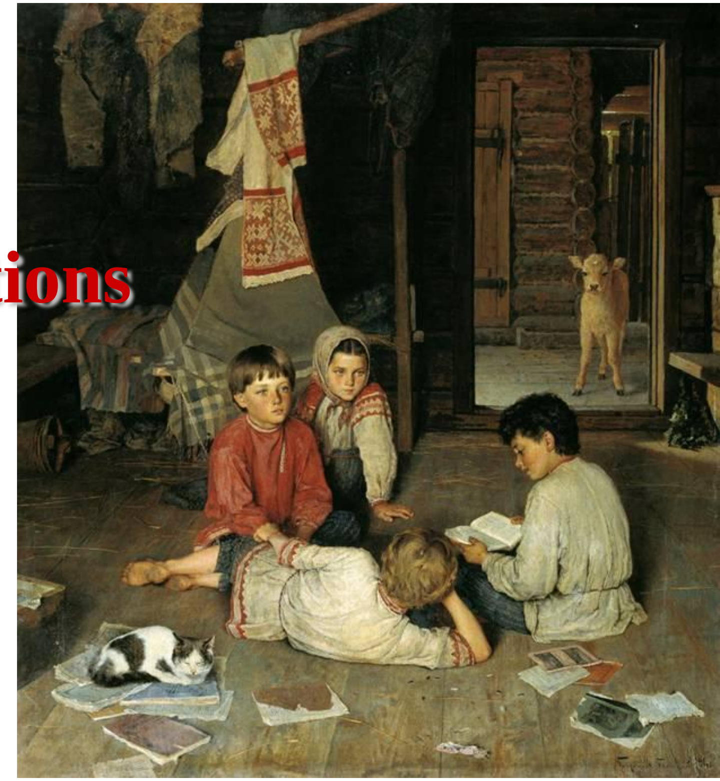
XX Baldin ISHEPP
October 8, 2010

Modification of fundamental interactions in strong electromagnetic fields

Alexander Titov
Bogoliubov Lab. of Theoretical Physics, JINR, Dubna

with Burkhard Kämpfer
Hideaki Takabe and Atsushi Hosaka

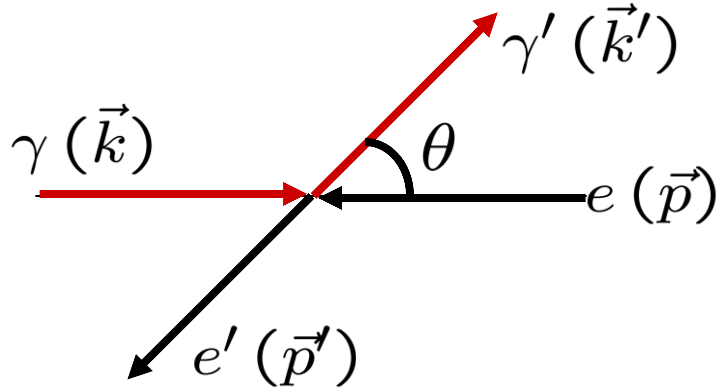


Bogdanov-Belsky
“New tale”

Outline

- Compton $\gamma e \rightarrow \gamma e$ scattering (Klein-Nishina equation)
- Volkov solution of Dirac equation in strong EM field
- Emission of photons by an electron in a strong EM field
- Reaction $e \rightarrow e + 2$ neutrinos in a strong EM field
- Weak decay of a neutron in a strong EM field
- Summary

Compton γ e → γ e Scattering (Klein-Nishina equation)

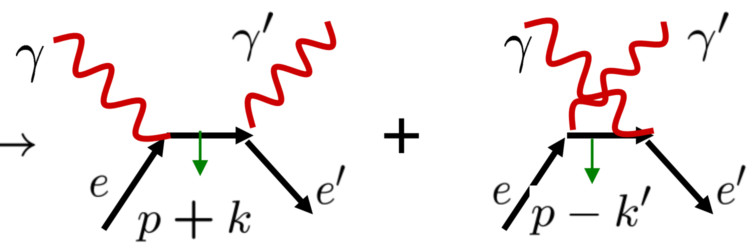


$$d\sigma = \frac{1}{16\pi(s - M_e^2)^2} |T|^2 \, dt$$

with $s = (p+k)^2 = M_e^2 + 2E_\gamma(E+p)$

$t = (p-p')^2 = (k'-k)^2$

$T^{\gamma e \to \gamma e} \longrightarrow$



$= e^2 \epsilon_\mu^*(\gamma') \epsilon_\nu(\gamma) \cdot [\bar{u}(e') M^{\mu\nu} u(p)]$

$\times (2\pi)^4 \delta(p + k - p' - k')$

$$M^{\mu\nu} = \gamma^\mu \frac{\gamma \cdot p + \gamma \cdot k + M_e}{2p \cdot k} \gamma^\nu + \gamma^\nu \frac{\gamma \cdot k - \gamma \cdot p' + M_e}{2p' \cdot k} \gamma^\mu$$

$$d\sigma = \frac{1}{16\pi(s - M_e^2)^2}|T|^2\, dt$$

$$|T|^2 = \frac{e^2}{4} \sum_{\lambda_\gamma,\lambda'_\gamma,m_e,m'_e} |\epsilon^*_\mu(\gamma')\,[\bar{u}(e')M^{\mu\nu}\, u(p)]\epsilon_\nu(\gamma)|^2$$

$$d\sigma/dt = \pi r_0^2\, F(p,p',k)M_e^2/(s - M_e^2)^2,$$

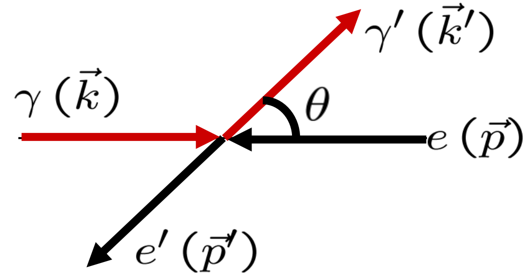
with

$$r_0 = \frac{e^2}{4\pi M_e} \equiv \frac{\alpha}{M_e} \simeq 2.82 \text{ fm}$$ *"electron radius"*

and function

$$F(p,p',k) = \left\{\left(\frac{M_e^2}{2k\cdot p} + \frac{M_e^2}{2k\cdot p'}\right)^2 + \left(\frac{M_e^2}{2k\cdot p} + \frac{M_e^2}{2k\cdot p'}\right) - \frac{1}{4}\left(\frac{k\cdot p}{k\cdot p'} + \frac{k\cdot p'}{k\cdot p}\right)\right\}$$

relation between cross section $d\sigma$ *to probability* dW



$$dW = J \cdot \frac{1}{V_\gamma} \cdot d\sigma$$

probability

flux factor

photon density ?

$$J = v(\gamma, e) = \frac{\sqrt{s}}{E_e}$$

$$\vec{A}_\gamma = \vec{a}_x \cos(\omega t - kz) + \vec{a}_y \sin(\omega t - kz)$$

circularly polarized photon field

$$\mathcal{E} = (E^2 + H^2)/2 = E^2$$

energy density of EM field

$$\vec{E} = -\frac{\partial \vec{A}_\gamma}{\partial t} = \omega(\vec{a}_x \sin(\omega t - kz) - \vec{a}_y \cos(\omega t - kz))$$

$$\langle \vec{E}^2 \rangle = \omega^2 a^2; \qquad \frac{1}{V_\gamma} = \frac{\mathcal{E}}{\omega} = \omega \cdot a^2 = \frac{\omega M_e^2 \xi^2}{4\pi\alpha}$$

$$a^2 = a_x^2 = a_y^2$$

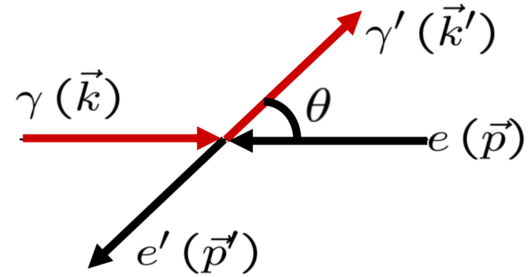
with $\xi^2 = \frac{e^2 a^2}{M_e^2}$

reduced intensity of EM field

$$dW = d\sigma \cdot \frac{\sqrt{s}}{E_e} \cdot \frac{M_e^2 \xi^2 \omega_\gamma}{4\pi\alpha}$$

Final expression for the Compton scattering probability ***dW***

$$dW = d\sigma \cdot \frac{\sqrt{s}}{E_e} \cdot \frac{M_e^2 \xi^2 \omega_\gamma}{4\pi\alpha}$$



square of the total energy $s = (p + k)^2$

square of the momentum transfer in cross channel $\bar{s} = (p - k')^2$

square of momentum transfer

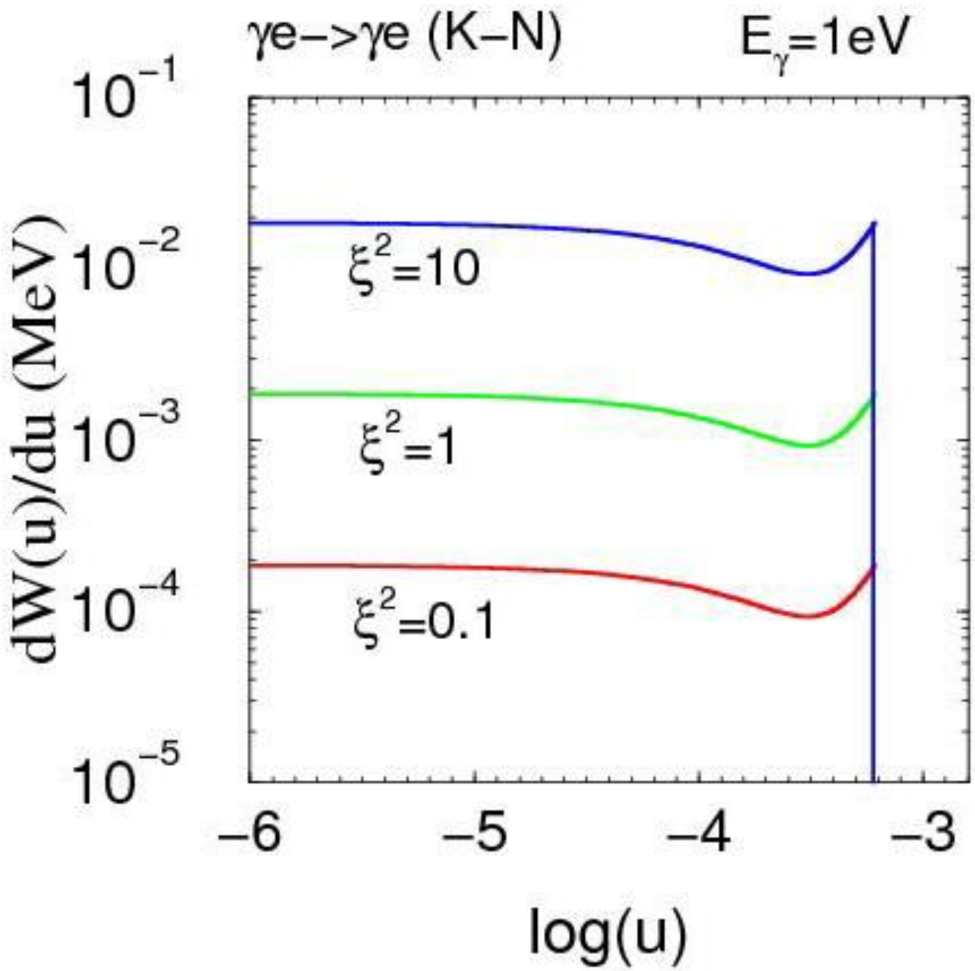
$$t = (k - k')^2 = -2k \cdot k' = -2\omega\omega'(1 - \cos\theta) \qquad (\text{or } \ \theta)$$

For further application it is convenient to use *new invariant variable* ***u***

$$u = \frac{k \cdot k'}{k \cdot p'} = \frac{\omega_c(1 - \cos\theta_c)}{E'_e + \omega_c \cos\theta_c} \qquad 0 \le u \le u_{\max} = \frac{2\omega_L}{M_e} \ (\ll 1)$$

$$dW = \pi r_0^2 \, F(p, p', k) \cdot \frac{M_e^2}{2E_e} \cdot \frac{M_e^2 \xi^2}{4\pi\alpha} \cdot \frac{du}{(1 + u)^2}$$

Compton γ e → γ e scattering (results)



Shape of differential distribution does not depend on field intensity because ξ^2 describes an overall factor BUT not matrix element!

Electron in a strong electromagnetic fields

D.M. Volkov, Z. Phys. **94,** 250 (1935)

Über eine Klasse von Lösungen der Diracschen Gleichung.

1. Der Fall eines sinusoidalen Feldes. — 2. Lösung für den Fall, daß das äußere Feld aus polarisierten, in einer bestimmten Richtung fortschreitenden Wellen besteht, die ein *abzählbares* Spektrum nach Frequenz und Anfangsphasen haben.

Dirac second order equation

$$[(\hat{p} - eA)^2 - m^2 - i\frac{1}{2}F_{\mu\nu}\sigma^{\mu\nu}]\psi = 0,$$

4-componenrt spinor

where $F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$ ***is EM field tensor***

$$\sigma_{\mu\nu} = \frac{i}{2}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu),\ \gamma_\mu \ - 4\times 4\ \textit{Dirac matricies}\quad \gamma_0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix},\ \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}$$

and $A = (0, \vec{A}_\gamma)$ ***is four vector of electromagnetic field with the special part chosen as***

$$\vec{A}_\gamma = \vec{a}_x \cos(\phi) + \vec{a}_y \sin(\phi);\quad \phi = k\cdot x = \omega t - kz$$

with $|\vec{a}_x| = |\vec{a}_y| = a$

Solution:

$$\psi_p = \left[1 + \frac{e(\gamma\cdot k)(\gamma\cdot A)}{2(k\cdot p)}\right] e^{iS'(\phi)} \frac{u_p}{\sqrt{2E_p}} e^{-ip\cdot x}$$

with

$$S'(\phi) = -\int_0^{kx}\left[\frac{e(p\cdot A)}{(k\cdot p)} - \frac{e^2A^2}{2(k\cdot p)}\right]d\phi'$$

when $\vec{A}\to 0$ ***or*** $(a_x, a_y \to 0)$

$$\psi_p \to \frac{u_p}{\sqrt{2E_p}} e^{-ip\cdot x}$$

Dirac solution for free electron

Properties of Volkov's solution

Effective "quasi" momentum

$$\langle \psi^*(\hat{p}^\mu - eA^\mu)\psi\rangle = q^\mu$$

$$\equiv p^\mu - \frac{e^2\bar{A}^2}{2(k\cdot p)}k^\mu = p^\mu + \frac{e^2a^2}{2(k\cdot p)}k^\mu = p^\mu + \frac{\xi^2 m_e^2}{2(k\cdot p)}k^\mu$$

$$\bar{A}^2 = -\frac{1}{2}(a_x^2 + a_y^2) = -a^2$$

Effective electron mass

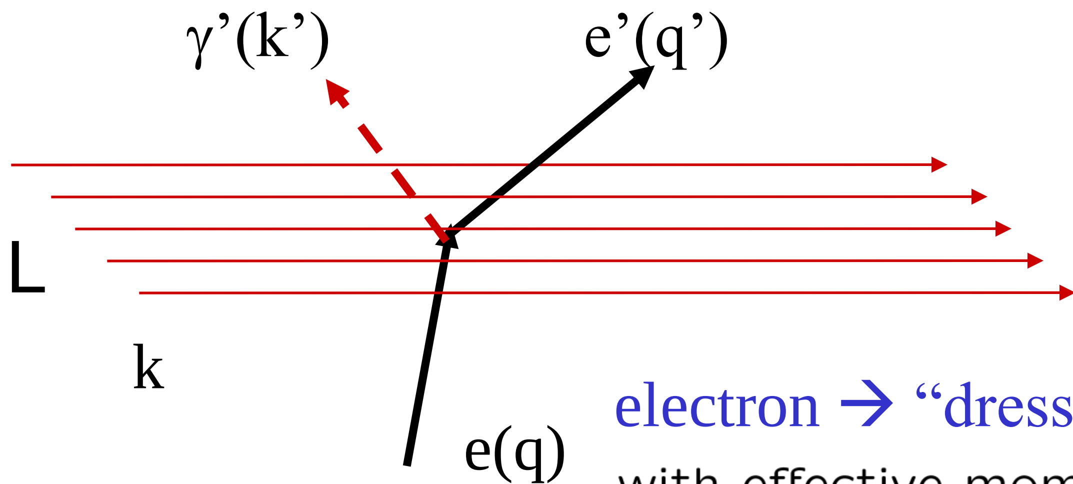
$$q^2 = m_*^2 \equiv m_e^2\left(1 - \frac{e^2\bar{A}^2}{m_e^2}\right) = m_e^2\left(1+\xi^2\right)$$

$$m_{e*}^2 = m^2(1+\xi^2)$$

with $\xi^2 = \frac{e^2a^2}{m_e^2} = \frac{e^2E^2}{m_e^2\omega^2}$

"quasi-momentum" and effective mass define momentum-energy conservation

Emission of a photon by an electron in the field of a strong electromagnetic wave



electron → "dressed" electron

with effective momentum $p \to q$

and mass $m_e^2 \to m_e^2(1+\xi^2)$

$$T_{fi} = -ie \int \psi_f^* (\gamma \cdot \varepsilon_f^*) \psi_i \mathrm{e}^{ik'x} \frac{d^4x}{\sqrt{2\omega'}}$$

Interaction of electron with an external field is considered non-perturbatively

Interaction of electron with outgoing photon is consider in first order of perturbation theory

Structure of matrix element

$$T_{fi} = -ie\int \psi_f^*(\gamma\cdot\varepsilon_f^*)\psi_i e^{ik'x}\frac{d^4x}{\sqrt{2\omega'}}$$

$$\frac{\bar{u}_{p'}}{\sqrt{2E_p'}}e^{-ip'\cdot x}\left[1+\frac{e(\gamma\cdot A)(\gamma\cdot k)}{2(k\cdot p')}\right]e^{iS'(k\cdot x,p_e')}$$

"non-perturbative" outgoing electron

$$\left[1+\frac{e(\gamma\cdot k)(\gamma\cdot A)}{2(k\cdot p)}\right]e^{iS'(k\cdot x,p_e)}\frac{u_p}{\sqrt{2E_p}}e^{-ip\cdot x}$$

"non-perturbative" incoming electron

$$\rightarrow \frac{-ie}{\sqrt{2q_0 2q_0' 2\omega'}}\int M(kx)e^{-i(q-q'-k')x}d^4x$$

with

$$M(kx) = [..]_f\,\bar{u}_{p'}(\gamma\cdot\varepsilon_f^*)[..]_i\,u_p e^{-i(S(kx)-S'(kx))}$$

In "regular" Compton scattering γe→>γe, one has

$$T \sim \int M(k,k',p,p')e^{-i(p+k-p'-k')x}d^4x$$

$$= (2\pi)^4\delta^4(p+k-p'-k')\,M(k,k',p,p')$$

$$\neq (2\pi)^4\delta^4(q+k-q'-k')\cdot M$$

Structure of matrix element (continuing)

$$\frac{-ie}{\sqrt{2q_0 2q_0' 2\omega'}} \int M(kx) \mathrm{e}^{-i(q-q'-k')x} d^4x$$

Fourier series

$$M(kx) = \sum_{n=-\infty}^{\infty} \mathrm{e}^{-in\,kx} M_n(k, k', q, q')$$

The amplitude becomes a sum of infinite numbers of partial harmonics

$$T_{fi} = -ie \sum_{n=-\infty}^{\infty} M_n \int \mathrm{e}^{-i(q+nk-q'-k')} d^4x$$

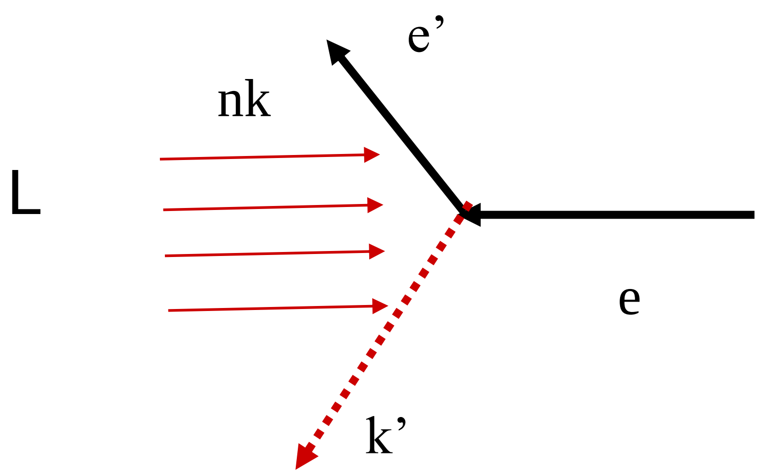
$$= \sum_n -ie\, M_n (2\pi)^4 \delta^4(q + nk - q' - k')$$

Each harmonic describes absorption (emitting) of n photons of external field A with wave vector k and emitting of outgoing photon with the wave vector k' with corresponding conservation low

Probability is a sum of partial contributions

$$dW = \sum_n dW_n$$

$$dW_n = \frac{1}{16\pi E_q} |T_n|^2 \frac{du}{(1+u)^2}$$

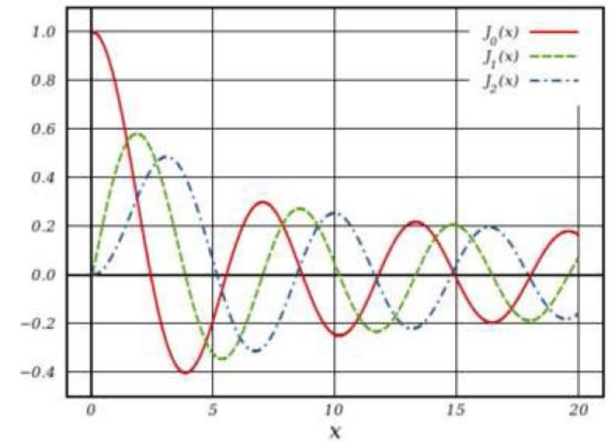


Properties of partial contributions

$$dW_n = \frac{\alpha}{4E_q}\frac{du}{(1+u)^2}\left\{-4J_n^2(z) + \xi^2(2+\frac{u^2}{1+u})\left(J_{n+1}^2(z) + J_{n-1}^2(z) - 2J_n^2(z)\right)\right\}$$

$$J_n(z) = \int_{-\pi}^{\pi} \mathrm{e}^{i(n\phi - z\sin\phi)}\,d\phi; \quad \phi = kx$$

$$u_{\max} = \frac{2n\omega_L(E_e+p)}{m_e^2(1+\xi^2)} \simeq \begin{cases} \frac{n\,\omega_\gamma^L}{m_e\sqrt{1+\xi^2}} & for\ E_e \simeq m_e \\ \frac{4nE_e\omega_\gamma^L}{m_e^2(1+\xi^2)} & for\ E_e \gg m_e \end{cases}$$

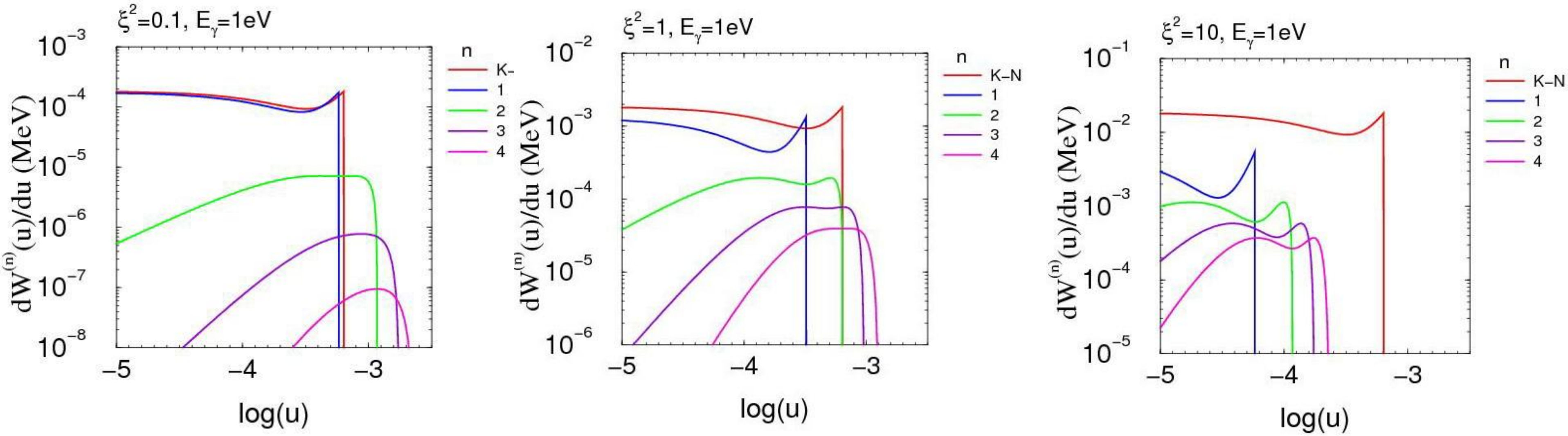


Kinematical limit (phase space) ***increases*** (n>0)

2 effects:
- Electron can interact with a few photons simultaneously ("cumulative" effect)
- Dressed electron mass exceeds free electron mass

Results in ***decrease*** of the phase space even for one photon absorbtion

Photon emission in strong EM (results)



At small field intensity $\xi^2 << 1$ *effect of mass modification is small, "cumulative" effect is large*

At large field intensity $\xi^2 >> 1$ *effect of mass modification is larger, than "cumulative" effect. However, the later one is also important.*

At $\xi^2 \geq 1$ *standard Klein-Nishina equation does not work even for* n=1.

pioneering works:

N.~D.~Sengupta, Bull. Calcutta Math. Soc. {\bf 44}, 175, (1952).

I.~I.~Goldman, Phys. Lett. {\bf 8}, 103 (1964)

L.~S.~Brown and T. W. B. Klibbe, Phys. Rev. A {\bf 133}, 705 (1964).

H.~R.~Reiss, J. Math. Phys. {\bf 3}, 59 (1962).

A.~I.~Nikishov and V.~I.~Ritus, Sov.\ Phys.\ JETP {(1964-79)

N.~B.~Narozhnyi, A.~I.~Nikishov, and V.~I.~Ritus,
Sov.\ Phys.\ JETP {\bf 20}, 622 (1965)
[Zh.\ Eksp.\ Teor.\ Fiz.\ {\bf 47}, 930 (1964)].

V.~A.~Lyulka, Zh.\ Eksp.\ Teor.\ Fiz.\ {\bf 69}, 800 (1975).

N.~P.~Merenkov, Yad.\ Fiz.\ {\bf 42}, 1484 (1985).

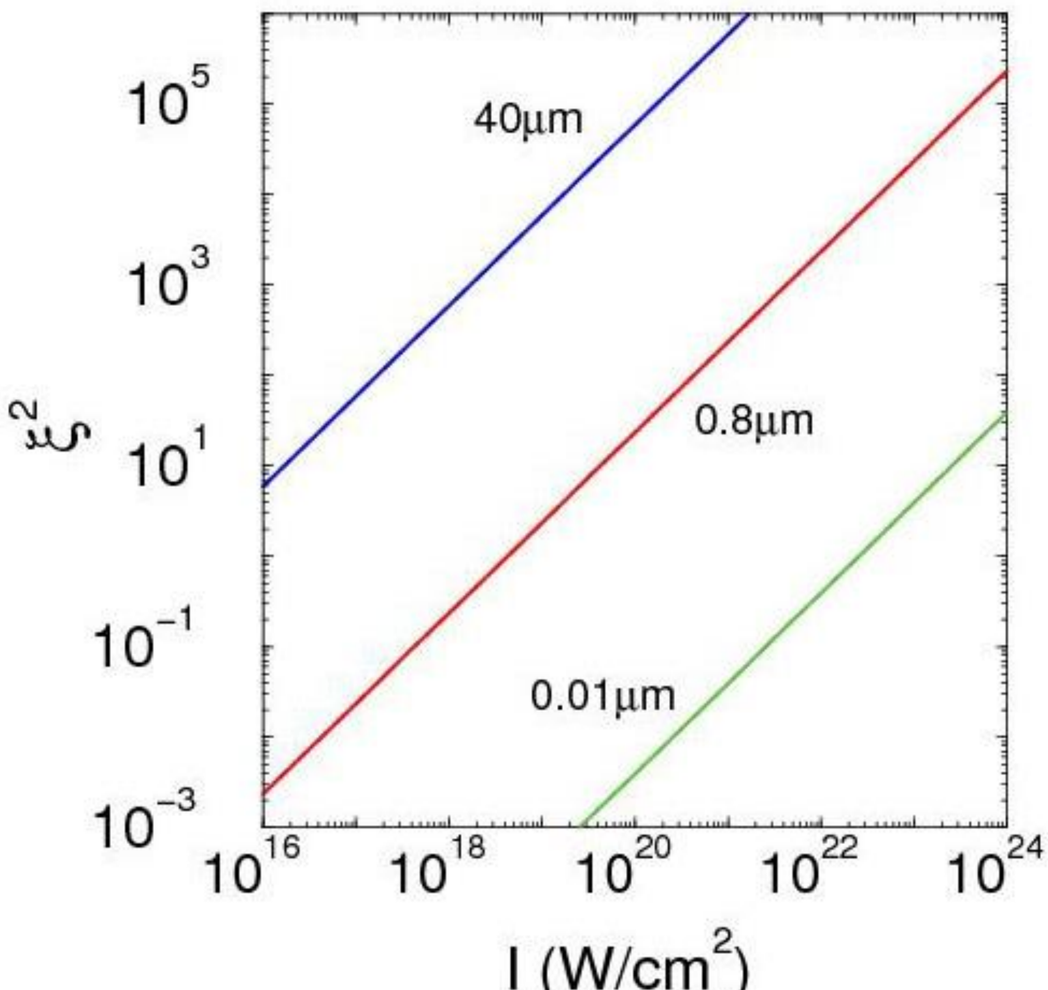
V.~V.~Skobelev, Yad.\ Fiz.\ {\bf 46}, 1738 (1987).

$$\xi^2 \ll 1$$

$$\xi^2 \to \infty; \qquad \xi \sim \frac{1}{\omega_\gamma}$$

$$\chi = \xi \frac{kp}{m_e^2}$$

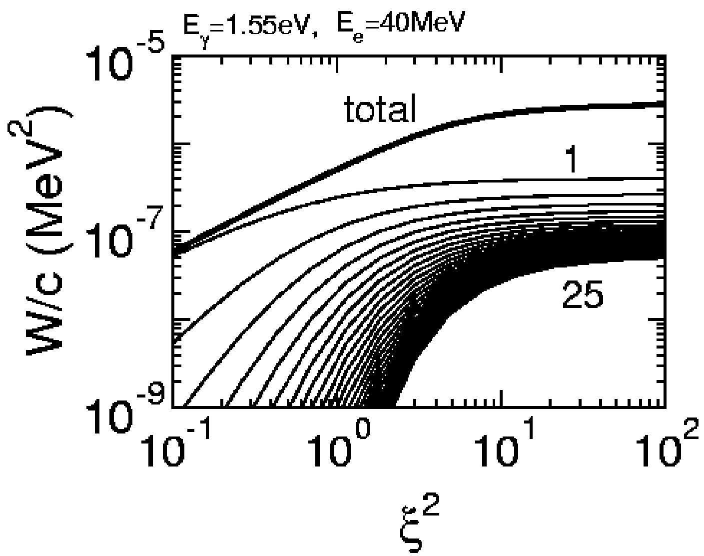
Dependance of reduced field strengh ξ^2 on laser pulse intensity I at different wavelength λ

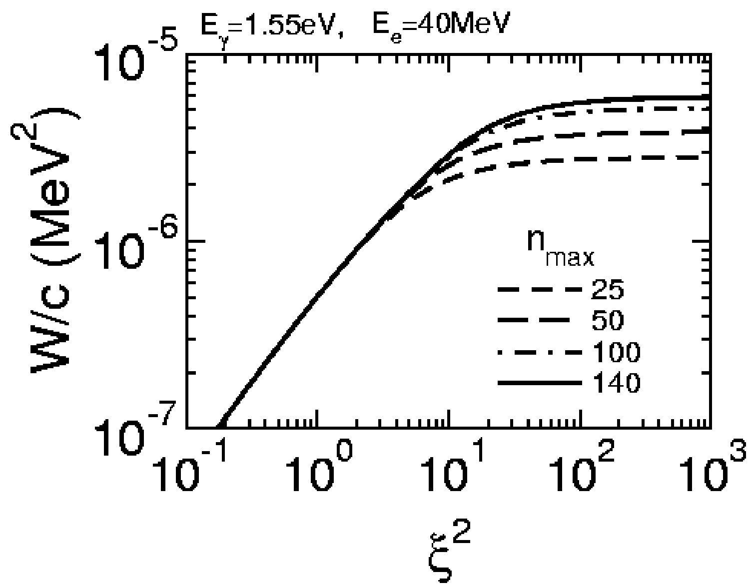


$$\xi^2 = \frac{4\pi\alpha(\hbar c)^2}{m_e^2\omega_\gamma^2\, c} I$$

$$\lambda = \frac{2\pi\hbar c}{\omega_\gamma}$$

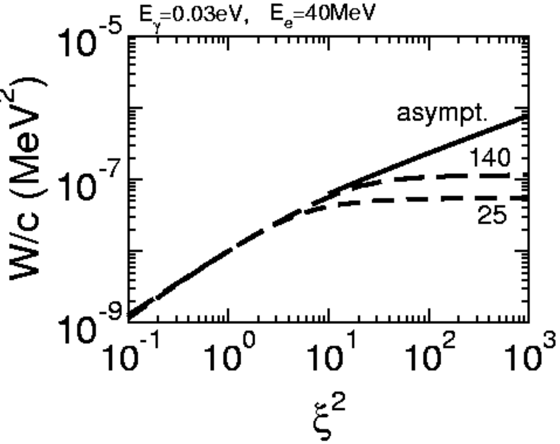
Problem of convergence

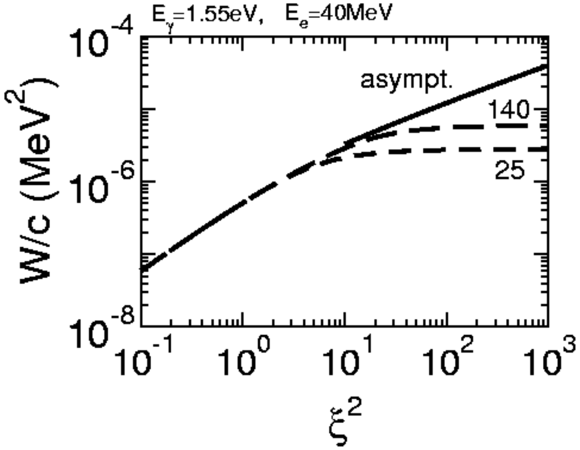


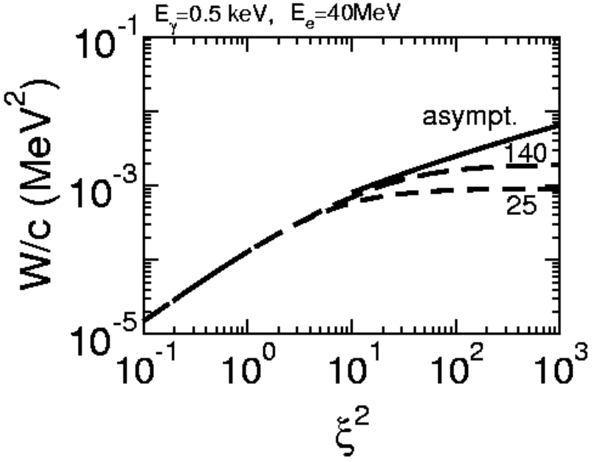


Asymptotic method

$$\sum_{n} \to \int_{n_{\mathrm{min}}(\xi)}^{\infty} dn$$







Summary of this part:

Non-perturbative effects of QED may be seen even at small $\xi^2 \simeq 0.1$

Perturbative QED does not work at finite values of $\xi^2 \geq 1$

Difference between predictions of perturbative QED and non-perturbative QED is large both qualitatively and quantitatively

Emission of neutrino pairs by an electron in a field of strong electromagnetic wave

Neutrino emission in $e \to e' + \bar{\nu}\nu$ *is forbidden by energy arguments*

$$P_i^2 = P_f^2 \to p_e = p_{e'} + p_{2\nu}$$
$$M_e^2 = M_e^2 + M_{2\nu}^2 + 2p_{e'} \cdot p_{2\nu}$$
$$E_{e'}E_{2\nu} + M_{2\nu}^2/2 = |\vec{p}_{e'}| \cdot |\vec{p}_{2\nu}| \cos\theta$$

It is wrong, because

$$E_{e'} > |\vec{p}_{e'}|, \;\; E_{2\nu} > |\vec{p}_{2\nu}|$$

and therefore, always

$$E_{e'}E_{2\nu} + M_{2\nu}^2/2 > |\vec{p}_{e'}| \cdot |\vec{p}_{2\nu}| \cos\theta$$

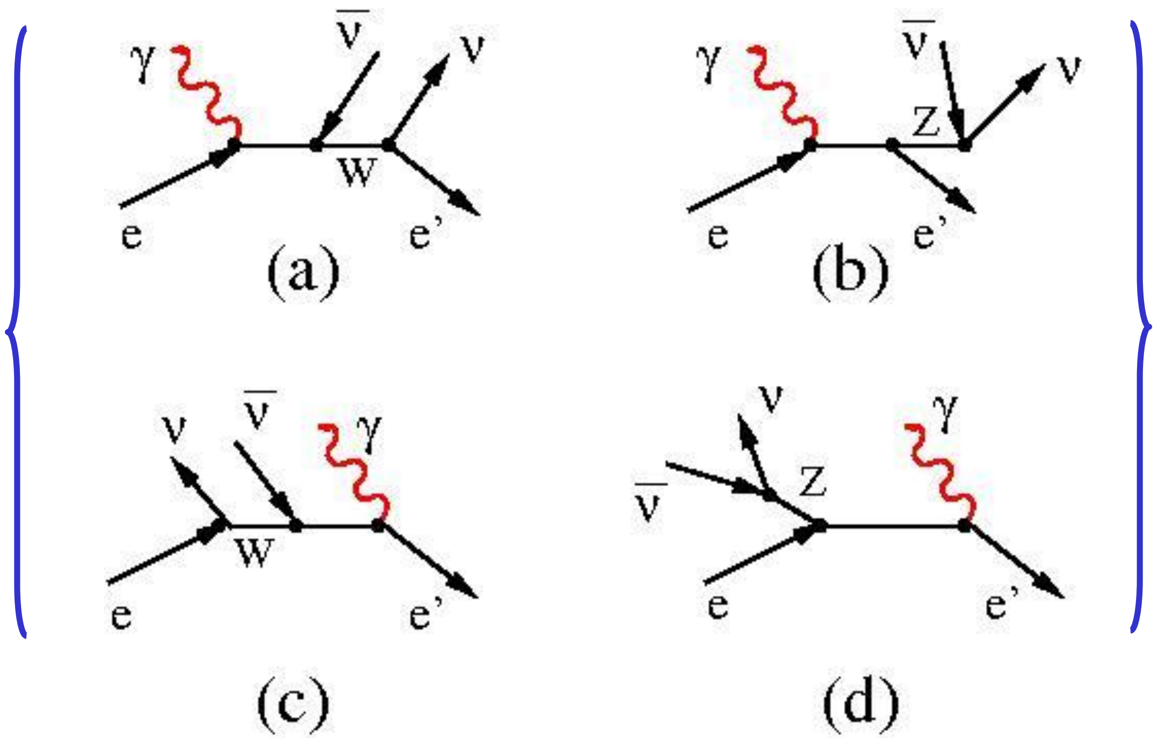
Reaction $\gamma e \to e' + \bar{\nu}\nu$ *is allowed. It is an analog to the Compton scattering*

Three types of neutrino may be emitted:

$$\gamma e \to e' + \bar{\nu}_e \nu_e \ (W, Z)$$
$$\gamma e \to e' + \bar{\nu}_\mu \nu_\mu \ (Z)$$
$$\gamma e \to e' + \bar{\nu}_\tau \nu_\tau \ (Z)$$

Only

$\bar{\nu}_e \nu_e$



All three types

$\bar{\nu}_e \nu_e$

$\bar{\nu}_\mu \nu_\mu$

$\bar{\nu}_\tau \nu_\tau$

Question: how is neutrino emission modified in strong EM field?

electron modification is important

$$\begin{cases} \psi^D \to \psi^V \\ p \to q \\ m_e^2 \to m_*^2 = m_e^2 + e^2 a^2 = m_e^2 + m_e^2 \xi^2 \end{cases}$$

$W^\pm$ modification is negligible:

$$\begin{cases} m_W^2 \to m_{W*}^2 = m_W^2 + e^2 a^2 = m_W^2 + m_e^2 \xi^2 \\ \Delta m_W = \dfrac{\Delta m_W^2}{2 m_W} = m_e \cdot \xi^2 \dfrac{m_e}{m_W} \sim m_e \times 10^{-5} \end{cases}$$

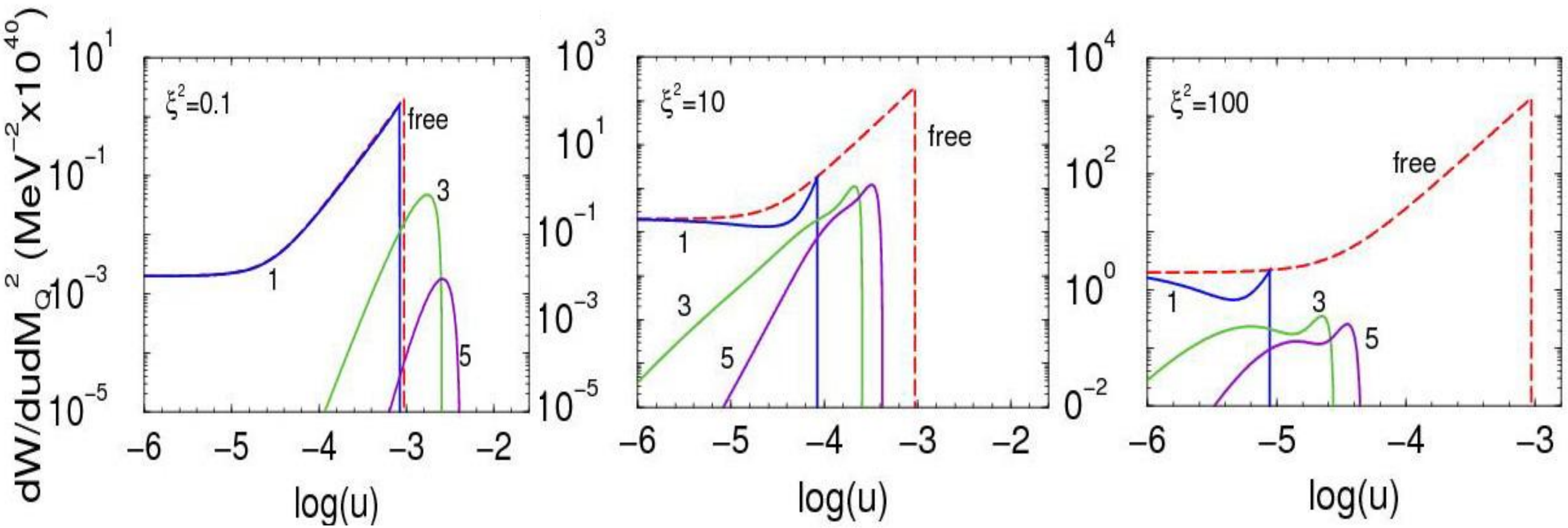
Amplitude(s)

$$T_{fi} = \frac{G_F}{\sqrt{2}} \int [\psi_f^* \gamma_\mu (a - b\gamma_5) \psi_i] \otimes [u_\nu \gamma^\mu (1 - \gamma_5) v_{\bar{\nu}}] \, \mathrm{e}^{i(k_\nu + k_{\bar{\nu}})x} \frac{d^4x}{\sqrt{2E_\nu 2E_{\bar{\nu}}}}$$

Probability

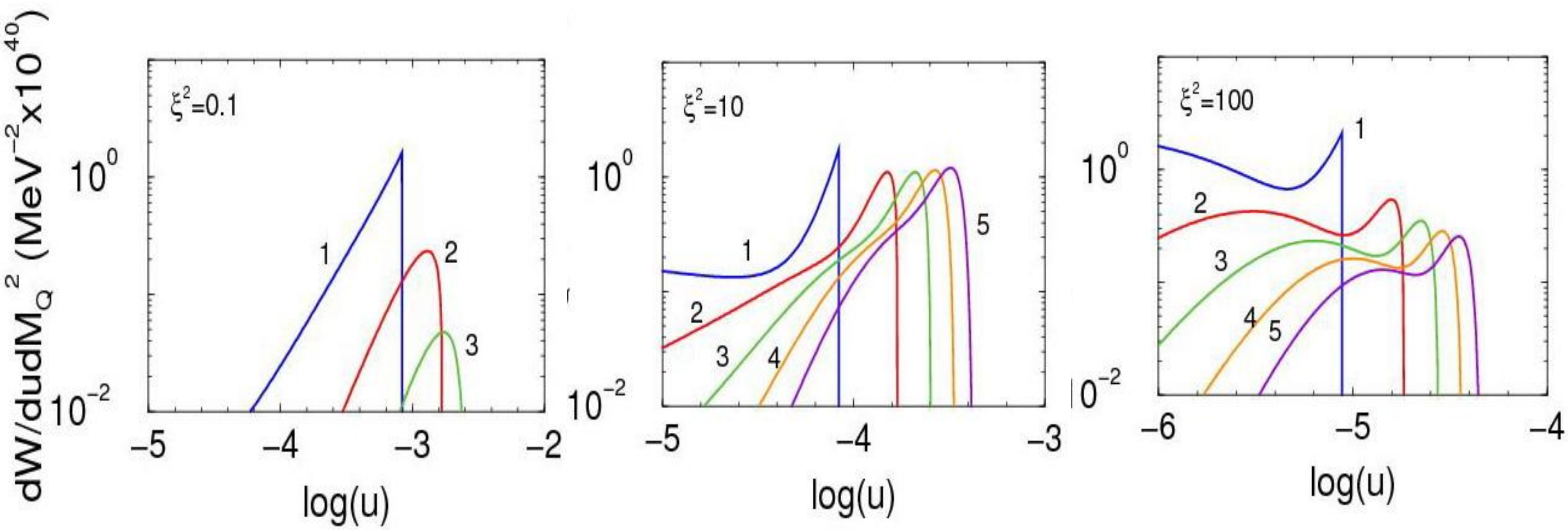
$$dW = \sum_{n>0} dW_n$$

2 neutrinos emission in strong EM (results)



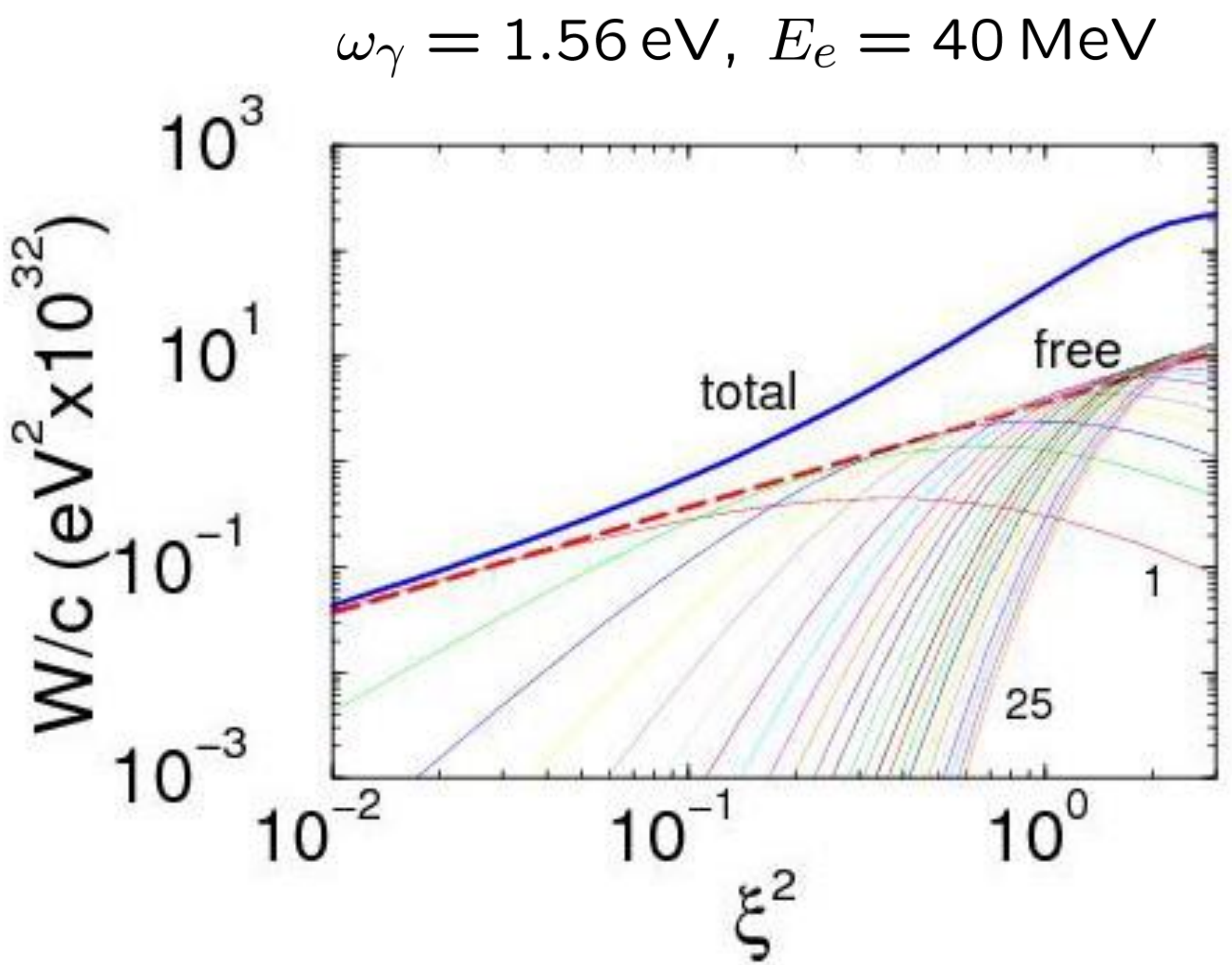
- *Variation of ξ^2 in $\gamma e \rightarrow e\nu_e\bar{\nu}_e$ reaction in vacuum does not change shape of distribution. It changes overall normalization*
- *Increase of ξ^2 leads to decrease of kinematical limit (see **n=1**)*
- *"Cumulative effect" – reactions with n>1 increases the phase space and the kinematical limit*

2 neutrinos emission in strong EM (results, without $\gamma e \to 2\nu$, continuing)



In general, higher harmonics are not suppressed at large ξ^2

Total yield of neutrino as a function of ξ^2



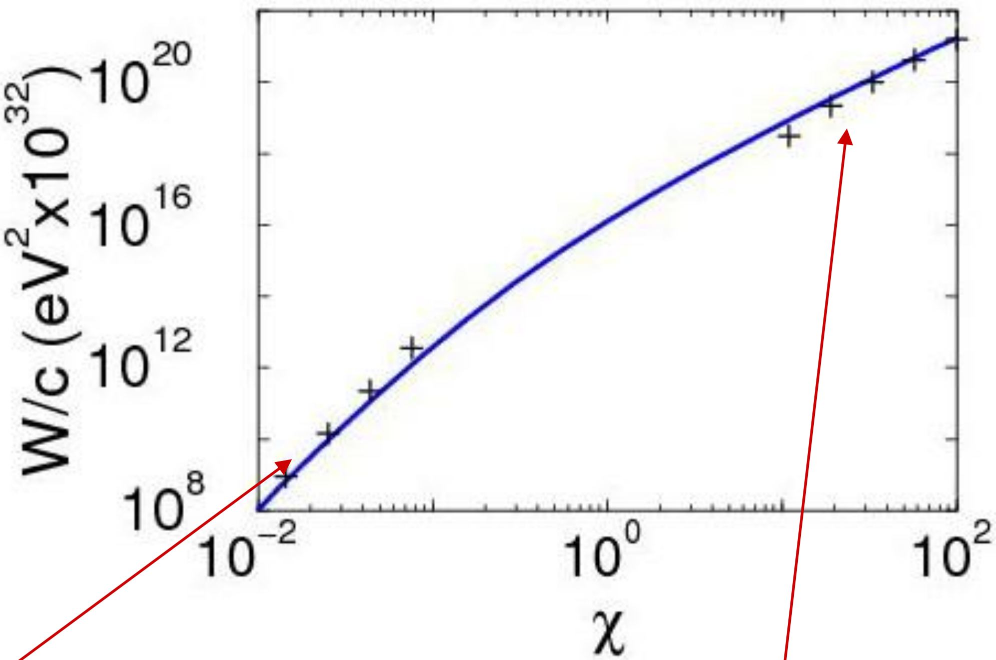
Higher harmonics dominate at large ξ^2 : $\quad n_{\max} = f(\xi^2)$

$$W(\chi, \xi^2)$$

$$\chi = \frac{e\sqrt{(\langle F_{\mu\nu} p_e{}^{\nu} \rangle)^2}}{m_e^3} \geq 0 \ , \qquad \xi^2 = -\frac{e^2 \langle A \rangle^2}{m_e^2} \geq 0$$

$$W(\chi, \xi^2 \to \infty) \to W_A(\chi)$$

Asymptotic total yield of neutrino in limit of $\xi^2 \to \infty$



$$\sim \frac{G_F^2 m_e^6}{192\sqrt{3}\pi^3}\chi^5, \;\; \chi \ll 1$$

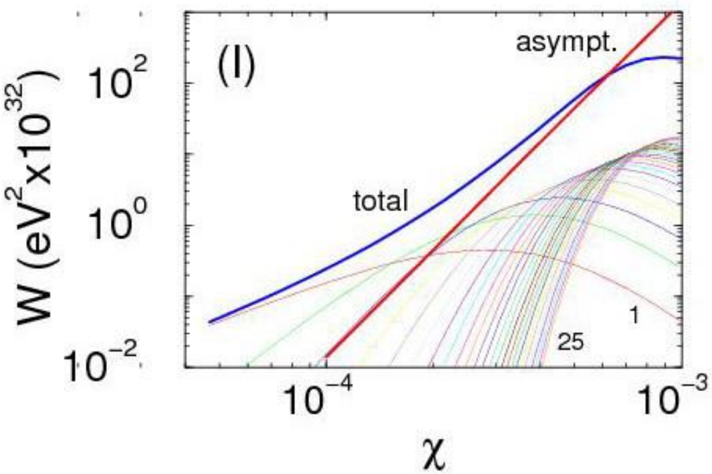
$$\sim \frac{G_F^2 m_e^6}{216\pi^3}\chi^2 \left(\ln\chi - 0.577 - \frac{1}{2}\ln 3 - \frac{5}{6}\right), \;\; \chi \gg 1$$

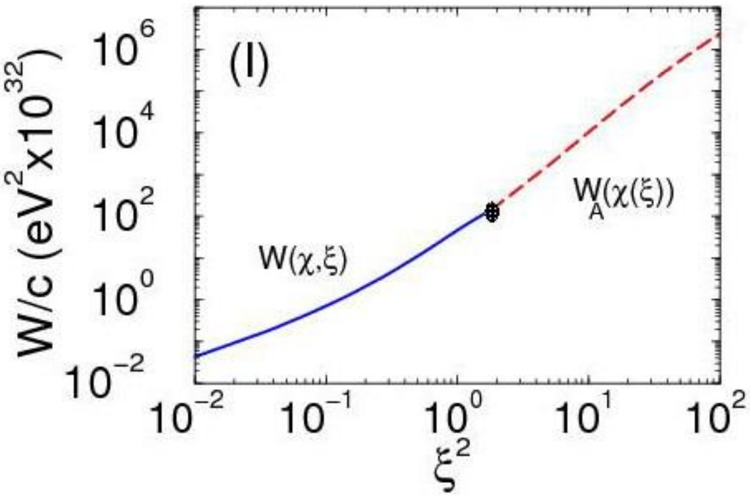
Merenkov, Sov.J.Nucl.Phys. **42** (1985**)**

How to stitch two solutions: asymptotic and sum of finite harmonics

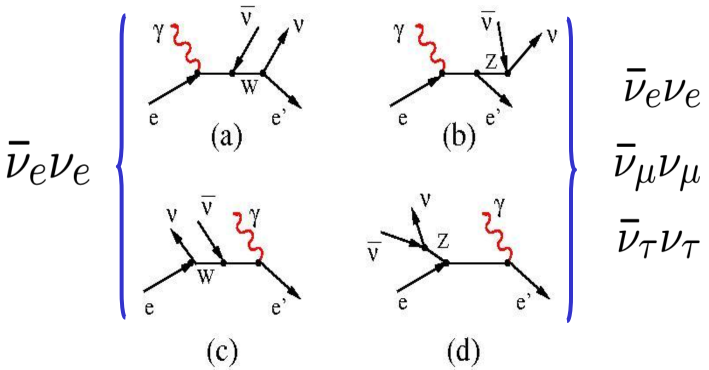
$$\chi^2 = \frac{\xi^2 (k\cdot p)^2}{m_e^4}$$

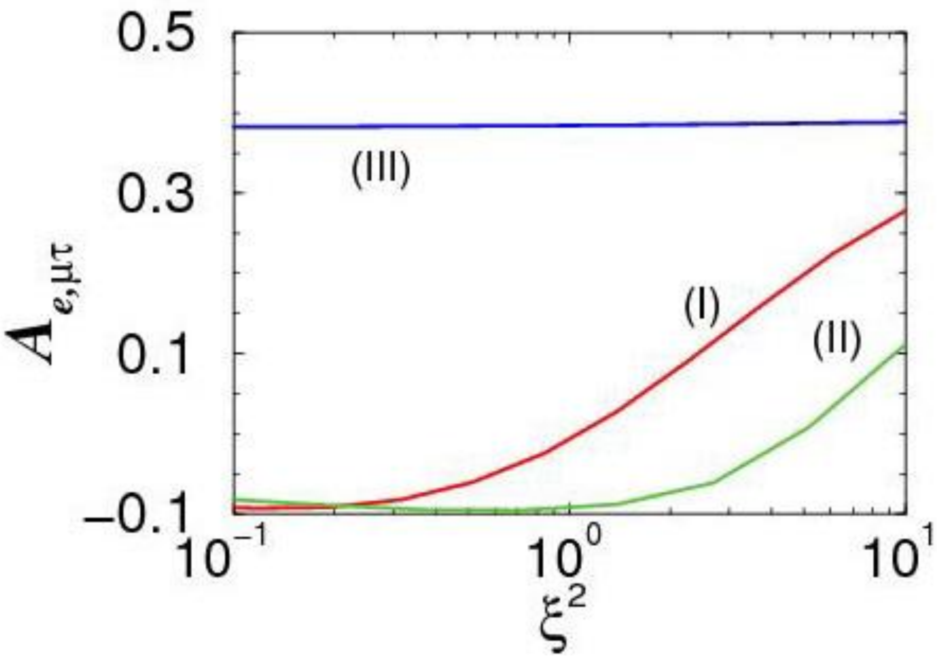
$$\omega_\gamma = 1.56\,\mathrm{eV},\ E_e = 40\,\mathrm{MeV}$$





Asymmetry of production of ν_e *and* $\nu_\mu + \nu_\tau$





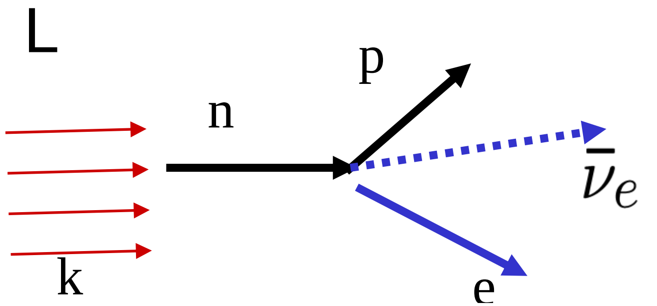
(I) $\omega_\gamma = 1.56\,\text{eV},\ E_e = 40\,\text{MeV}$

(II) $\omega_\gamma = 1.0\,\text{KeV},\ E_e = 40\,\text{MeV}$

(III) $\omega_\gamma = 1.0\,\text{KeV},\ E_e = 10\,\text{GeV}$

Neutron decay in the field of a strong electromagnetic wave

$$n \to p + e + \bar{\nu}_e$$



electron modification

$$\begin{cases} \psi^D \to \psi^V \\ p \to q \\ m_e^2 \to m_*^2 = m_e^2 + e^2a^2 = m_e^2 + m_e^2\xi^2 \end{cases}$$

proton modification

$$\begin{cases} p_\mu \to q_\mu = p_\mu + \dfrac{e^2a^2}{2\omega_\gamma M_p} k_\mu \sim p_\mu \\ M_p \to M_{p*} = \sqrt{M_p^2 + m_e^2\xi^2} \simeq M_p(1 + \dfrac{m_e^2}{M_p^2}) \simeq M_p \\ \psi^D \to \psi^V \simeq \psi^D \end{cases}$$

neutron modification

$$\{\ \psi^D \to \psi^V \simeq \psi^D$$

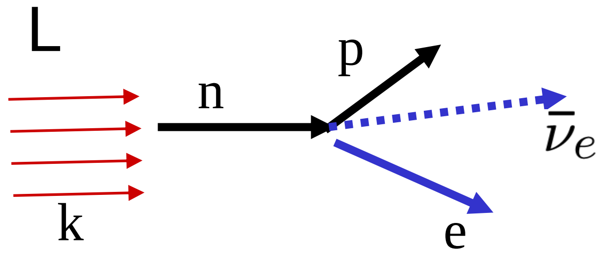
Amplitude

$$T_{fi} = \frac{G_F}{\sqrt{2}} \int [\bar{u}_p \gamma_\mu (1 - g_A \gamma_5) u_n] \otimes [\psi_e^* \gamma^\mu (1 - \gamma_5) v_{\bar{\nu}}] \, \mathrm{e}^{-i(p_n - p_p - p_{\bar{\nu}})x} \frac{d^4x}{\sqrt{2E_n 2E_p 2E_{\bar{\nu}}}}$$

$$\left.\begin{aligned} m_{e*}^2 &= m_e^2 + \xi^2 m_e^2 \\ m_{e*\mathrm{max}} &\simeq M_n - M_p \end{aligned}\right\} \longrightarrow \quad \xi^2_{\mathrm{max}} = \frac{(M_n - M_p)^2}{m_e^2} - 1 \simeq 5.4$$

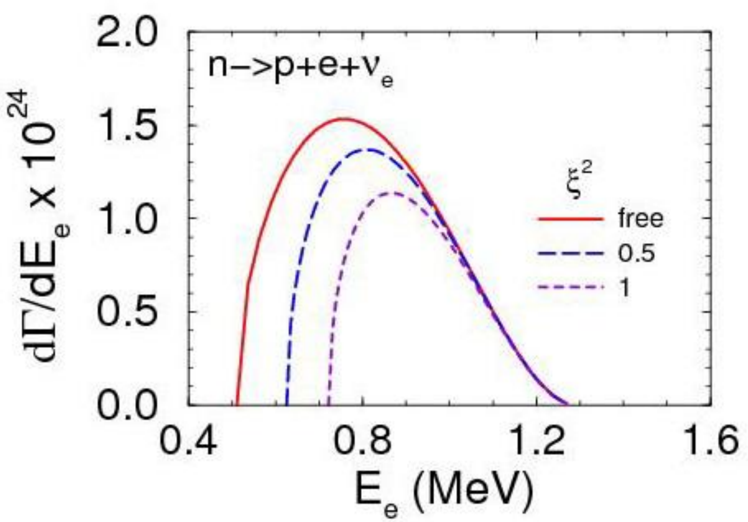
Weak decay in the field of a strong electromagnetic wave

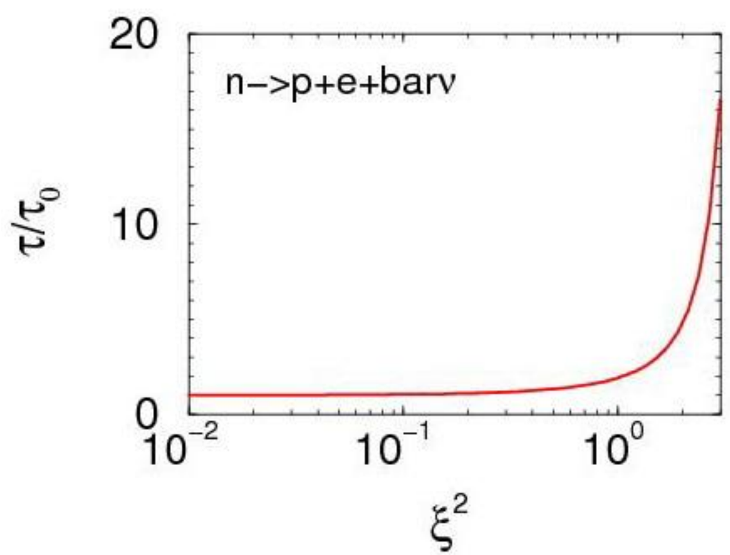
$$n \to p + e + \bar{\nu}_e$$



$$m_*^2 = m_e^2(1 + \xi^2)$$

$$\xi_{\rm max}^2 = \frac{(M_n - M_p)^2}{m_e^2} - 1 \simeq 5.4$$





Summary

- *Strong electromagnetic fields modify basic/fundamental interactions and result in non-trivial nonlinear effects.*

- *Coherent interactions with n- field photons*

- *Modification of kinematical limits because (a) electron dressing and (b) coherent interactions with several photons (cumulative effect)*

- *Leads to some not-trivial dynamical effects, like*

  ν_e *and* $\nu_\mu + \nu_\tau$ *asymmetries …*

THE END

Thank you very much for attention !