SPACE STRUCTURE, FLUCTUATIONS AND CORRELATIONS OF ELECTROMAGNETIC CASCADES PRODUCED BY 100 MeV - 100 GeV GAMMA QUANTA IN HEAVY AMORPHOUS MEDIA

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OUTLINE OF THE TALK

- **MOTIVATION**
- **ABSTRACT**
- **SHORT HISTORY**
- **RESULTS OF MODELING**
- **SUMMARY AND CONCLUSION**

Motivation

- Longitudinal profiles of electromagnetic cascades (ECs) produced in heavy amorphous media by high enough energy gamma quanta and electrons (or positrons) are the basic characteristics of the phenomenon both from cognitive and application viewpoints.
- Such are the relevant fluctuations of these profiles since the process of EC is of strongly expressed stochastic nature that is especially perceptible at not too high energies (i.e. hundredths MeV to several GeV).
- The fluctuations determine the energy resolution and accuracy of flight direction of particles initiating EMC.
- Information about fluctuations and correlations of ECs is needed for electromagnetic calorimeters under construction as PANDA (GSI), as well for radiation shielding construction and radiation material physics needs.

Abstract

- We study the average longitudinal and transverse **profiles** of electromagnetic cascades (ECs) created in most popular dense amorphous media (liquid xenon, PWO, CdWO4, GaAs, NaI, Pb, lead glass and BGO) by gamma quanta of energy $E\gamma$ =100MeV÷3500MeV at three different cut-off energies (0.6, 1.2, 3.0 MeV). The work has been performed using the EGS4 & GEANT4 modeling codes. (For BGO we investigated the region $E\gamma$ =100MeV÷100GeV at one cut-off 1.2 MeV)
- We analyzed longitudinal and transverse **fluctuations** in two ways. One of them is to estimate the value of standard deviation of the mean energy deposited to a given distance *t* from the shower start point. The other is the distribution of the shower depth, up to which an amount of energy exceeding the so-called threshold energy (TE) was deposited.
- Estimate are also the correlation parameters of energy deposition
- The results are compared with available experiment. The ultimate objective of this investigation is to obtain concise information about average profiles and fluctuations in ECs suitable for practical purposes.

Basic steps in the investigation of electromagnetic cascades:

1. **Rossi B. Phys. Zs., 1932, vol.33, p.304 - discovery of the phenomenon.**
2. **Rossi B. High-Energy Particles. Prentice-Hall, New York, 1952 - one-dimensional theory of EMC.**
3. **Longo E., Sestili J. Nucl. Instr. Meth., 1975, vol.128, p.283 – computer model of EMC** *(neither EGS4, nor GEANT)***.**
4. **De Angelis A. Nucl. Instr. Meth. A., 1988, vol.271, p.455 - computer model of EMC** *(neither EGS4, nor GEANT)***.**
5. **Słowiński B. Phys. Part. Nucl. 25 (2), March-April 1994 - overview of experimental and theoretical description of EMC.**
6. **Modern description of EMC is needed urgently** *(with EGS4 and GEANT confronted with experiment)***.**

What is the EMC in short?

$$\gamma \rightarrow e^+ + e^-$$

$$e^+ \rightarrow e^+ + \gamma$$

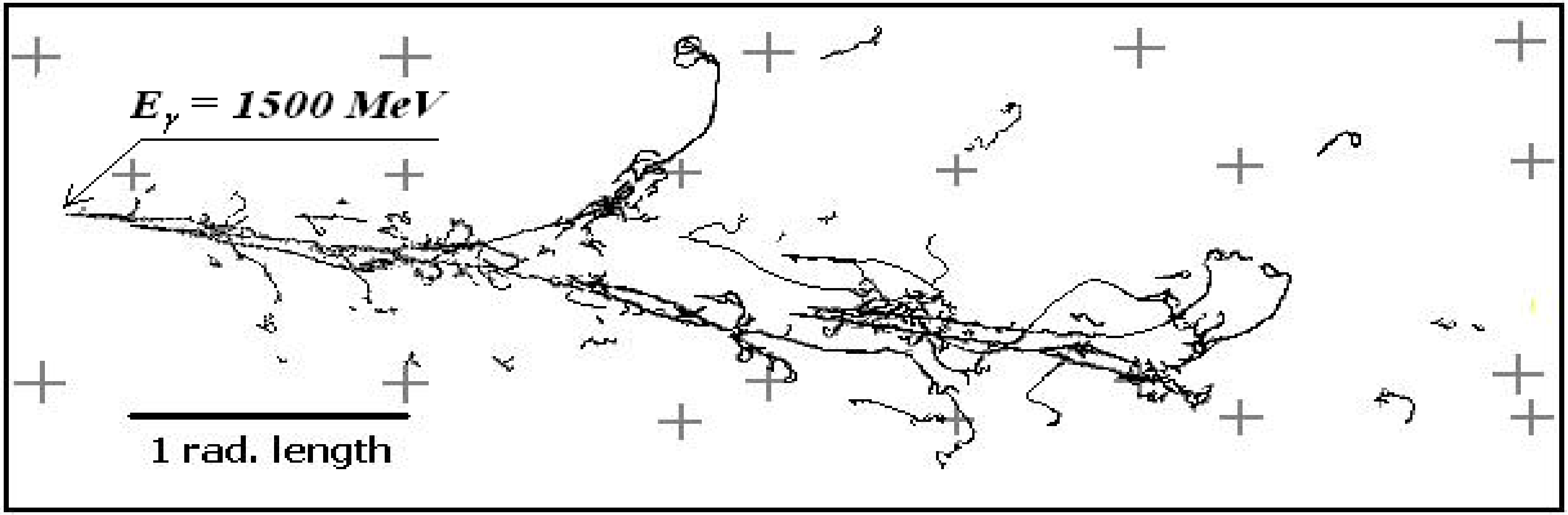
$$e^- \rightarrow e^- + \gamma$$

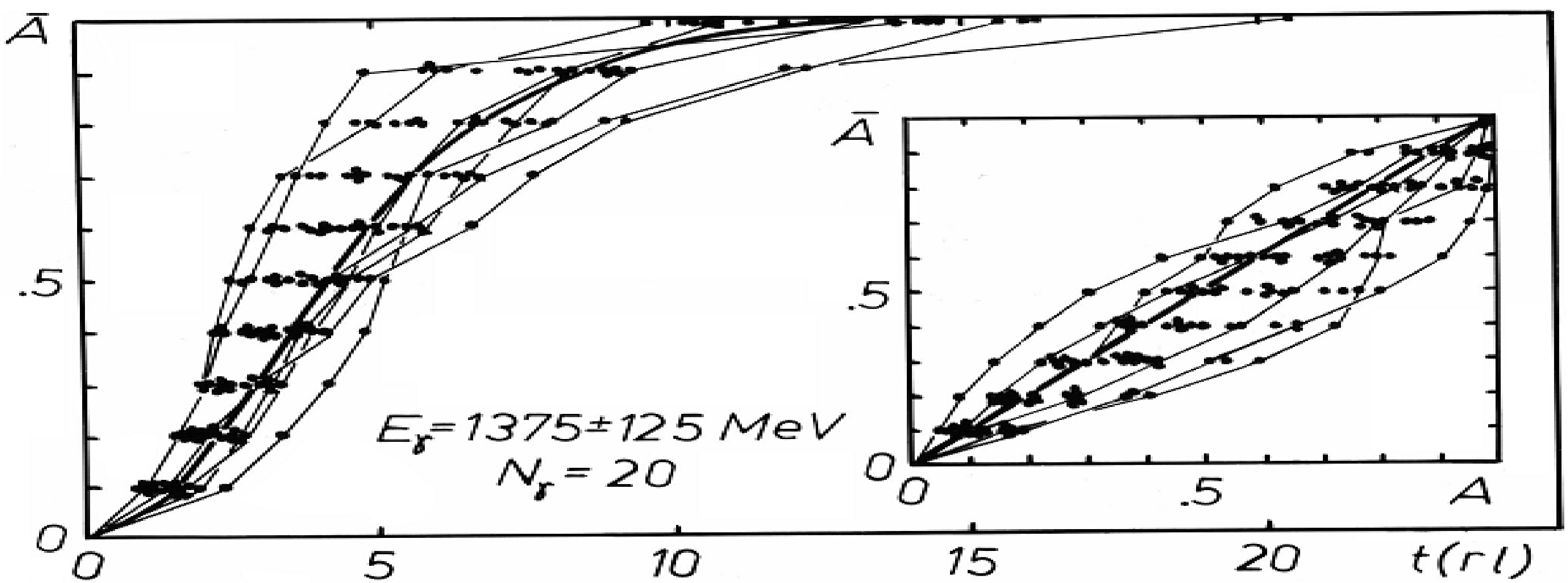
...

Leading elementary processes:

1. Pair creation
2. Bremstrahlung (radiation emission)
3. Ioniztion
4. Multiple Coulomb scattering

A picture of EMC from 26 liter Xenon Bubble Chamber (LHE JINR)





Average description of EC profiles

Profiles

- longitudinal: $(-dE/dt) = A \cdot t^{\alpha} \cdot \exp(-bt)$

- lateral (or radial): $(-dE/dr) = B \cdot \exp(-\mu(t) \cdot r)$

- three-dimensional EMC picture:

$$(-d^2E/dt \cdot dr) = C \cdot t^{\alpha} \cdot \exp\{-[bt + \mu(t) \cdot r]\}$$

and fluctuation

(as an estimation of energy resolution when a cascade totally develops inside a target matrial)

$$\frac{\sigma}{E} \propto \frac{\sqrt{t}}{\sqrt{E}},$$

t is the step of sampling in units of r.l.

(C.Grupen. Particle detectors. Cambridge University Press. 1996)

$$\frac{\sigma_E}{E} = \left(\frac{a^2}{E^2} + \frac{b^2}{E} + c^2\right)^{1/2}$$

E is in GeV *(for $PbWO_4$ as PWO; ALICE experiment)*

Modeling of EC

Programs: EGS4 and GEANT4

GQ energies: E_γ = 100, 210, 555, 875, 1625, 2375, 3125 MeV

(for BGO: 5, 10, 20, 50, 100 GeV)

Cut-off energies: $E_{c.o.}$ = 0.6, 1.2, 3.0 MeV

Materials:

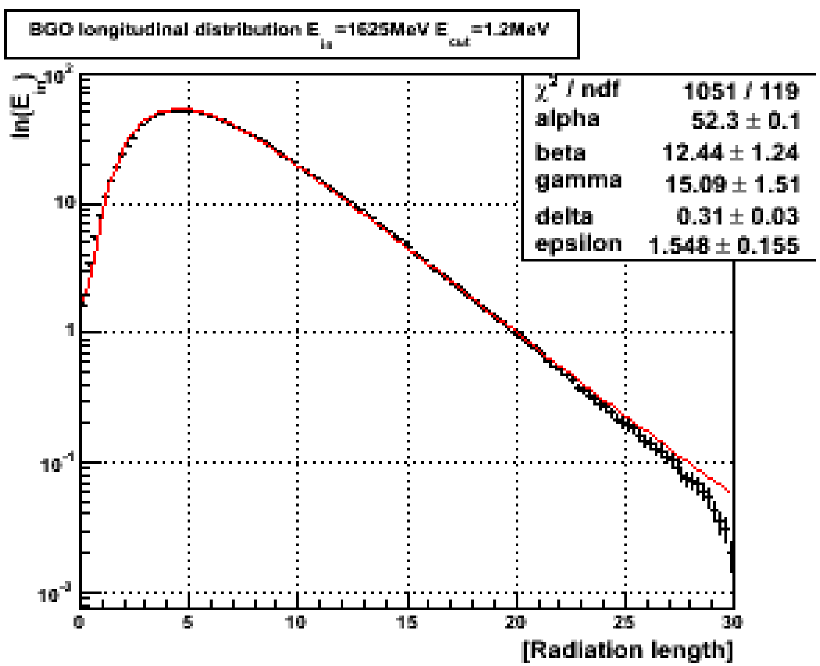
- liquid xenon
- PWO
- $CdWO_4$
- GaAs
- NaI
- Pb
- lead glass
- BGO

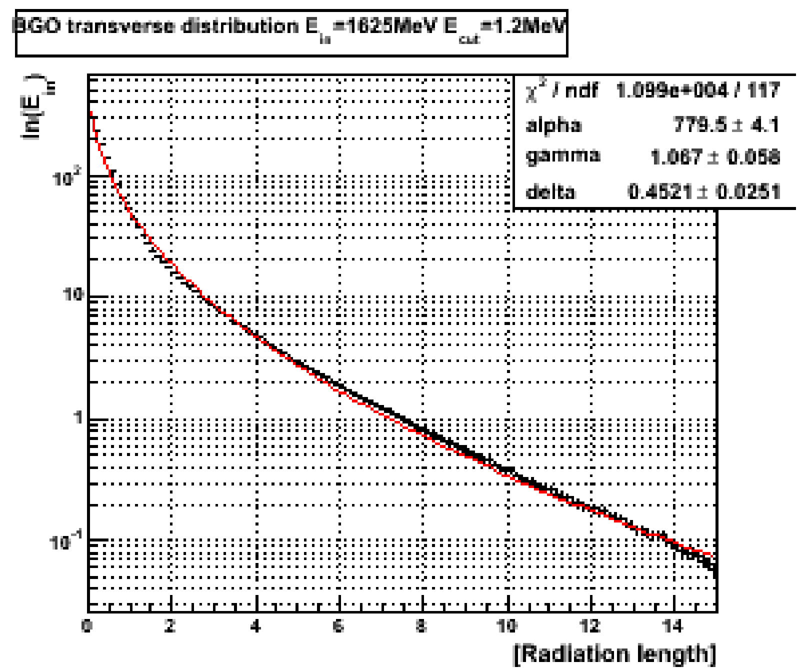
For every set of parameters: E_γ, E_{co} and material we modeled 20000 events (histories).

LONGITUDINAL & TRANSVERSE PROFILES

An example for illustration:

Average longitudinal and transverse profile of EC produced in BGO by gammas of energy 1625 MeV (E_{co}=1.2 MeV).





FITTING LONGITUDINAL PROFILES

to the approximating function:

$$(-dE/dt) = \alpha_t \left(t - \varepsilon_t\right)^{\beta_t} \exp\left(-\gamma_t\, t^{\delta_t}\right)$$

α_t, β_t, δ_t, γ_t, ,ε_t are the fit parameters depending on cut-off energy $E_{c.o.}$, E_γ and material properties.

Dependence of parameters:

β_t, δ_t, γ_t *and* ε_t

on

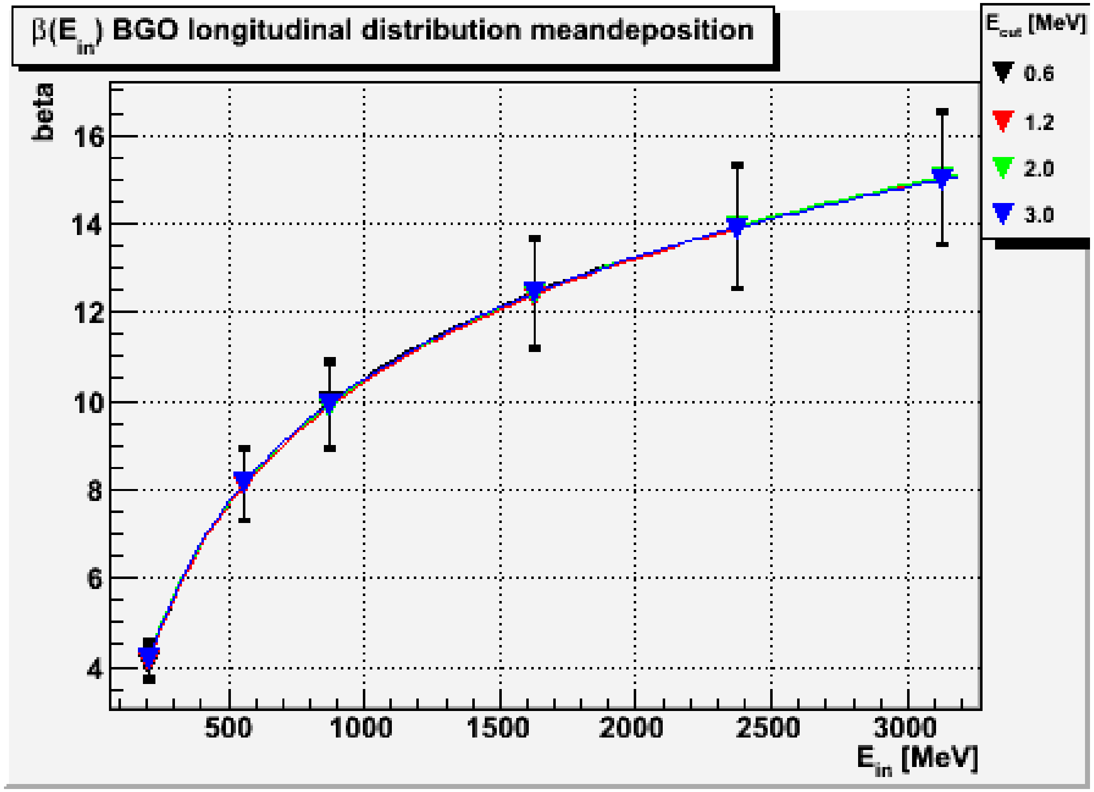
E_γ, $E_{c.o.}$, Z/A.

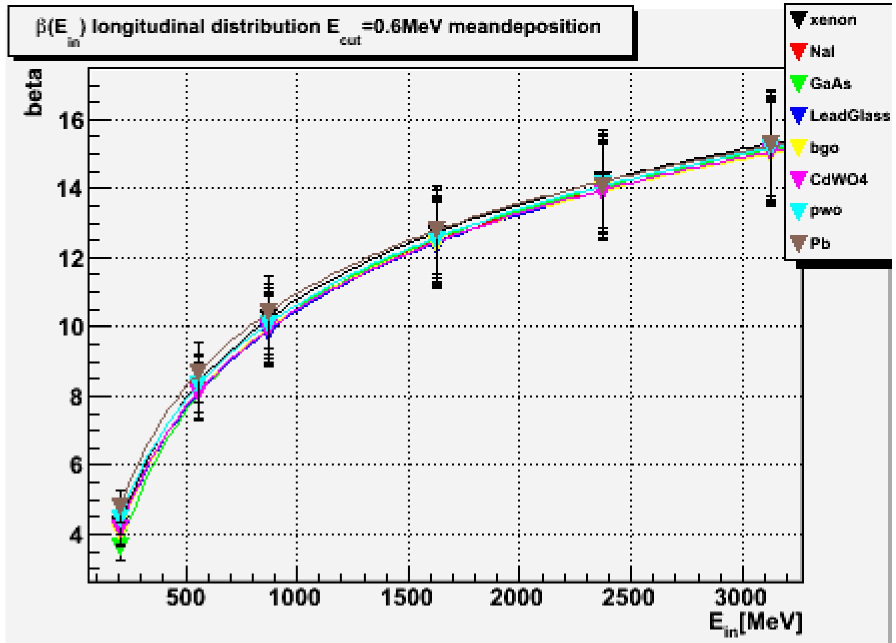
Parameter

β_t

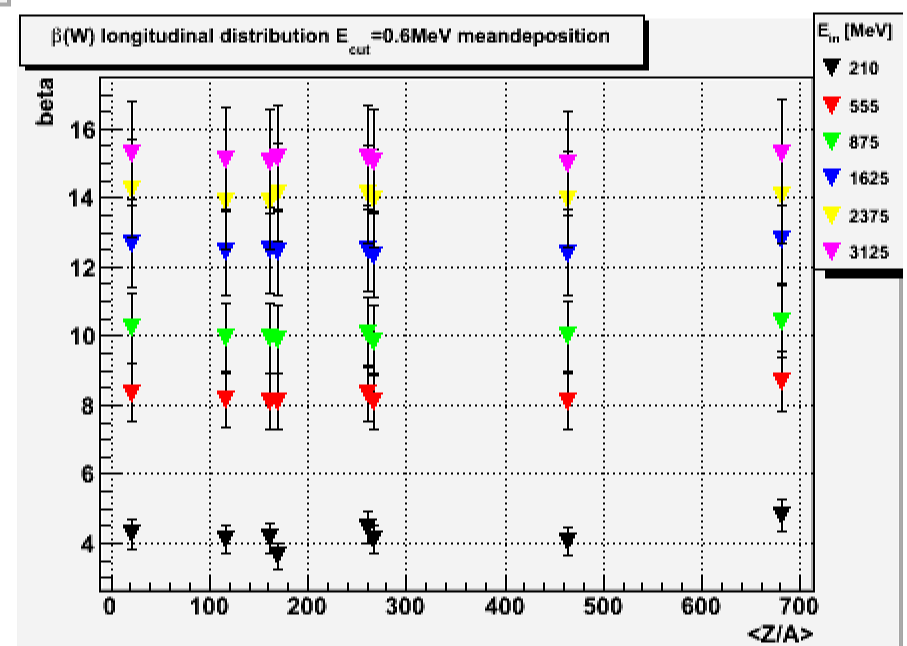
BGO

$$\beta_t(E_\gamma) = a \cdot E_\gamma{}^b + c$$





Dependencies on E_γ energy for all materials (E_c=0.6MeV)



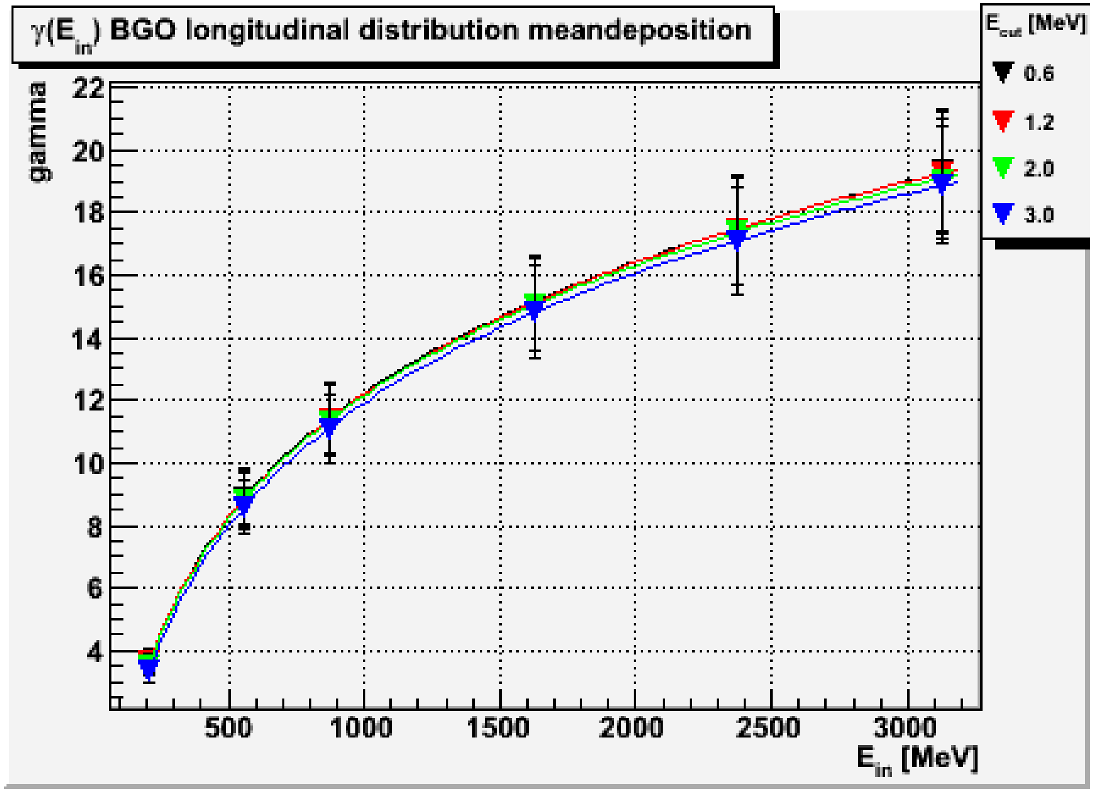
Dependencies on material parameter W=Z/A for all energies (E_c=0.6MeV)

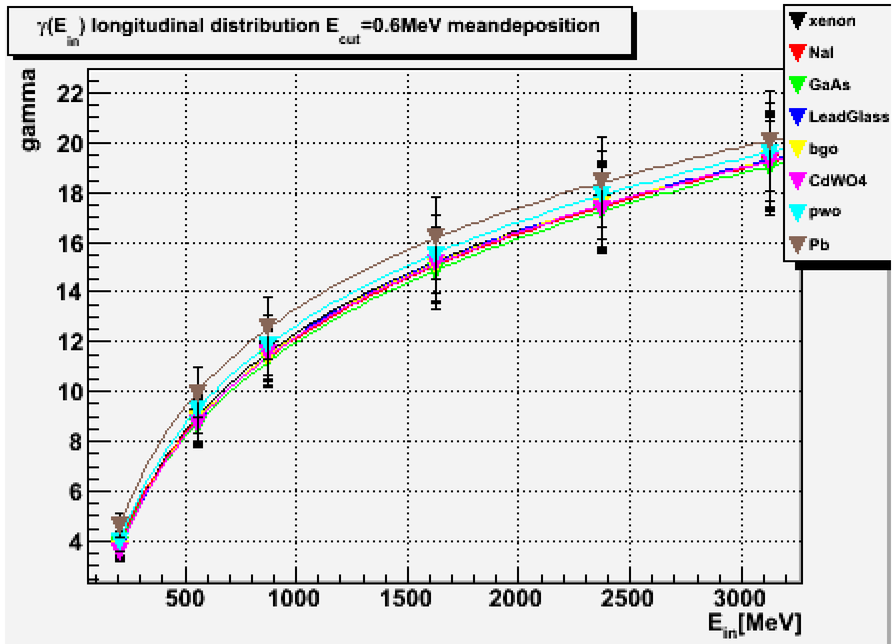
Parameter

γ_t

BGO

$$\gamma_t(E_\gamma) = a \cdot E_\gamma{}^b + c$$





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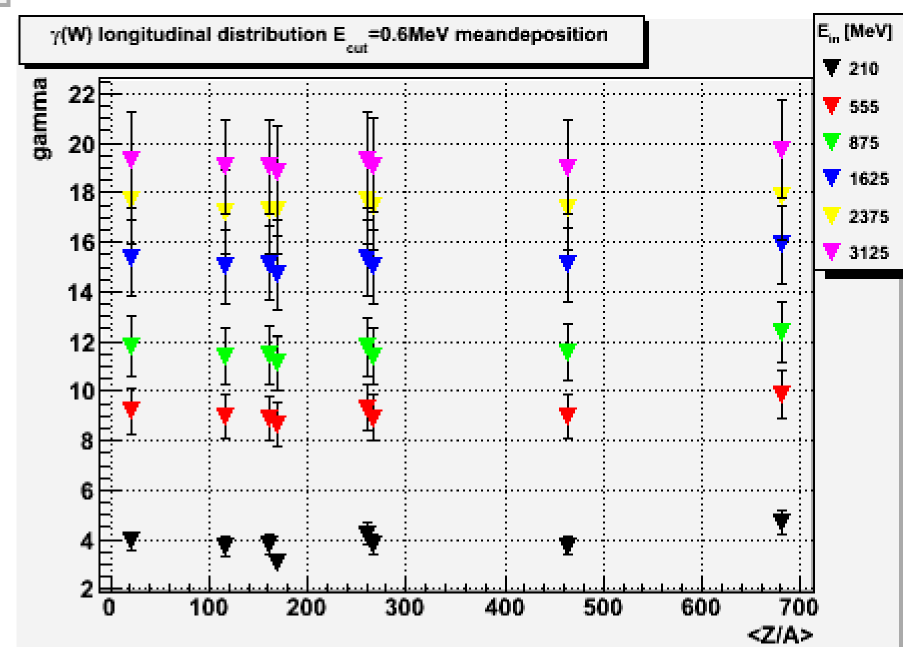


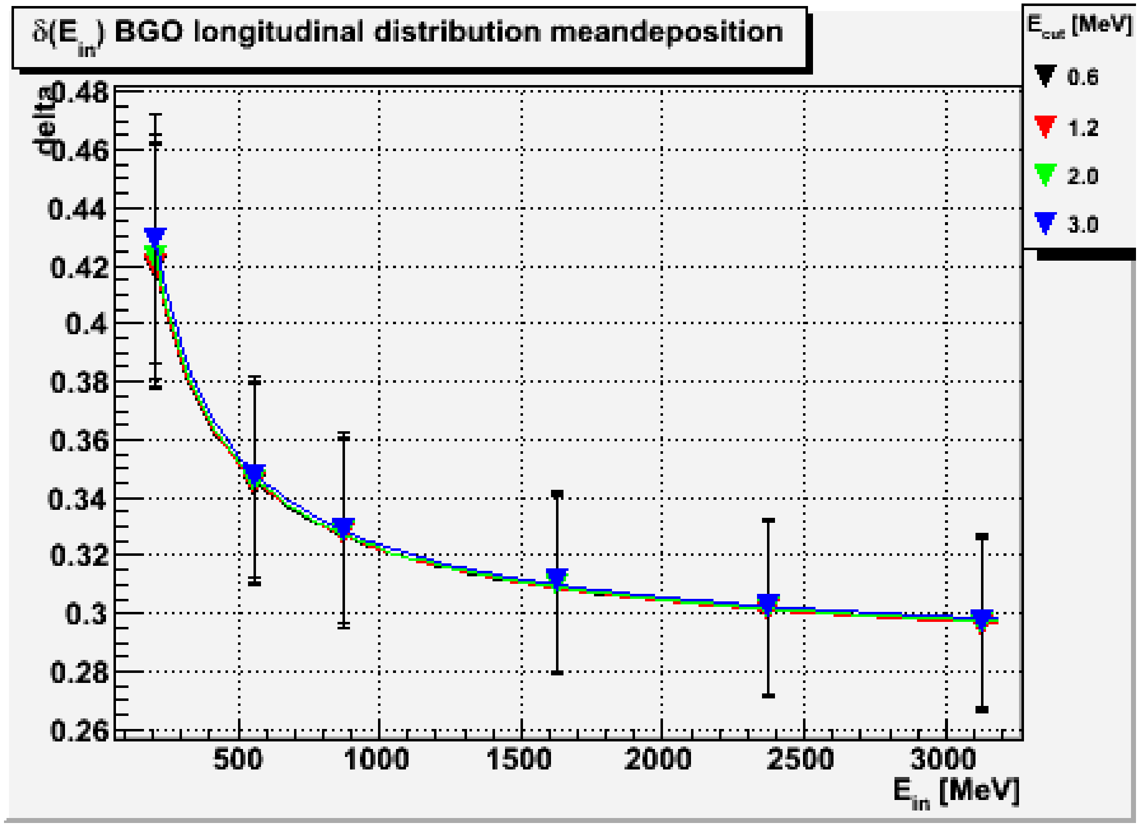
Dependencies on the material parameter W=Z/A for all energies (E_c=0.6MeV)

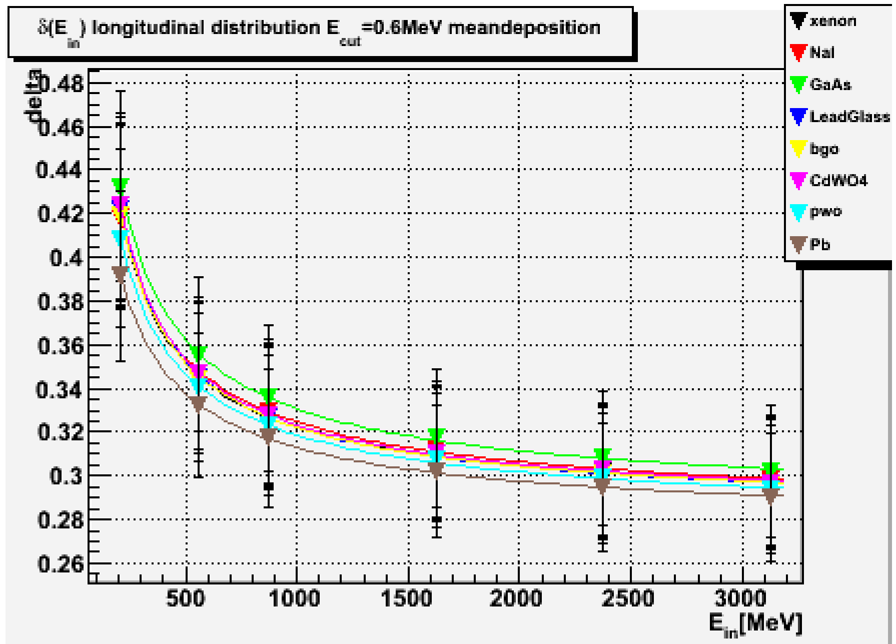
Parameter

δ_t

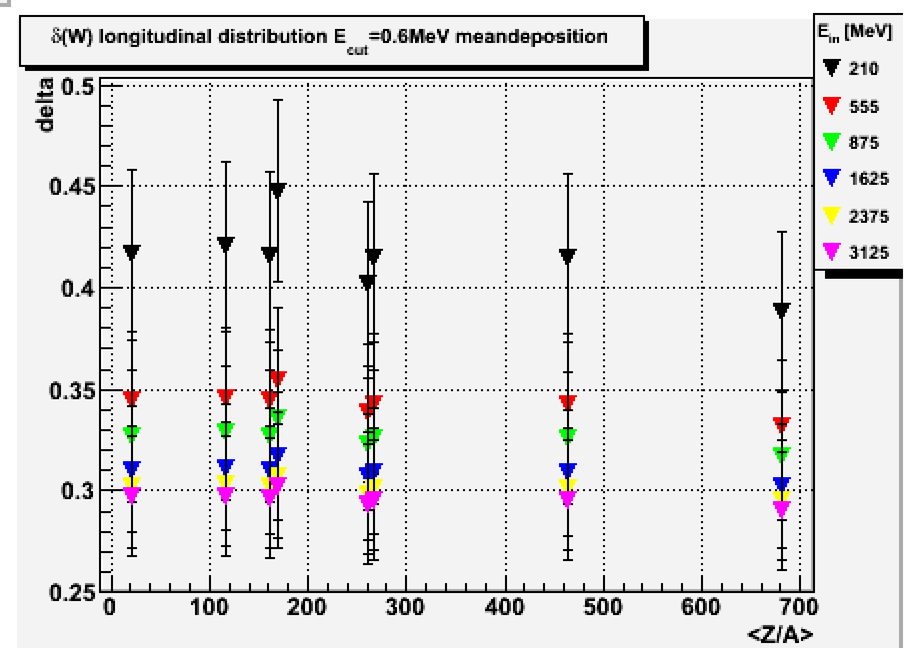
BGO

$$\delta_t(E_\gamma) = a \cdot E_\gamma^{\ b} + c$$





Dependencies on E_γ energy for all materials (E_c=0.6MeV)



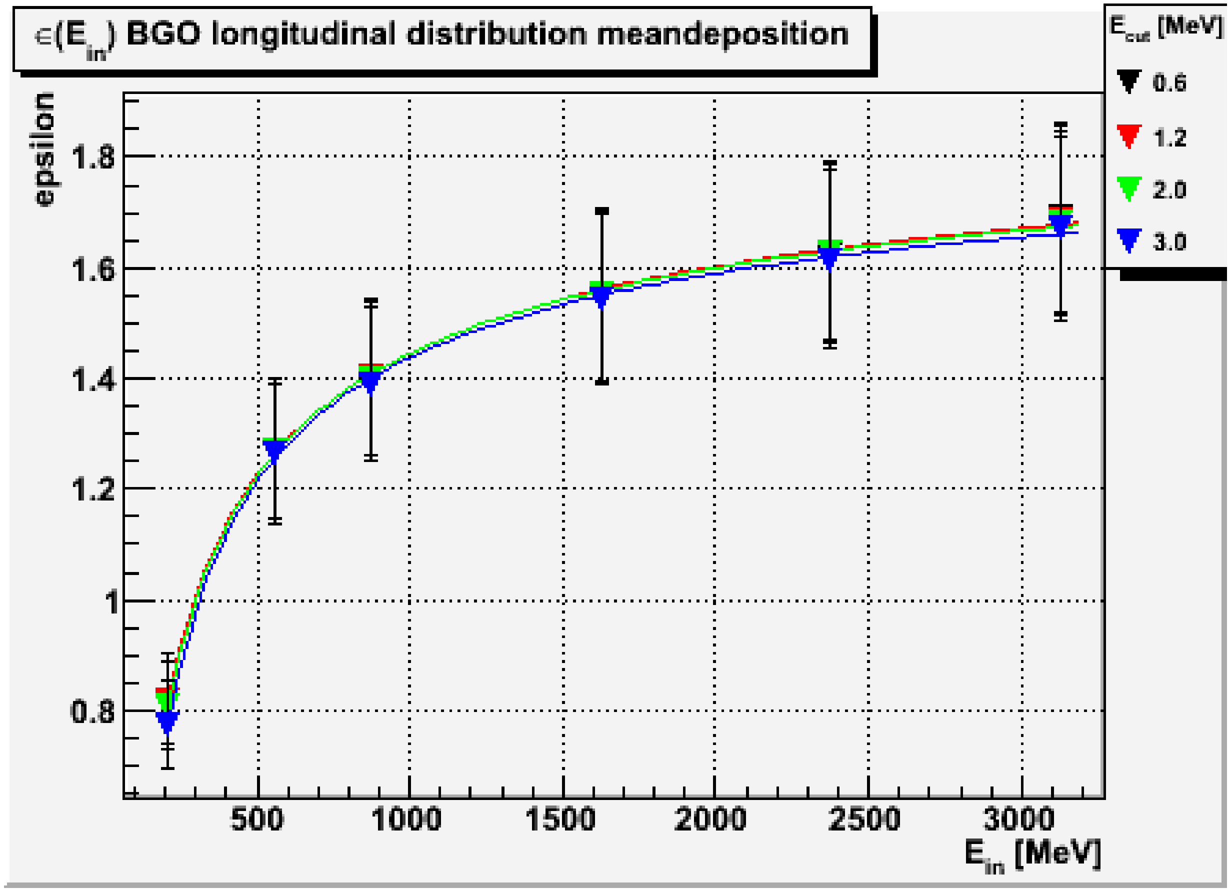
Dependencies on the material parameter W=Z/A for all energies (E_c=0.6MeV)

Parameter

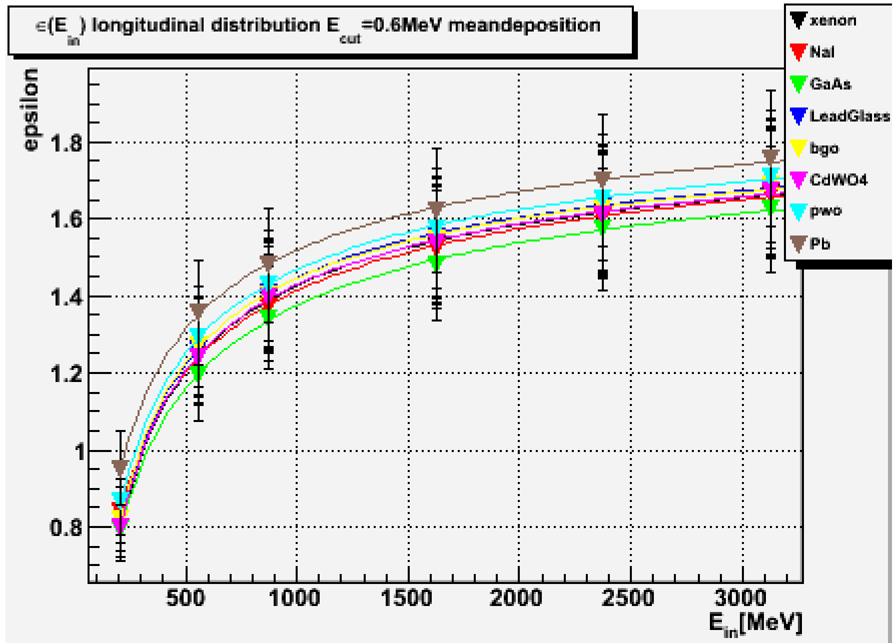
ε_t

BGO

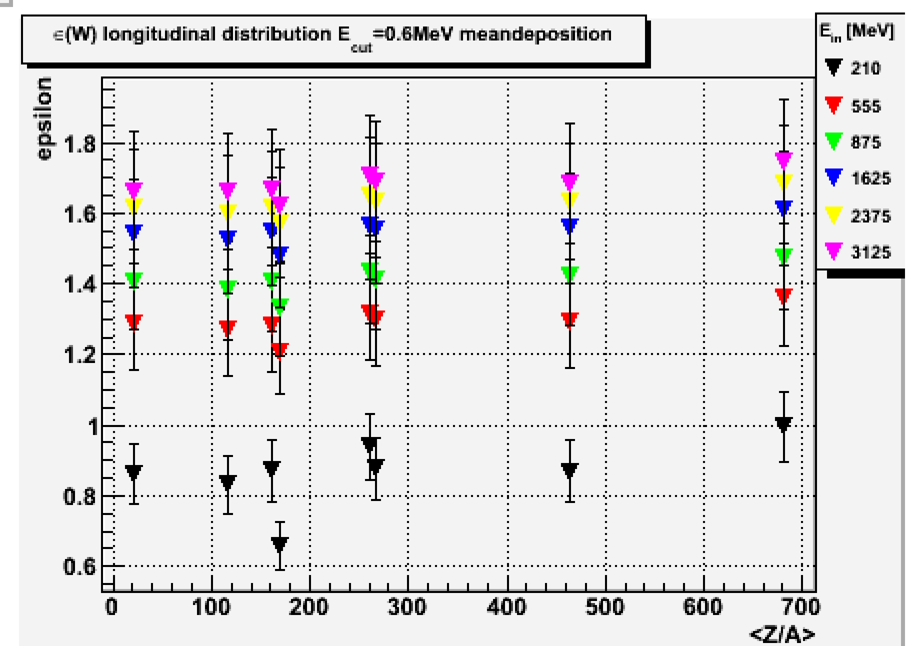
$$\varepsilon_t(E_\gamma) = a \cdot E_\gamma{}^b + c$$



Dependencies on E_γ energy for all materials (E_c=0.6MeV)



Dependencies on the material parameter W=Z/A for all energies (E_c=0.6MeV)



LONGITUDINAL PROFILES

to the approximating function:

$$(-dE/dr) = \alpha_r \exp\left(-\gamma_r\, r^{\delta_r}\right)$$

α_r, δ_r, γ_r are the fit parameters depending on cut-off energy $E_{c.o.}$, E_γ and material properties Z/A.

Dependence of parameters

γ_r and δ_r

on

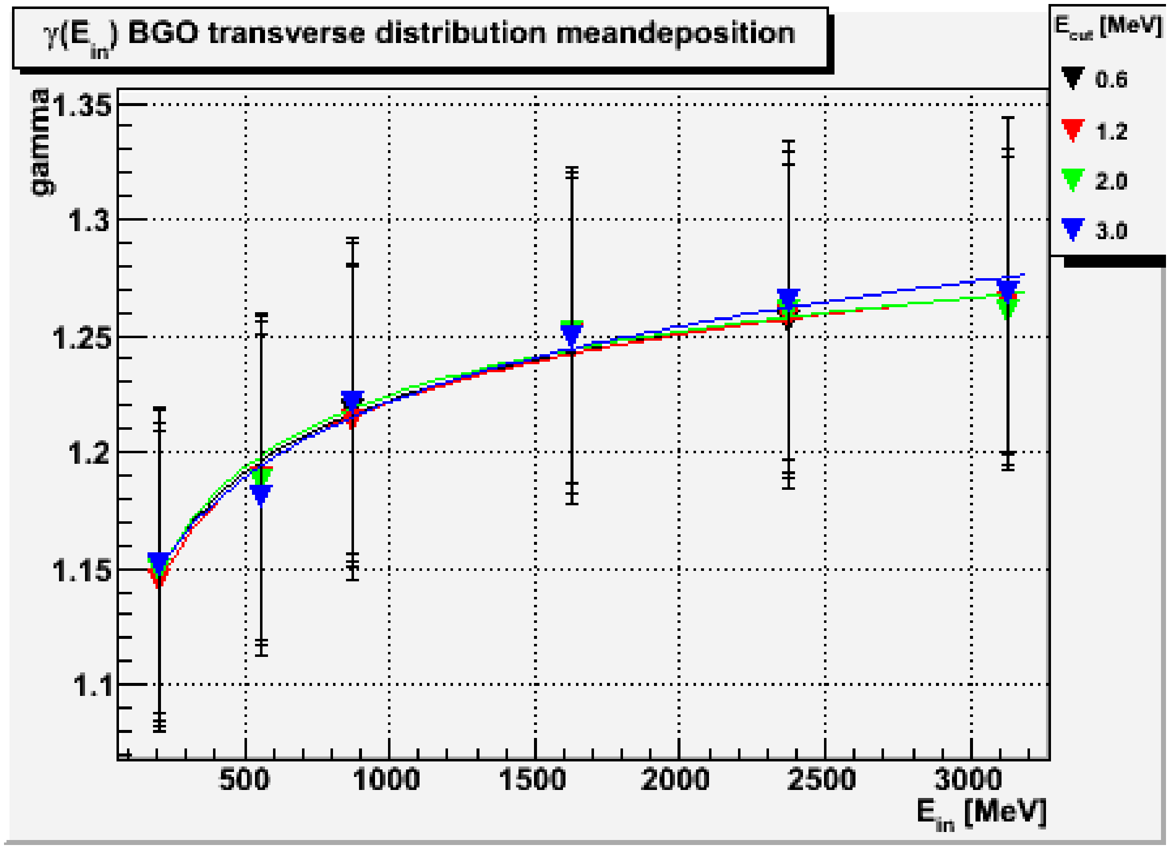
E_γ, $E_{c.o.}$, Z/A.

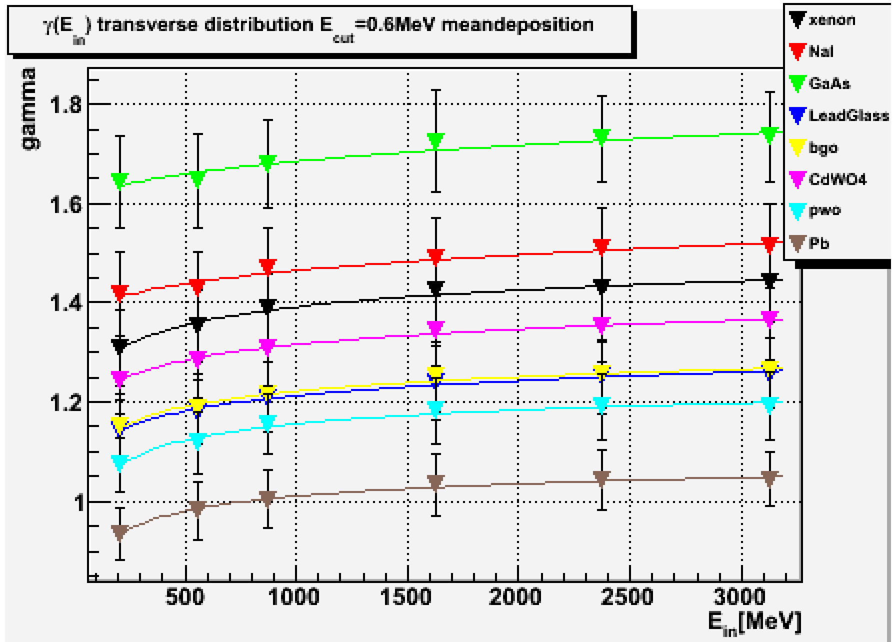
Parameter

γ_r

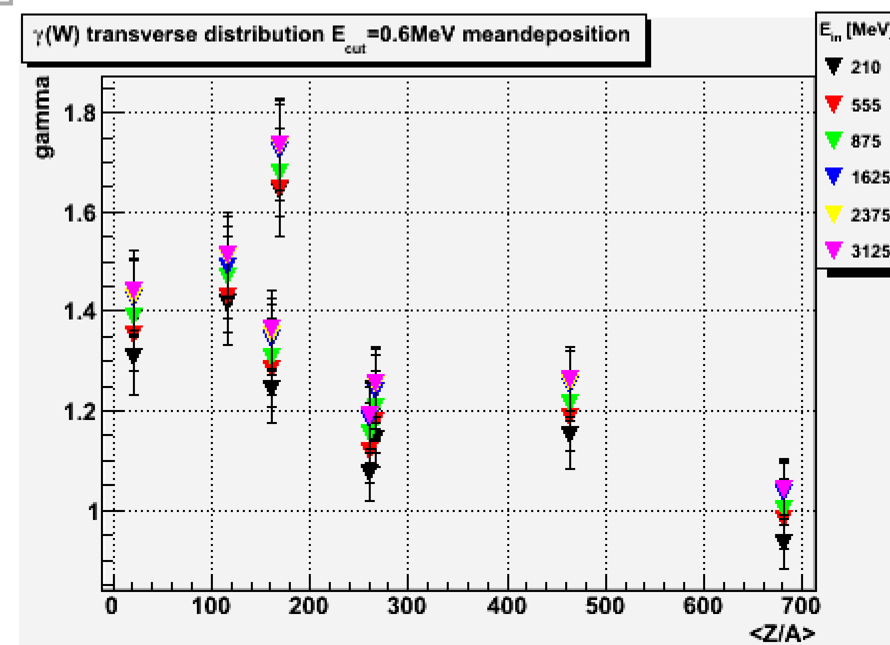
BGO

$$\gamma_r(E_\gamma) = a \cdot E_\gamma{}^b + c$$





Dependencies on E_γ energy for all materials (E_c=0.6MeV)



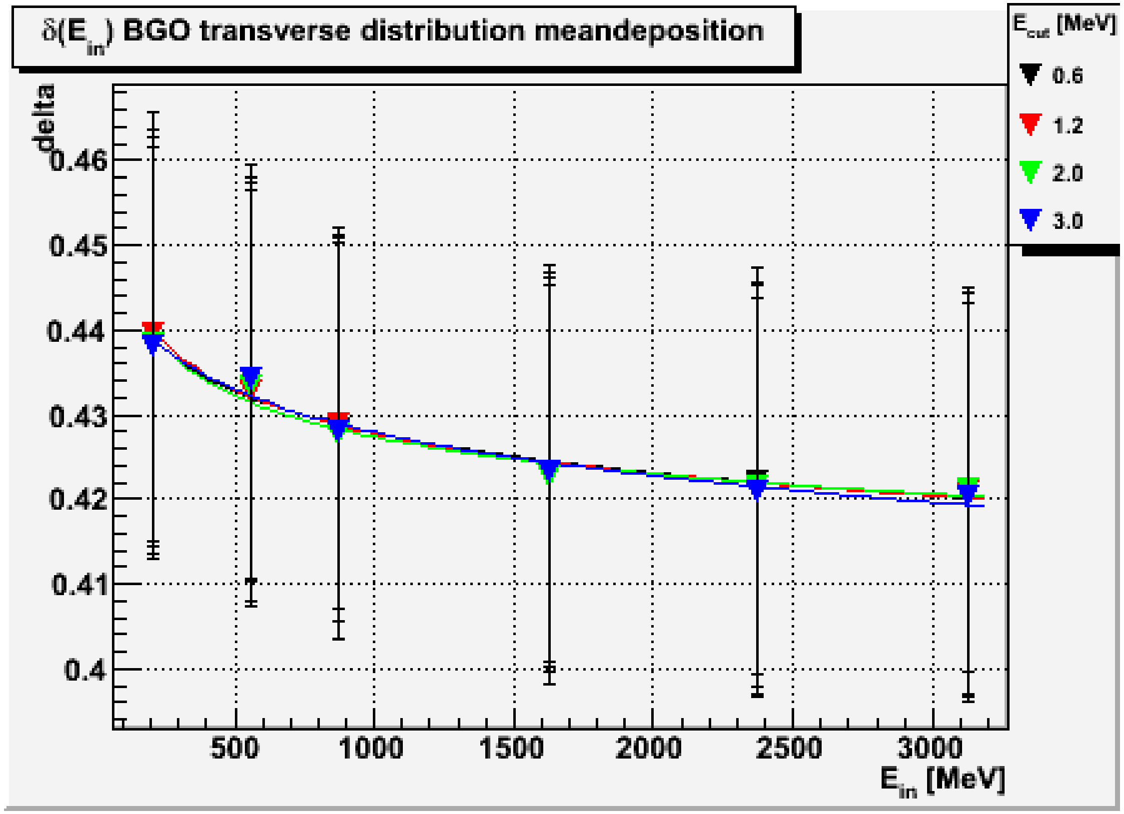
Dependencies on the material parameter W=Z/A for all energies (E_c=0.6MeV)

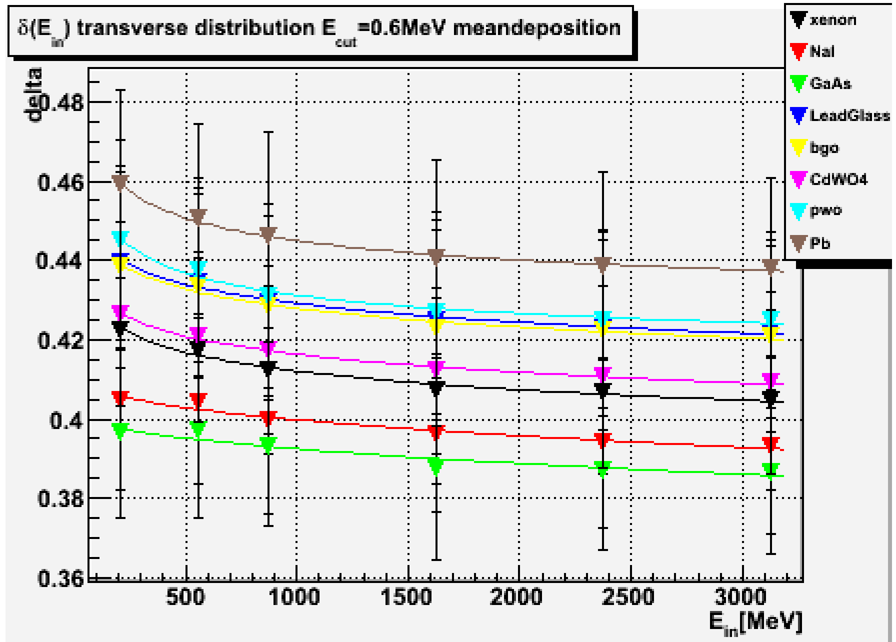
Parameter

δ_t

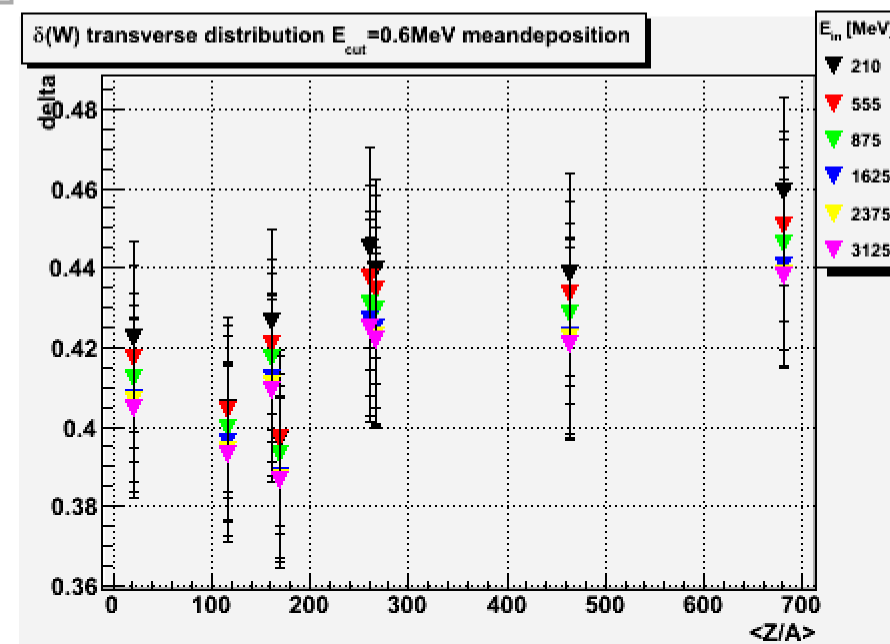
BGO

$$\delta_r(E_\gamma) = a \cdot E_\gamma^{\ b} + c$$





Dependencies on E_γ energy for all materials (E_c=0.6MeV)



Dependencies on the material parameter W=Z/A for all energies (E_c=0.6MeV)

FLUCTUATIONS

Distributions of the depth t at which a fraction A of the total EMC energy is deposited

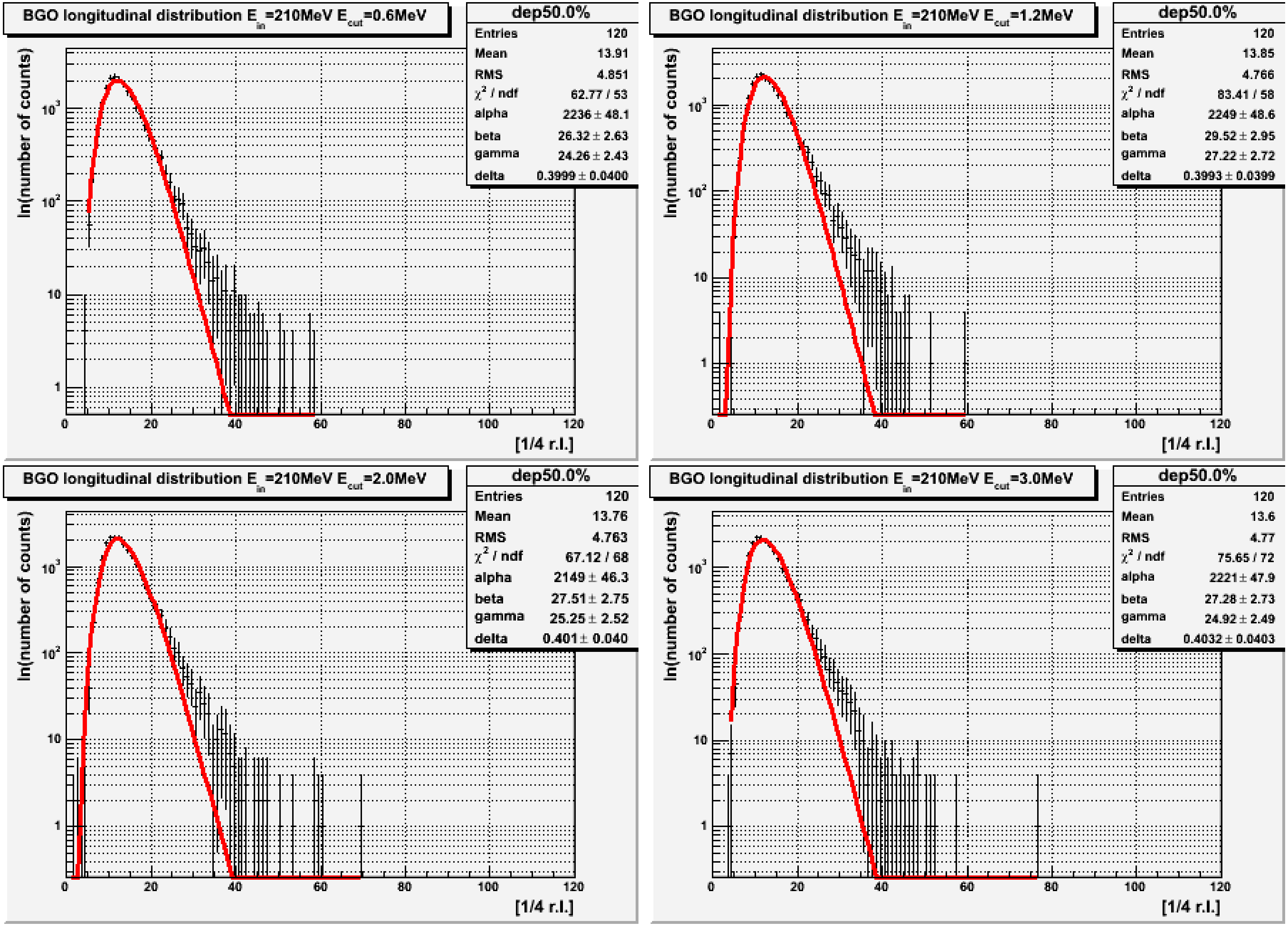
Fitting function for longitudinal and transverse fluctuation of EMC:

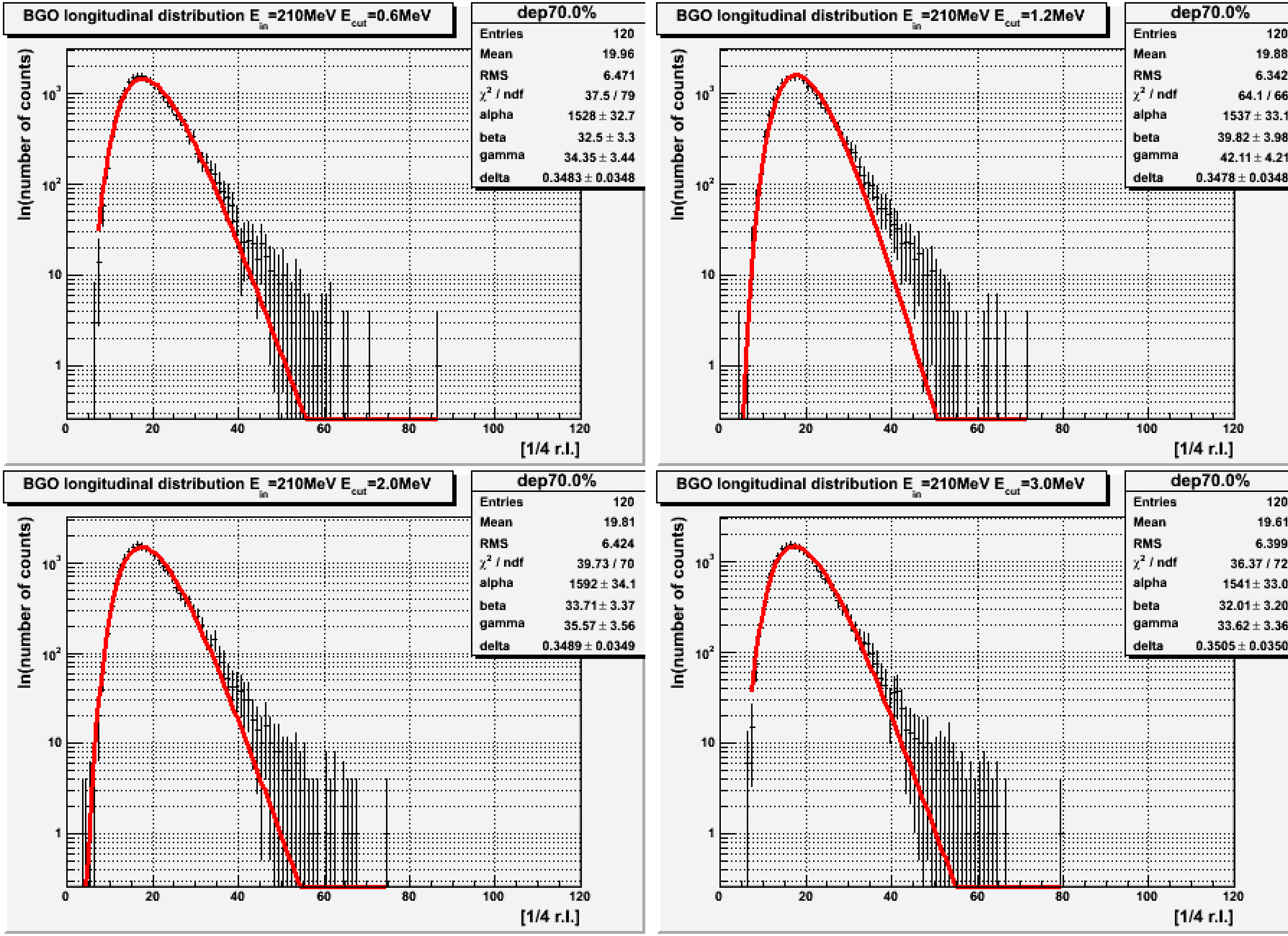
$$P(t_A) = \alpha\, t_A{}^{\beta} \exp\left(-\gamma\, t_A{}^{\delta}\right)$$

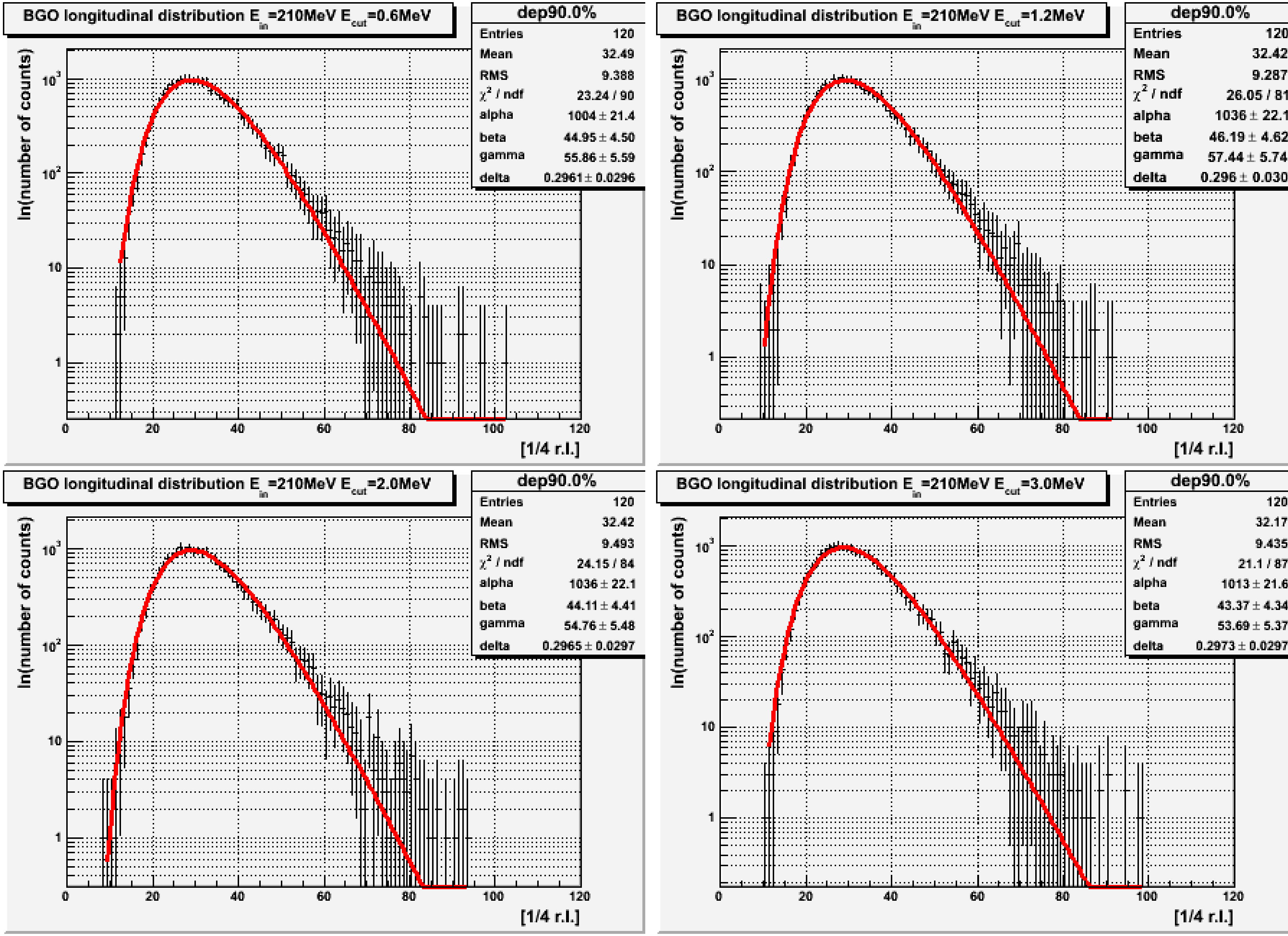
Illustration for $E_\gamma = 210$ MeV in BGO

Distributions of the shower depth t_A at which a fixed part A of average cascade energy is released when the cascade is initiated by gamma quanta of energy $E_\gamma = 1625$ MeV and detected with the cut-off energies $E_{c.o.} = 0.6$, 1.2, 2.0 and 3.0 MeV.

LONGITUDINAL


BGO longitudinal distribution E_in=210MeV E_cut=0.6MeV
dep50.0%
Entries 120
Mean 13.91
RMS 4.851
χ² / ndf 62.77 / 53
alpha 2236 ± 48.1
beta 26.32 ± 2.63
gamma 24.26 ± 2.43
delta 0.3999 ± 0.0400
ln(number of counts)
[1/4 r.l.]
BGO longitudinal distribution E_in=210MeV E_cut=1.2MeV
dep50.0%
Entries 120
Mean 13.85
RMS 4.766
χ² / ndf 83.41 / 58
alpha 2249 ± 48.6
beta 29.52 ± 2.95
gamma 27.22 ± 2.72
delta 0.3993 ± 0.0399
ln(number of counts)
[1/4 r.l.]
BGO longitudinal distribution E_in=210MeV E_cut=2.0MeV
dep50.0%
Entries 120
Mean 13.76
RMS 4.763
χ² / ndf 67.12 / 68
alpha 2149 ± 46.3
beta 27.51 ± 2.75
gamma 25.25 ± 2.52
delta 0.401 ± 0.040
ln(number of counts)
[1/4 r.l.]
BGO longitudinal distribution E_in=210MeV E_cut=3.0MeV
dep50.0%
Entries 120
Mean 13.6
RMS 4.77
χ² / ndf 75.65 / 72
alpha 2221 ± 47.9
beta 27.28 ± 2.73
gamma 24.92 ± 2.49
delta 0.4032 ± 0.0403
ln(number of counts)
[1/4 r.l.]

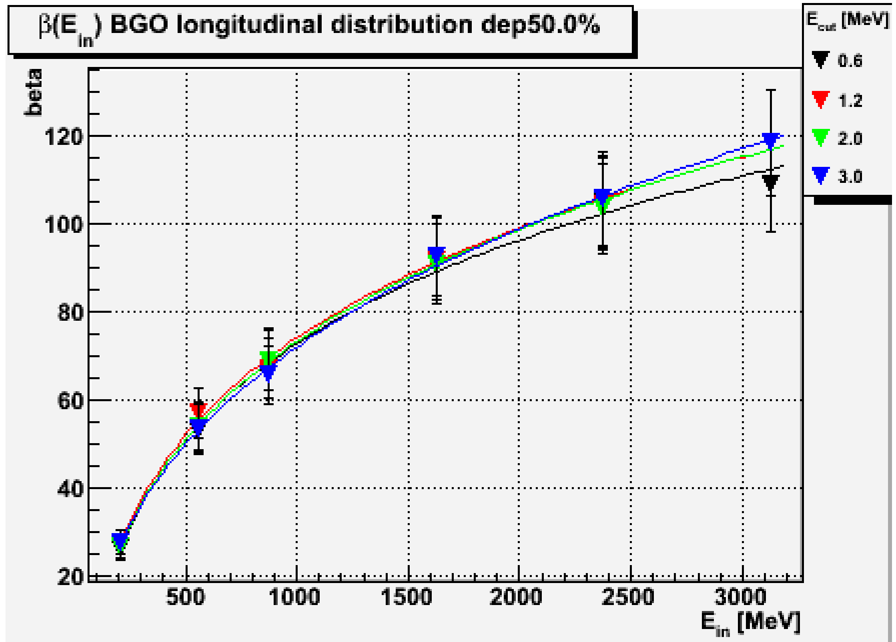

BGO longitudinal distribution E_in=210MeV E_cut=0.6MeV
dep70.0%
Entries 120
Mean 19.96
RMS 6.471
χ² / ndf 37.5 / 79
alpha 1528 ± 32.7
beta 32.5 ± 3.3
gamma 34.35 ± 3.44
delta 0.3483 ± 0.0348
ln(number of counts)
[1/4 r.l.]
BGO longitudinal distribution E_in=210MeV E_cut=1.2MeV
dep70.0%
Entries 120
Mean 19.88
RMS 6.342
χ² / ndf 64.1 / 66
alpha 1537 ± 33.1
beta 39.82 ± 3.98
gamma 42.11 ± 4.21
delta 0.3478 ± 0.0348
ln(number of counts)
[1/4 r.l.]
BGO longitudinal distribution E_in=210MeV E_cut=2.0MeV
dep70.0%
Entries 120
Mean 19.81
RMS 6.424
χ² / ndf 39.73 / 70
alpha 1592 ± 34.1
beta 33.71 ± 3.37
gamma 35.57 ± 3.56
delta 0.3489 ± 0.0349
ln(number of counts)
[1/4 r.l.]
BGO longitudinal distribution E_in=210MeV E_cut=3.0MeV
dep70.0%
Entries 120
Mean 19.61
RMS 6.399
χ² / ndf 36.37 / 72
alpha 1541 ± 33.0
beta 32.01 ± 3.20
gamma 33.62 ± 3.36
delta 0.3505 ± 0.0350
ln(number of counts)
[1/4 r.l.]

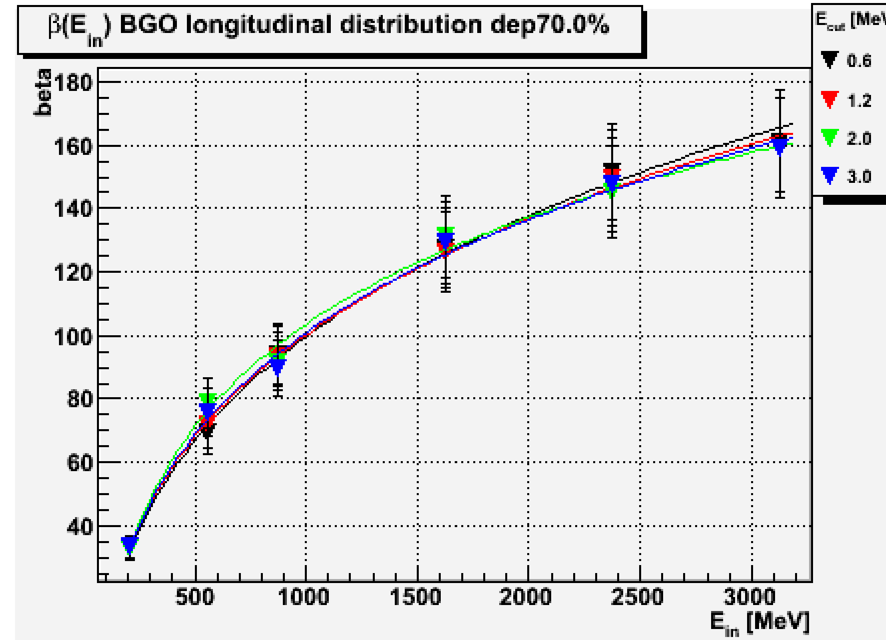


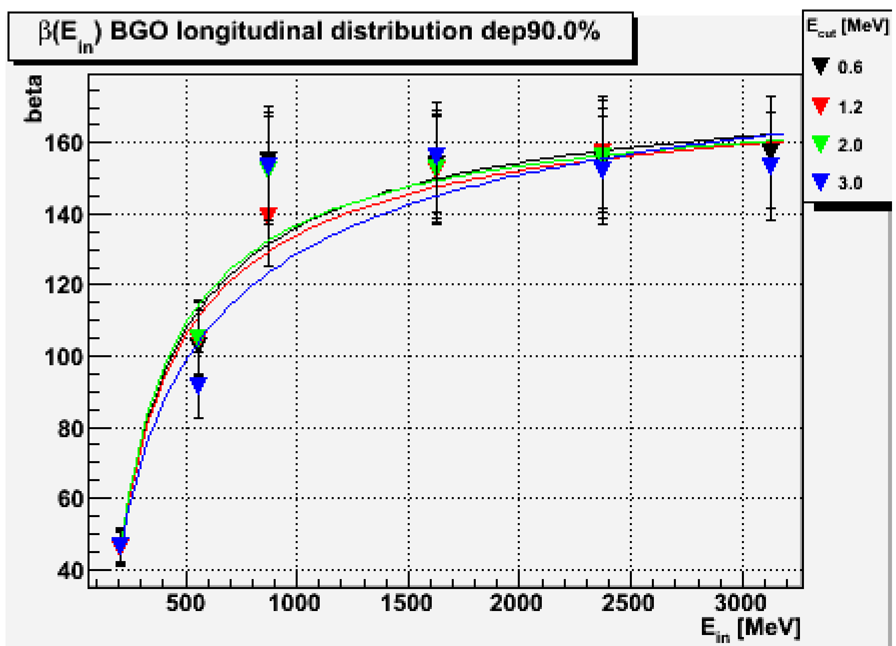
BGO longitudinal distribution E_in=210MeV E_cut=0.6MeV
dep90.0%
Entries 120
Mean 32.49
RMS 9.388
χ2 / ndf 23.24 / 90
alpha 1004 ± 21.4
beta 44.95 ± 4.50
gamma 55.86 ± 5.59
delta 0.2961 ± 0.0296
ln(number of counts)
[1/4 r.l.]
BGO longitudinal distribution E_in=210MeV E_cut=1.2MeV
dep90.0%
Entries 120
Mean 32.42
RMS 9.287
χ2 / ndf 26.05 / 81
alpha 1036 ± 22.1
beta 46.19 ± 4.62
gamma 57.44 ± 5.74
delta 0.296 ± 0.030
ln(number of counts)
[1/4 r.l.]
BGO longitudinal distribution E_in=210MeV E_cut=2.0MeV
dep90.0%
Entries 120
Mean 32.42
RMS 9.493
χ2 / ndf 24.15 / 84
alpha 1036 ± 22.1
beta 44.11 ± 4.41
gamma 54.76 ± 5.48
delta 0.2965 ± 0.0297
ln(number of counts)
[1/4 r.l.]
BGO longitudinal distribution E_in=210MeV E_cut=3.0MeV
dep90.0%
Entries 120
Mean 32.17
RMS 9.435
χ2 / ndf 21.1 / 87
alpha 1013 ± 21.6
beta 43.37 ± 4.34
gamma 53.69 ± 5.37
delta 0.2973 ± 0.0297
ln(number of counts)
[1/4 r.l.]

Dependence of fit parameters
β, γ, δ

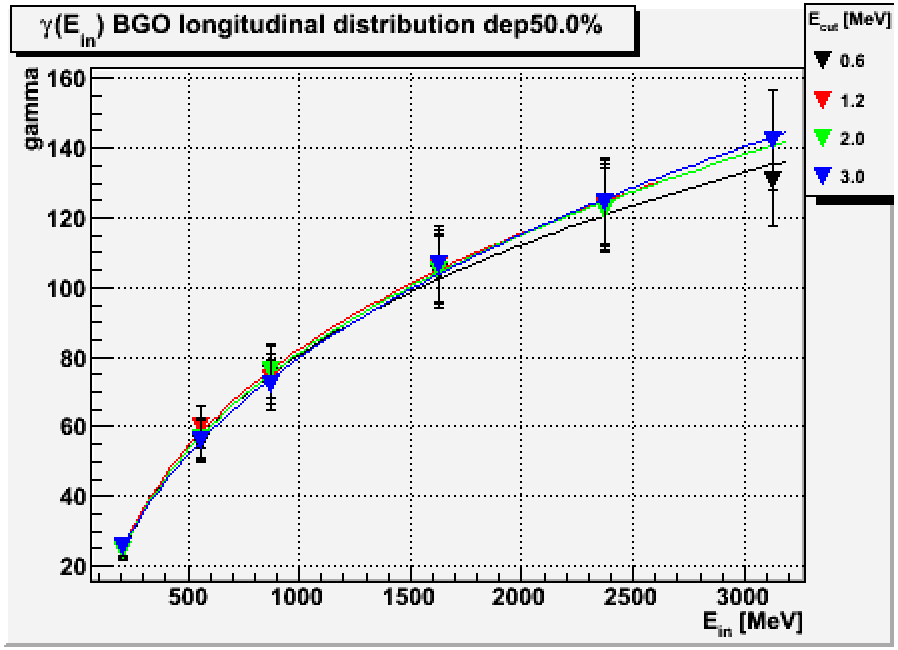
on E_γ, $E_{c.o.}$ *and Z/A*

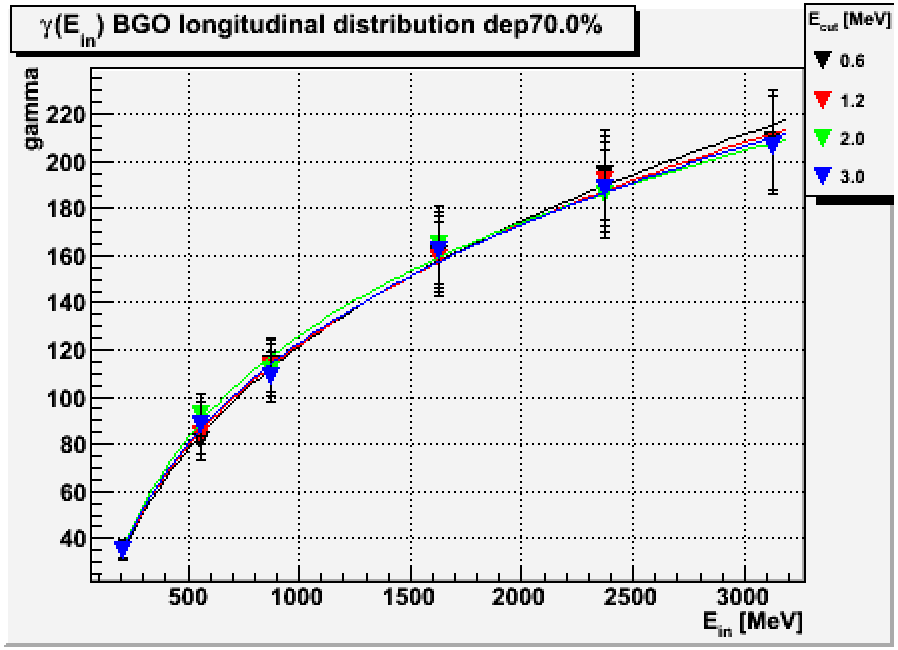


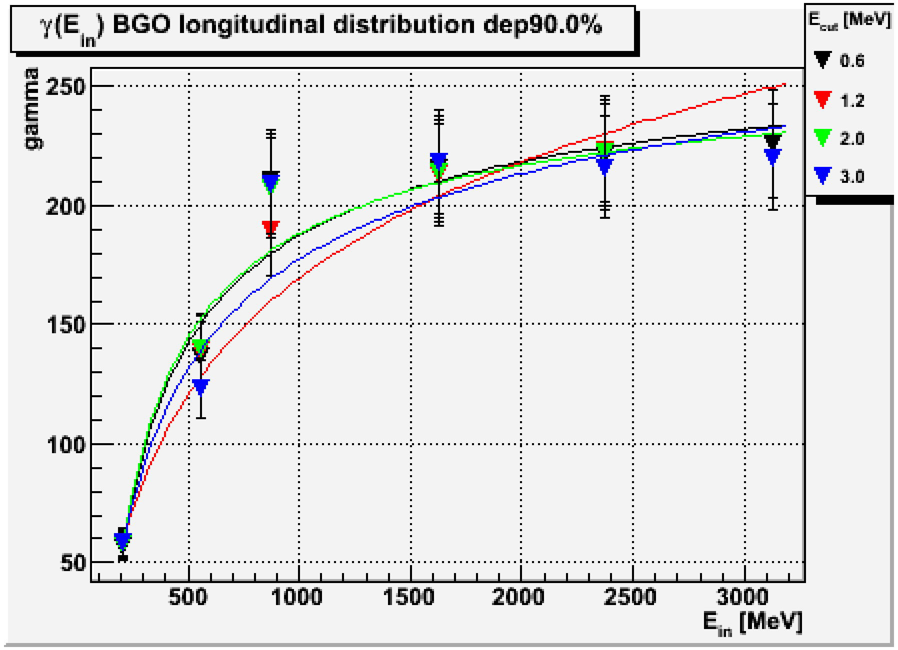




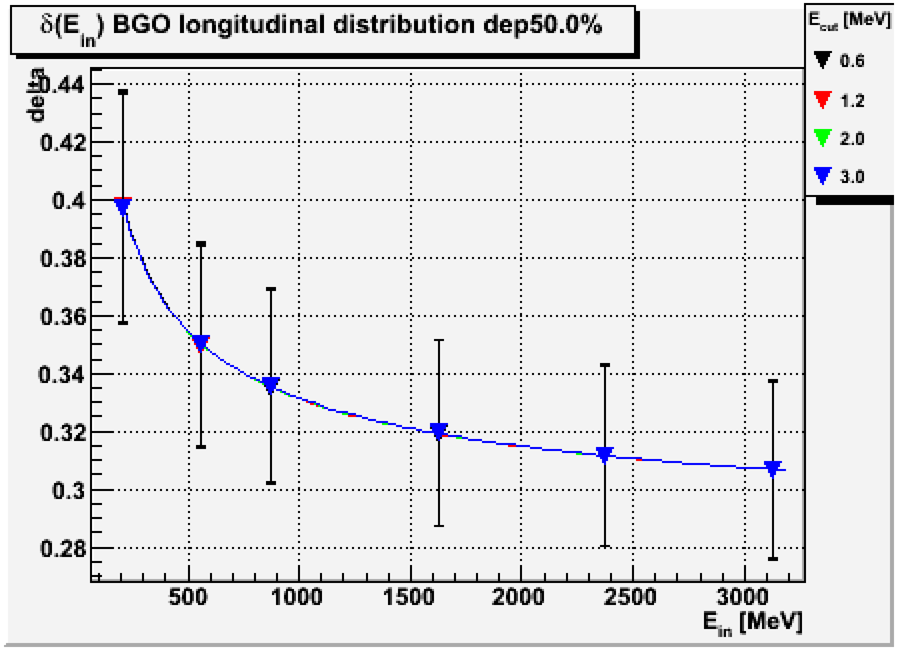
$$\beta(E_\gamma) = a \cdot E_\gamma{}^{b} + c$$

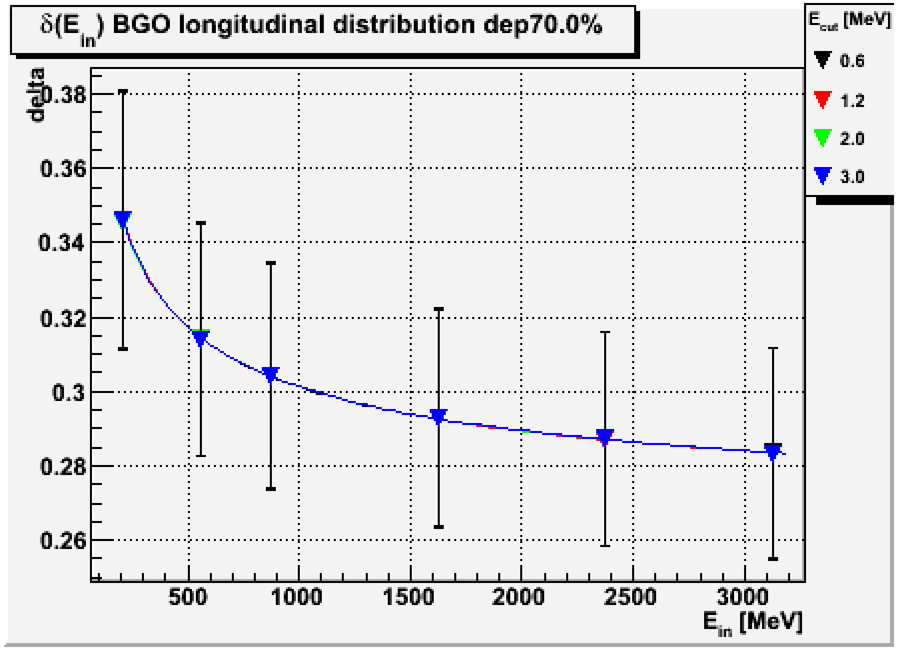


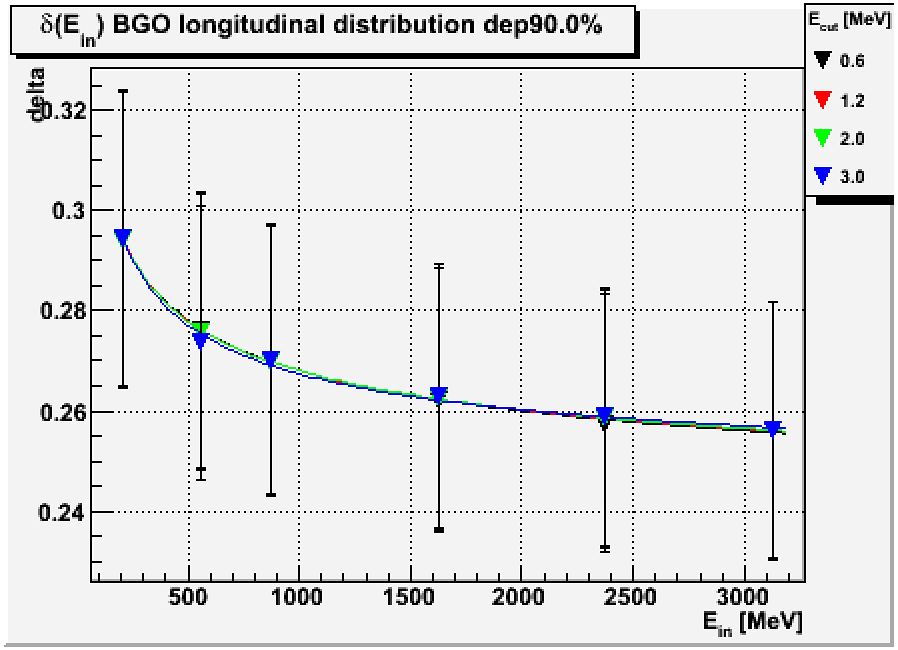




$$\gamma(E_\gamma) = a \cdot {E_\gamma}^b + c$$

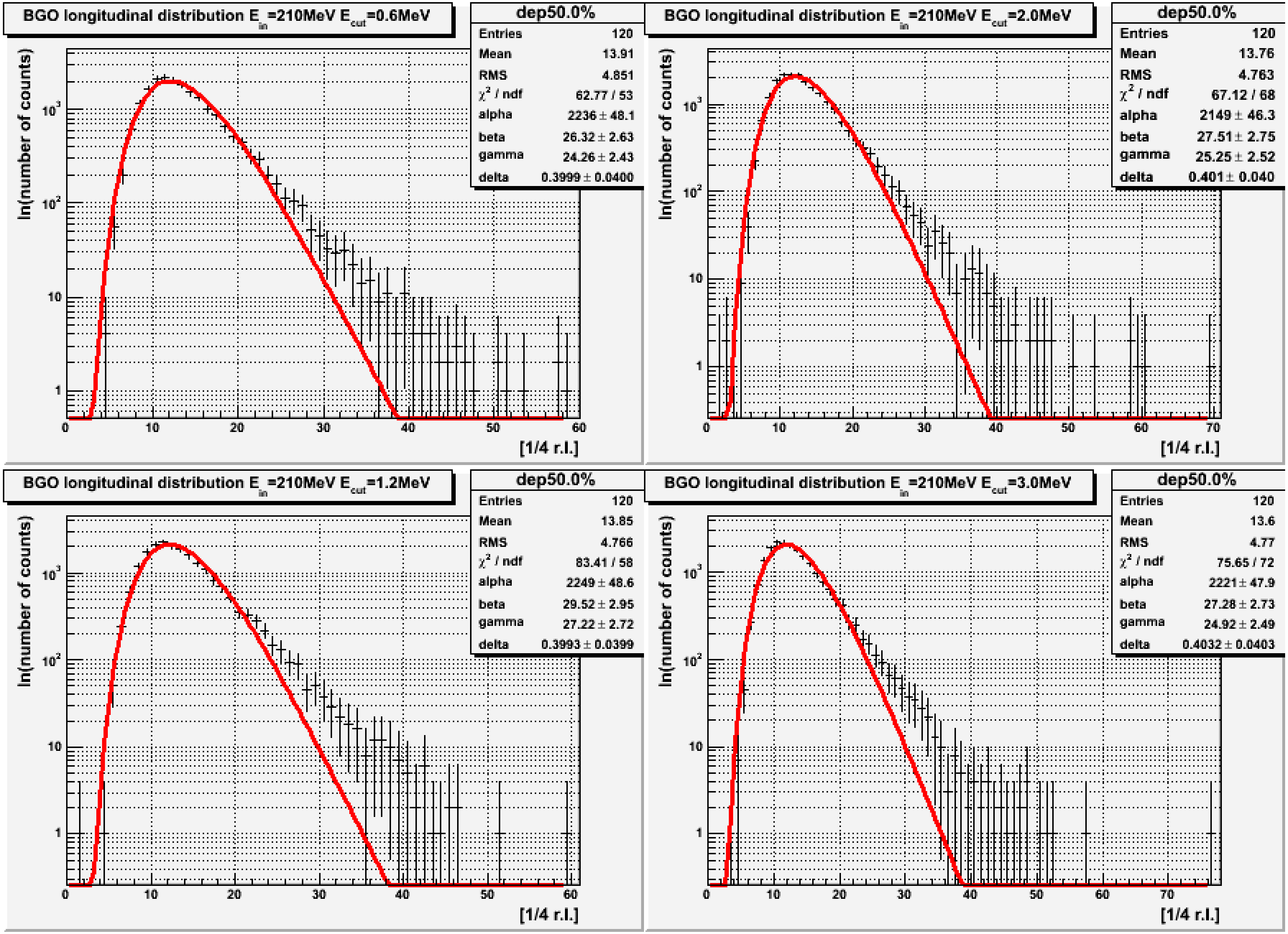






$$\delta(E_\gamma) = a \cdot E_\gamma^{\,b} + c$$

TRANSVERSE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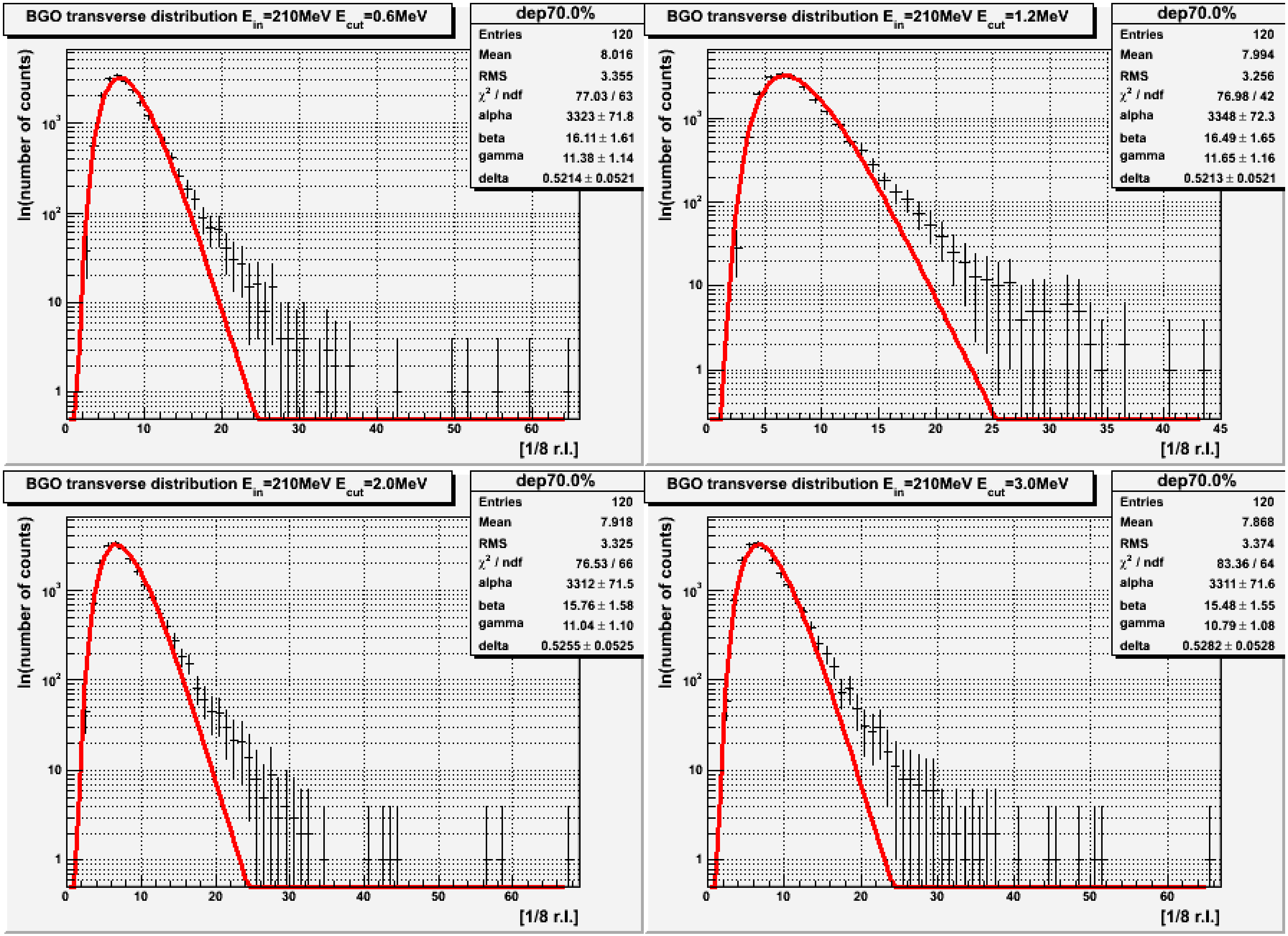
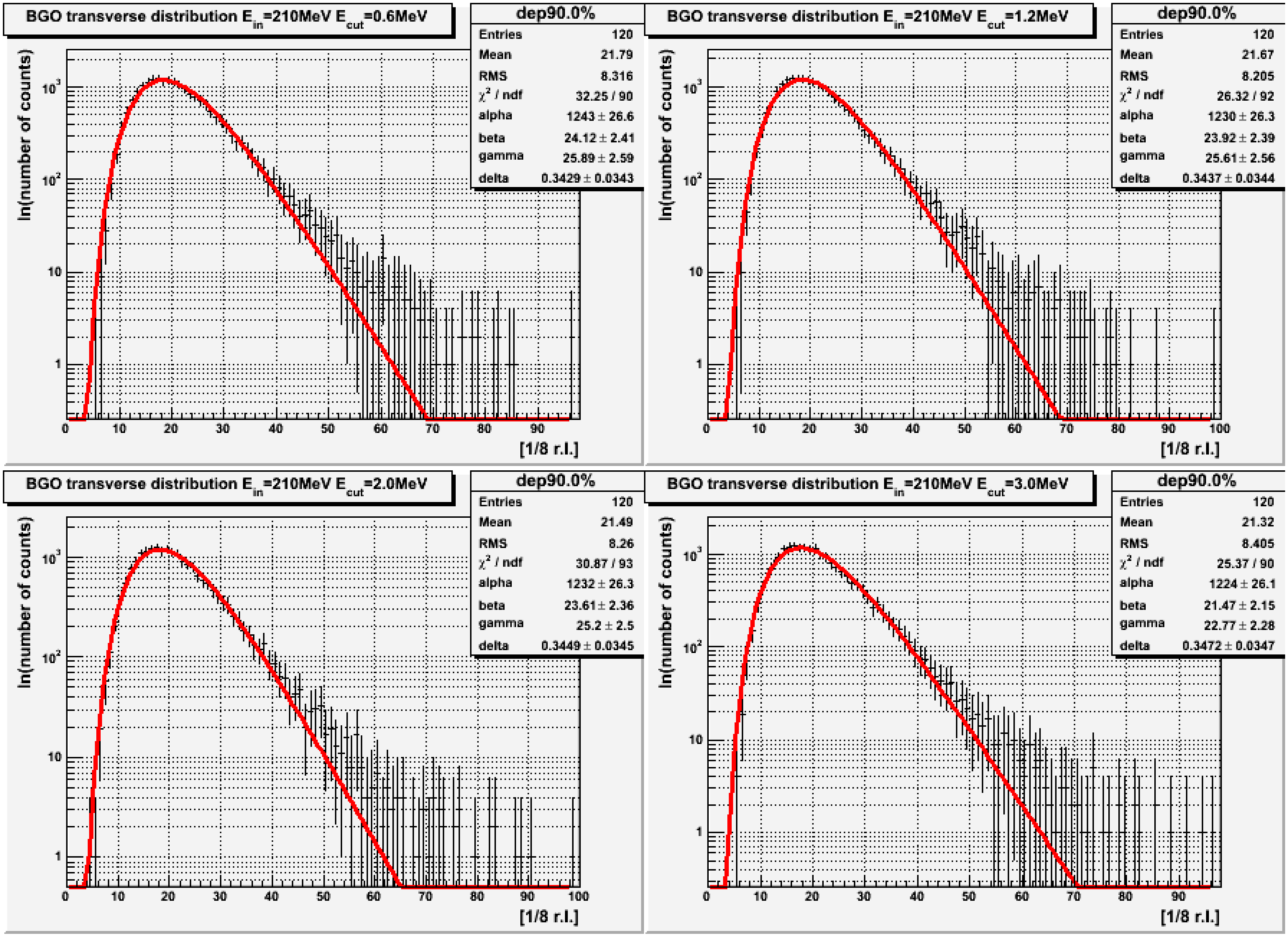


BGO transverse distribution E_in=210MeV E_cut=0.6MeV
dep70.0%
Entries 120
Mean 8.016
RMS 3.355
χ² / ndf 77.03 / 63
alpha 3323 ± 71.8
beta 16.11 ± 1.61
gamma 11.38 ± 1.14
delta 0.5214 ± 0.0521
ln(number of counts)
[1/8 r.l.]
BGO transverse distribution E_in=210MeV E_cut=1.2MeV
dep70.0%
Entries 120
Mean 7.994
RMS 3.256
χ² / ndf 76.98 / 42
alpha 3348 ± 72.3
beta 16.49 ± 1.65
gamma 11.65 ± 1.16
delta 0.5213 ± 0.0521
ln(number of counts)
[1/8 r.l.]
BGO transverse distribution E_in=210MeV E_cut=2.0MeV
dep70.0%
Entries 120
Mean 7.918
RMS 3.325
χ² / ndf 76.53 / 66
alpha 3312 ± 71.5
beta 15.76 ± 1.58
gamma 11.04 ± 1.10
delta 0.5255 ± 0.0525
ln(number of counts)
[1/8 r.l.]
BGO transverse distribution E_in=210MeV E_cut=3.0MeV
dep70.0%
Entries 120
Mean 7.868
RMS 3.374
χ² / ndf 83.36 / 64
alpha 3311 ± 71.6
beta 15.48 ± 1.55
gamma 10.79 ± 1.08
delta 0.5282 ± 0.0528
ln(number of counts)
[1/8 r.l.]

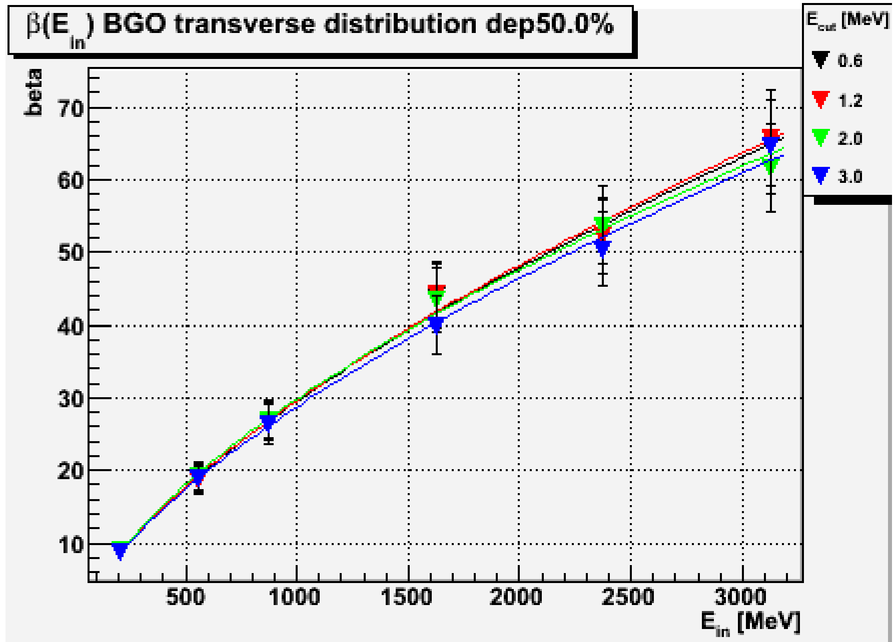


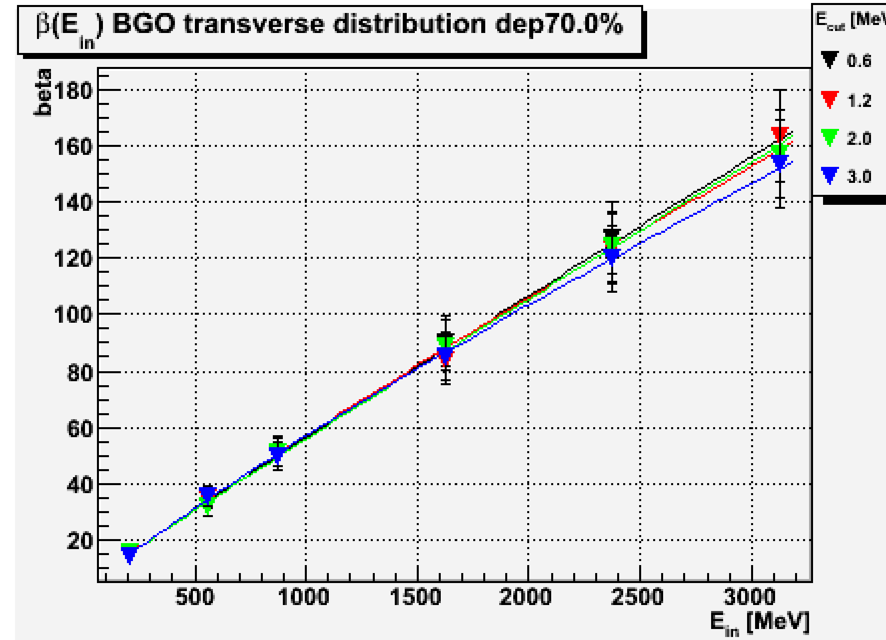
BGO transverse distribution E_in=210MeV E_cut=0.6MeV
dep90.0%
Entries 120
Mean 21.79
RMS 8.316
χ² / ndf 32.25 / 90
alpha 1243 ± 26.6
beta 24.12 ± 2.41
gamma 25.89 ± 2.59
delta 0.3429 ± 0.0343
ln(number of counts)
[1/8 r.l.]
BGO transverse distribution E_in=210MeV E_cut=1.2MeV
dep90.0%
Entries 120
Mean 21.67
RMS 8.205
χ² / ndf 26.32 / 92
alpha 1230 ± 26.3
beta 23.92 ± 2.39
gamma 25.61 ± 2.56
delta 0.3437 ± 0.0344
ln(number of counts)
[1/8 r.l.]
BGO transverse distribution E_in=210MeV E_cut=2.0MeV
dep90.0%
Entries 120
Mean 21.49
RMS 8.26
χ² / ndf 30.87 / 93
alpha 1232 ± 26.3
beta 23.61 ± 2.36
gamma 25.2 ± 2.5
delta 0.3449 ± 0.0345
ln(number of counts)
[1/8 r.l.]
BGO transverse distribution E_in=210MeV E_cut=3.0MeV
dep90.0%
Entries 120
Mean 21.32
RMS 8.405
χ² / ndf 25.37 / 90
alpha 1224 ± 26.1
beta 21.47 ± 2.15
gamma 22.77 ± 2.28
delta 0.3472 ± 0.0347
ln(number of counts)
[1/8 r.l.]

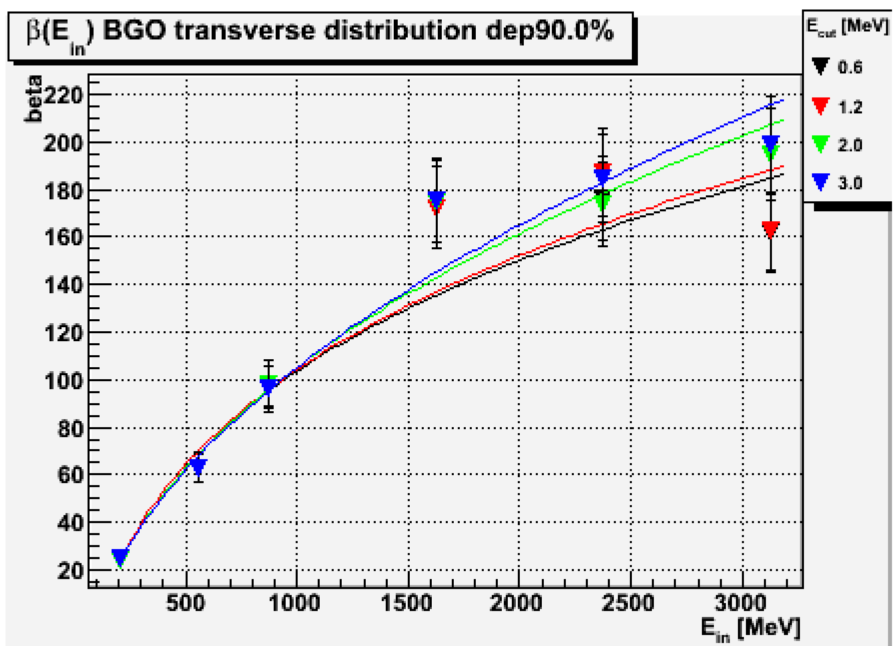
Dependence of the fit parameters

β, γ, δ

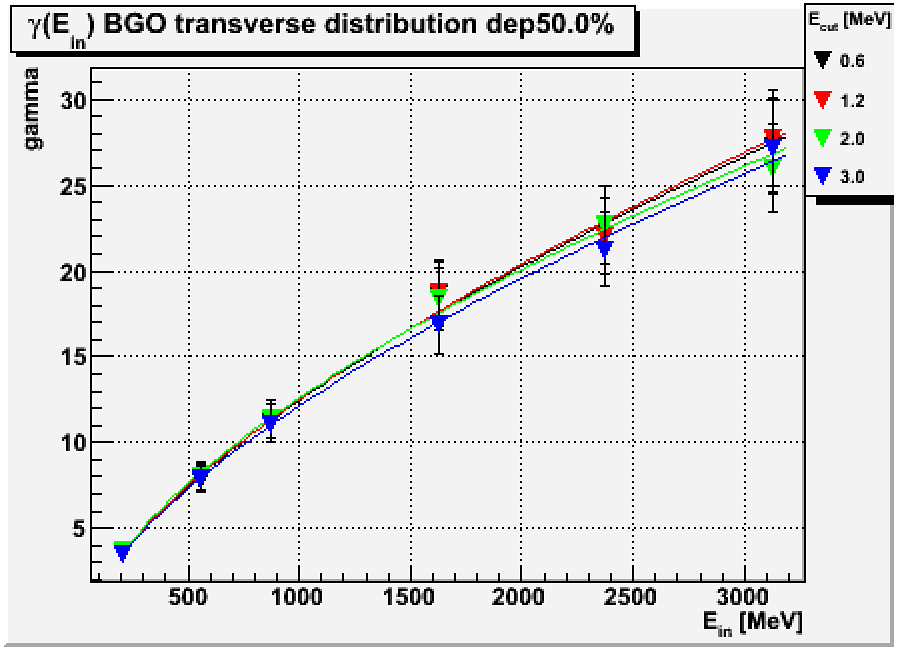
on E_γ, $E_{c.o.}$ and Z/A

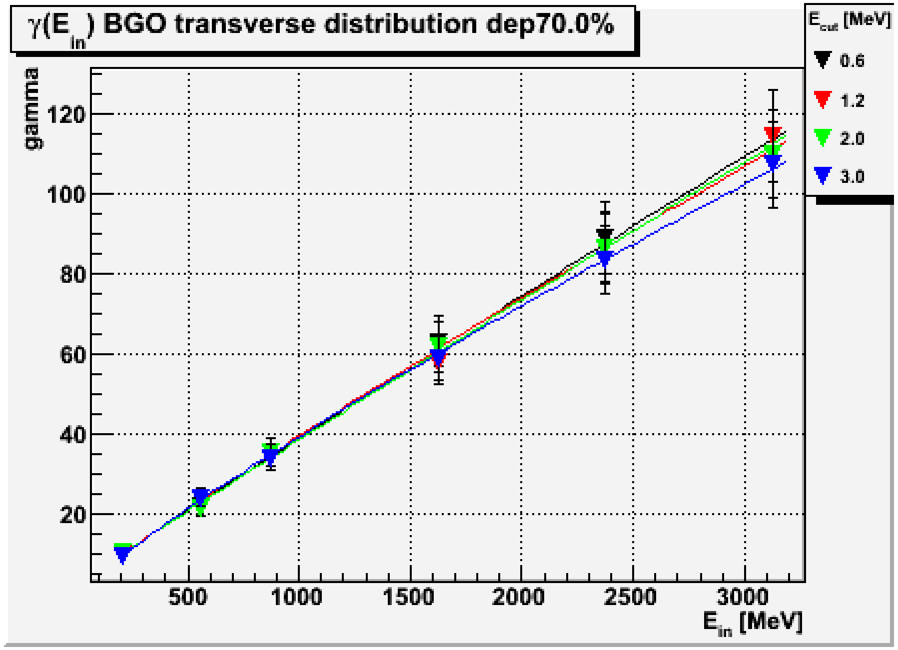


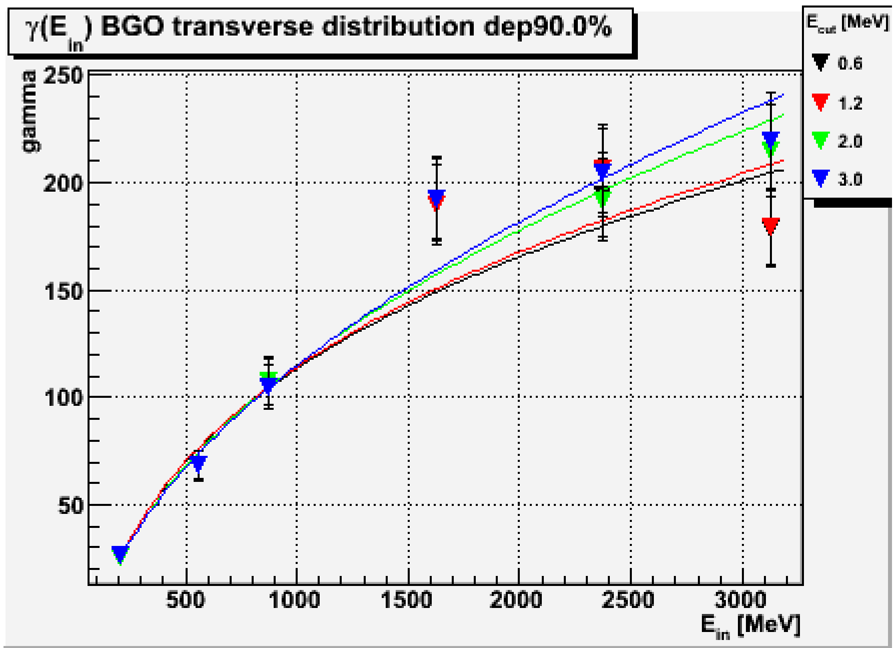




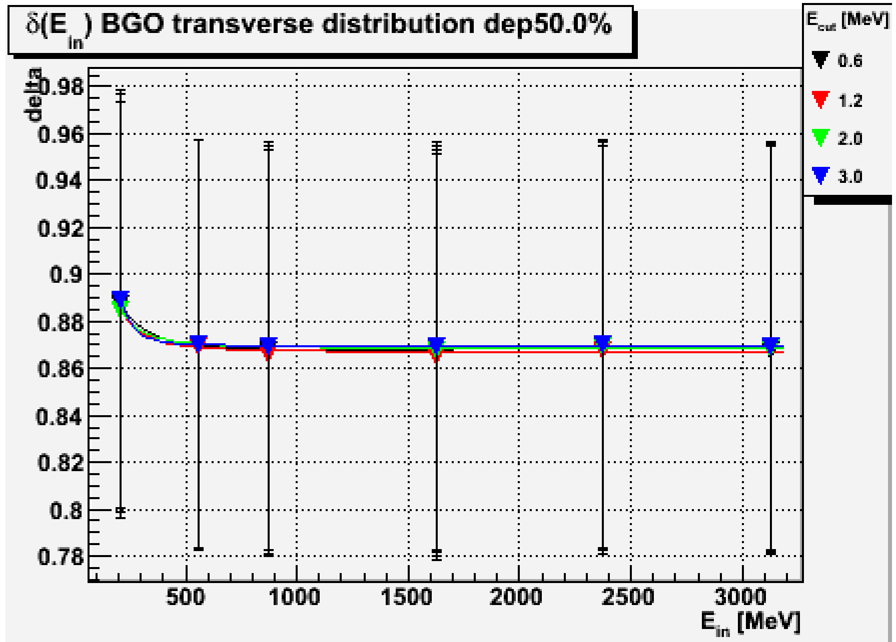
$$\beta(E_\gamma) = a \cdot E_\gamma^{\ b} + c$$

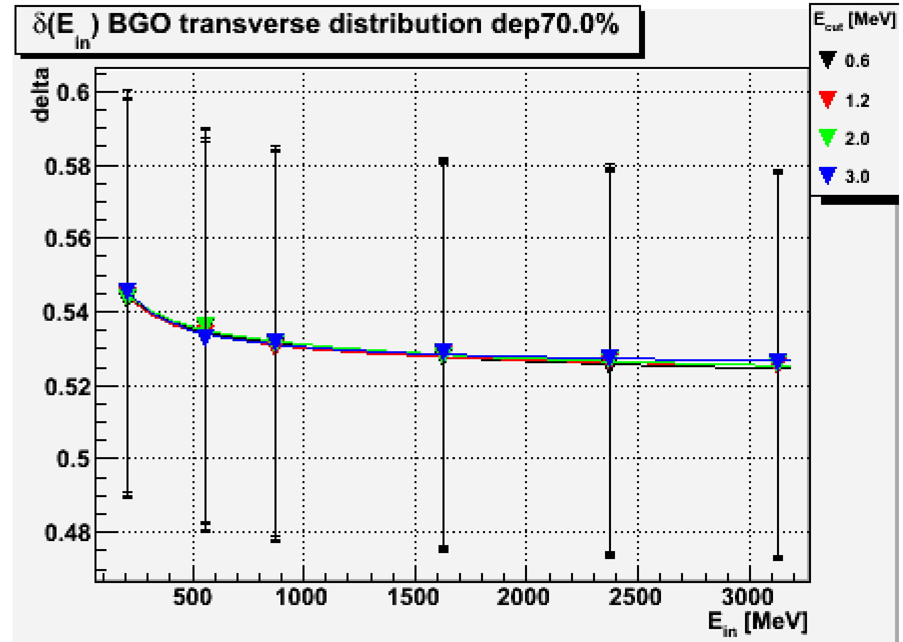


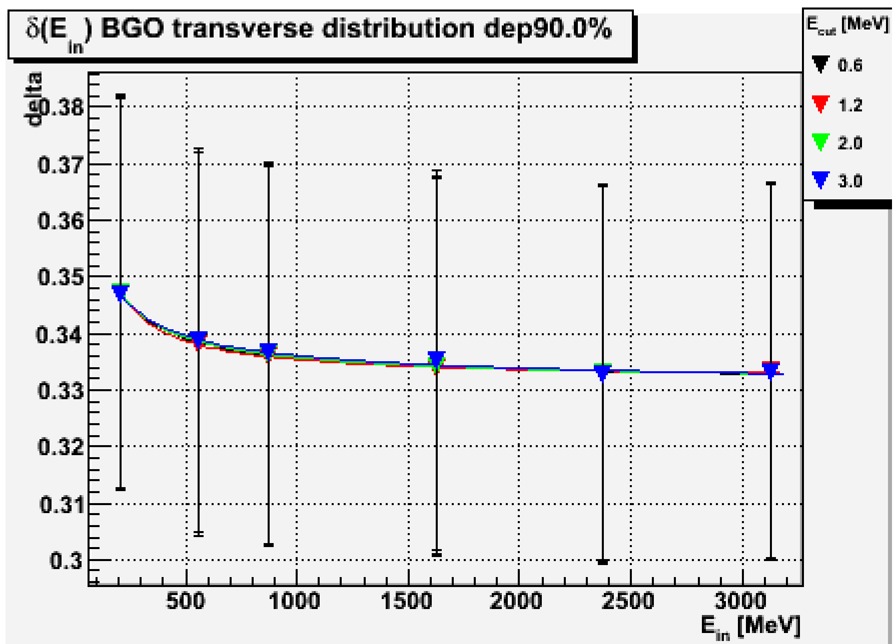




$$\gamma(E_\gamma) = a \cdot E_\gamma{}^b + c$$





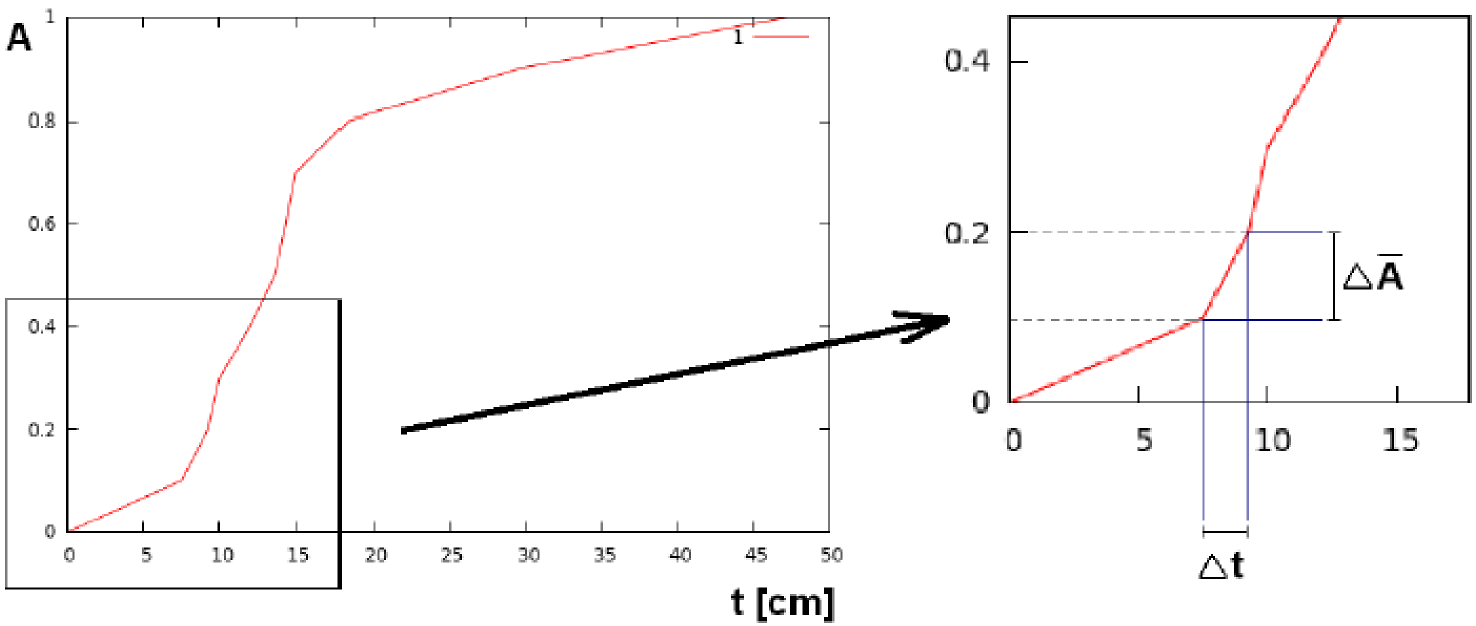


$$\delta(E_\gamma) = a \cdot E_\gamma^{\;b} + c$$

CORRELATIONS OF LONGITUDINAL ENERGY DEPOSITION

(in liquid xenon)

Random Variable



$$X_i = \frac{\Delta \bar{A}}{\Delta t_i}$$

i = 2,3 … 10

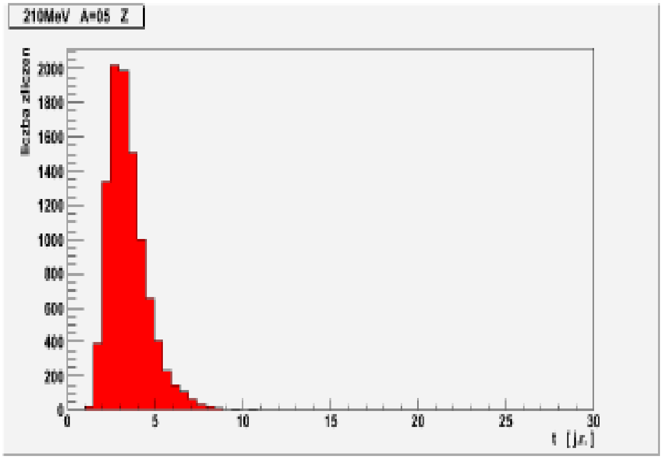
ΔA – part of the energy absorbed in the layer
$\Delta t_i = t_i - t_{i-1}$ – layers' thickness for ΔA=0.1

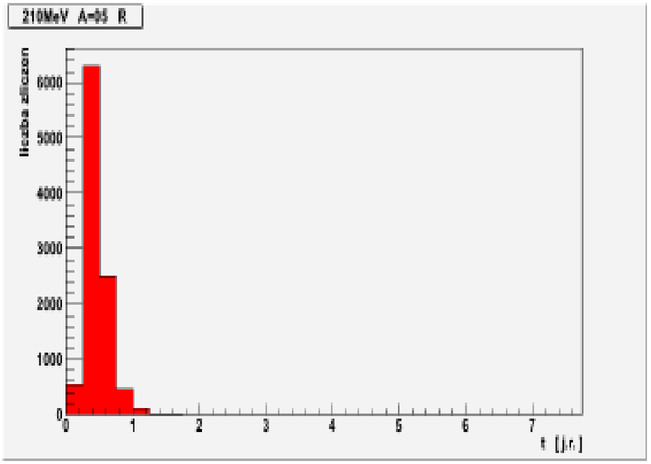
Correlations

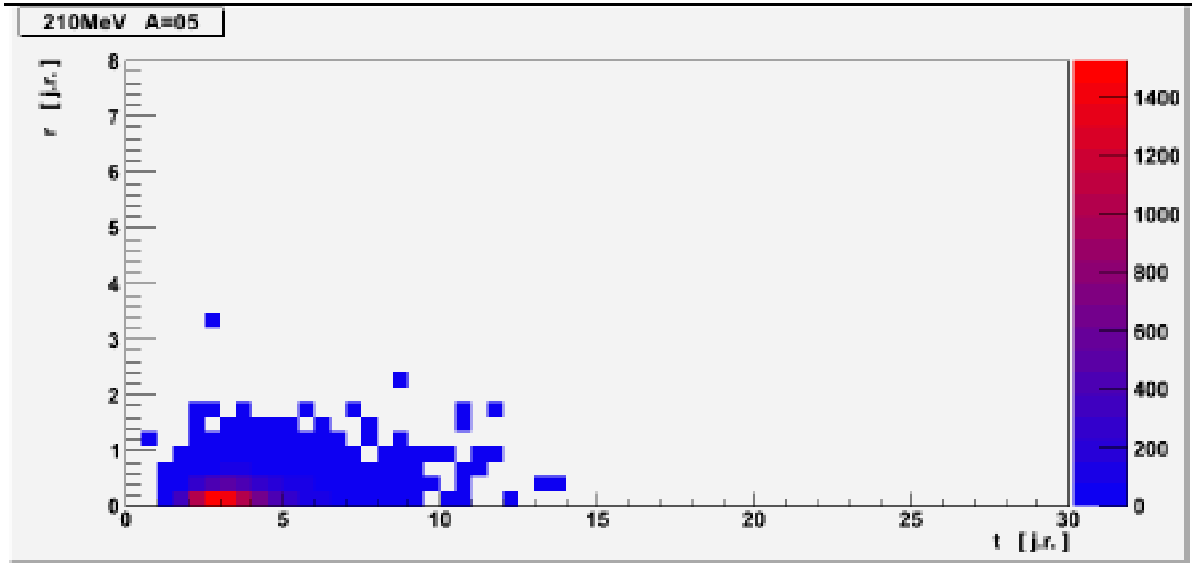
$$r_{x_i x_j} = \frac{\operatorname{cov}(X_i, X_j)}{\sigma(X_i)\sigma(X_j)}$$

$\operatorname{cov}(X_i,X_j)$ – covariance
$\sigma(X_i)$, $\sigma(X_j)$ - standard deviations

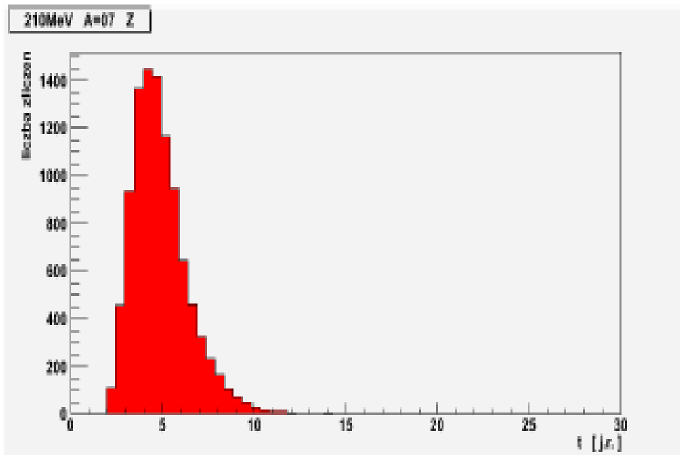
Correlations (A=50%, E_γ=210MeV, E_c=1MeV)

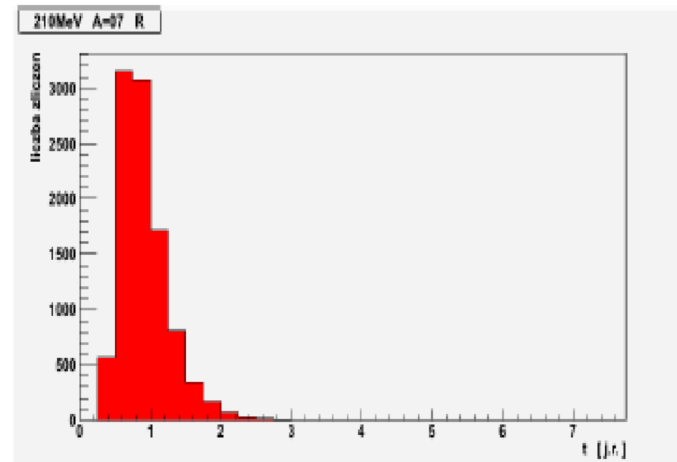


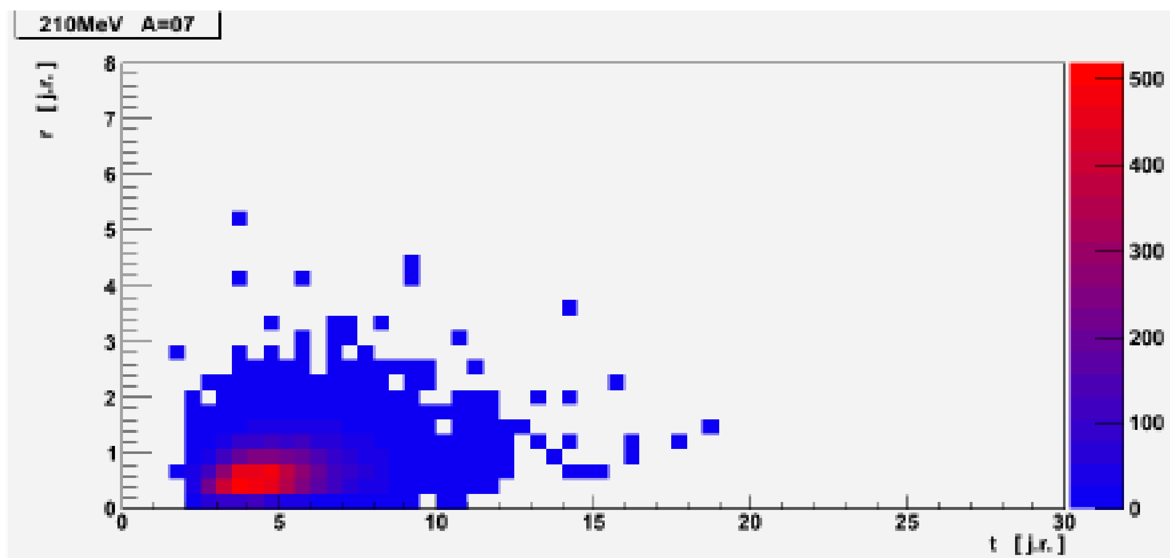




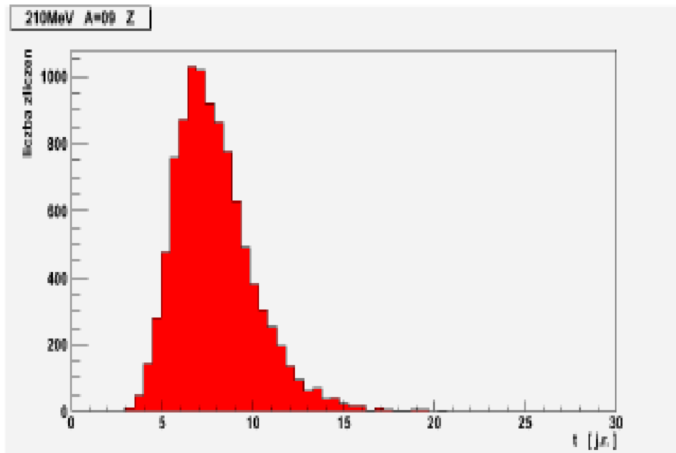
Correlations (A=70%, E_γ=210MeV, E_c=1MeV)

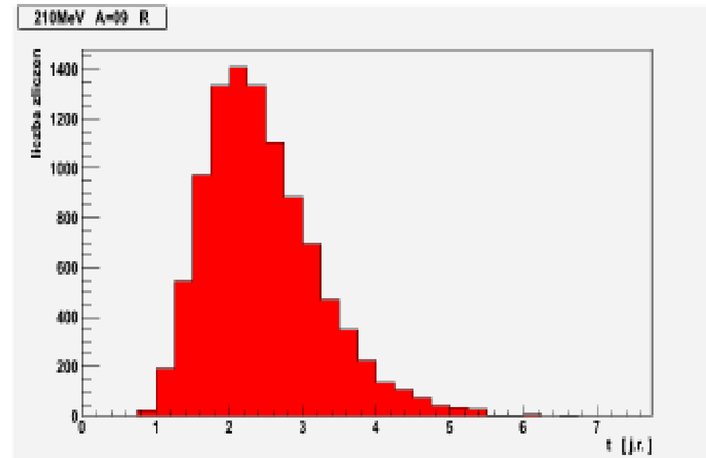


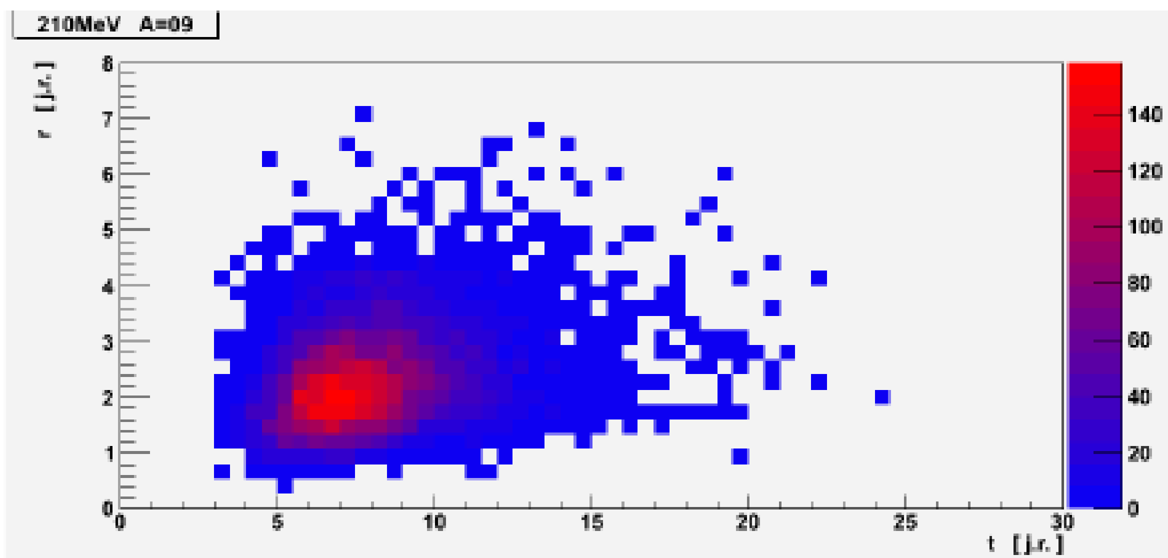




Correlations (A=90%, E_{γ}=210MeV, E_c=1MeV)



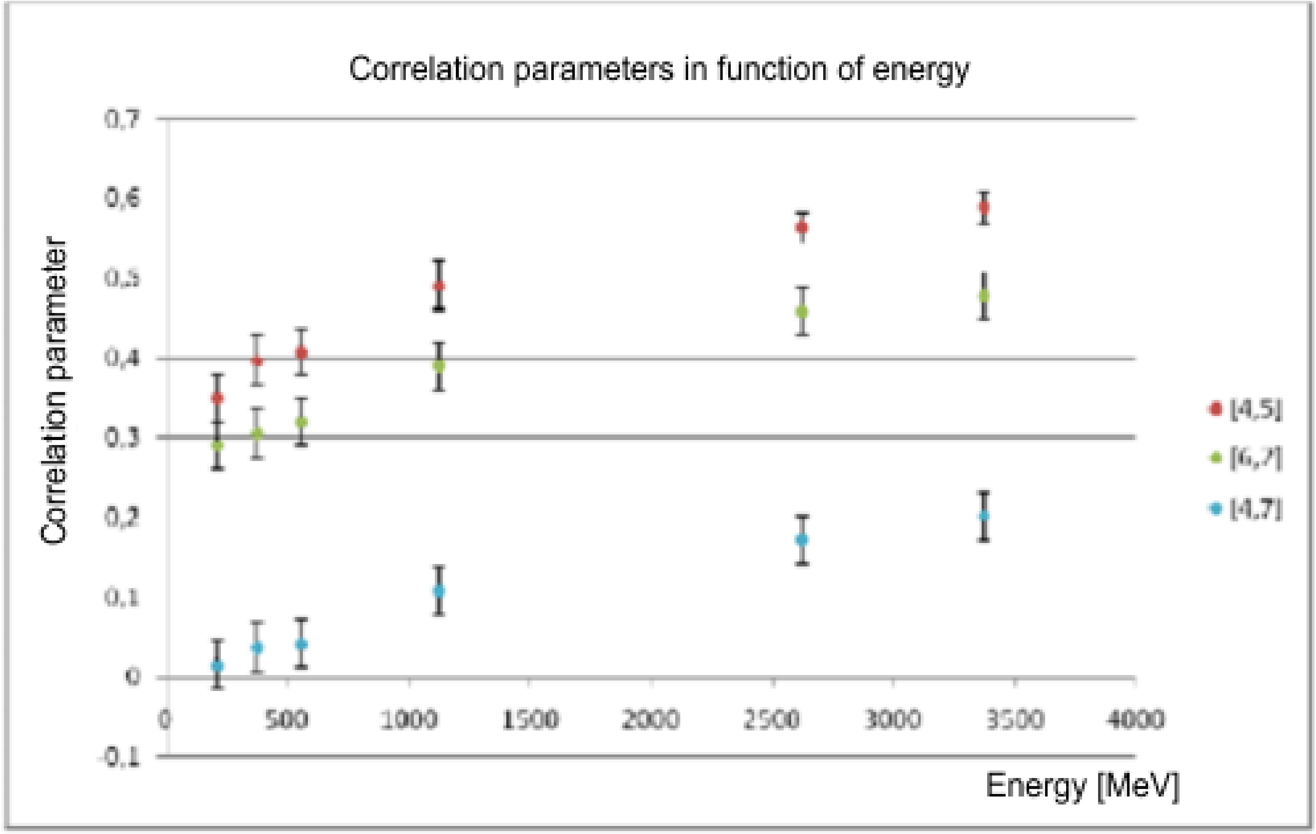




Correlation coefficients r_{ij}

i/j	3	4	5	6	7	8	9	10
2	0,29 ± 0,03	0,10 ± 0,03	0,04 ± 0,03	0,02 ± 0,03	0,01 ± 0,03	0,00 ± 0,03	-0,02 ± 0,03	0,01 ± 0,03
3		0,32 ± 0,03	0,13 ± 0,03	0,06 ± 0,03	0,02 ± 0,03	-0,01 ± 0,03	-0,02 ± 0,03	-0,01 ± 0,03
4			0,35 ± 0,03	0,10 ± 0,03	0,02 ± 0,03	-0,02 ± 0,03	-0,02 ± 0,03	-0,02 ± 0,03
5				0,33 ± 0,03	0,07 ± 0,03	-0,02 ± 0,03	-0,04 ± 0,03	-0,02 ± 0,03
6					0,29 ± 0,03	0,02 ± 0,03	-0,06 ± 0,03	-0,03 ± 0,03
7						0,20 ± 0,03	-0,07 ± 0,03	-0,05 ± 0,03
8							0,06 ± 0,03	-0,06 ± 0,03
9								-0,08 ± 0,03

i,j – numbers of layers

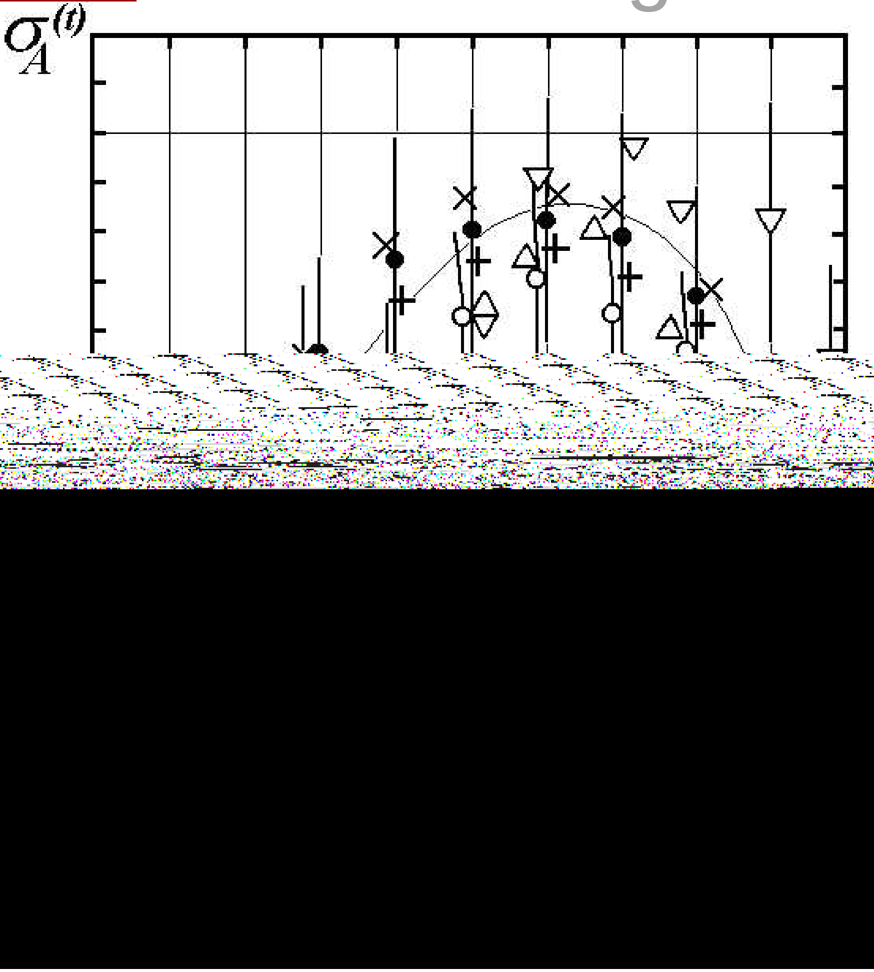


Correlation parameters in function of energy
Correlation parameter
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
-0,1
0
500
1000
1500
2000
2500
3000
3500
4000
Energy [MeV]
[4,5]
[6,7]
[4,7]

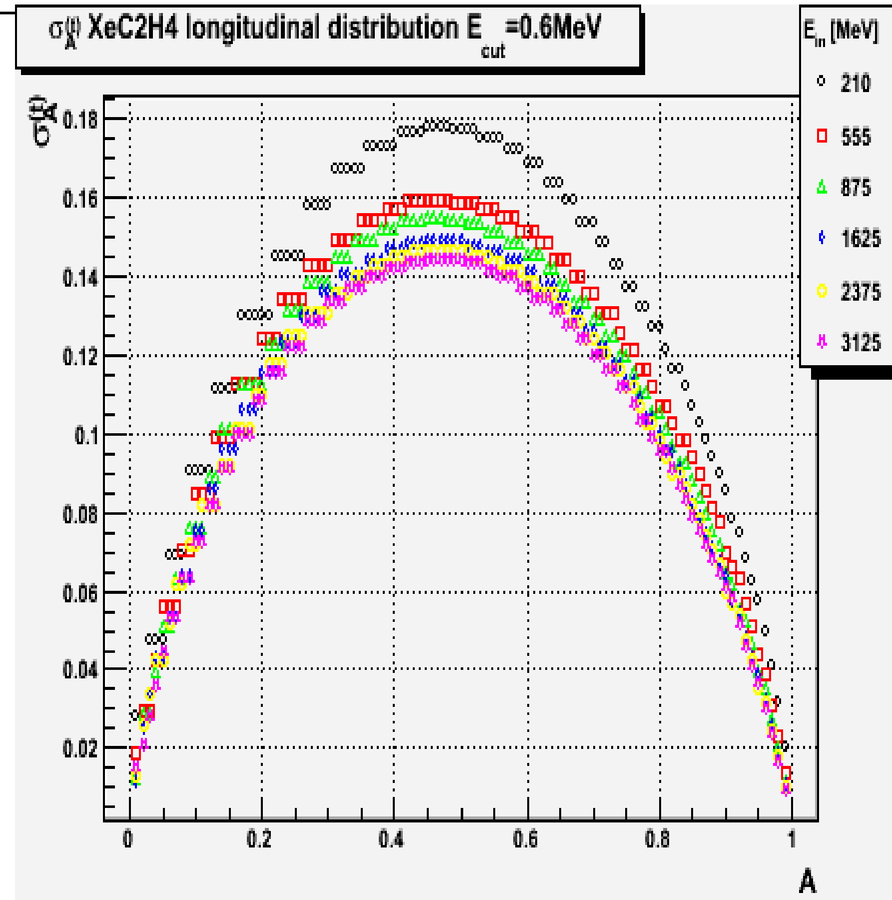
COMPARISON EGS4 & GEANT4 WITH EXPERIMENT

(material: liquid xenon)

Longitudinal fluctuation [EGS4]

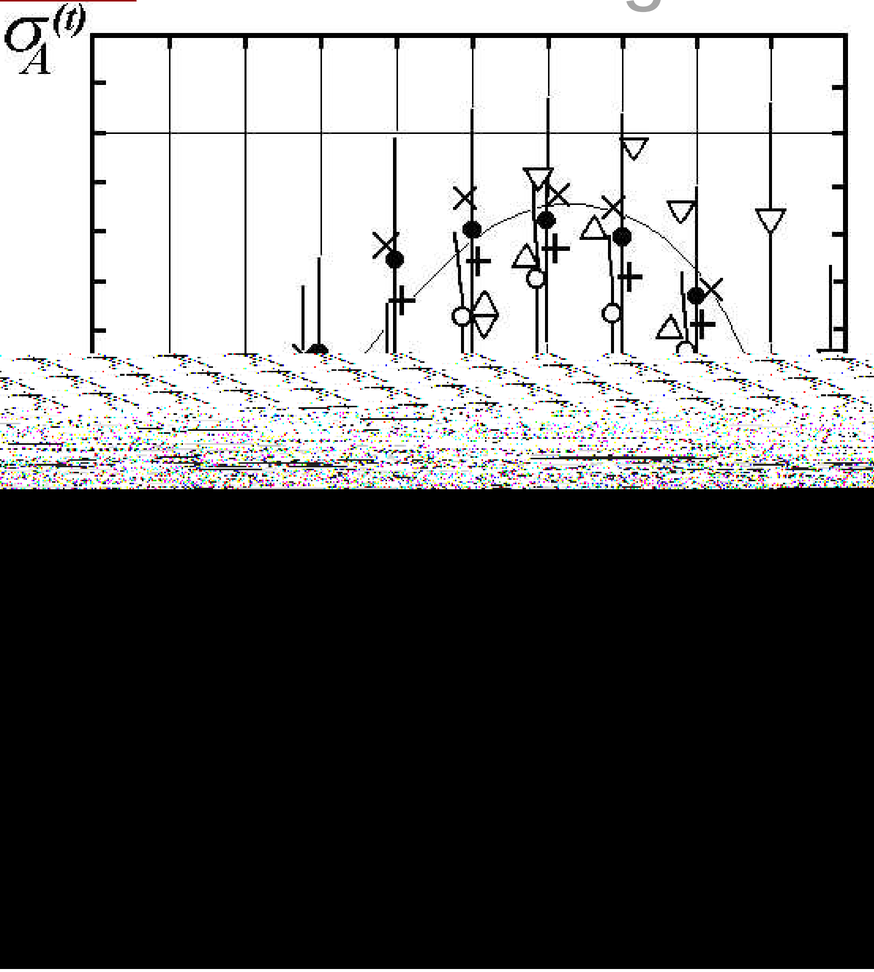


Experiment

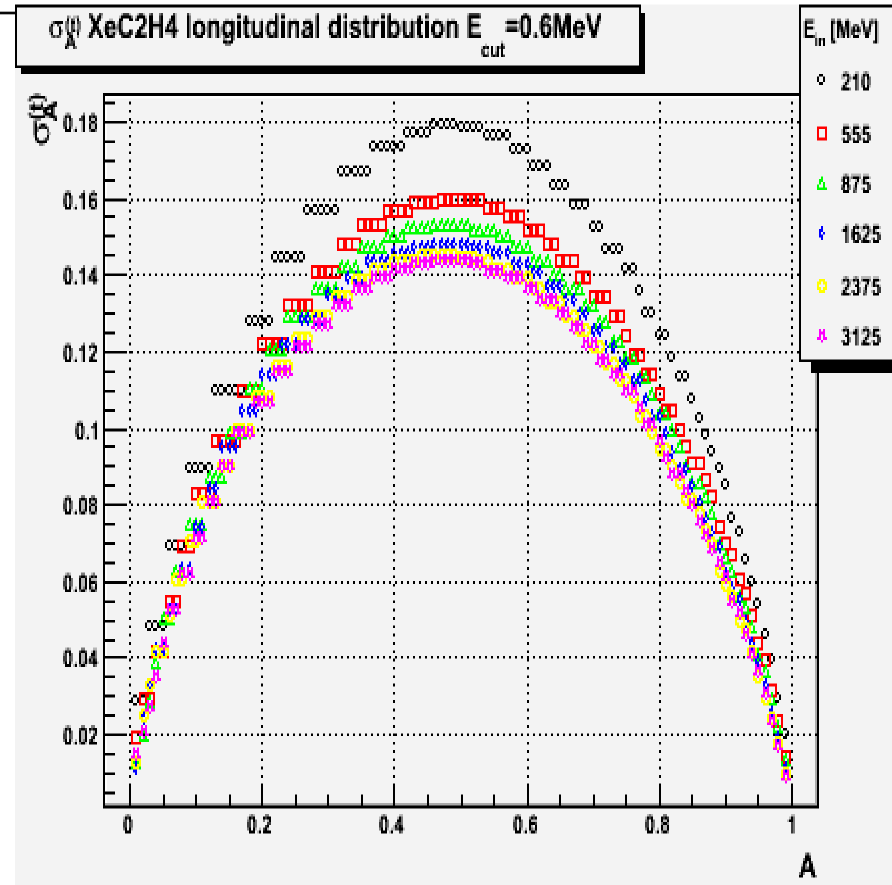


Simulation

Longitudinal fluctuation [GEANT4]

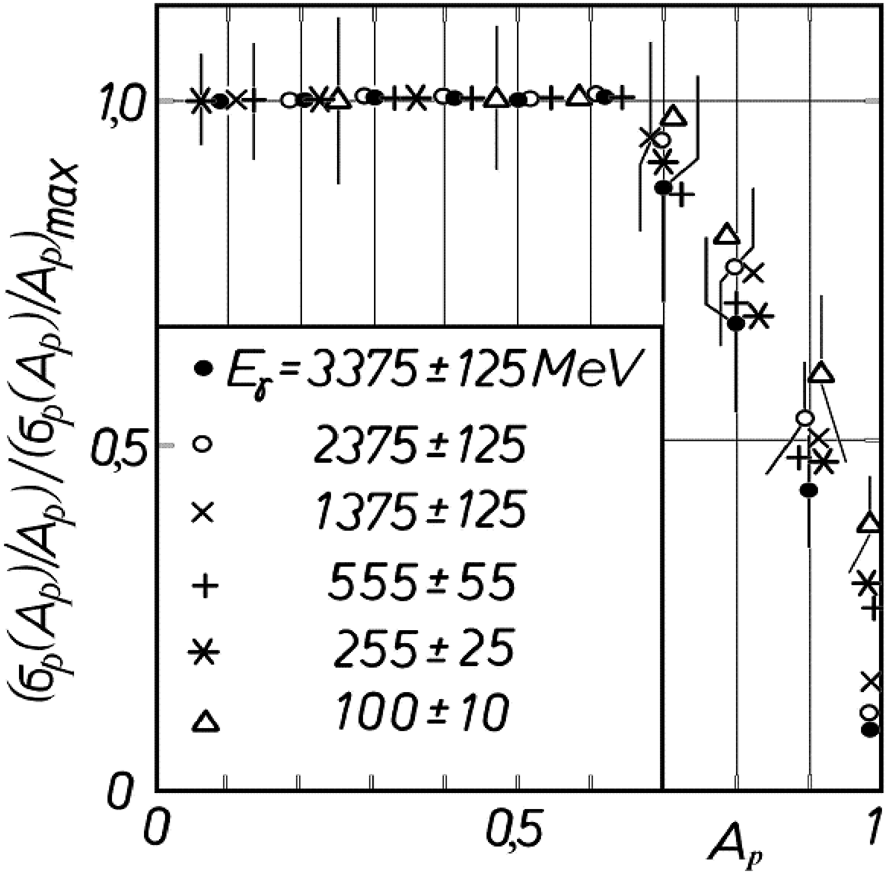


Experiment

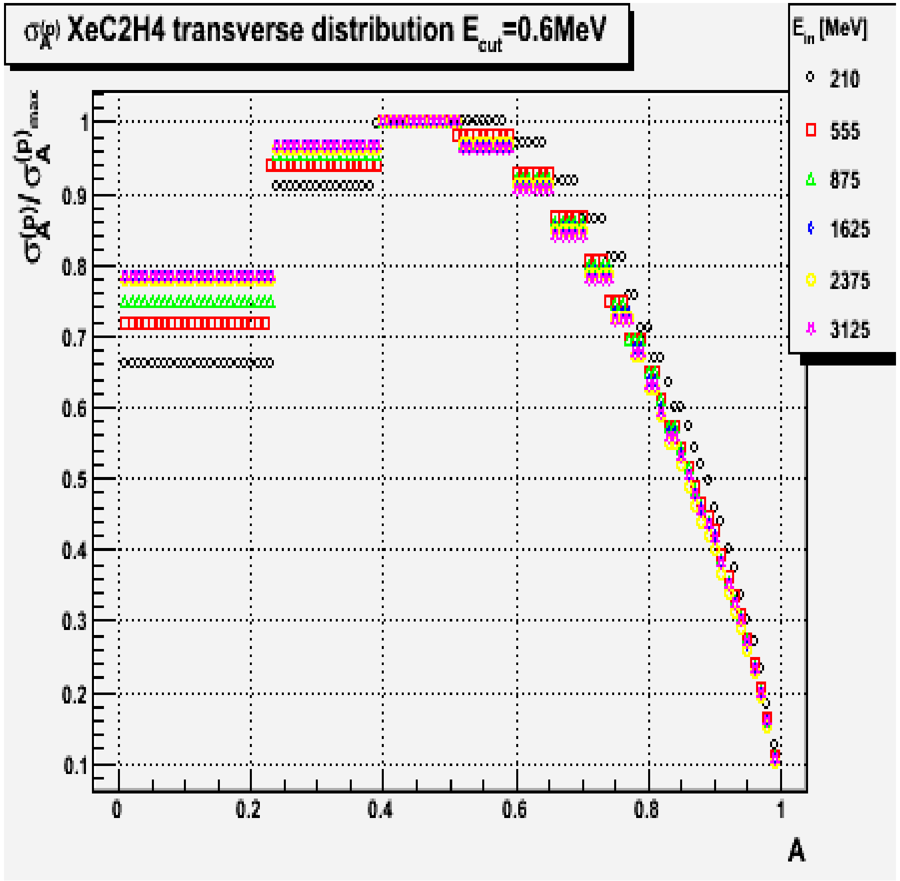


Simulation

Transverse fluctuation [EGS4]

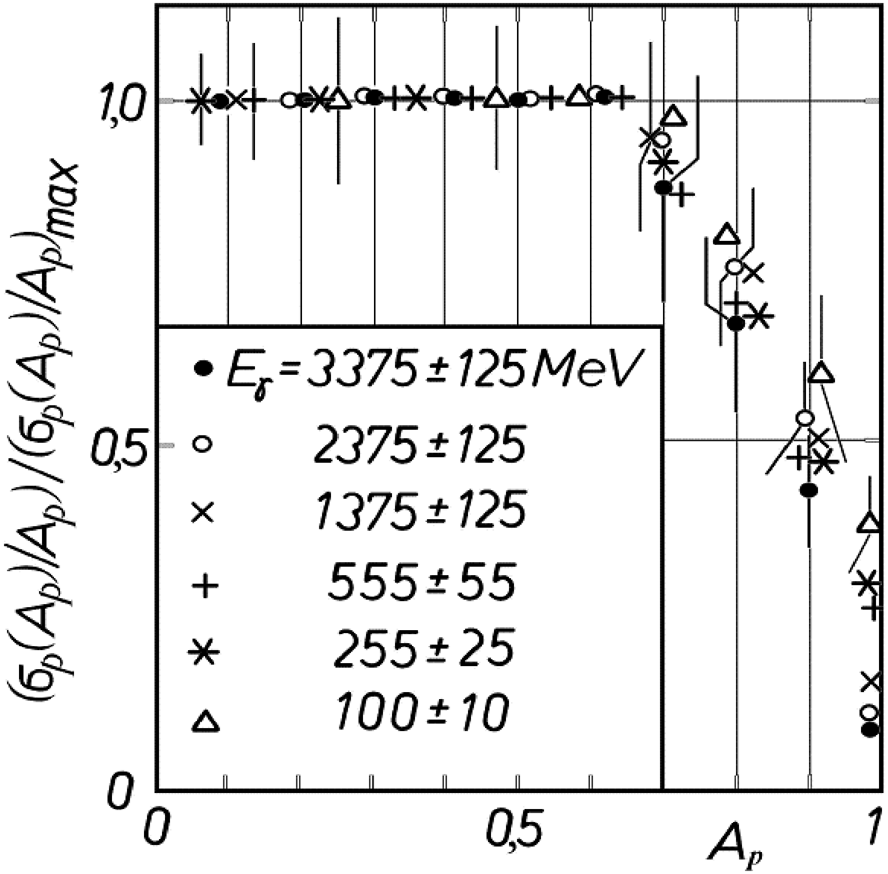


Experiment

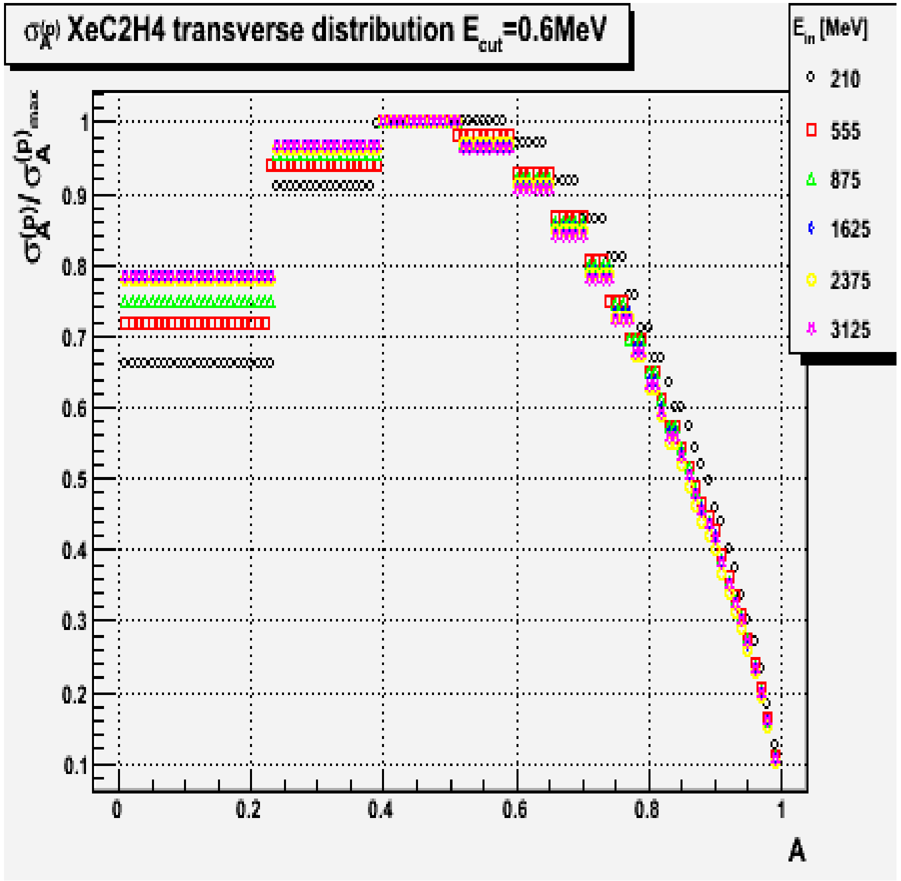


Simulation

Transverse fluctuation [GEANT4]



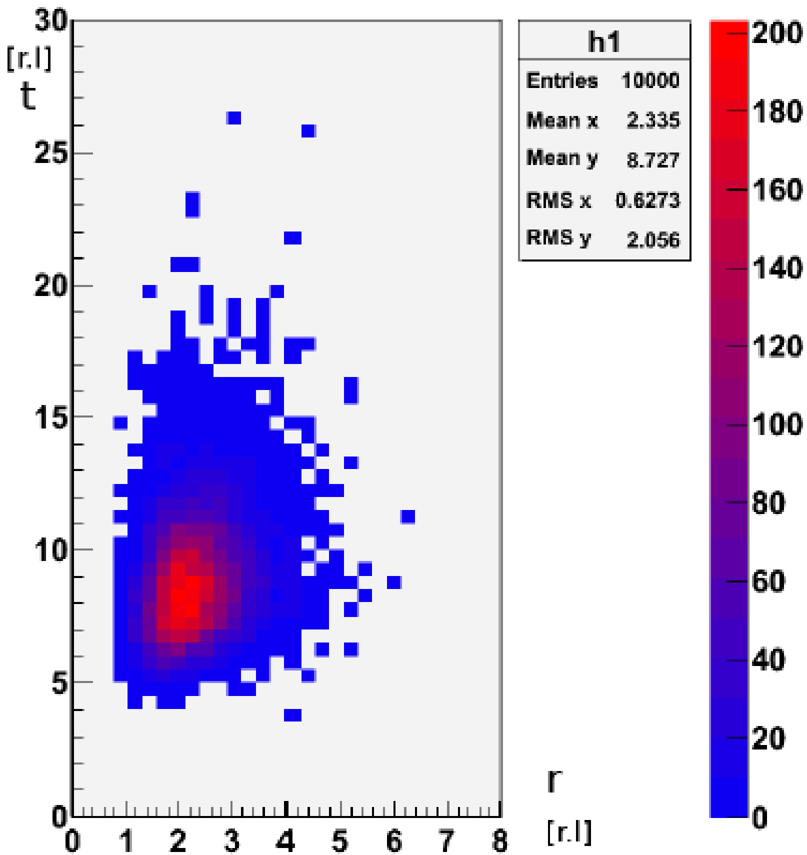
Experiment



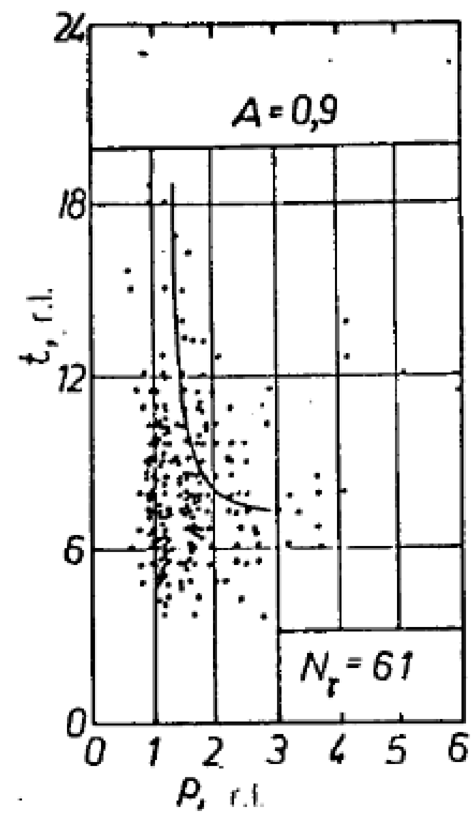
Simulation

Correlations

Eγ=375 MeV
A=70%



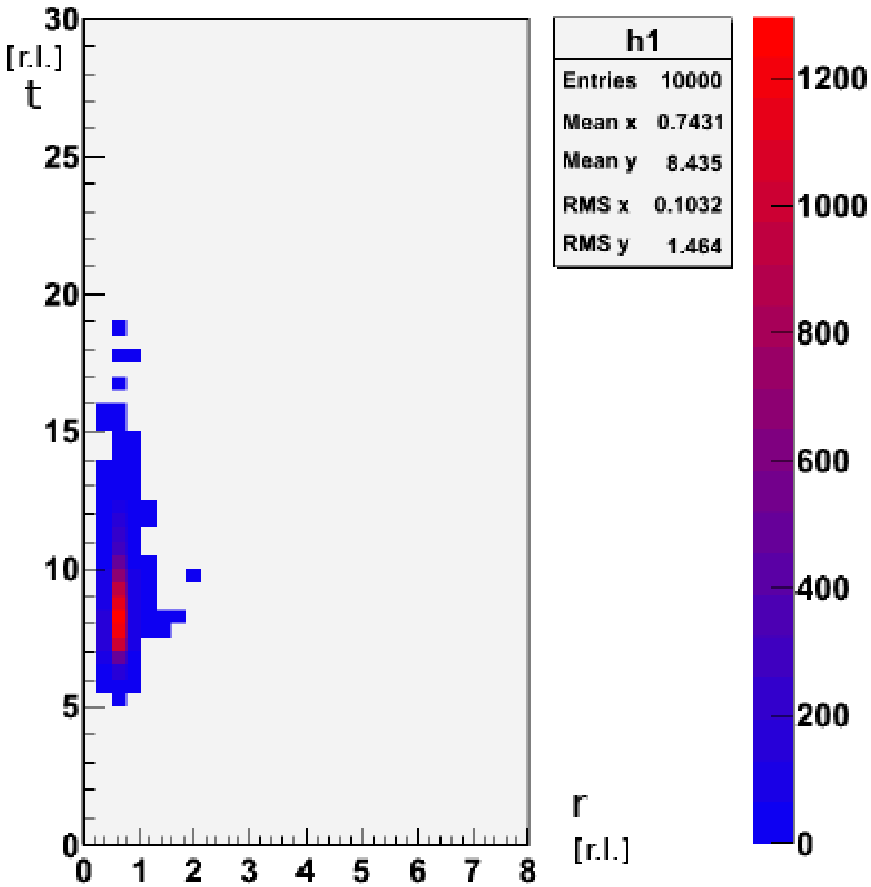
Simulation



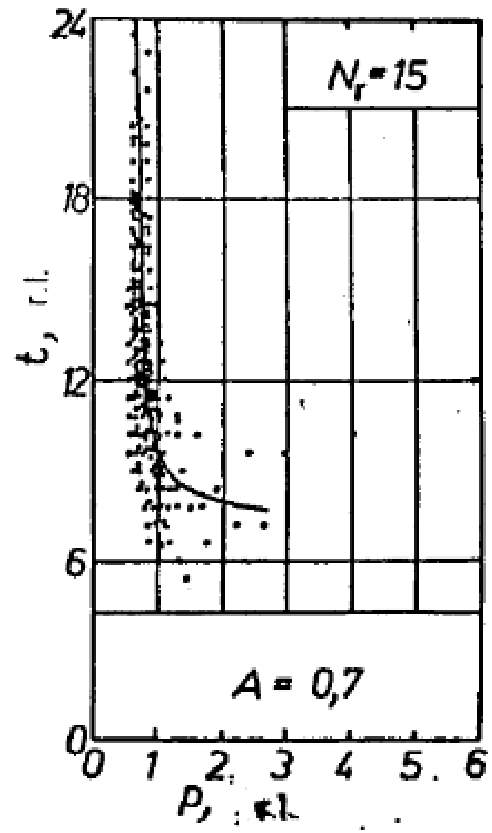
Experiment

Correlations

Eγ=3375 MeV
A=70%



Simulation



Experiment

SUMMARY AND CONCLUSION

- The comprehensive analysis of the longitudinal and transverse profiles of EMC initiated in 8 various amorphous materials by qamma quanta of energy E_γ= 210, 555, 875, 1625, 2375 and 3125 MeV has been performed by using EGS4 and GEANT4 code at four values of cut-off energy $E_{c.o.}$= 0.6, 1.2, 2.0 and 3.0 MeV.
- All the obtained approximating formulas in the form of simple functions reveal a quite acceptable scaling description of the electromagnetic cascade process initiated by gamma quanta of transitional energy interval 100÷3500 MeV in the most often used dense materials. They can be applied both for hard gamma detection and radiation shielding construction.
- Knowledge of correlations can be used when we want to reconstruct the electromagnetic cascade having a fragment of the EMC event only.

THANK YOU

FOR

YOUR ATTANTION