIVE QUARK MASS SHIFT, GLUONIC and other QCD effects

Instanton effects

- In order to verify how much energy we need to subtract from the total energy of the system, we use the Lagrangean of the instanton model given in A. E. Dorokhov, N. I. Kochelev, and Y.A. Zubov, Sov. J. Part. Nucl. 23 (1992) 522.
- □ The difference between *u* and *d* quark interactions in the nucleon are considered as due instantons, which are spin-flavor dependent.
- □ There is no instanton contributions for spin 3/2 particles, such as Δ (1232)(uud). And, for the *nucleon, the contribution to the energy is negative.*

	Ν	$\Delta(uud)$	$\Omega(sss)$
$M_0 \; ({\rm MeV})$	1232	1232	1676
$\varepsilon_I \ (MeV)$	-267	0	0
$M \ (MeV)$	$M_N = 965$	1232	1676



	u	d	8
$\lambda ~({\rm MeV/fm})$	239.05	239.15	357.35
$m \; (MeV)$	0	0	104
$\varepsilon_0 \; (MeV)$	1232/3=410.67	410.67	1676/3 = 558.67

Parametrization of the confining potential at zero temperature



Instanton effects and Effective light quark mass shift

Therefore, the difference between the interaction of u and d quarks is supposed to come from instanton effects, which are flavour-spin dependent, as given by *A. E. Dorokhov, N. I. Kochelev, and Y.A. Zubov, Sov. J. Part. Nucl.* 23 (1992) 522. By taking this effect into account in our model for the nucleon, we should fit the Delta 3/2, instead of the nucleon. As the instanton effect will be responsible for the mass shift between these two particles.

Also, to consider the difference of u and d quark interactions in an effective way, we note that d-quarks in the proton have a more attractive channel, with the energy lower than of the u-quarks. The initial confining conditions are the same in our model, a simple mathematical trick is used to implement such conditions, by a displacement of the distributions over the x scale, in such a way that u(x) and d(x)(and corresponding antiparticle) distributions have their respective maxima at different x positions. Note that working with both, particle and antiparticle distributions, one can obtain at once all ratios between structure functions.



Light quark mass shift

□ With help of the Delta function, we can implement this condition in the following way:

$$q(x) = \delta\left(\frac{M_q}{M_n} - x\right),$$

$$u(x) = \delta\left(\frac{M_u}{M_q}\frac{M_q}{M_n} - x\right) = \frac{M_q}{M_u}\delta\left(\frac{M_q}{M_n} - \frac{M_q}{M_u}x\right) = \frac{M_q}{M_u}q\left(\frac{M_q}{M_u}x\right)$$

It end up that we use $M_q = M_{d'} M_u = 1.25 M_q$ for the proton.

We show this effect in our results for the structure function.



<u>GLUON SPLITTING PROCESS</u>: Perturbative QCD process of gluon emmision by quarks are considered following *[Halzen, Martin, Quarks and Leptons, Wiley, NY, 1984*].

The probability of particle-antiparticle pairs from gluon emission is given by the Gribov-Altarelli-Parisi equations.

The joint probability density to obtain a quark confining from the decays of quark to quark and gluons, and gluons to quark + antiquark at some fixed low Q is given by

$$q_g(x) = \frac{N\alpha_s^2\left(Q_v^2\right)}{(2\pi)^2} \int_x^1 \mathrm{d}y \frac{P_{qg}\left(\frac{x}{y}\right)}{y} \int_y^1 \frac{\mathrm{d}z}{z} P_{gq}\left(\frac{y}{z}\right) f_T(z) \,, \qquad \qquad \frac{\bar{d}(x)}{\bar{u}(x)} = \frac{\bar{d}_T(x) + q_g(x)}{\bar{u}_T(x) + q_g(x)} \,.$$

$$f_T(x) = u_T(x) + \bar{u}_T(x) + d_T(x) + \bar{d}_T(x)$$
$$P_{gq}(z) = \frac{4}{3} \frac{1 + (1 - z)^2}{z}, \quad P_{qg}(z) = \frac{1}{2} \left(1 - 2z + 2z^2\right)$$

(The relevant parameter in the model will be the coupling constant alpha)



QUARK SUBSTRUCTURE:

By considering the quarks as effective degrees of freedom, one should consider that they can have substructure. In this case, the structure function of the consitutent quark/antiquark can be extracted from the pion structure function P(x) with the assumption that it is dominated by the asymptotic behavior, as considered by [*Frederico and Miller, PRD 50 (1994) 210*]:

$$v^{\pi}(x,Q^2) = \int_x^1 rac{\mathrm{d}y}{y} P(y,Q^2) F_{q\overline{q}}^{\pi}\left(rac{x}{y}
ight),$$

 $F_{q\bar{q}}^{\pi}$ is the pion structure function for constituent quarks from the valence wave function. Assuming $F_{q\bar{q}}^{\pi} = 1$ for the asymptotic form of the valence wave function.

$$v^{\pi}(x,Q^2) = \int_x^1 rac{\mathrm{d}y}{y} P(y,Q^2), \qquad \qquad P(x,Q^2) = -x rac{\partial}{\partial x} v^{\pi}(x,Q^2),$$



A parametrization for the structure function of a valence quark can be obtained from [Gluck et al, EPJC 10 (1999) 313]:

$$\begin{aligned} xv^{\pi}(x,Q^2) &\equiv xv^{\pi}(x) = N_{\pi}x^a \left(1 + A\sqrt{x} + Bx\right) (1-x)^D \\ N_{\pi} &= 1.212 + 0.498s + 0.009s^2, \qquad A = -0.037 - 0.578s, \qquad B = 0.241 + 0.251s, \qquad D = 0.383 + 0.624s, \\ a &= 0.517 - 0.020s, \qquad \qquad s = \ln \frac{\ln(Q^2/0.204^2)}{\ln(\mu^2/0.204^2)}. \end{aligned}$$

Using the above, the antiquark $\overline{q}(x)$ structure function in the nucleon from the constituent substructure is given by

$$ar{q}_{ ext{const}}(x) = - \int_x^1 \left. rac{\partial}{\partial z} v^\pi(z,Q^2)
ight|_{oldsymbol{z}=x/y} ar{q}(y) \, \mathrm{d} y \, ,$$

(Here, s is the relevant parameter in the model.)



MAIN RESULTS

Behavior of the model parametrization with respect to changes in the temperature "T", as well as the corresponding mass changes.

As explained, T is being used to fix the GSR violation, in a consistent way, such that the chemical potentials can keep the valence quark numbers of the nucleon

We have study the behavior of the model parametrization with and also without considering instanton effects.

The instanton effects are applied mainly in the fundamental state.





MAIN RESULTS

Gottfried sum rule violation versus temperature (MeV).

We reproduce the value of the GSR violation, given in *Towell et al.*, *Phys. Rev. D 64 (2002) 052002*, with T=104 MeV and the chemical potentials λ_u =131 MeV and λ_d =76 MeV, when taking instanton effects. Without instanton effect, we have T=107 MeV λ_u =136 MeV and λ_d =80 MeV.





The u and d quark structure functions for the proton in the statistical quark model, as functions of the Bjorken momentum scale x.

The results for the d-quark distribution are presented with the short-dashed line. For the uquark distribution, we show 3 plots: solid red line assuming equal masses (Mu =Md \equiv Mq), the dashed green line assuming Mu/Mq =1.25, the dashed orange line assuming Mu/Mq =1.43, having the maximum shifted to the right-hand side.





The model results for the antiquark ratio as a function of x in the proton, compared with data results. Without mass-scaling displacement, we obtain the *constant long-dashed line*. With the small-dashed green line curve, we present the results with $\alpha s = 2.4$; and, with solid red line, the results with $\alpha s = 2.1$ and s = 0.7.





Effect of the number of states in the Fitting, considering M_d/M_u =1.25.



from s=0.7 till s=0.3.

 M_d/M_u =1.35 (blue and pink).





Dependence of αs in the model

Results for the difference, as functions of x in the proton. The model results are compared with experimental data. With the green dashed line we have the results with α s=1.72; and, with the red solid line, with α s =2.1 and s =0.7



Behavior with the number of states, considering M_d/M_u =1.25 and changing the parameter *s*. As the number of states increase, s is reduced.

Changing the mass ratio, from M_d/M_u =1.25 to M_d/M_u =1.35



$$\frac{F_2^n}{F_2^p} = \frac{[u_{val}(x) + \bar{u}(x)] + 4[d_{val}(x) + \bar{d}(x)]}{4[u_{val}(x) + \bar{u}(x)] + [d_{val}(x) + \bar{d}(x)]}$$

$$F_2^p - F_2^n = \frac{x}{3} [u_{val}(x) - d_{val}(x)] + \frac{2x}{3} [\overline{u}(x) - \overline{d}(x)]$$







For the difference F2p-F2n, the model provides good fits to the available experimental data. In the model we consider M d/M u=1.25. With solid (red) line the results are with or without gluon splitting. With dashed (blue) line we show the sea antiquark contribution for the difference. The quark valences u and d are related to $F2p-F2n = (x/3)[u \{val\} - d \{val\}] +$ $(2x/3)[bar{u}-bar{d}].$





The same in the linear scale for M_d/M_u =1.25. With red line we also have α_s =2.1 and *s*=0.7.



The Nucleon strangeness

[Eur. Phys. J. C. 11, 351 (1999)]

□ The strangeness content of the nucleon the chemical potential for the strange quark is zero: μ_s =0. We have the same quark and antiquark content.

$$\int_0^1 \left[s(x) - \bar{s}(x) \right] dx = 0$$

□ With T satisfying the GSR violation, we can obtain the following:

$$\eta = \frac{2\int_0^1 x s(x)dx}{\int_0^1 x (u(x) + d(x)) \, dx}, \quad \kappa = \frac{2\int_0^1 x s(x)dx}{\int_0^1 x \left(\bar{u}(x) + \bar{d}(x)\right) \, dx} \quad \frac{\eta}{\kappa} = \frac{\int_0^1 \left(\bar{u}(x) + \bar{d}(x)\right) \, xdx}{\int_0^1 (u(x) + d(x)) \, xdx}$$

□ m_s=104 MeV, m_u=m_d=0



Número de	Tempera-	η	κ	η/κ	Gottfried	Gottfried
estados	tura				inicial	normalizado
Instanton (1)						
23	108	0.102021	0.77138	0.13226	0.11840	0.11589
35	105	0.101358	0.75735	0.13383	0.11961	0.11690
40	104	0.100801	0.73364	0.13740	0.12168	0.11966
45	104	0.099791	0.69837	0.14289	0.12687	0.12729

Quantidade	Intervalo x	Valor experimental		
$\int \left[\bar{d}(x) - \bar{u}(x)\right] \mathrm{d}x$	0-1	0.118 ± 0.012 E866/NuSea (2002) [58]		
η	0-1	$0.099\substack{+0.009\\-0.006}$		
κ	0-1	$0.477\substack{+0.063\\-0.053}$		
η/κ	0-1	0.2075		

Tabela 7.1: Valores experimentais de $\eta \in \kappa$ desde a refs. [11, 50, 51].



Final Considerantions:

- light quarks with different flavors (u, d, and antiparticles) have should have different energies in the model. A simple mathematical trick was used to obtain the shift of the given structure functions (which are equal, a priori), producing a quite good fit to the ratios and differences of u and d quark distributions.

- The constituent quark structure should include all possible gluonic splitting processes.

- We observe, from our parameterization, that the probability of gluon emission decreases when we consider the quark substructure (parameter s). It implies that a non-vanishing gluon splitting contribution is necessary to compensate in part for the simplified assumptions of the nucleon and pion wave functions.

-To extract the constituent quark structure we have considered the asymptotic form of the pion wave function, with the assumption that it will be the same in the nucleon.

-By considering a more realistic model, one can diminish further the relevance of explicit gluon contributions.



Even considering measurement processes for the dbar/ubar and d/u to be different, our considerations are quite good, with numerical results approaching experimental data.

The difference between the confining potentials for the u and d quarks in the nucleon (due to instanton induced Interactions) and the contributions from the gluon splitting process are found all relevant in to our fit to the data.

To improve the model, we may need to consider different strengths in the confining potentials of u and d quarks.



With my THANKS to your attention!