# Role of Vacuum Polarization in the Annihilation Channel at the Presence of a Strong Laser Field

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# Outline

## Kinetic equation for the single quasi - particle distribution function $f(\bar{P},t) =< 0|a^{\dagger}_{\bar{P}}(t)a_{\bar{P}}(t)|0 >$:

$$\frac{df_{\pm}(\bar{P},t)}{dt} = \frac{\partial f_{\pm}(\bar{P},t)}{\partial t} + eE(t)\frac{\partial f_{\pm}(\bar{P},t)}{\partial P_{\parallel}(t)} =$$

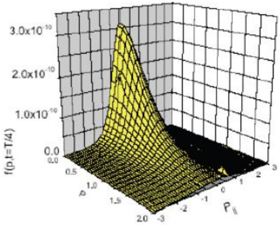
$$\frac{1}{2}\mathcal{W}_{\pm}(t)\int_{-\infty}^{t} dt'\mathcal{W}_{\pm}(t')[1 \pm 2f_{\pm}(\bar{P},t')]\cos[x(t',t)]$$

Kinematic momentum $\bar{P} = (p_1, p_2, p_3 - eA(t))$,

$$\mathcal{W}_{-}(t) = \frac{eE(t)\varepsilon_{\perp}}{\omega^2(t)},$$

where $\omega(t) = \sqrt{\varepsilon_{\perp}^2 + P_{\parallel}^2(t)}$, with
$\varepsilon_{\perp} = \sqrt{m^2 + \bar{p}_{\perp}^2}$ and
$x(t',t) = 2[\Theta(t) - \Theta(t')]$:

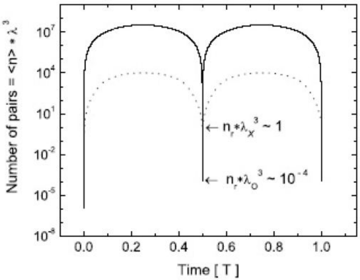
$$\Theta(t) = \int_{-\infty}^{t} dt'\omega(t') \ .$$



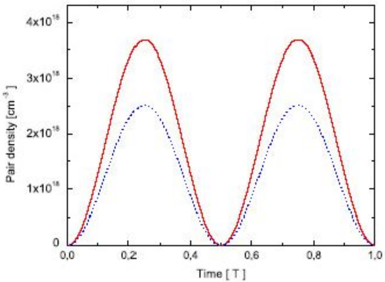

Schmidt et al: Phys. Rev. D**59** (1999) 094005

The particle number density:

$$n(t) = 2\int \frac{d\mathbf{p}}{(2\pi)^2} f(\mathbf{p}, t).$$



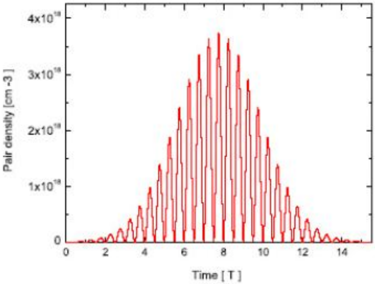

Number of $e^-e^+$ pairs in the volume $\lambda^3$ for a weak field (Jena $Ti : AlO_3$ laser, solid line) and for near - critical field $E_m/E_{crit} = 0.24$, $\lambda = 0.15nm$ (X - FEL, dashed line)




Time dependence of the density $n(t)$ for a monochromatic field

$$E(t) = E_m \sin \omega t,\ 0 < t < NT,\ \ T = 2\pi/\omega$$

.




Time dependence of the density $n(t)$ for a Gaussian wave packet

$$E(t) = E_m e^{-(t/\tau_L)^2} \sin \omega t$$

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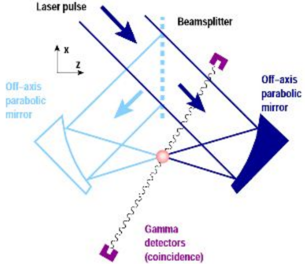
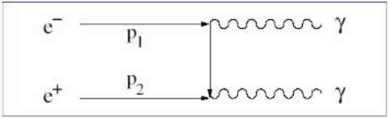



Observable: $e^- + e^+ \to \gamma + \gamma$

Project: G.Gregori et al. (2008) at RAL Astra - Gemini Laser.




$$\frac{d\nu}{dVdt} = \int d\mathbf{p}_1 d\mathbf{p}_2\, \sigma(\mathbf{p}_1, \mathbf{p}_2) f_1(\mathbf{p}_1, t) f_2(\mathbf{p}_2, t) \times \sqrt{(\mathbf{v}_1 - \mathbf{v}_2)^2 - |\mathbf{v}_1 \times \mathbf{v}_2|^2},$$

cross- section $\sigma$ of two photon annihilation

$$\sigma(\mathbf{p}_1, \mathbf{p}_2) = \frac{\pi e^4}{2m^2\tau^2(\tau-1)}\left[(\tau^2 + \tau - 1/2) \times \ln\left\{\frac{\sqrt{\tau}+\sqrt{\tau-1}}{\sqrt{\tau}-\sqrt{\tau-1}}\right\} - (\tau+1)\sqrt{\tau(\tau-1)}\right].$$

Result: some annihilation events per laser pulse for the laser with $I = 10^{20} W/cm^2$, $\lambda = 795nm$ and pulse duration $\tau = 85fs$.

Deficiencies:

- $f(\mathbf{p}, t)$- the distribution function of electrons and positrons considered as short - lived quasi - particles, i.e. the vacuum fluctuations with quantum numbers of an electron and positron
  D.B.Blaschke, S.M.Schmidt, S.A. Smolyansky, A.V. Tarakanov, Izvestiya VUZ Applied Nonlinear Dynamics, **17**, N5, 17 (2009).
- The using of S - matrix methods for calculation of the effective cross section of the two photon annihilation process is not correct in the given situation in the presence of the strong subcritical fields because the free in- and out- states of electron and positron are absent here.

# Outline

The total electromagnetic field acting in the system is

$$\hat{\mathbf{A}}_{tot} = \mathbf{A}_{qc} + \hat{\mathbf{A}} = \mathbf{A}_{ex} + \mathbf{A}_{in} + \hat{\mathbf{A}}$$

The interaction Hamiltonian density

$$H_{in} = e : \bar{\Psi}\gamma^{\mu}\hat{A}_{\mu}\Psi :$$

is described in the holomorphic representation in the nonstationary spinor basis.

D.B. Blaschke, S.M. Schmidt, S.A. Smolyansky, A.V. Tarakanov, contribution to Proceedings of the International Bogolyubov Conference, Dubna (Russia), August 21-27, (2009), arXiv:0912.0381.

The basic differences from the standard QED consists that all forbidden processes (in the framework of the standard QED) are became allowed one in the presence of a strong external field (the multi - photon processes with participation of the quasiclassical photon reservoir).

# Outline

The single - time photon two - point correlation function

$$F_{rr'}(\mathbf{k},\mathbf{k}',t) =< A_r^{(+)}(\mathbf{k},t)A_{r'}^{(-)}(\mathbf{k}',t) >,$$

with following approximations

$$< A_r^{(+)}(\mathbf{k},t)A_{r'}^{(-)}(\mathbf{k}',t) >= \delta_{rr'}\delta(\mathbf{k}-\mathbf{k}')F_r(\mathbf{k},t)$$

and

$$< A_r^{(-)}(\mathbf{k},t)A_{r'}^{(+)}(\mathbf{k}',t) >= \delta_{rr'}\delta(\mathbf{k}-\mathbf{k}')\{1+F_r(\mathbf{k},t)\}.$$

The first equation of the BBGKY chain:

$$
\begin{aligned}
\dot{F}_{rr'}(\mathbf{k},\mathbf{k}',t) = ie(2\pi)^{-3/2}\textstyle\sum_{\alpha,\beta}\int d^3p_1 d^3p_2 \Big\{ -\frac{1}{\sqrt{2k}}\delta(\mathbf{p}_1-\mathbf{p}_2-\mathbf{k})\cdot \\
\cdot\Big[ [\bar{u}u]^{r}_{\beta\alpha}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k};t) < a^{+}_{\alpha}(\mathbf{p}_1,t)a_{\beta}(\mathbf{p}_2,t)A^{(-)}_{r'}(\mathbf{k}',t) > + \\
+[\bar{u}v]^{r}_{\beta\alpha}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k};t) < a^{+}_{\alpha}(\mathbf{p}_1,t)b^{+}_{\beta}(-\mathbf{p}_2,t)A^{(-)}_{r'}(\mathbf{k}',t) > + \\
+[\bar{v}u]^{r}_{\beta\alpha}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k};t) < b_{\alpha}(-\mathbf{p}_1,t)a_{\beta}(\mathbf{p}_2,t)A^{(-)}_{r'}(\mathbf{k}',t) > + \\
+ [\bar{v}v]^{r}_{\beta\alpha}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k};t) < b_{\alpha}(-\mathbf{p}_1,t)b^{+}_{\beta}(-\mathbf{p}_2,t)A^{(-)}_{r'}(\mathbf{k}',t) > \Big] + \\
+\frac{1}{\sqrt{2k'}}\delta(\mathbf{p}_1-\mathbf{p}_2+\mathbf{k}')\Big[ [\bar{u}u]^{r'}_{\beta\alpha}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k}';t) < a^{+}_{\alpha}(\mathbf{p}_1,t)a_{\beta}(\mathbf{p}_2,t)A^{(+)}_{r}(\mathbf{k},t) > + \\
+[\bar{u}v]^{r'}_{\beta\alpha}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k}';t) < a^{+}_{\alpha}(\mathbf{p}_1,t)b^{+}_{\beta}(-\mathbf{p}_2,t)A^{(+)}_{r}(\mathbf{k},t) > + \\
+[\bar{v}u]^{r'}_{\beta\alpha}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k}';t) < b_{\alpha}(-\mathbf{p}_1,t)a_{\beta}(\mathbf{p}_2,t)A^{(+)}_{r}(\mathbf{k},t) > + \\
+ [\bar{v}v]^{r'}_{\beta\alpha}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k}';t) < b_{\alpha}(-\mathbf{p}_1,t)b^{+}_{\beta}(-\mathbf{p}_2,t)A^{(+)}_{r}(\mathbf{k},t) > \Big]\Big\} + i(k-k')F_{rr'}(\mathbf{k},\mathbf{k}',t).
\end{aligned}
$$

D.B. Blaschke, S.M. Schmidt, S.A. Smolyansky, A.V. Tarakanov, contribution to Proceedings of the International Bogolyubov Conference, Dubna (Russia), August 21-27, (2009), arXiv:0912.0381.

Thus, the kinetics of the photon states is defined by the different forced processes of either one photon scattering of electron and positron (considered as quasiparticles) or their creation and annihilation. Some processes forbidden in absence of an external field become possible here: in the lowest order of the perturbation theory it is the one photon annihilation, the simultaneous creation of an electron - positron pair and a photon.
V.I.Ritus, Trudi FIAN SSSR, **111**, 5 (1979).
In the highest orders of the perturbation theory the number of the such kind forbidden processes are increased abruptly. In the general case, the necessity of taking into account of such kind processes strongly complicates the problem and leads to the idea of selection of the correllation function.

We limit ourselves to the consideration of the annihilation channel only:

$$\dot{F}_{rr'}(\mathbf{k},\mathbf{k}',t) = ie(2\pi)^{-3/2}\int d^3p_1 d^3p_2 \Big\{ \frac{1}{\sqrt{2k}}\delta(\mathbf{p}_1-\mathbf{p}_2-\mathbf{k})$$
$$\cdot[\bar{u}v]^r_{\beta\alpha}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k};t) < b^+_\beta(-\mathbf{p}_2,t)a^+_\alpha(\mathbf{p}_1,t)A^{(-)}_{r'}(\mathbf{k}',t) > +$$
$$+\frac{1}{\sqrt{2k'}}\delta(\mathbf{p}_1-\mathbf{p}_2+\mathbf{k}')[\bar{v}u]^{r'}_{\beta\alpha}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k}';t) < b_\alpha(-\mathbf{p}_1,t)a_\beta(\mathbf{p}_2,t)A^{(+)}_r(\mathbf{k},t) > \Big\}.$$

For obtaining of a closed photon KE it is necessary to fulfill some truncation procedure for the correlators entering in this equation. The simplest truncation of the type

$$< b_\alpha(-\mathbf{p}_1,t)a_\beta(\mathbf{p}_2,t)A^{(\pm)}_r(\mathbf{k},t) > \simeq < b_\alpha(-\mathbf{p}_1,t)a_\beta(\mathbf{p}_2,t) > < A^{(\pm)}_r(\mathbf{k}',t) > = 0$$

is not effective due to the definition of the photon vacuum $< A^{(\pm)}_r(\mathbf{k},t) > = 0$.

The equations of the second level for the correlators:

$$\left\{\frac{\partial}{\partial t} + i[\omega(\mathbf{p}_1,t) + \omega(\mathbf{p}_2,t) - k]\right\} < b_\alpha(-\mathbf{p}_1,t)a_\beta(\mathbf{p}_2,t)A_r^{(+)}(\mathbf{k},t) >$$

$$= -ie(2\pi)^{-3/2}\int d^3p'\,\frac{d^3k'}{\sqrt{2k'}}\Big\{\delta(\mathbf{p}'-\mathbf{p}_1+\mathbf{k}')$$

$$\cdot\Big[[\bar{u}v]^{r'}_{\alpha\beta'}(\mathbf{p}',\mathbf{p}_1,\mathbf{k}';t) < a^+_{\beta'}(\mathbf{p}',t)a_\beta(\mathbf{p}_2,t)A_{r'}(\mathbf{k}',t)A_r^{(+)}(\mathbf{k},t) >$$

$$+ [\bar{v}v]^{r'}_{\alpha\beta'}(\mathbf{p}',\mathbf{p}_1,\mathbf{k}';t) < b_{\beta'}(-\mathbf{p}',t)a_\beta(\mathbf{p}_2,t)A_{r'}(\mathbf{k}',t)A_r^{(+)}(\mathbf{k},t) >\Big]$$

$$-\delta(\mathbf{p}_2-\mathbf{p}'+\mathbf{k}')\cdot\Big[[\bar{u}u]^{r'}_{\beta'\beta}(\mathbf{p}_2,\mathbf{p}',\mathbf{k}';t) < b_\alpha(-\mathbf{p}_1,t)a_{\beta'}(\mathbf{p}',t)A_{r'}(\mathbf{k}',t)A_r^{(+)}(\mathbf{k},t) >$$

$$+ [\bar{u}v]^{r'}_{\beta'\beta}(\mathbf{p}_2,\mathbf{p}',\mathbf{k}';t) < b_\alpha(-\mathbf{p}_1,t)b^+_{\beta'}(-\mathbf{p}',t)A_{r'}(\mathbf{k}',t)A_r^{(+)}(\mathbf{k},t) >\Big]\Big\}$$

$$+S^r_{\alpha\beta}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k};t) + U^r_{\alpha\beta}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k};t).$$

The vacuum polarization effects at the presence of the quantized electromagnetic field are described by the term

$$\begin{aligned} U^r_{\alpha\beta}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k};t) &= U^{(1)}_{\beta'\alpha}(\mathbf{p}_1,t) < b_{\beta'}(-\mathbf{p}_1,t)a_\beta(\mathbf{p}_2,t)A^{(+)}_r(\mathbf{k},t) > - \\ &- U^{(1)}_{\beta\beta'}(\mathbf{p}_2,t) < b_\alpha(-\mathbf{p}_1,t)a_{\beta'}(\mathbf{p}_2,t)A^{(+)}_r(\mathbf{k},t) > + \\ &+ U^{(2)}_{\beta'\alpha}(\mathbf{p}_1,t) < a^+_{\beta'}(\mathbf{p}_1,t)a_\beta(\mathbf{p}_2,t)A^{(+)}_r(\mathbf{k},t) > - \\ &- U^{(2)}_{\beta\beta'}(\mathbf{p}_2,t) < b_\alpha(-\mathbf{p}_1,t)b_{\beta'}(-\mathbf{p}_2,t)A^{(+)}_r(\mathbf{k},t) > \end{aligned}$$

which is absent in the standard QED without a strong field. The other set of terms

$$\begin{aligned} S^r_{\alpha\beta}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k};t) = -ie(2\pi)^{-3/2}\frac{1}{\sqrt{2k}}\int d^3p'_1 d^3p'_2\delta(\mathbf{p}'_1-\mathbf{p}'_2-\mathbf{k})\,\cdot \\ \cdot\ \Big\{[\bar{u}u]^r_{\alpha'\beta'}(\mathbf{p}'_1,\mathbf{p}'_2,\mathbf{k};t) < b_\alpha(-\mathbf{p}_1,t)a_\beta(\mathbf{p}_2,t)a^+_{\alpha'}(\mathbf{p}'_1,t)a_{\beta'}(\mathbf{p}'_2,t) > + \\ + [\bar{u}v]^r_{\alpha'\beta'}(\mathbf{p}'_1,\mathbf{p}'_2,\mathbf{k};t) < b_\alpha(-\mathbf{p}_1,t)a_\beta(\mathbf{p}_2,t)a^+_{\alpha'}(\mathbf{p}'_1,t)b^+_{\beta'}(-\mathbf{p}'_2,t) > + \\ + [\bar{v}u]^r_{\alpha'\beta'}(\mathbf{p}'_1,\mathbf{p}'_2,\mathbf{k};t) < b_\alpha(-\mathbf{p}_1,t)a_\beta(\mathbf{p}_2,t)b_{\alpha'}(-\mathbf{p}'_1,t)a_{\beta'}(\mathbf{p}'_2,t) > + \\ + [\bar{v}v]^r_{\alpha'\beta'}(\mathbf{p}'_1,\mathbf{p}'_2,\mathbf{k};t) < b_\alpha(-\mathbf{p}_1,t)a_\beta(\mathbf{p}_2,t)b_{\alpha'}(-\mathbf{p}'_1,t)b^+_{\beta'}(-\mathbf{p}'_2,t) > \Big\} \end{aligned}$$

are connected with different transformations in the $e^-e^+\gamma$- plasma without participation of the quasiparticle photon excitations.

# Outline

Research region: All processes of the 2-nd order relatively of the $e\gamma$-interaction.

Approximations: (on the phone of nonperturbative calculations $f$)
— weak "laser field" $E_0 \ll E_c = \dfrac{m^2}{e}, E(t) = E_0 sin\nu t$
— ladge adibatical parameter $\gamma = \frac{E_c}{E_0}\frac{\nu}{m} \gg 1$ or $\frac{m}{\nu} \gg \frac{E_0}{E_c}$ - the limitation on $\nu$. It corresponds to multiphoton process.
— RPA of the type (truncation procedure)

$$< a^{+}_{\beta'}(\mathbf{p}', t) a_{\beta}(\mathbf{p}_2, t) A_{r'}(\mathbf{k}', t) A^{(+)}_{r}(\mathbf{k}, t) > \simeq$$
$$\simeq < a^{+}_{\beta'}(\mathbf{p}', t) a_{\beta}(\mathbf{p}_2, t) >< A_{r'}(\mathbf{k}', t) A^{(+)}_{r}(\mathbf{k}, t) >$$

— the diagonalization respect to the spin and polarization indexes, e.q.

$$< a^{+}_{\alpha}(\mathbf{p},t) a_{\beta}(\mathbf{p},t) > \simeq \delta_{\alpha\beta}\delta(\mathbf{p}-\mathbf{p}')f(\mathbf{p},t),$$

— low density $f(\mathbf{p},t), F(\mathbf{k},t) \ll 1$,
— the multiphoton decomposition

$$f(\mathbf{p},t) = \sum f_{n,l}(\mathbf{p}) e^{in\nu t + imlt},$$

and the leading approximation in the resonance condition (N is the photon number involved in the considered process) $lm - N\nu = 0$,

— "isotropic approximation" (select by the angular dependences).

## The photon production rate in these approximations

$$\dot{F}(k) \simeq \frac{5\alpha}{2k}\xi(k, n_0+1)J_{n_0+1}(a_0)[J_{n_0+3}(a_0) + J_{n_0-1}(a_0)][f_{2,0}(p_0) + f_{2,0}(p_0+k)]$$

where

$$\xi(k, n_0+1) = p_0(n_0+1)\frac{m\omega_0(k)}{m+\omega_0(k)}$$

$k$ is the frequency of radiated photon,

$$p_0(n) = \left[\frac{(n\nu)^2(n\nu-2k)^2}{4(n\nu-k)^2} - m^2\right]^{1/2},$$

where $n_0 = [\Omega_0/\nu]$ and $\Omega_0 = \omega_0(p) + \omega_0(p+k) - k$ $(\omega_0(p) = \omega(p)|_{A=0})$ are the number of photons which are necessary for clamp of the gap ([x] is the integer part x); $n_0 \sim p_0(n_0) = 0$. Finally,

$$a_0 = \frac{4\pi\alpha E_0}{\nu^2}\left[\frac{p_0}{\omega(p_0)} + \frac{p_0+k}{\omega(p_0+k)}\right]$$

Infrared and optic region $k \ll m$ (essentially multiphoton region, $\nu \sim k$)

$$\frac{1}{m}\dot{F}(k) = 10\alpha f_{2,0}\frac{1}{n_0^2}\left[\frac{n_0 E_0}{4E_c}\right]^{2n_0}\sqrt{\frac{m}{k}},$$

$n_0 = [\frac{2m}{k}] \gg 1$, the photon number necessary for clamp of the gap 2m.

Example: $E_0 = 3\cdot 10^{-5}E_c \sim f_{2,0} \sim 10^{-11}$; for $k_0 = 10eV$, $n_0 \sim 10^5$ photons. Then $\frac{1}{m}\dot{F}(k) \sim 10^{-20}$, i.e. the effect is very small, if

$$\eta = \frac{n_0 E_0}{4E_c} = 1.$$

However the result depends from "the play" of large and small numbers and very sensitive to chose of the parameters in the vicinity of $\eta = 1$, where the behaviour of this function is similar to the $\Theta$-function.

## $\gamma$ – ray region $\nu \sim m$.

In this case the number of the clanp photons can be small, $n_0 \sim 1$. Let $n_0 = 1, \nu \sim m, k \ll m$.

$$\frac{1}{m}\dot{F}(k) = \frac{\alpha k}{8m}\left(\frac{E_0}{E_c}\right)^2\left[\left(\frac{\nu}{2m}\right)^2 - 1\right]^{1/2} f_{2,0}(0) = \frac{k}{m}\left[\left(\frac{\nu}{2m}\right)^2 - 1\right]^{1/2} A,$$

$$A = \frac{\alpha}{8}\left(\frac{E_0}{E_c}\right)^2 f_{2,0}(0)$$

Features:

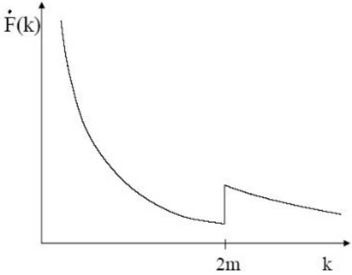
- threshold at $\nu = 2m$
- linear growth with k

Example: for $E_0 = 3 \cdot 10^{-5} E_c$, $A \sim 10^{-12} f_{2,0}(0)$.

## Conclusions

Features of the annihilation photon spectrum:

- the continuity
- the threshold increase at $k = 2m$
- the flicker noise in the infrared region.




The photon production rate jump at $k = 2m$.

# Astra Gemini Laser System