New Manifestations of Axial Anomalies

XX International Baldin Seminar

RELATIVISTIC NUCLEAR PHYSICS & QUANTUM CHROMODYNAMICS

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Outline

Two new distinct manifestations

Anomaly for virtual photons and transition formfactors

in collaboration with

Yaroslav Klopot (JINR), Armen Oganesian (ITEP)

Anomaly for medium velocity and Chiral Vortaic Effect for neutrons

in collaboration with

Oleg Rogachevsky, Alexandr Sorin (JINR)

Anomaly for virtual photons

- Anomaly as a collective effect of Transition Formfactors of infinite number of states
- Anomaly Sum Rule vs quark hadron duality
- Amplification of correction to lower states at large Q

Anomaly and voriticity

- Coupling to 4-dimensional velocity (variable whose role was always stressed by A.M. Baldin – talks of A. Malakhov, A. Baldin)
- Fluid vorticity plays a role of magnetic field
- Possible tests in neutron asymmetries at NICA

Symmetries and conserved operators

- (Global) Symmetry -> conserved current ($\partial^{\mu} J_{\mu} = 0$)
- Exact:
- U(1) symmetry – charge conservation - electromagnetic (vector) current
- Translational symmetry – energy momentum tensor $\partial^{\mu} T_{\mu\nu} = 0$

Massless fermions (quarks) – approximate symmetries

- Chiral symmetry (mass flips the helicity)

$$\partial^{\mu} J^{5}{}_{\mu} = 0$$

- Dilatational invariance (mass introduce dimensional scale – c.f. energy-momentum tensor of electromagnetic radiation)

$$T_{\mu\mu} = 0$$

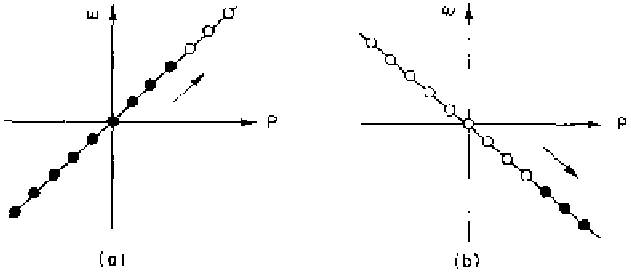
Quantum theory

- Currents -> operators
- Not all the classical symmetries can be preserved -> anomalies
- Enter in pairs (triples?...)
- Vector current conservation <-> chiral invariance
- Translational invariance <-> dilatational invariance

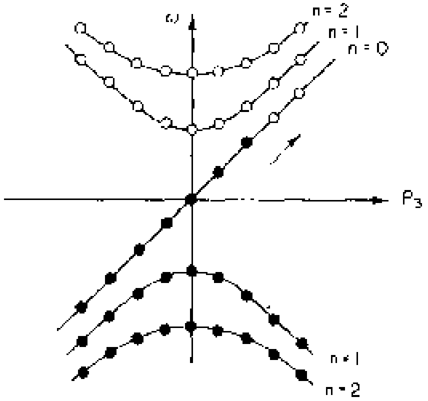
Calculation of anomalies

- Many various ways
- All lead to the same operator equation

$$\partial^\mu j^{(0)}_{5\mu} = 2i\sum_q m_q \bar{q}\gamma_5 q - \left(\frac{N_f\alpha_s}{4\pi}\right) G^a_{\mu\nu}\tilde{G}^{\mu\nu,a}$$



- UV vs IR languages- understood in physical picture (Gribov, Feynman, Nielsen and Ninomiya) of Landau levels flow (E||H)



Manifestation of anomalies

- Pion and other preudoscalars decays (generalization: consider virtual photon)
- Nucleon spin problem
- $U_A(1)$
- Low energy theorems
- Chiral (and other effective theories) anomalies
- …

Anomaly and virtual photons

- Often assumed that only manifested in real photon amplitudes
- Not true – appears at any Q^2
- Natural way – dispersive approach to anomaly (Dolgov, Zakharov'70) - anomaly sum rules
- One real and one virtual photon – Horejsi,OT'95

$$\int_{4m^2}^{\infty} A_3(t;q^2,m^2)dt = \frac{1}{2\pi}$$

- where

$$F_j(p^2) = \frac{1}{\pi}\int_{4m^2}^{\infty} \frac{A_j(t)}{t-p^2}dt, \qquad j = 3,4$$

$$\begin{aligned} T_{\alpha\mu\nu}(k,q) &= F_1\, \varepsilon_{\alpha\mu\nu\rho}k^\rho + F_2\, \varepsilon_{\alpha\mu\nu\rho}q^\rho \\ &+ F_3\, q_\nu \varepsilon_{\alpha\mu\rho\sigma}k^\rho q^\sigma + F_4\, q_\nu \varepsilon_{\alpha\mu\rho\sigma}k^\rho q^\sigma \\ &+ F_5\, k_\mu \varepsilon_{\alpha\nu\rho\sigma}k^\rho q^\sigma + F_6\, q_\mu \varepsilon_{\alpha\nu\rho\sigma}k^\rho q^\sigma \end{aligned}$$

Dispersive derivation

- Axial WI $F_2 - F_1 = 2mG + \frac{1}{2\pi^2}$
- GI $F_2 - F_1 = (q^2 - p^2)F_3 - q^2 F_4$
- No anomaly for imaginary parts

$$(q^2 - t)A_3(t) - q^2 A_4(t) = 2mB(t) \qquad F_j(p^2) = \frac{1}{\pi}\int_{4m^2}^{\infty} \frac{A_j(t)}{t - p^2} dt, \qquad j = 3, 4$$

- Anomaly as a finite subtraction

$$F_2 - F_1 - 2mG = \frac{1}{\pi}\int_{4m^2}^{\infty} A_3(t)dt \qquad \int_{4m^2}^{\infty} A_3(t; q^2, m^2)dt = \frac{1}{2\pi}$$

Properties of anomaly sum rules

- Valid for any Q^2 (and quark mass)
- No perturbative QCD corrections (Adler-Bardeen theorem)
- No non-perturbative QCD correctioons (t'Hooft consistency principle)
- Exact – powerful tool

Mesons contributions (Klopot, Oganesian, OT)

arXiv:1009.1120 [hep-ph]

- Pion – saturates sum rule for real photons $ImF_3 = \sqrt{2} f_\pi \pi F_{\pi\gamma\gamma*}(Q^2)\delta(s-m_\pi^2)$ $F_{\pi\gamma^*\gamma}(0) = \frac{1}{2\sqrt{2}\pi^2 f_\pi}$
- For virtual photons – pion contribution is rapidly decreasing $F^{\rm asymp}_{\pi\gamma\gamma^*}(Q^2) = \frac{\sqrt{2} f_\pi}{Q^2} + \mathcal{O}(1/Q^4)$
- This is also true also for axial and higher spin mesons (longitudianl components are dominant)
- Heavy PS decouple in a chiral limit

Anomaly as a collective effect

- One can never get constant summing finite number of decreasing function
- Anomaly at finite Q^2 is a collective effect of meson spectrum
- For quantitative analysis – quark-hadron duality

Mesons contributions within quark hadron duality

- Pion: $F_{\pi\gamma\gamma*}(Q^2) = \frac{1}{2\sqrt{2}\pi^2 f_\pi} \frac{s_0}{s_0+Q^2}$
- Cf Brodsky&Lepage, Radyushkin – comes now from anomaly!
- Axial mesons contribtion to ASR

$$\int_0^\infty A_3(s;Q^2)ds = \frac{1}{2\pi} = I_\pi + I_{a_1} + I_{cont}. \qquad I_{a_1} = \frac{1}{2\pi} Q^2 \frac{s_1 - s_0}{(s_1+Q^2)(s_0+Q^2)}$$

Content of Anomaly Sum Rule ("triple point")

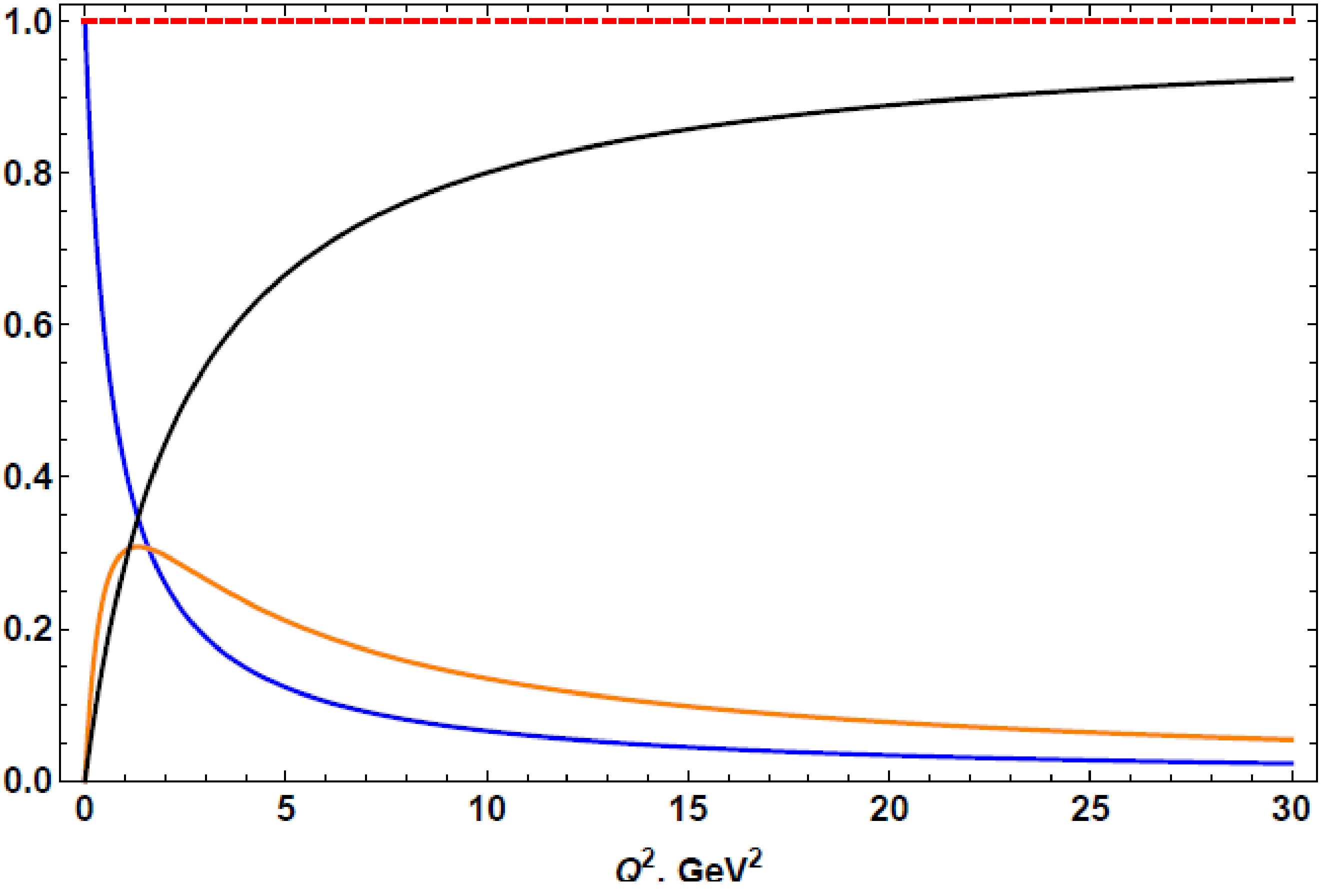


Figure 1: Relative contributions of π (blue line) and a_1 (orange line) mesons, intervals of duality are $s_0 = 0.7\ GeV^2$ and $s_1 - s_0 = 1.8\ GeV^2$ respectively, and continuum (black line), continuum threshold is $s_1 = 2.5\ GeV^2$

ASR and BaBar data

- In the BaBar(2009) region – main contribution comes from the continuum
- Small relative correction to continuum –due to exactness of ASR must be compensated by large relative contributions to lower states!
- Amplification of corrections

$$\frac{\delta I_{cont}/I^0_{cont}}{\delta I_\pi/I^0_\pi} = \frac{s_0}{Q^2} \simeq \frac{1}{30} \qquad Q^2 = 20\ GeV^2,\ s_0 = 0.7\ GeV^2$$

- Smaller for eta because of larger duality interval (supported by BaBar)

Corrections to Continuum

- Perturbative – zero at 2 loops level (massive-Pasechnik&OT – however cf Melnikov; massless-Jegerlehner&Tarasov)
- Massless - may be matched with factorization approach of Mikhailov et al!
- Non-perturbative (e.g. instantons)
- The general properties of ASR require decrease at asymptotically large Q^2 (and $Q^2=0$)

$$I_{cont} = \frac{1}{2\pi}\frac{Q^2}{s_0+Q^2} - cs_0\frac{\ln(Q^2/s_0)+b}{Q^2},$$

$$I_\pi = \frac{1}{2\pi}\frac{s_0}{s_0+Q^2} + cs_0\frac{\ln(Q^2/s_0)+b}{Q^2}.$$

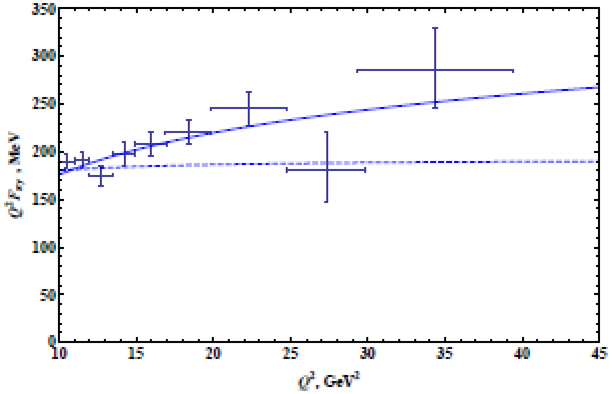
- Corresponds to logarithmically growing pion contribution (cf Radyushkin, Polyakov, Dorokhov).

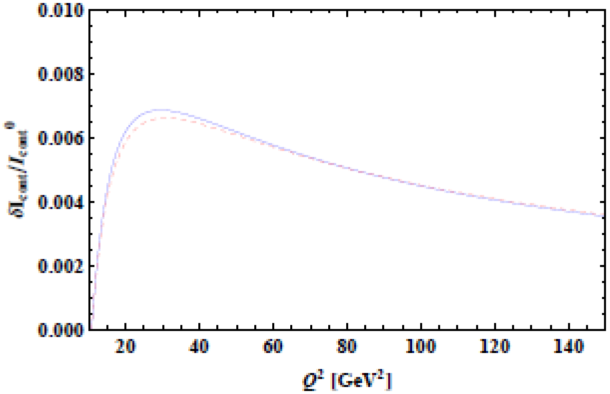
Modelling of corrections

$$I_{cont} = \frac{1}{2\pi}\frac{Q^2}{s_0+Q^2} - cs_0\frac{\ln(Q^2/s_0)+b}{Q^2},$$

$$I_\pi = \frac{1}{2\pi}\frac{s_0}{s_0+Q^2} + cs_0\frac{\ln(Q^2/s_0)+b}{Q^2}.$$

- Continuum vs pion
- Fit $b = -2.74, \quad c = 0.045.$
- Continuum contribution similar for Radyushkin’s approach





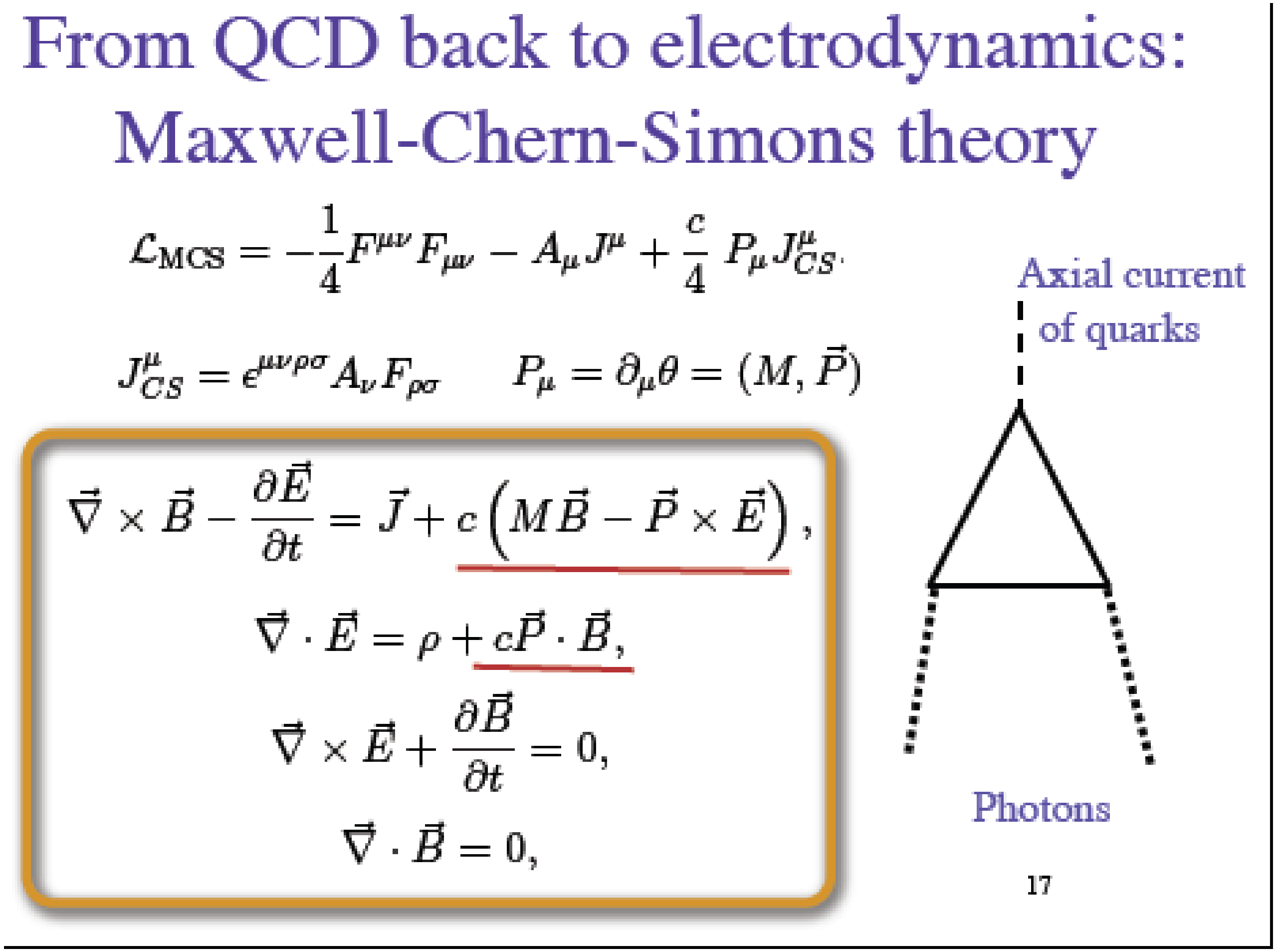
Interplay of pion with lower resonances

- Small (NP) corrections to continuum – interplay of pion with higher states
- A1 – decouples for real photons
- Relation between transiktion FF’s of pion and A1 (testable!)
- Role of A1 in anomaly manifestation – first discussed by N.N. Achasov

Conclusions/Discussion-I

- New manifetsation of Axial Anomaly - Anomaly Sum Rule – exact NPQCD tool- do not require QCD factorization
- Anomaly for virtual photons – collective effect (with fast excitation of collective mode)
- Exactness of ASR – very unusual situation when small pion contribution can be studied on the top of large continuum – amplification of corrections to continuum
- BaBar data – small negative correction to continuum
- For eta – effect of amlification is smaller
- If continuum is precisely described by Born term– interplay with A1 (TO BE STUDIED THEORETICALLY AND EXPERIMENTALLY)

Anomaly in Heavy Ion Collisions - Chiral Magnetic Effect D. Kharzeev – this and two next slides



From QCD back to electrodynamics: Maxwell-Chern-Simons theory

$$\mathcal{L}_{\rm MCS} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - A_\mu J^\mu + \frac{c}{4}\,P_\mu J^\mu_{CS}.$$

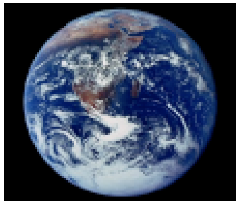
$$J^\mu_{CS} = \epsilon^{\mu\nu\rho\sigma}A_\nu F_{\rho\sigma} \qquad P_\mu = \partial_\mu\theta = (M, \vec{P})$$

$$\boxed{\begin{aligned} \vec{\nabla}\times\vec{B} - \frac{\partial\vec{E}}{\partial t} &= \vec{J} + \underline{c\left(M\vec{B} - \vec{P}\times\vec{E}\right)}, \\ \vec{\nabla}\cdot\vec{E} &= \rho + \underline{c\vec{P}\cdot\vec{B}}, \\ \vec{\nabla}\times\vec{E} + \frac{\partial\vec{B}}{\partial t} &= 0, \\ \vec{\nabla}\cdot\vec{B} &= 0, \end{aligned}}$$

Axial current of quarks

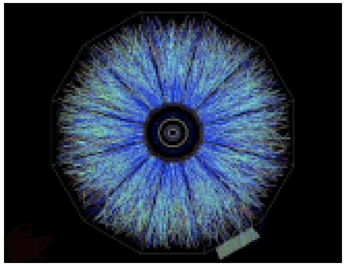
Photons

Comparison of magnetic fields



The Earths magnetic field	0.6 Gauss
A common, hand-held magnet	100 Gauss
The strongest steady magnetic fields achieved so far in the laboratory	4.5×10^5 Gauss
The strongest man-made fields ever achieved, if only briefly	10^7 Gauss
Typical surface, polar magnetic fields of radio pulsars	10^{13} Gauss
Surface field of Magnetars	10^{15} Gauss

http://solomon.as.utexas.edu/~duncan/magnetar.html

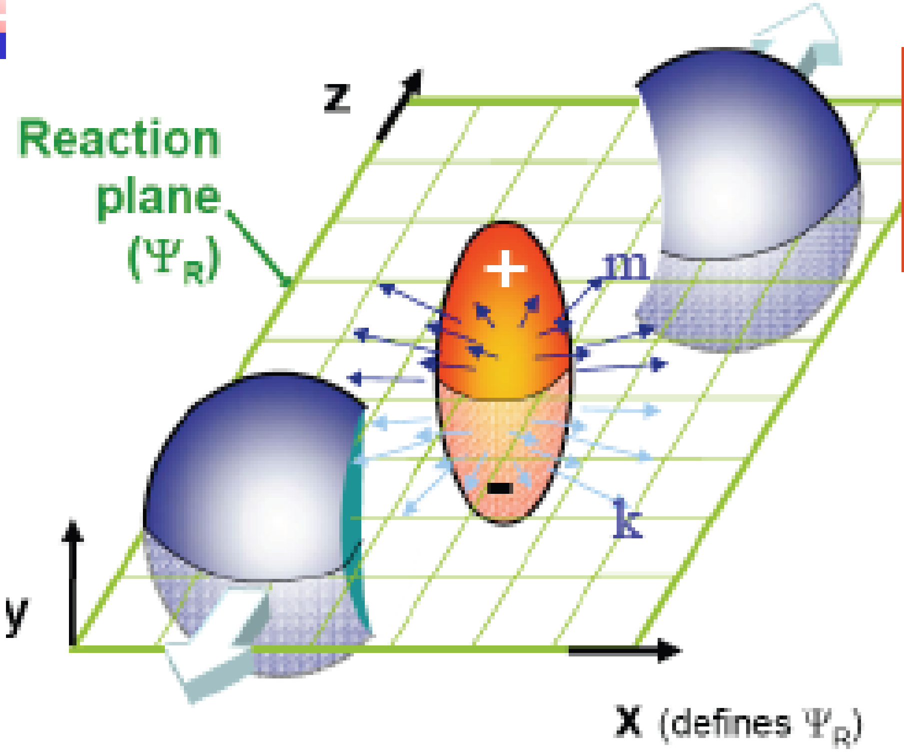


At BNL we beat them all!

Off central Gold-Gold Collisions at 100 GeV per nucleon

$$e\,B(\tau = 0.2\,\mathrm{fm}) \;=\; 10^3 \sim 10^4\ \mathrm{MeV}^2 \sim 10^{17}\,\mathrm{Gauss}$$

Charge asymmetry w.r.t. reaction plane: how to detect it?



$$\begin{aligned} &\langle \cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP}) \rangle = \\ &= \langle \cos\Delta\phi_\alpha \, \cos\Delta\phi_\beta \rangle - \langle \sin\Delta\phi_\alpha \, \sin\Delta\phi_\beta \rangle \\ &= [\langle v_{1,\alpha} v_{1,\beta} \rangle + B^{in}] - [\boxed{\langle a_\alpha a_\beta \rangle} + B^{out}]. \end{aligned}$$

S.Voloshin, hep-ph/0406311

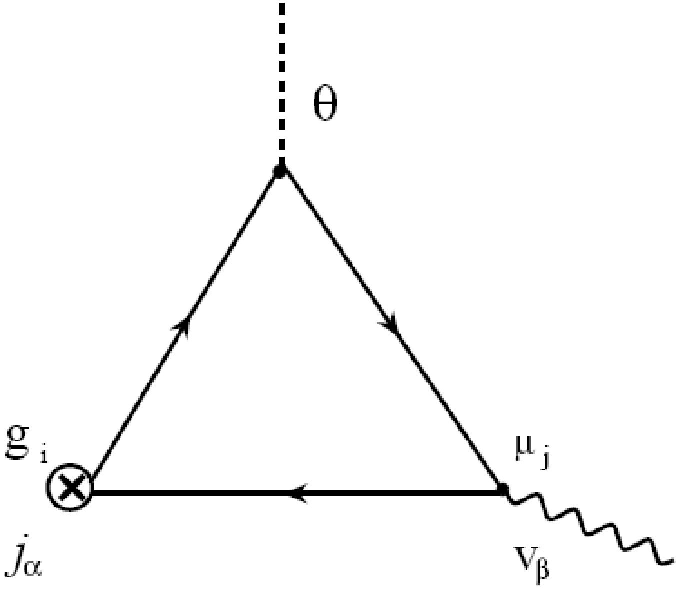
A sensitive measure of the asymmetry:

$$a^k a^m = \langle \sum_{ij} \sin(\varphi_i^k - \Psi_R) \, \sin(\varphi_j^m - \Psi_R) \rangle$$

Expect $a^+ a^+ = a^- a^- > 0; \quad a^+ a^- < 0$

Anomaly in medium – new external lines in VVA graph

- Gauge field -> velocity
- CME -> CVE
- Kharzeev, Zhitnitsky (07) – EM current
- Straightforward generalization: any (e.g. baryonic) current – neutron asymmeries@NICA - Rogachevsky, Sorin, OT - Arxive 1006.1331 (hep-ph)



Baryon charge with neutrons – (Generalized) Chiral Vortaic Effect

- Coupling: $e_j A_\alpha J^\alpha \Rightarrow \mu_j V_\alpha J^\alpha$
- Current: $J_e^\gamma = \frac{N_c}{4\pi^2 N_f}\varepsilon^{\gamma\beta\alpha\rho}\partial_\alpha V_\rho \partial_\beta(\theta \sum_j e_j \mu_j)$
- - Uniform chemical potentials: $J_i^\nu = \frac{\sum_j g_{i(j)}\mu_j}{\sum_j e_j \mu_j} J_e^\nu$
- - Rapidly (and similarly) changing chemical potentials:

$$J_i^0 = \frac{|\vec{\nabla} \sum_j g_{i(j)}\mu_j|}{|\vec{\nabla} \sum_j e_j \mu_j|} J_e^0$$

Comparing CME and CVE

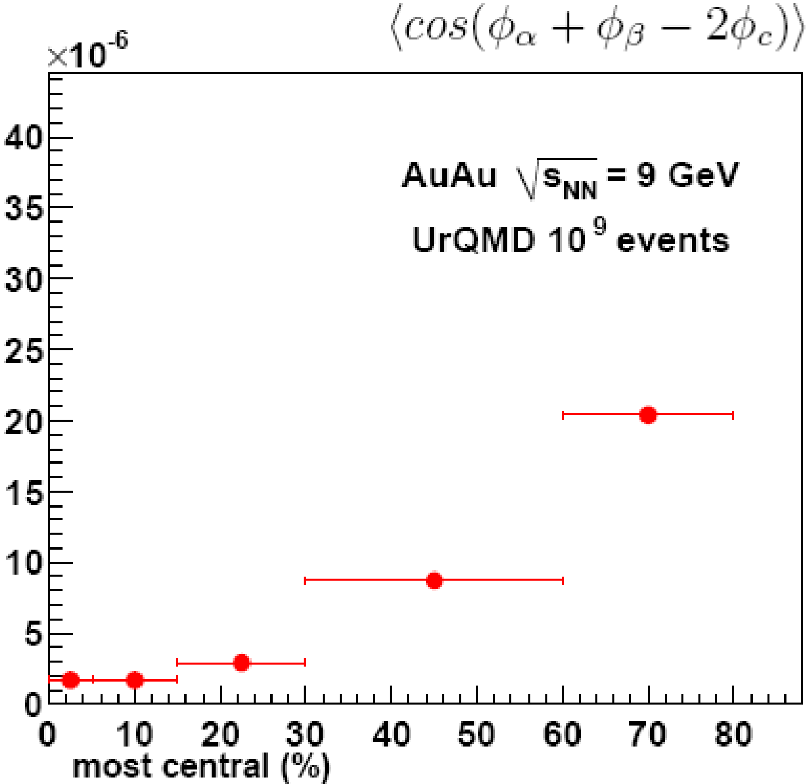
- Orbital Angular Momentum and magnetic moment are proportional – Larmor theorem
- Vorticity for uniform rotation – proportional to OAM
- Same scale as magnetic field
- Tests are required

Observation of GCVE

- Sign of topological field fluctuations unknown – need quadratic (in induced current) effects
- CME – like-sign and opposite-sign correlations – S. Voloshin
- No antineutrons, but like-sign baryonic charge correlations possible
- Look for neutron pairs correlations!
- MPD may be well suited for neutrons!

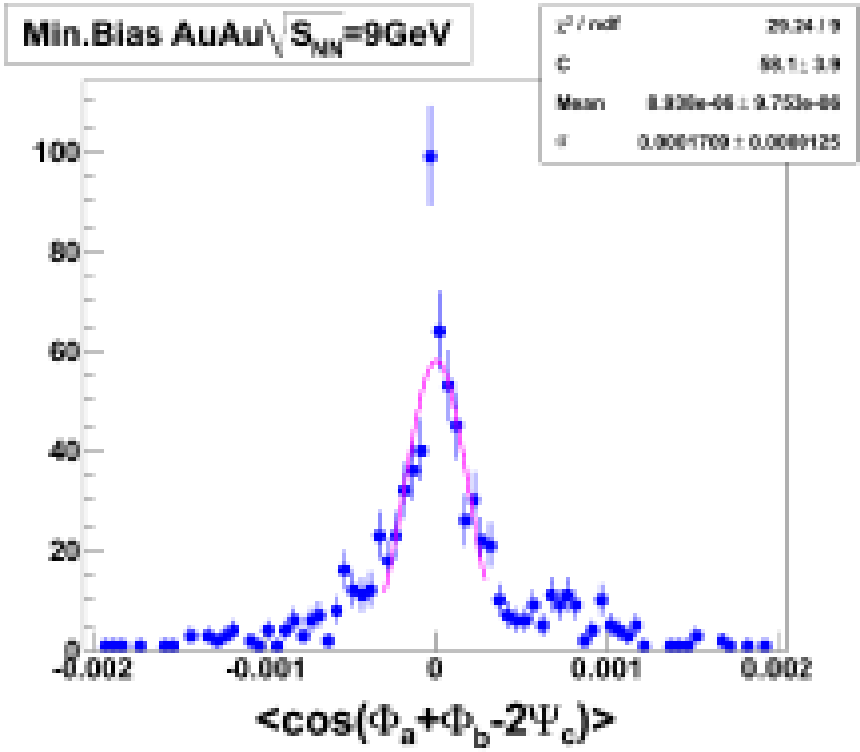
Estimates of statistical accuracy at NICA MPD (months of running)

- UrQMD model : $Au + Au$ at $\sqrt{s_{NN}} = 9$ GeV
- 2-particles -> 3-particles correlations
  no necessity to fix the event plane
- 2 neutrons from mid-rapidity $(|\eta| < 1)$
- +1 from ZDC $(|\eta| > 3)$



Background effects

- Can correlations be simulated by UrQMD generator?



Other sources of quadratic effects

- Quadratic effect of induced currents – not necessary involve (C)P-violation
- May emerge also as C&P even quantity
- Complementary probes of two-current correlators desirable
- Natural probe – dilepton angular distributions

Observational effects of current correlators in medium

- McLerran Toimela'85 $W^{\mu\nu} = \int d^4x\, e^{-iq\cdot x} \langle J^\mu(x) J^\nu(0) \rangle$
- Dileptons production rate

$$\frac{d(R/V)}{d^4q\, d^3p\, d^3p'} = -\frac{1}{E_p E_{p'}} e^4 \frac{1}{(2\pi)^6} \times \delta^{(4)}(p+p'-q) L^{\mu\nu}(p,p') \times (1/q^4) W_{\mu\nu}(q) \,.$$

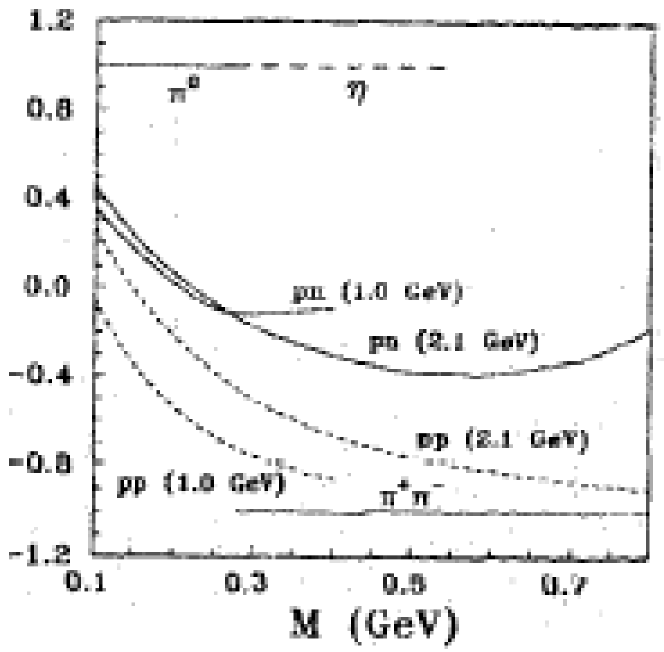
- Structures –similar to DIS F1, F2 (p ->v)

Tensor polarization of in-medium vector mesons (Bratkovskaya, Toneev, OT'95)

- Hadronic in-medium tensor – analogs of spin-averaged structure functions: p -> v
- Only polar angle dependence
- Tests for production mechanisms - recently performed by HADES in Ar+KCl at 1.75 A GeV !

$$W^{\mu\nu} = W_1(q^2, vq)\ (g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}) + W_2(q^2, vq)\ (v^\mu - q^\mu \frac{vq}{q^2})(v^\nu - q^\nu \frac{vq}{q^2})$$

$$\frac{d\sigma}{d\cos\theta} \sim 1 + \frac{|v|^2}{2W_1/W_2 + 1 - (vq)^2/q^2} \cos^2\theta$$



General hadronic tensor and dilepton angular distribution

- Angular distribution

$$d\sigma \propto 1 + \lambda \cos^2\theta + \mu \sin 2\theta \cos\phi + \frac{\nu}{2} \sin^2\theta \cos 2\phi + \rho \sin 2\theta \sin\phi + \sigma \sin^2\theta \sin 2\phi$$

- Positivity of the matrix (= hadronic tensor in dilepton rest frame)

$$\begin{pmatrix} \frac{1-\lambda}{2} & \mu & \rho \\ \mu & \frac{1+\lambda+\nu}{2} & \sigma \\ \rho & \sigma & \frac{1+\lambda-\nu}{2} \end{pmatrix}$$

$$|\lambda| \le 1,\ |\nu| \le 1+\lambda,\ \mu^2 \le \frac{(1-\lambda)(1+\lambda-\nu)}{4}$$

$$\rho^2 \le \frac{(1-\lambda)(1+\lambda+\nu)}{4},\ \sigma^2 \le \frac{(1-\lambda)^2-\nu^2}{4}$$

- + cubic – det M> 0
- 1st line – Lam&Tung by SF method

Magnetic field conductivity and asymmetries

- zz-component of conductivity (~hadronic) tensor dominates
- $\lambda = -1$
- Longitudinal polarization with respect to magnetic field axis
- Effects of dilepton motion – work in progress

Other signals of rotation

- Hyperons (in particular, Λ) polarization (self-analyzing in weak decay)
- Searched at RHIC (S. Voloshin et al.) – oriented plane (slow neutrons) - no signal observed
- No tensor polarizations as well

Why rotation is not seen?

- Possible origin – distributed orbital angular momentum and local spin-orbit coupling
- Only small amount of collective OAM is coupled to polarization
- The same should affect lepton polarization
- Global (pions) momenta correlations (handedness)

New sources of Λ polarization coupling to rotation

- Bilinear effect of vorticity – generates quark axial current (Son, Surowka)
- Strange quarks - should lead to Λ polarization
- Proportional to square of chemical potential – small at RHIC – may be probed at FAIR & NICA

$$j_A^{\mu} \sim \mu^2 \left(1 - \frac{2\,\mu\,n}{3\,(\epsilon + P)}\right) \epsilon^{\mu\nu\lambda\rho}\, V_\nu\, \partial_\lambda V_\rho$$

Conclusions/Discussion - II

- Anomalous coupling to fluid vorticity – new source of neutron asymmetries
- Two-current effects – dilepton tensor polarization
- New source of hyperon polarization in heavy ions collisions

What do we test (question of A.M. Baldin)?

- Fundamental field-theoretical property of anomaly manifested in the new effects:
- - Non-perturbative exact sum rule controlling the meson spectrum as a whole
- - Medium velocity and vorticity as an effective fields coupled to anomaly