IMPACT PARAMETER ESTIMATION IN HADRON-NUCLEUS INTERACTIONS AT INTERMEDIATE ENERGIES

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OUTLINE OF THE TALK

I. MOTIVATION

II. ABSTRACT

III. CONFRONTATION OF THE JAM CODE WITH EXPERIMENT

IV. RESULTS OF CALCULATION USING JAM

1. Average multiplicity vs impact parameter
2. Average rapidity vs impact parameter
3. Average rapidity vs average multiplicity
4. Correlation between multiplicity, rapidity and impact parameter
5. Quasibinary reactions

V. SUMMARY AND CONCLUSION

I. M O T I V A T I O N

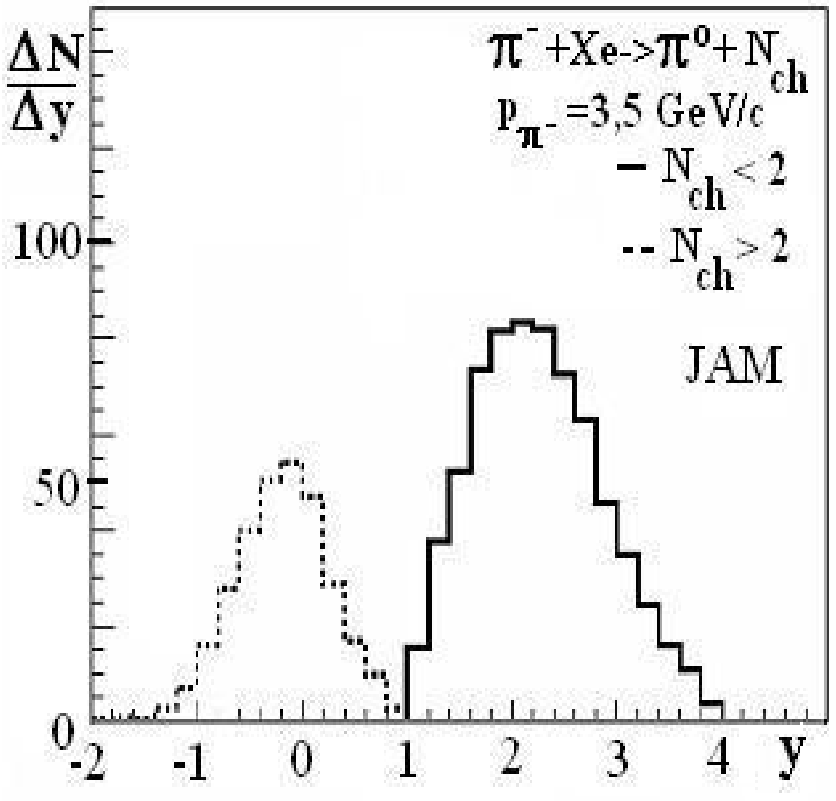
The impact parameter (IP) is one of the basic characteristics determining the initial geometry of the interaction of hadrons and nuclei with nuclear targets at high enough energies. It appears across the relevant theoretical formalism and, in particular, in the commonly used Glauber model. However, experimentally measured are only such quantities as multiplicity (M) of particles of definite kind, their energies, momenta and emission angles. By means of these observables one can construct the characteristics enabling categorization of experimental data like various and numerous scaling variables, longitudinal rapidity (LR), four-velocity transfer and centrality. But the classification of this kind is by its nature of statistical meaning and always the question arises about the reliability and ambiguity of this procedure.

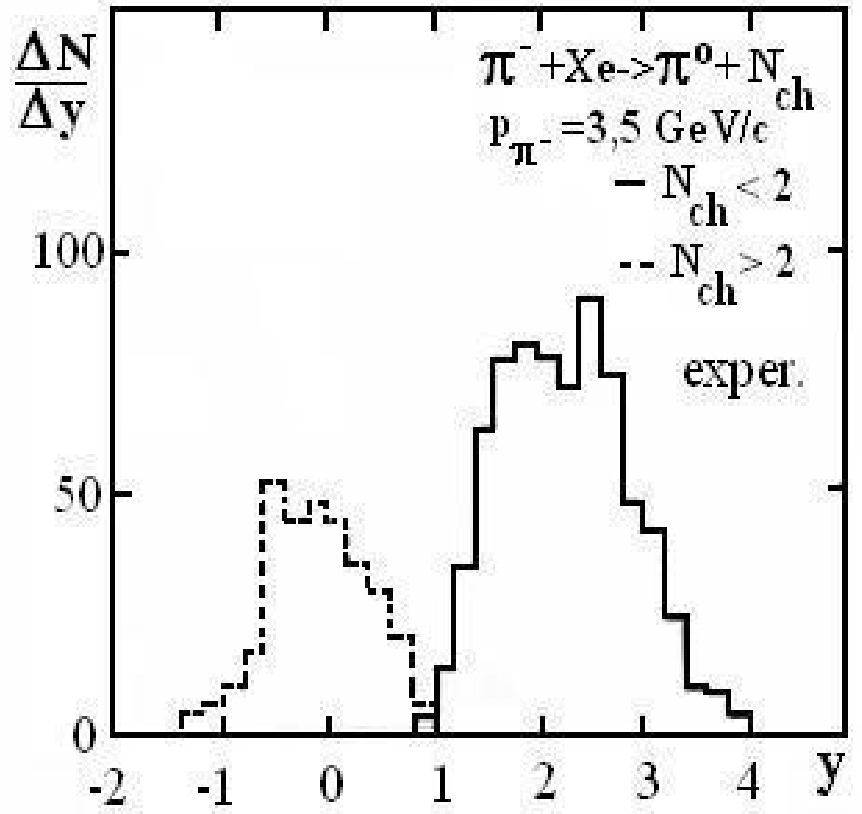
II. A B S T R A C T

We study the correlation between the impact parameter, multiplicity and longitudinal rapidity of neutral and charged pions, protons and neutrons produced/emitted in the interactions of charged pions with xenon nuclei at momenta of 2.34, 3.5, 9 and 30 GeV/c.

The correlation is investigated using JAM modeling code. These reactions have been chosen since just in this case the corresponding experimental information is available, especially on neutral pions, registered within 4π geometry and practically without limitation on their energy, except the case at 30 GeV/c which is selected in order to observe the energy behavior of the quantity under study. Partcular attention is given to quasibinary reactions of π mesons with xenon nuclei at 2.34 and 3.5 GeV/c.

III. Confrontation of JAM code with experiment: longitudinal rapidity distribution of neutral pions from the reaction $\pi^-+Xe \rightarrow \pi^0+N_{ch}$ at 3.5GeV/c





IV. Results of calculation by using JAM code corrlations between:

▲ multiplicity of emitted particles (pions, protons and neutrons) and the impact parameter

▲ average rapidity of these particles and the impact parameter

▲ multiplicity of emitted particles (pions, protons and netrons) and their average rapidity

for the reaction

$\pi^{+/-}$ + Xe $\rightarrow$ pions + protons + neutrons

at 2.34, 3.5, 9 and 30 GeV/c.

Average multiplicity (of emitted particles: pions, protons and neutrons) *versus* impact parameter

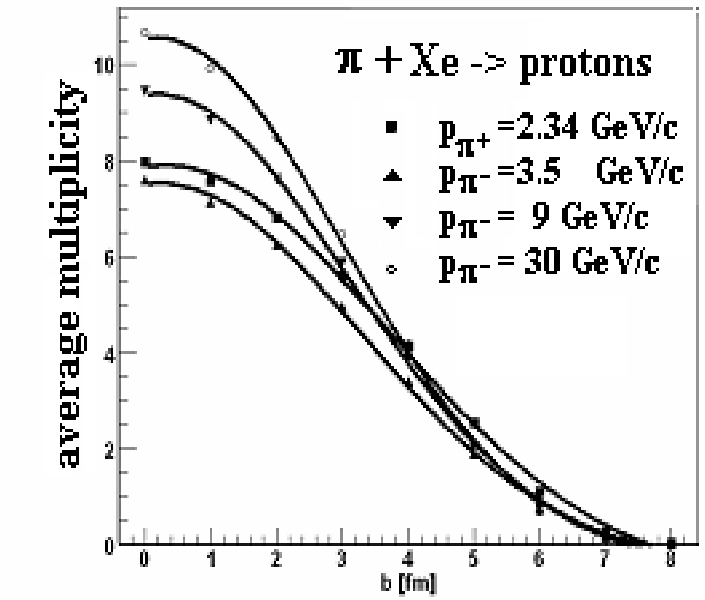
Reaction $\pi^{+/-}$ + Xe

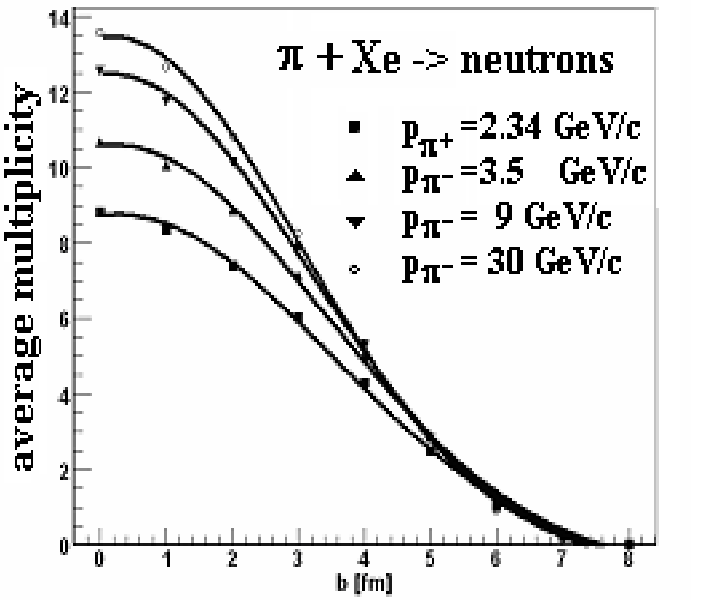
at

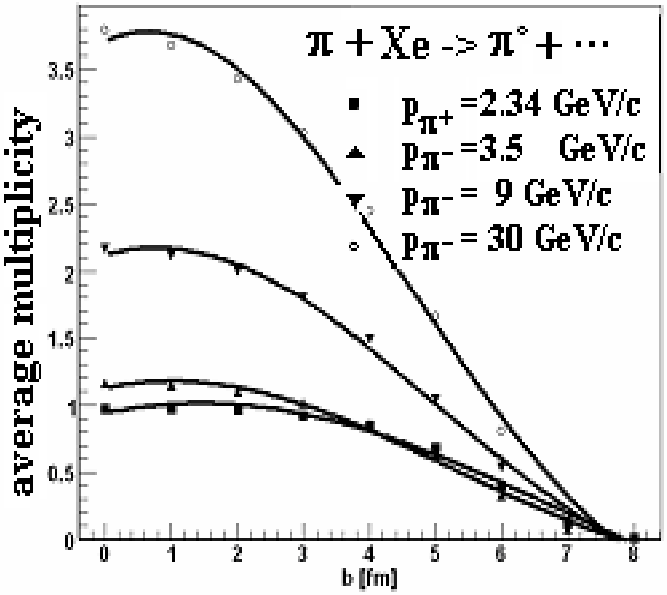
2.34, 3.5, 9 and 30 GeV/c

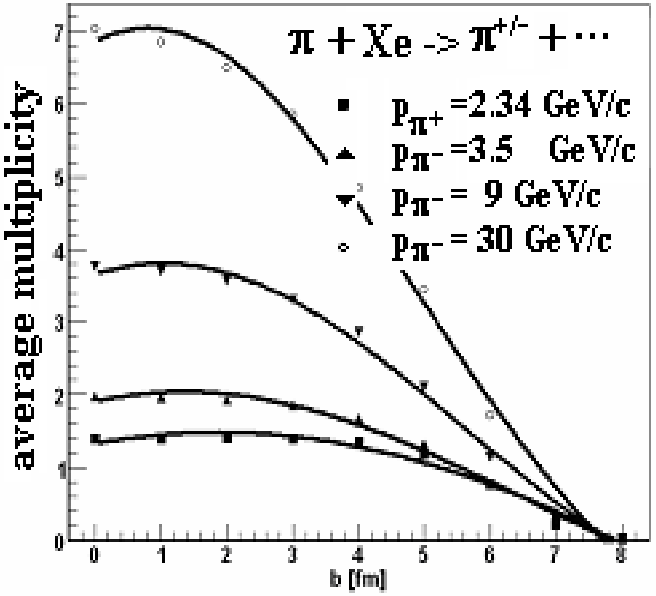
Simulation after JAM

Average multiplicity *versus* impact parameter









Approximating function
for distributions of the

average multiplicity vs impact parameter

$$f(b) = p_0 + p_1 * \exp\left(-0.5 * \left(\frac{(b - p_2)}{p_3}\right)^2\right)$$

Here p_0, p_1, p_2 and p_3 are parameters of distributions, b is the impact parameter.

Values of parameters of the function $f(b) = p_0 + p_1 * \exp\left(-0.5 * \left(\frac{(b - p_2)}{p_3}\right)^2\right)$

fitting the calculated distribution of the average multiplicity vs impact parameter

particle	momentum GeV/c	p_0	p_1	p_2	p_3	χ^2
π^0	2.34	-1.1 +/- 1.1	2.1 +/- 1.1	1.5 +/- 0.4	5.6 +/- 2.1	0.0055
π^0	3.5	-0.4 +/- 0.3	1.6 +/- 0.3	1.0 +/- 0.4	4.2 +/- 0.8	0.0044
π^0	9.0	-0.7 +/- 0.4	2.8 +/- 0.4	0.8 +/- 0.4	4.1 +/- 0.7	0.0113
π^0	30.0	-0.9 +/- 0.5	4.7 +/- 0.5	0.7 +/- 0.3	3.9 +/- 0.6	0.0302
π^{+-}	2.34	-11.9 +/- 0.5	13.4 +/- 0.5	1.9 +/- 0.4	12.5 +/- 1.0	0.0190
π^{+-}	3.5	-1.8 +/- 1.7	3.8 +/- 1.6	1.4 +/- 0.4	5.3 +/- 1.8	0.0208
π^{+-}	9.0	-1.9 +/- 1.3	5.7 +/- 1.3	1.0 +/- 0.4	4.5 +/- 1.0	0.0497
π^{+-}	30.0	-2.2 +/- 1.3	9.2 +/- 1.3	0.8 +/- 0.4	4.1 +/- 0.7	0.1339
protons	2.34	-0.8 +/- 0.4	8.7 +/- 0.4	0.3 +/- 0.2	3.4 +/- 0.3	0.0528
protons	3.5	-0.5 +/- 0.2	8.0 +/- 0.3	0.23 +/- 0.21	3.1 +/- 0.2	0.0391
protons	9.0	-0.5 +/- 0.2	9.8 +/- 0.3	0.18 +/- 0.18	2.9 +/- 0.2	0.0478
protons	30.0	-0.5 +/- 0.2	11.0 +/- 0.3	0.18 +/- 0.19	2.9 +/- 0.2	0.0722
neutrons	2.34	-0.7 +/- 0.3	9.5 +/- 0.4	0.27 +/- 0.22	3.2 +/- 0.2	0.0591
neutrons	3.5	-0.8 +/- 0.3	11.4 +/- 0.4	0.23 +/- 0.21	3.2 +/- 0.2	0.0777
neutrons	9.0	-0.6 +/- 0.3	13.1 +/- 0.4	0.17 +/- 0.19	3.0 +/- 0.2	0.0872
neutrons	30.0	-0.6 +/- 0.3	14.0 +/- 0.4	0.19 +/- 0.19	2.9 +/- 0.2	0.1139

Average rapidity (of emitted particles: pions, protons and neutrons) *versus* impact parameter

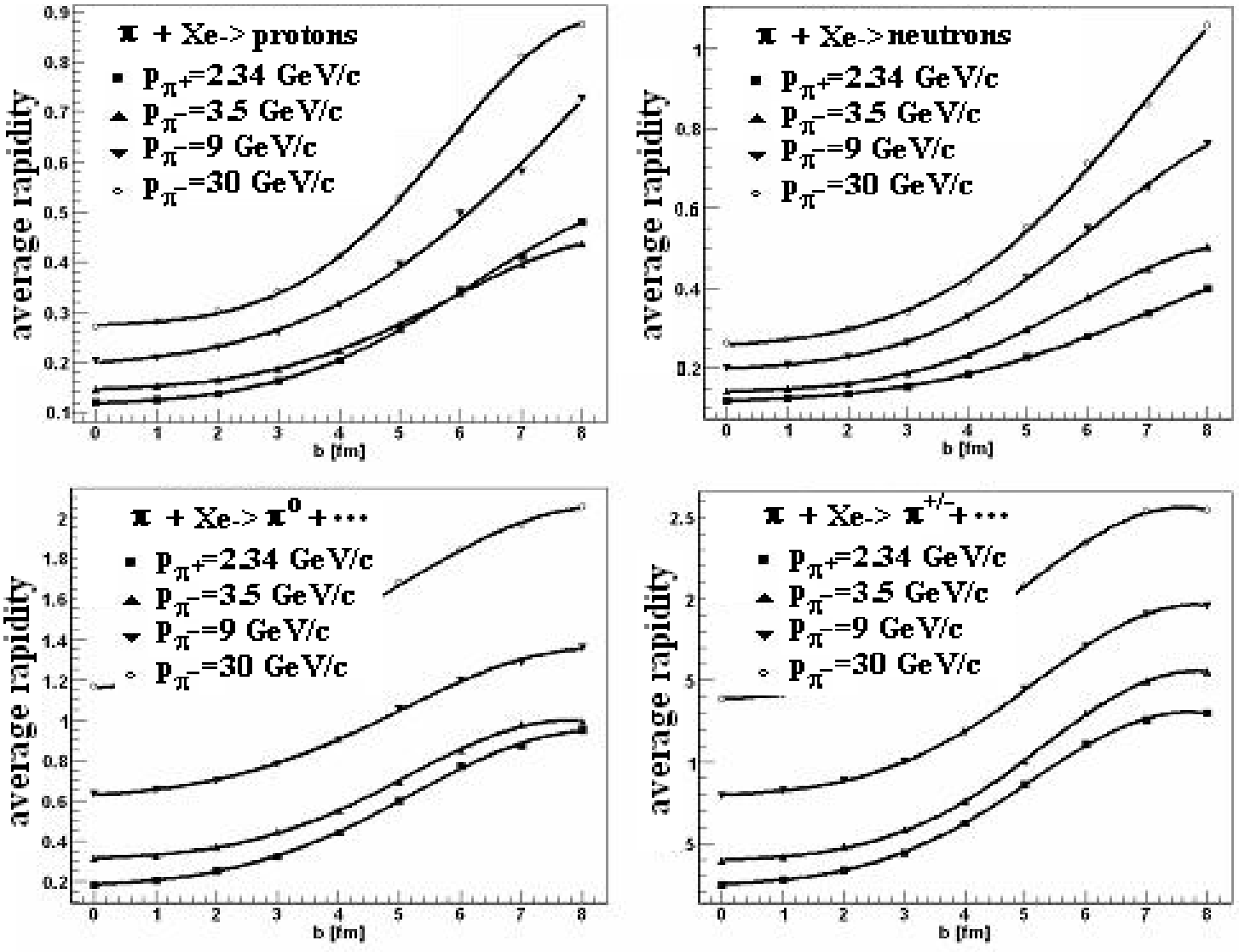
Reaction $\pi^{+/-}$ + Xe

at

2.34, 3.5, 9 and 30 GeV/c

Simulation after JAM

Average rapidity *versus* impact parameter



Approximating function
for distributions of the

average rapidity vs impact parameter

$$f(b) = p_0 + p_1 * \exp\left(-0.5 * \left(\frac{(b - p_2)}{p_3} \right)^2 \right)$$

Here p_0, p_1, p_2 and p_3 are parameters of distributions, b is the impact parameter.

Valeus of parameters of the gaussian fuction

$$f(b) = p_0 + p_1 * \exp\left(-0.5 * \left(\frac{(b - p_2)}{p_3}\right)^2\right)$$

fitting the distribution of the average rapidity *vs* impact parameter

particle	*momentum GeV/c*	p_0	p_1	p_2	p_3	χ^2
π^0	2,34	0,17 ± 0,01	0,77 ± 0,02	8,15 ± 0,14	2,89 ± 0,12	0,0001
π^0	3,5	0,31 ± 0,01	0,69 ± 0,01	7,8 ± 0,1	2,6 ± 0,1	0,0001
π^0	9,0	0,61 ± 0,01	0,74 ± 0,02	8,13 ± 0,19	3,05 ± 0,17	0,0002
π^0	30,0	1,13 ± 0,02	0,93 ± 0,03	8,29 ± 0,19	3,18 ± 0,17	0,0002
$\pi^{+/-}$	2,34	0,238 ± 0,008	1,07 ± 0,01	7,69 ± 0,06	2,60 ± 0,06	0,0001
$\pi^{+/-}$	3,5	0,389 ± 0,007	1,17 ± 0,01	7,82 ± 0,06	2,53 ± 0,05	0,0001
$\pi^{+/-}$	9,0	0,786 ± 0,005	1,18 ± 0,01	7,81 ± 0,04	2,61 ± 0,03	0,00004
$\pi^{+/-}$	30,0	1,373 ± 0,007	1,19 ± 0,01	7,59 ± 0,04	2,54 ± 0,04	0,0001
protons	2,34	0,114 ± 0,003	0,39 ± 0,01	9,18 ± 0,24	3,03 ± 0,15	0,00002
protons	3,5	0,142 ± 0,001	0,306 ± 0,002	8,82 ± 0,04	2,98 ± 0,02	0,000001
protons	9,0	0,18 ± 0,01	0,85 ± 0,18	11,9 ± 1,3	4,1 ± 0,5	0,0002
protons	30,0	0,271 ± 0,005	0,61 ± 0,01	8,35 ± 0,12	2,53 ± 0,09	0,00005
neutrons	2,34	0,113 ± 0,002	0,37 ± 0,01	10,7 ± 0,2	3,69 ± 0,11	0,000003
neutrons	3,5	0,139 ± 0,001	0,374 ± 0,004	8,70 ± 0,08	2,83 ± 0,05	0,000004
neutrons	9,0	0,194 ± 0,007	0,61 ± 0,03	9,2 ± 0,3	3,02 ± 0,19	0,0001
neutrons	30,0	0,25 ± 0,01	1,0 ± 0,1	10,3 ± 0,7	3,4 ± 0,3	0,0002

Average rapidity (of emitted particles: pions, protons and neutrons) *versus* average multiplicity

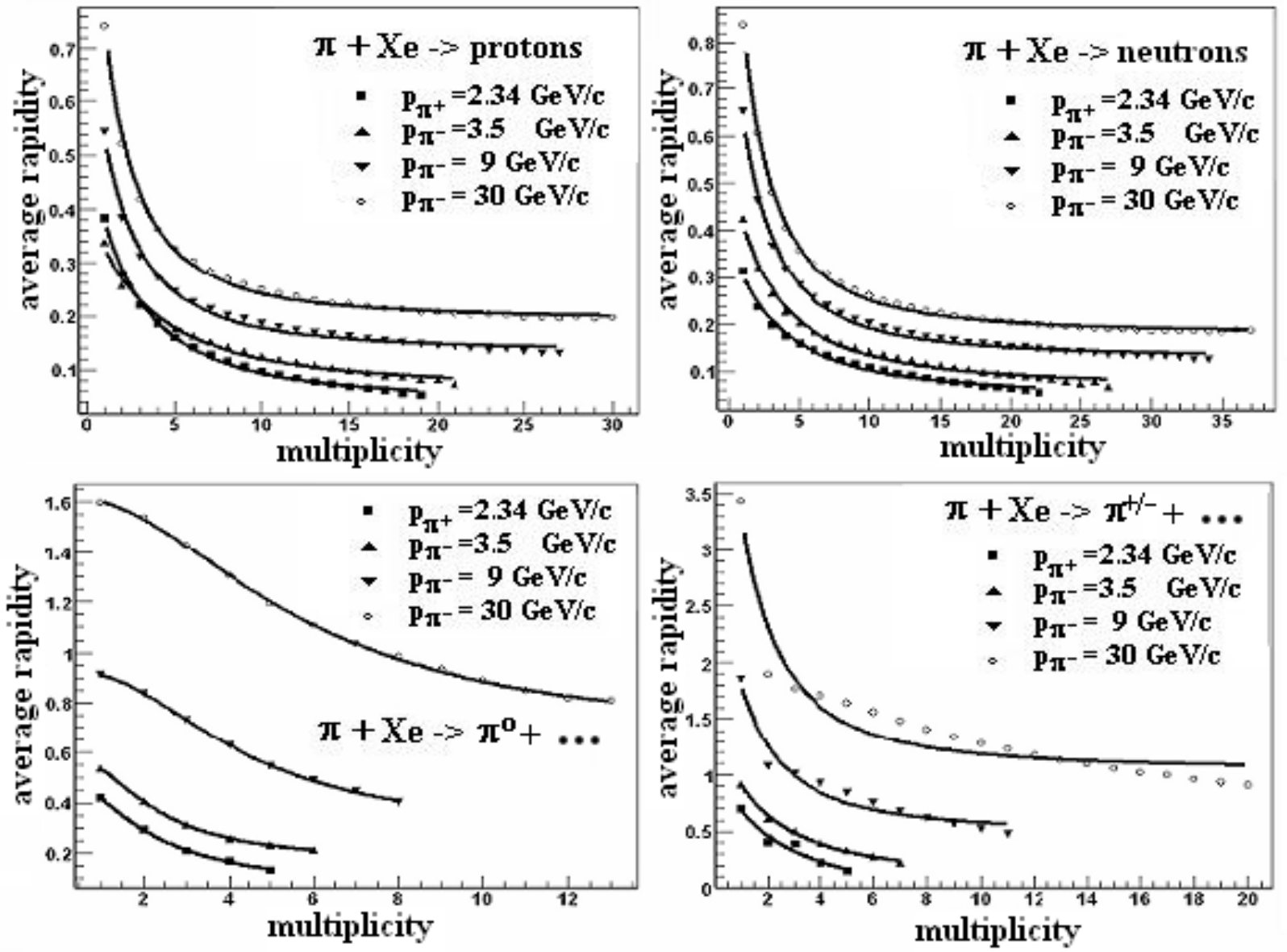
Reaction $\pi^{+/-}$ + Xe

at

2.34, 3.5, 9 and 30 GeV/c

Simulation after JAM

Average rapidity *versus* multiplicity of the reaction π + Xe



Approximation function

of the dependence of the average rapidity of particles on their multiplicity:

$$f(N) = p_0 + p_1 \frac{1}{\pi}\left[\frac{p_2}{(N - p_3)^2 + {p_2}^2}\right]$$

Here p_0, p_1, p_2 and p_3 are free parameters and N is the multiplicity of particles

Values of parameters of approximating function

$$f(N) = p_0 + p_1 \frac{1}{\pi}\left[\frac{p_2}{(N - p_3)^2 + {p_2}^2}\right]$$

particle	momentum $[GeV/c]$	p_0	p_1	p_2	p_3	χ^2
π^0	2,34	0,07 ± 2,03	3,3 ± 104,5	2,2 ± 28,7	–0,3 ± 60,3	0,000008
π^0	3,5	0,169 ± 0,009	2,6 ± 0,4	2,0 ± 0,2	0,3 ± 0,3	0,00001
π^0	9,0	0,27 ± 0,02	8,1 ± 0,8	3,9 ± 0,3	0,5 ± 0,2	0,000023
π^0	30,0	0,66 ± 0,02	16,3 ± 1,2	5,4 ± 0,3	0,3 ± 0,2	0,0002
$\pi^{+/-}$	2,34	0,05 ± 3,84	$(0,04 \pm 3)\cdot 10^6$	–53 ± 775	11 ± 145	0,003
$\pi^{+/-}$	3,5	0,20 ± 0,06	$(1,5 \pm 4,3)\cdot 10^5$	–56 ± 19	12 ± 3	0,001
$\pi^{+/-}$	9,0	0,56 ± 0,09	$(1,5 \pm 3,9)\cdot 10^8$	–80 ± 30	13 ± 5	0,03
$\pi^{+/-}$	30,0	1,11 ± 0,09	$(0,5 \pm 1,4)\cdot 10^{10}$	–103 ± 46	16 ± 7	0,26
protons	2,34	0,065 ± 0,005	$(4 \pm 5)\cdot 10^6$	–117 ± 13	20,7 ± 1,8	0,0006
protons	3,5	0,085 ± 0,005	$(2,1 \pm 2,1)\cdot 10^6$	–150 ± 19	26,6 ± 2,9	0,0006
protons	9,0	0,150 ± 0,004	$(7,5 \pm 9,5)\cdot 10^7$	–117 ± 13	19,1 ± 1,8	0,002
protons	30,0	0,211 ± 0,004	$(6,5 \pm 5,2)\cdot 10^7$	–101 ± 9	–16,7 ± 1,3	0,003
neutrons	2,34	0,067 ± 0,005	$(2,1 \pm 2,2)\cdot 10^6$	–156 ± 19	28 ± 3	0,0006
neutrons	3,5	0,089 ± 0,005	$(2,4 \pm 2,2)\cdot 10^7$	–167 ± 19	28 ± 3	0,002
neutrons	9,0	0,148 ± 0,005	$(1,1 \pm 1,2)\cdot 10^8$	–133 ± 13	22 ± 2	0,004
neutrons	30,0	0,201 ± 0,004	$(1,9 \pm 1,7)\cdot 10^9$	–140 ± 11	21,3 ± 1,4	0,004

CORRELATION

between multiplicity, rapidity and impact parameter for the reaction

$\Pi^{+/-} + Xe \rightarrow protons + neutrons + pions$

at 2.34, 3.5, 9 and 30 GeV/c

calculated by using the JAM code

Values of coefficients of the correlation between multiplicity, rapidity and impact parameter for the reaction π+Xe calculated by using the JAM code

momentum $[GeV/c]$	secondary particles	coeficients of correlation		
		multiplicity vs impact parameter	rapidity vs impact parameter	rapidity vs multiplicity
$2,34$	π^0	$-0,083 \pm 0,009$	$0,215 \pm 0,009$	$-0,098 \pm 0,009$
$2,34$	$\pi^{+/-}$	$-0,08 \pm 0,01$	$0,28 \pm 0,01$	$-0,15 \pm 0,01$
$2,34$	*protons*	$-0,539 \pm 0,006$	$0,13 \pm 0,01$	$-0,171 \pm 0,009$
$3,5$	π^0	$-0,124 \pm 0,009$	$0,19 \pm 0,01$	$-0,11 \pm 0,01$
$3,5$	$\pi^{+/-}$	$-0,14 \pm 0,01$	$0,25 \pm 0,01$	$-0,18 \pm 0,01$
$3,5$	*protons*	$-0,500 \pm 0,007$	$0,10 \pm 0,01$	$-0,139 \pm 0,008$
$9,0$	π^0	$-0,200 \pm 0,009$	$0,18 \pm 0,01$	$-0,134 \pm 0,009$
$9,0$	$\pi^{+/-}$	$-0,236 \pm 0,009$	$0,210 \pm 0,009$	$-0,20 \pm 0,01$
$9,0$	*protons*	$-0,511 \pm 0,006$	$0,10 \pm 0,01$	$-0,134 \pm 0,009$
$30,0$	π^0	$-0,275 \pm 0,009$	$0,17 \pm 0,01$	$-0,161 \pm 0,009$
$30,0$	$\pi^{+/-}$	$-0,313 \pm 0,009$	$0,18 \pm 0,01$	$-0,20 \pm 0,01$
$30,0$	*protons*	$-0,517 \pm 0,006$	$0,09 \pm 0,01$	$-0,126 \pm 0,009$

Quasibinary reactions
(preliminary results)

- $\pi^+ + Xe \rightarrow \pi^0 + p + A'$ at 2,34 GeV/c

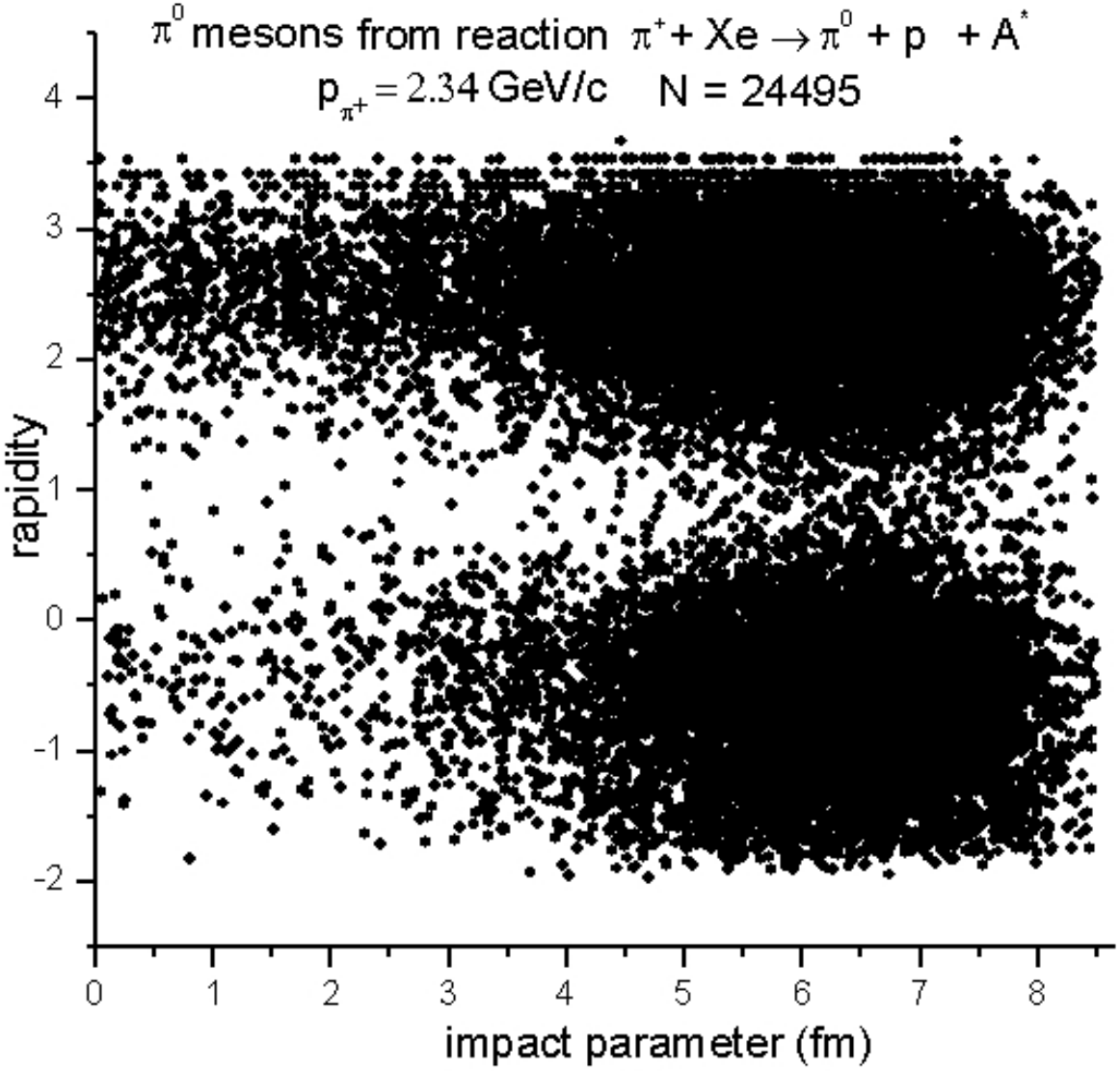
- $\pi^- + Xe \rightarrow \pi^0 + n + A'$

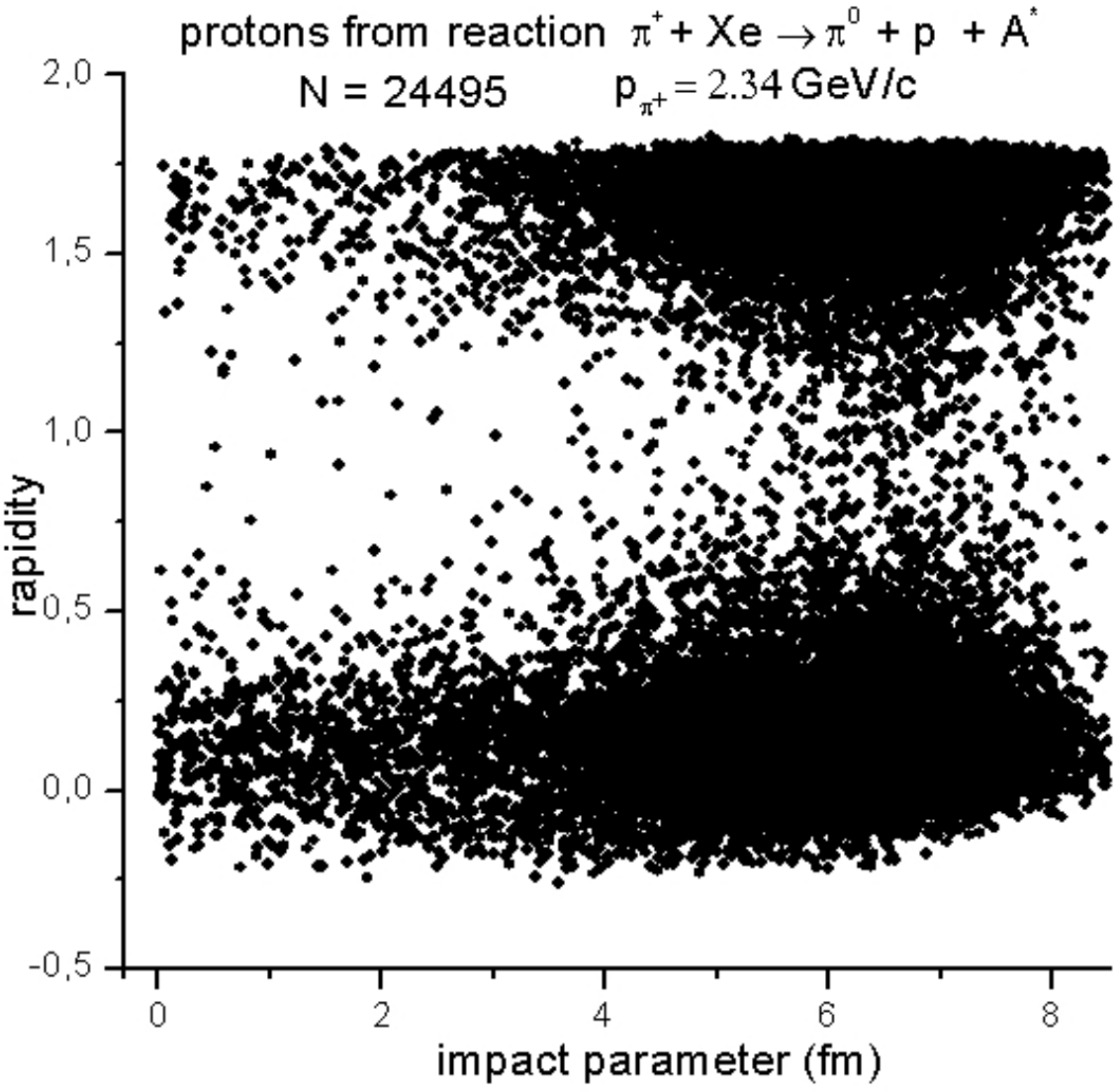
$\pi^0 + n + (\pi^{+/-}, p) + A'$ at 3,5 GeV/c

($\pi^{+/-}$, p): $E_\pi \lesssim 5$ MeV, $E_p \lesssim 10$ MeV

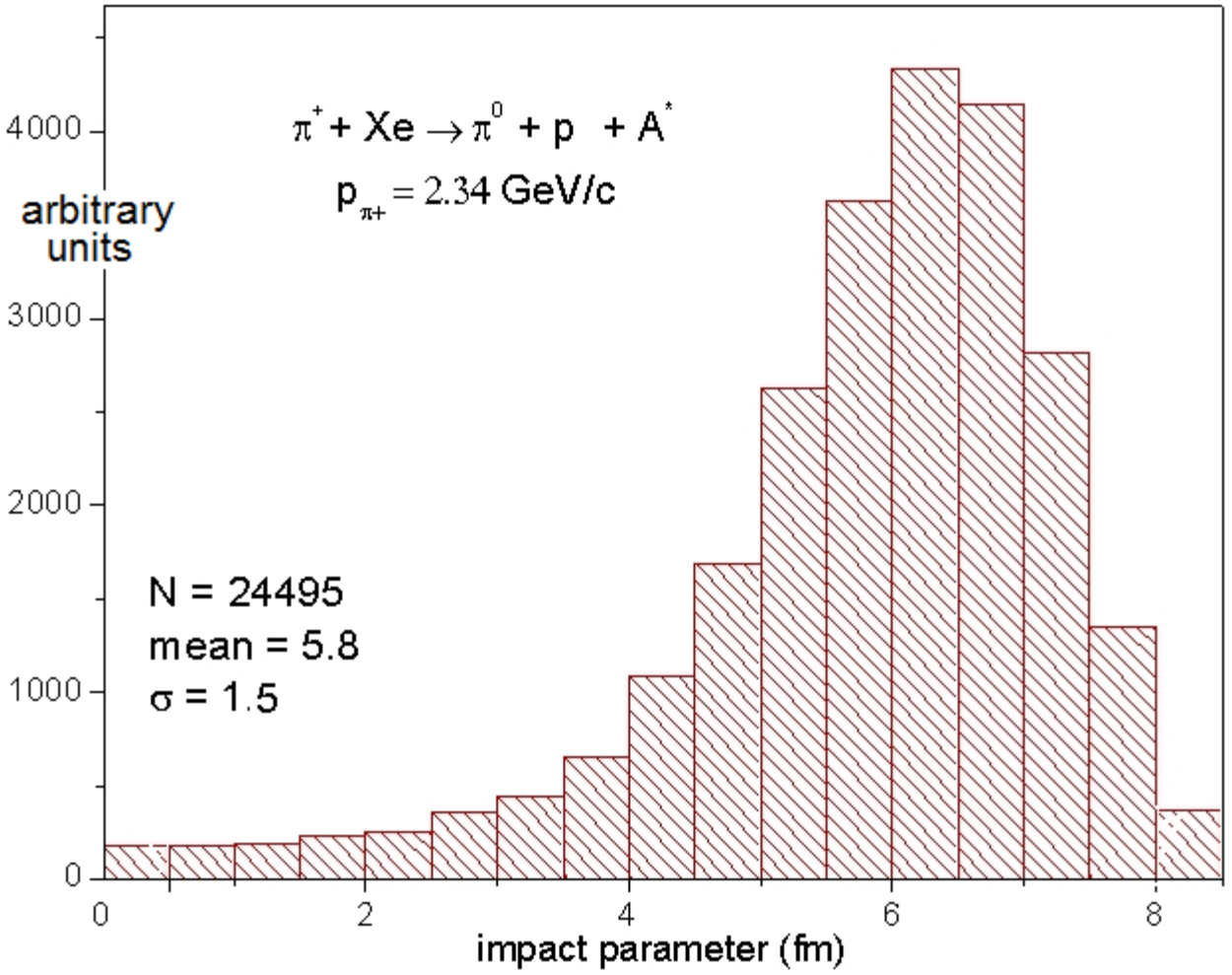
$$\pi^+ + Xe \rightarrow \pi^0 + p + A'$$

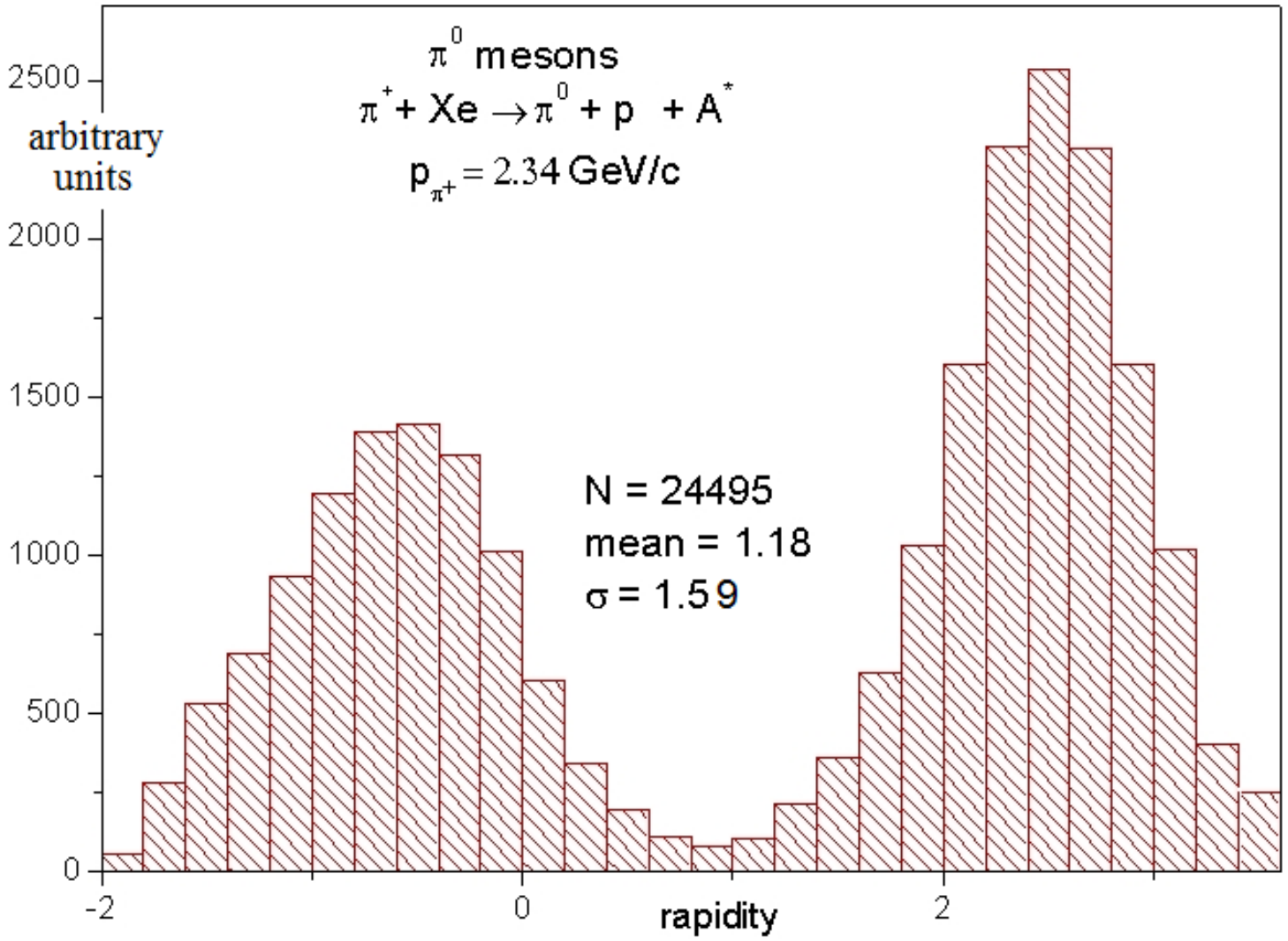
at 2,34 GeV/c


π^0 mesons from reaction $\pi^+ + Xe \rightarrow \pi^0 + p + A^*$
$p_{\pi^+} = 2.34$ GeV/c N = 24495
rapidity
impact parameter (fm)
4
3
2
1
0
-1
-2
0
1
2
3
4
5
6
7
8


protons from reaction $\pi^+ + Xe \rightarrow \pi^0 + p + A^*$
N = 24495
$p_{\pi^+} = 2.34$ GeV/c
rapidity
2,0
1,5
1,0
0,5
0,0
-0,5
0
2
4
6
8
impact parameter (fm)

Boundary distributions

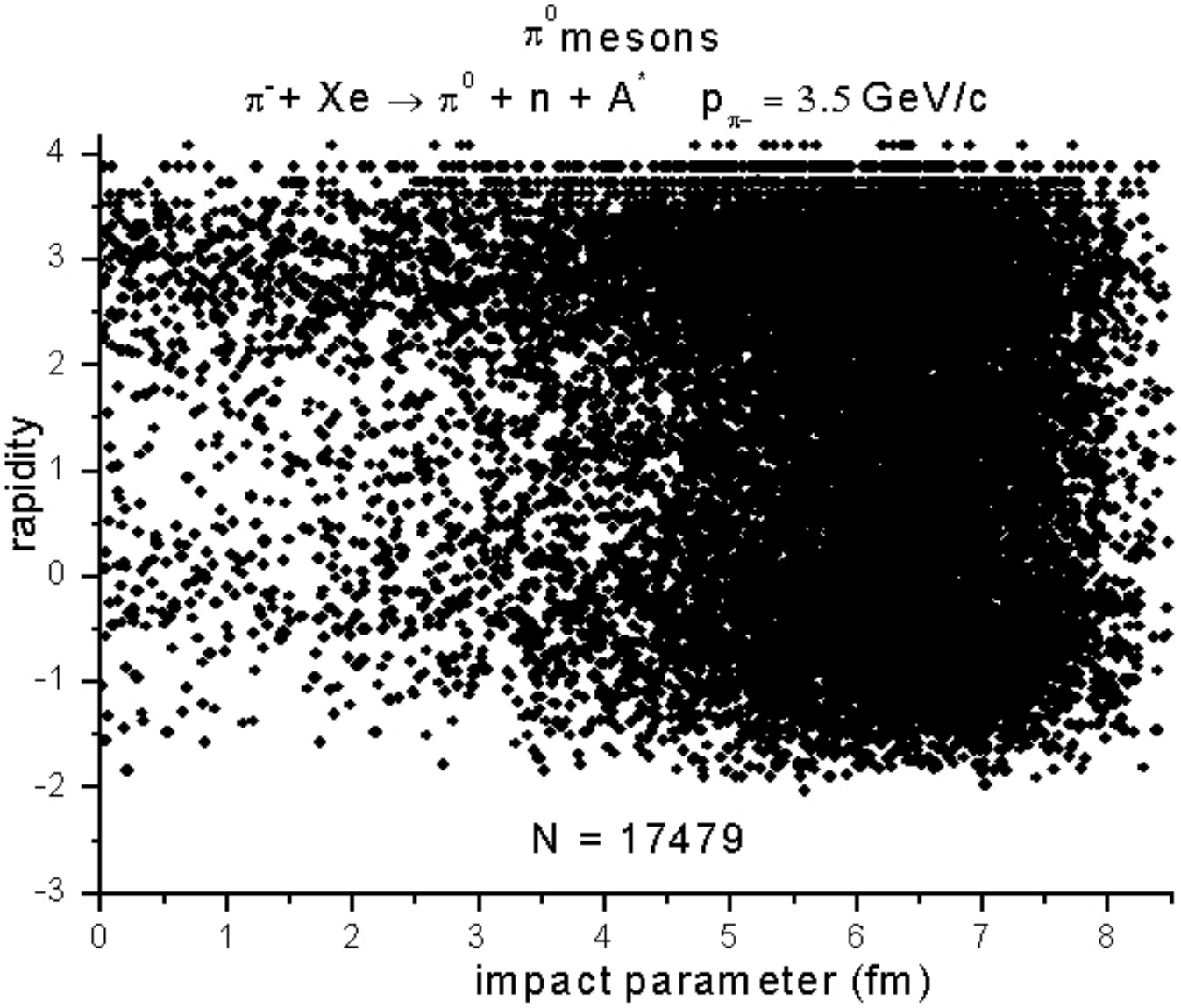

$\pi^+ + Xe \rightarrow \pi^0 + p + A^*$
$p_{\pi+} = 2.34$ GeV/c
arbitrary units
N = 24495
mean = 5.8
σ = 1.5
4000
3000
2000
1000
0
0
2
4
6
8
impact parameter (fm)

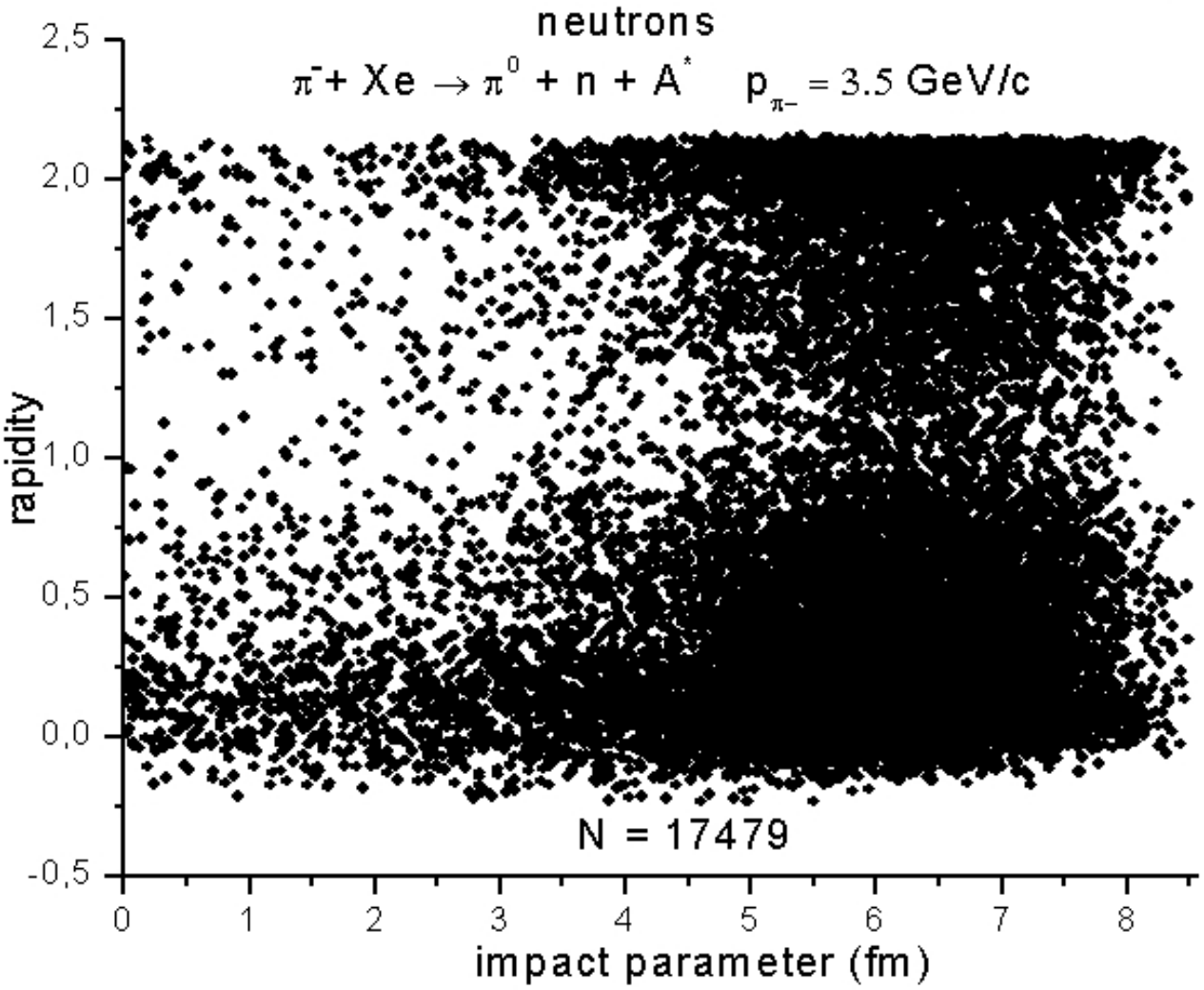

π^0 mesons
$\pi^+ + Xe \rightarrow \pi^0 + p + A^*$
$p_{\pi^+} = 2.34$ GeV/c
arbitrary units
2500
2000
1500
1000
500
0
N = 24495
mean = 1.18
σ = 1.59
-2
0
2
rapidity

$$\pi^- + Xe \rightarrow \pi^0 + n + A'$$

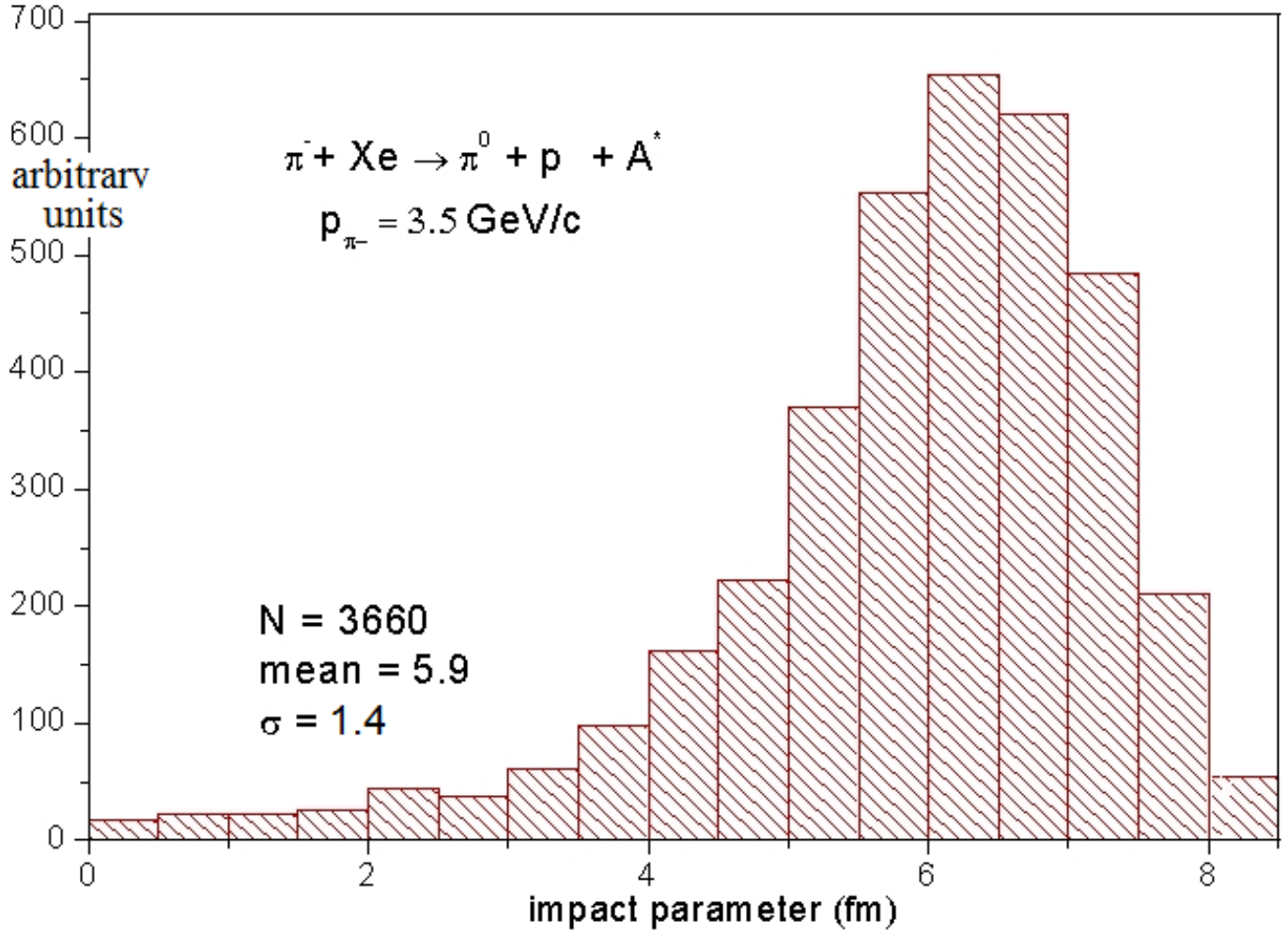
$$\pi^0 + n + (\pi^{+/-}, p) + A'$$

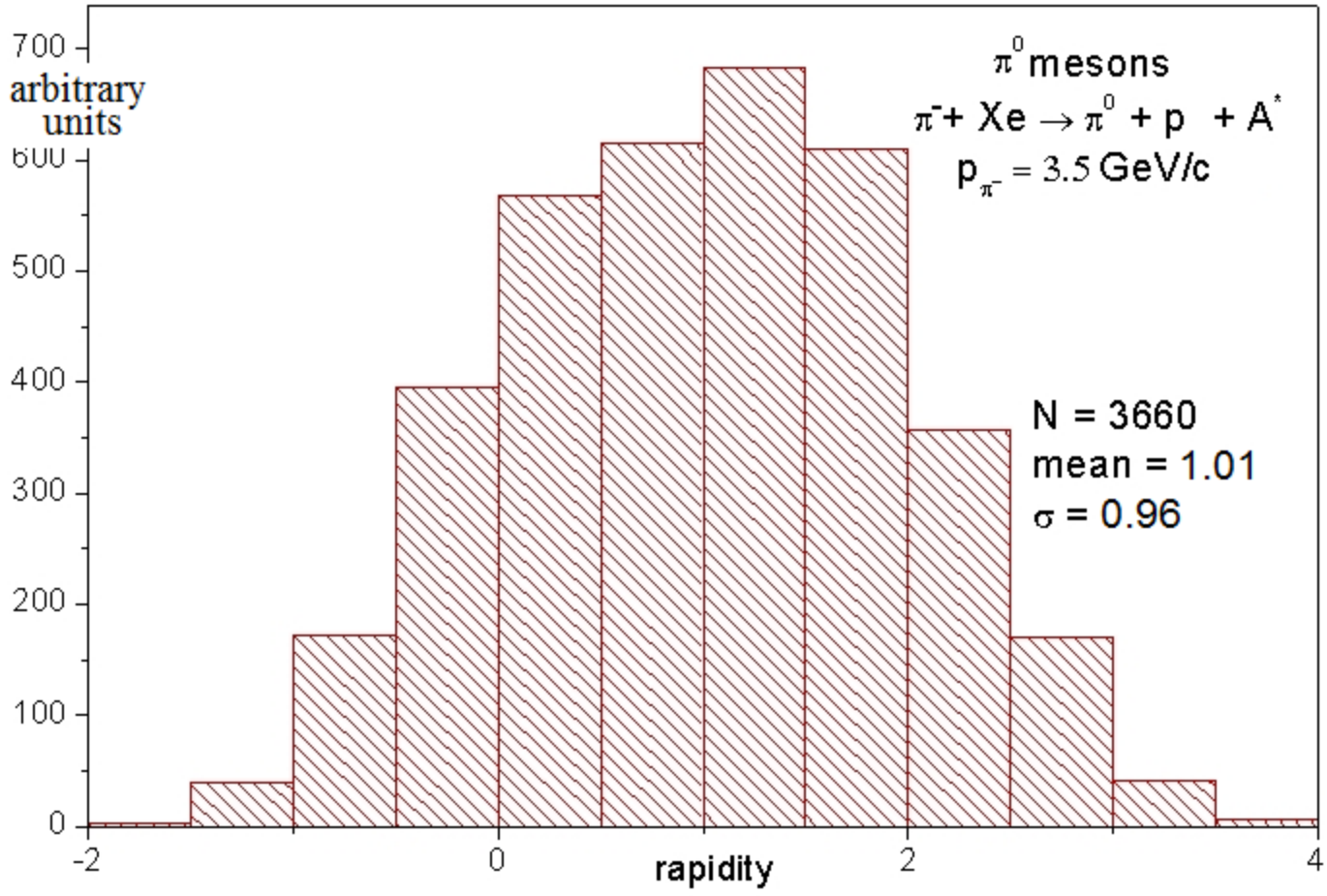
at 3,5 GeV/c

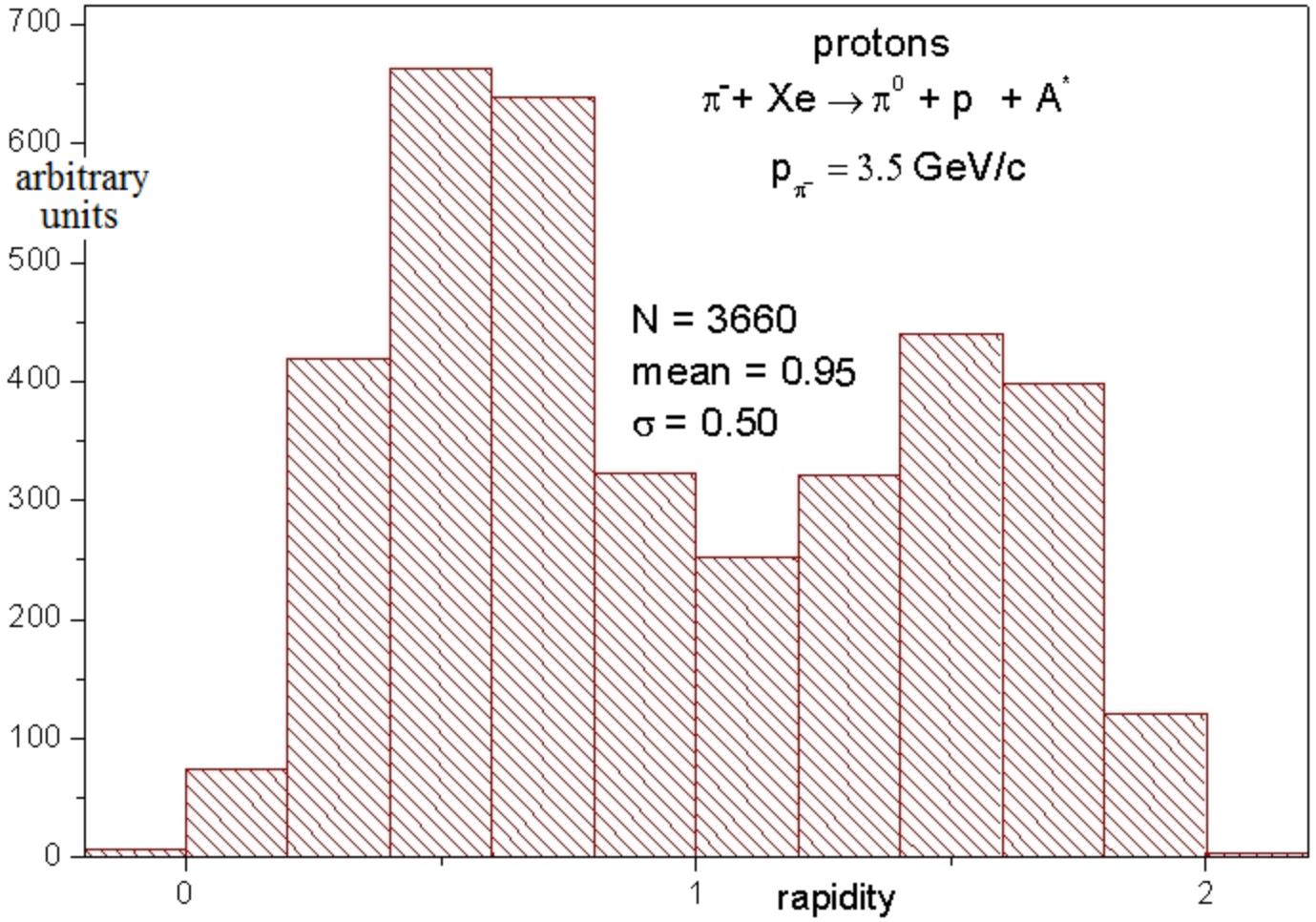

π^0 mesons
$\pi^- + Xe \rightarrow \pi^0 + n + A^*$ $p_{\pi^-} = 3.5$ GeV/c
rapidity
impact parameter (fm)
N = 17479
4
3
2
1
0
-1
-2
-3
0
1
2
3
4
5
6
7
8

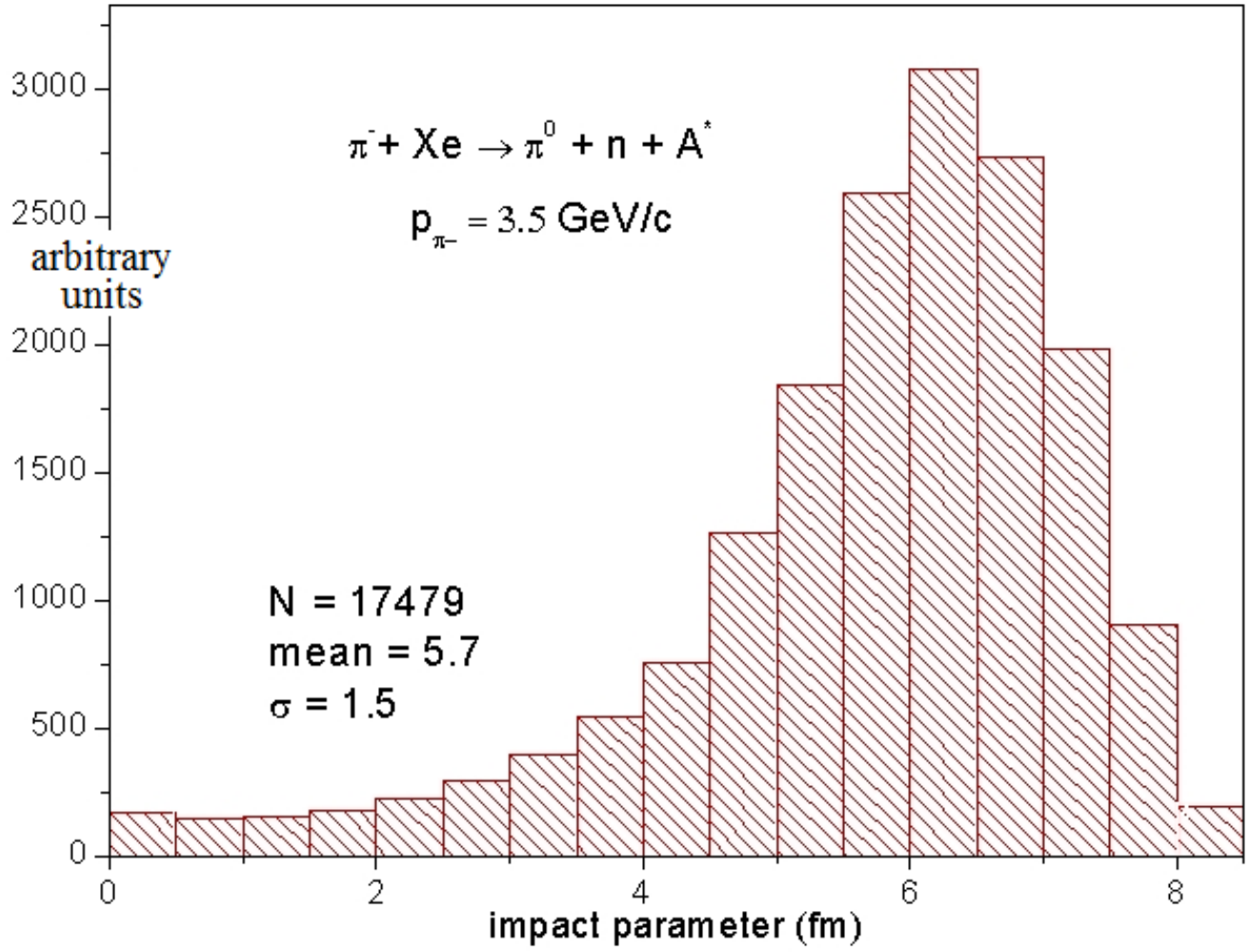

neutrons
$\pi^- + Xe \rightarrow \pi^0 + n + A^*$ $p_{\pi^-} = 3.5$ GeV/c
N = 17479
rapidity
impact parameter (fm)
2,5
2,0
1,5
1,0
0,5
0,0
-0,5
0
1
2
3
4
5
6
7
8

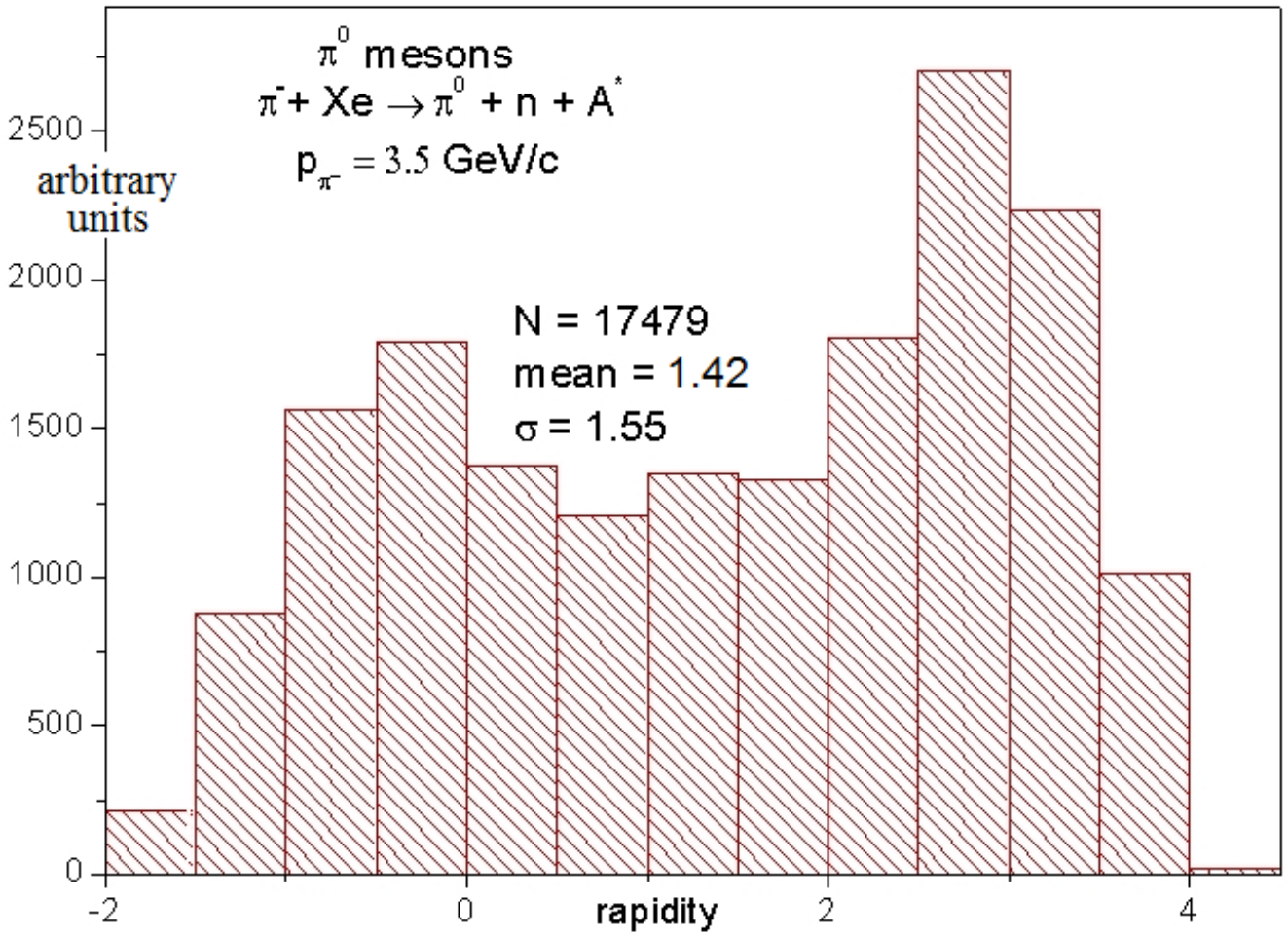
Boundary distributions

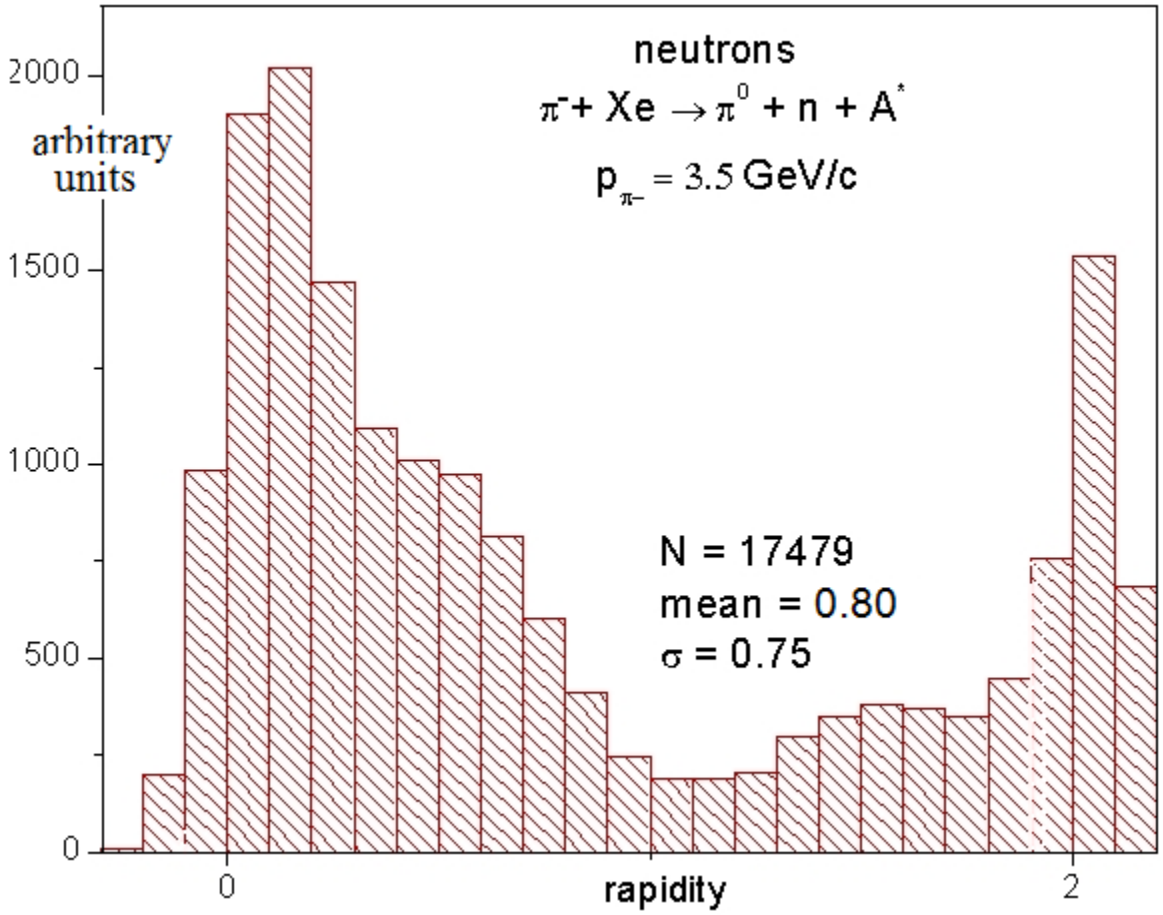

700
600
500
400
300
200
100
0
arbitrary
units
$\pi^- + Xe \rightarrow \pi^0 + p + A^*$
$p_{\pi^-} = 3.5$ GeV/c
N = 3660
mean = 5.9
σ = 1.4
0
2
4
6
8
impact parameter (fm)

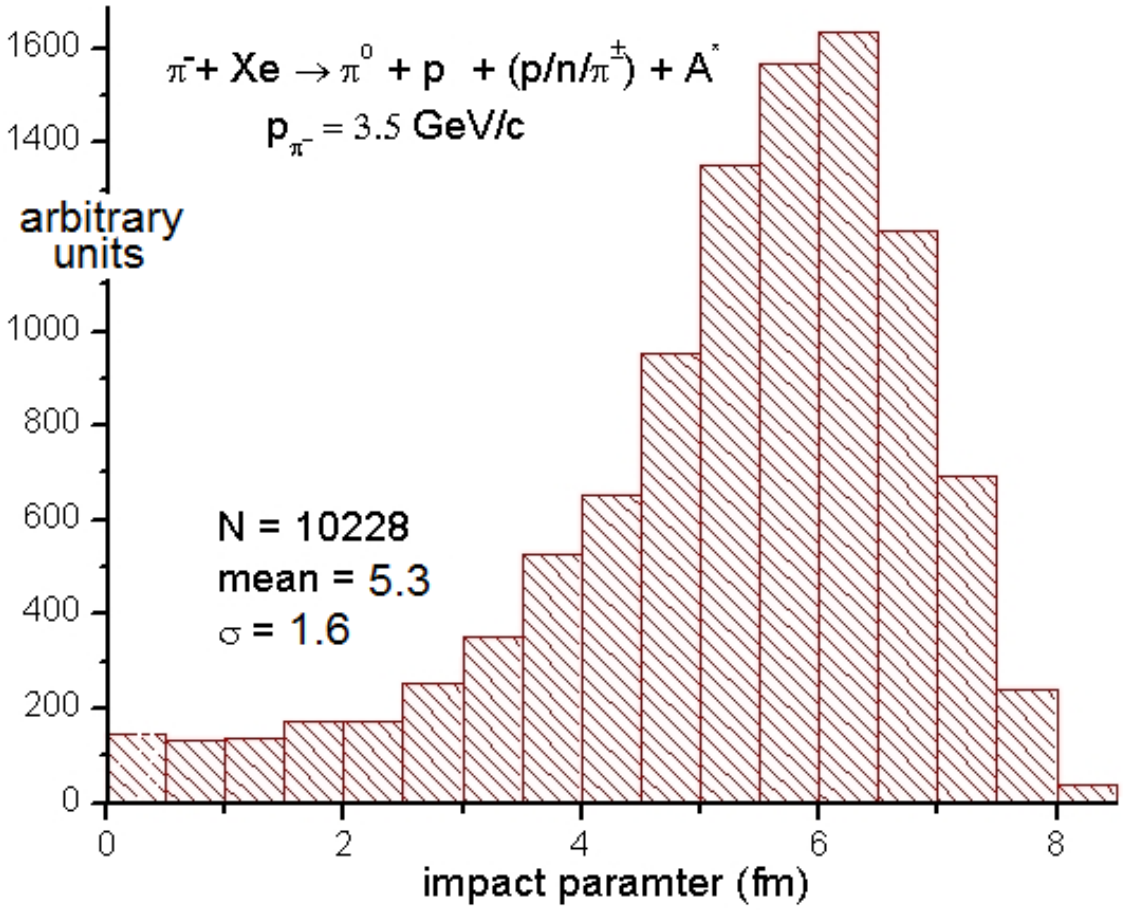

700
arbitrary
units
600
500
400
300
200
100
0
-2
0
2
4
rapidity
π^0 mesons
$\pi^- + Xe \rightarrow \pi^0 + p + A^*$
$p_{\pi^-} = 3.5$ GeV/c
N = 3660
mean = 1.01
σ = 0.96

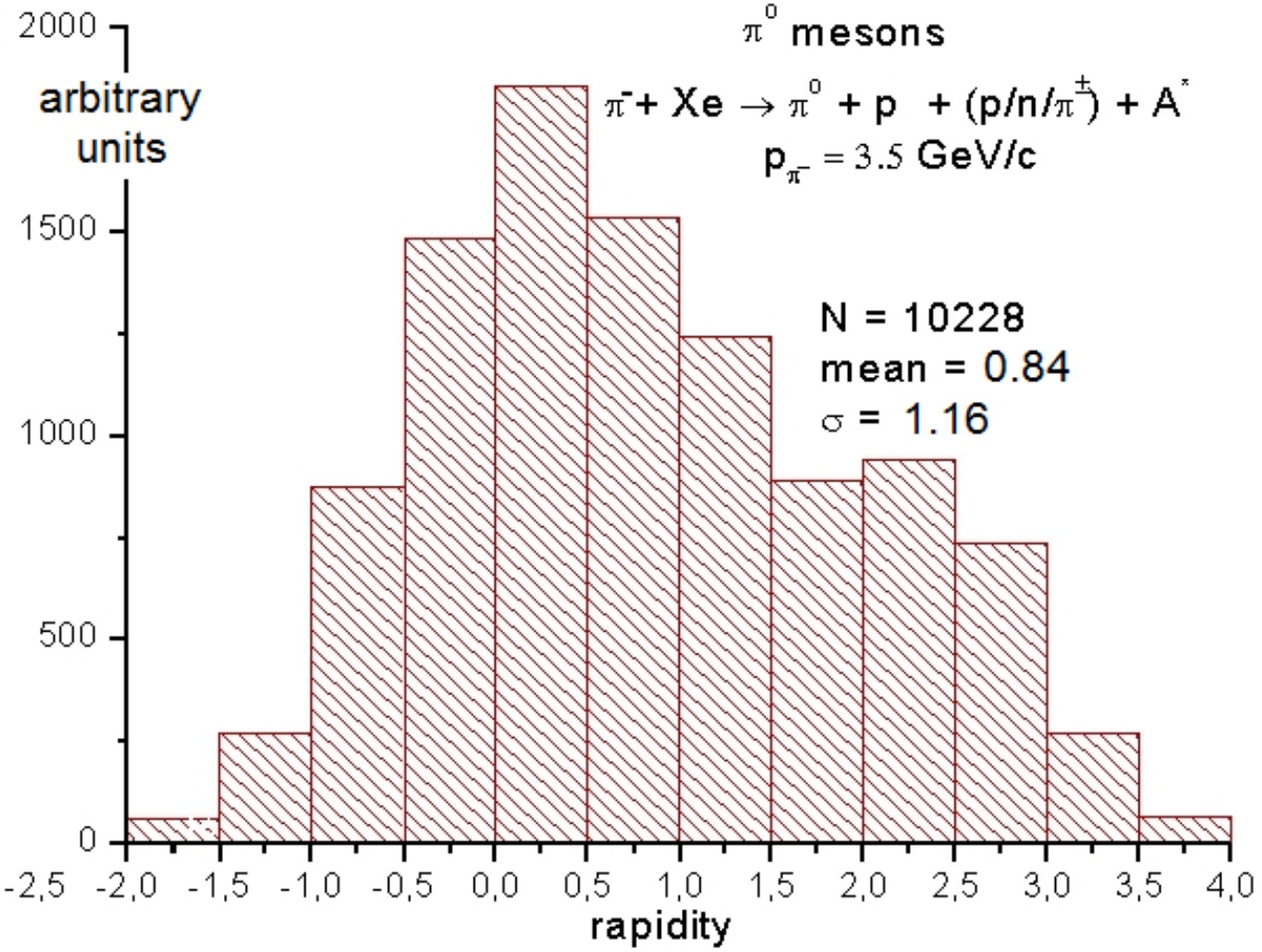

protons
$\pi^- + Xe \rightarrow \pi^0 + p + A^*$
$p_{\pi^-} = 3.5$ GeV/c
N = 3660
mean = 0.95
σ = 0.50
arbitrary units
rapidity
700
600
500
400
300
200
100
0
0
1
2

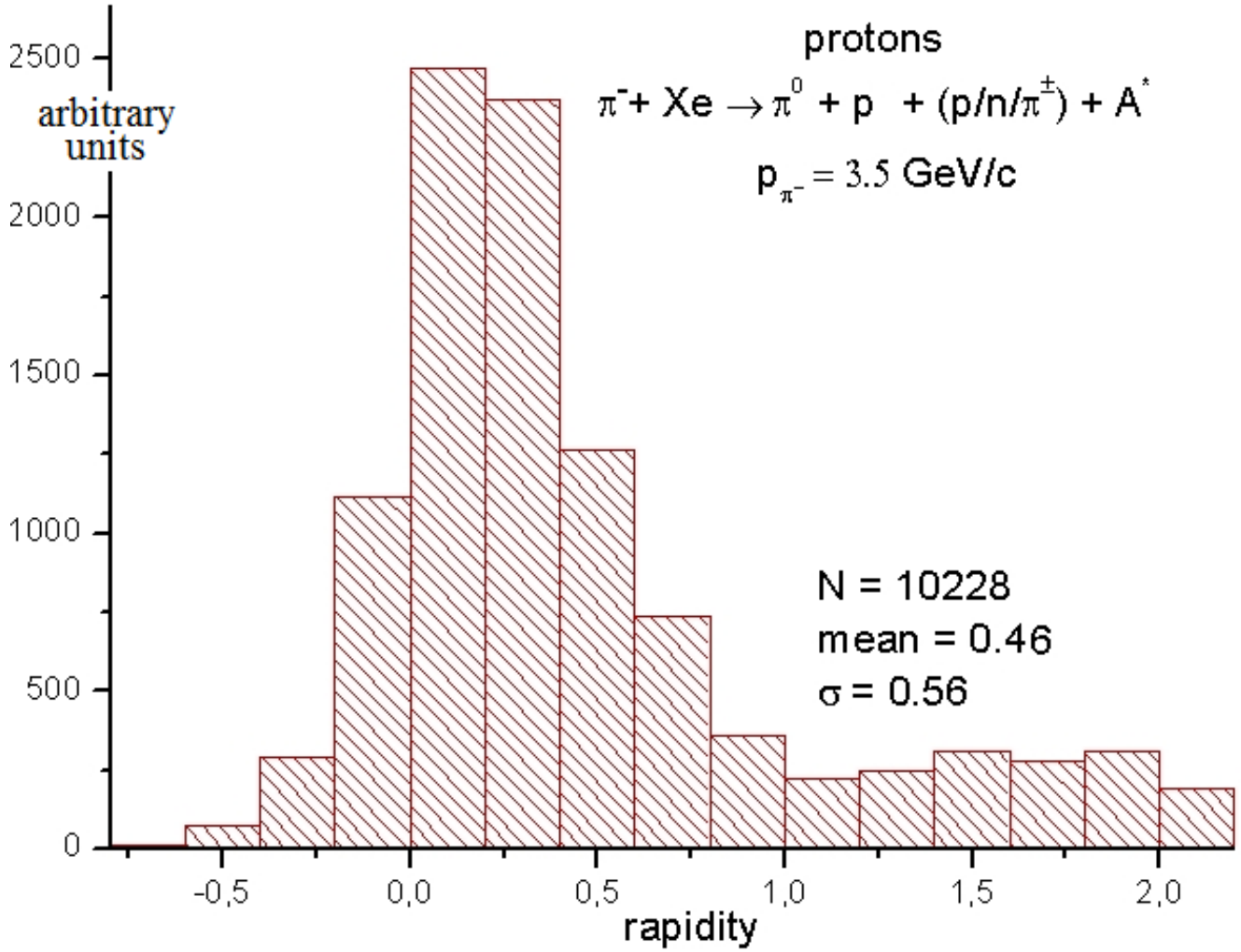

$\pi^- + Xe \rightarrow \pi^0 + n + A^*$
$p_{\pi^-} = 3.5$ GeV/c
N = 17479
mean = 5.7
σ = 1.5
arbitrary units
impact parameter (fm)
0
500
1000
1500
2000
2500
3000
0
2
4
6
8

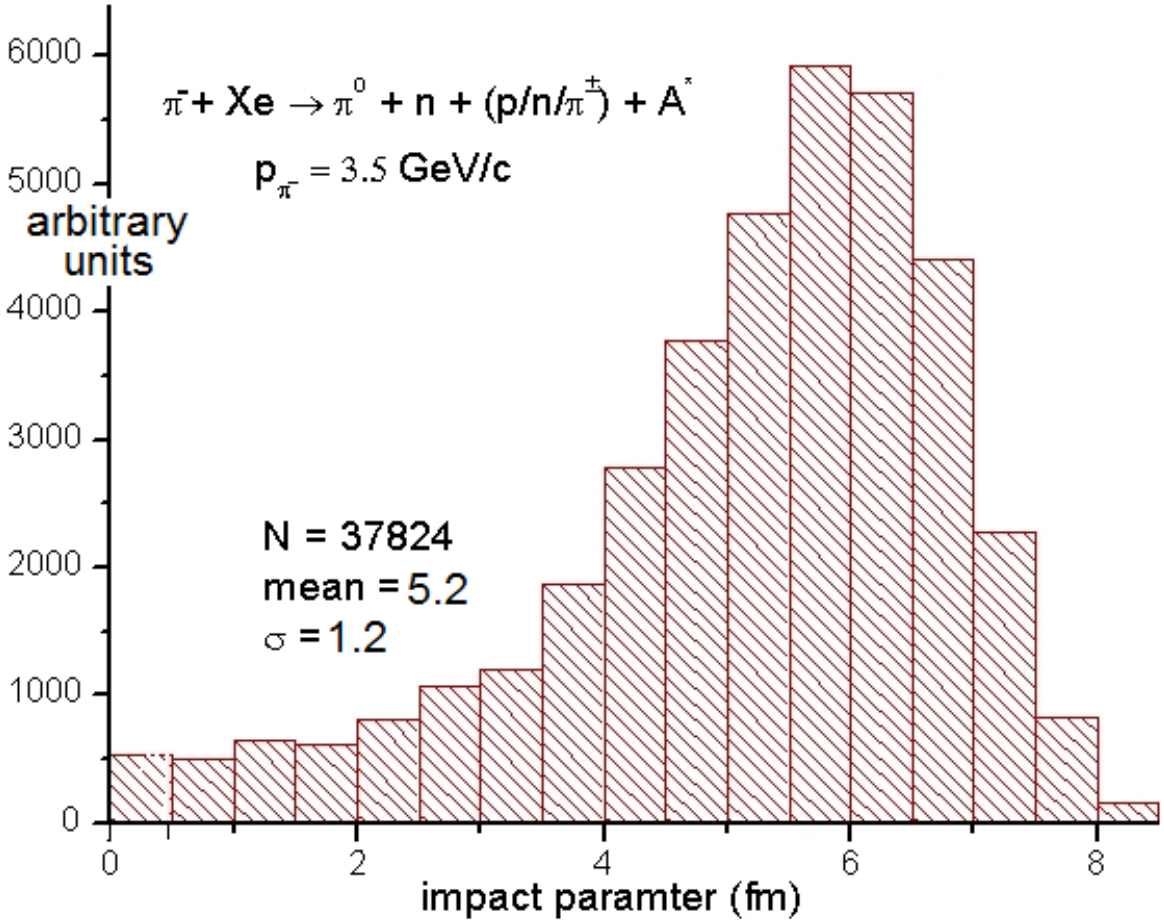

π^0 mesons
$\pi^- + Xe \rightarrow \pi^0 + n + A^*$
$p_{\pi^-} = 3.5$ GeV/c
N = 17479
mean = 1.42
σ = 1.55
arbitrary units
2500
2000
1500
1000
500
0
-2
0
2
4
rapidity

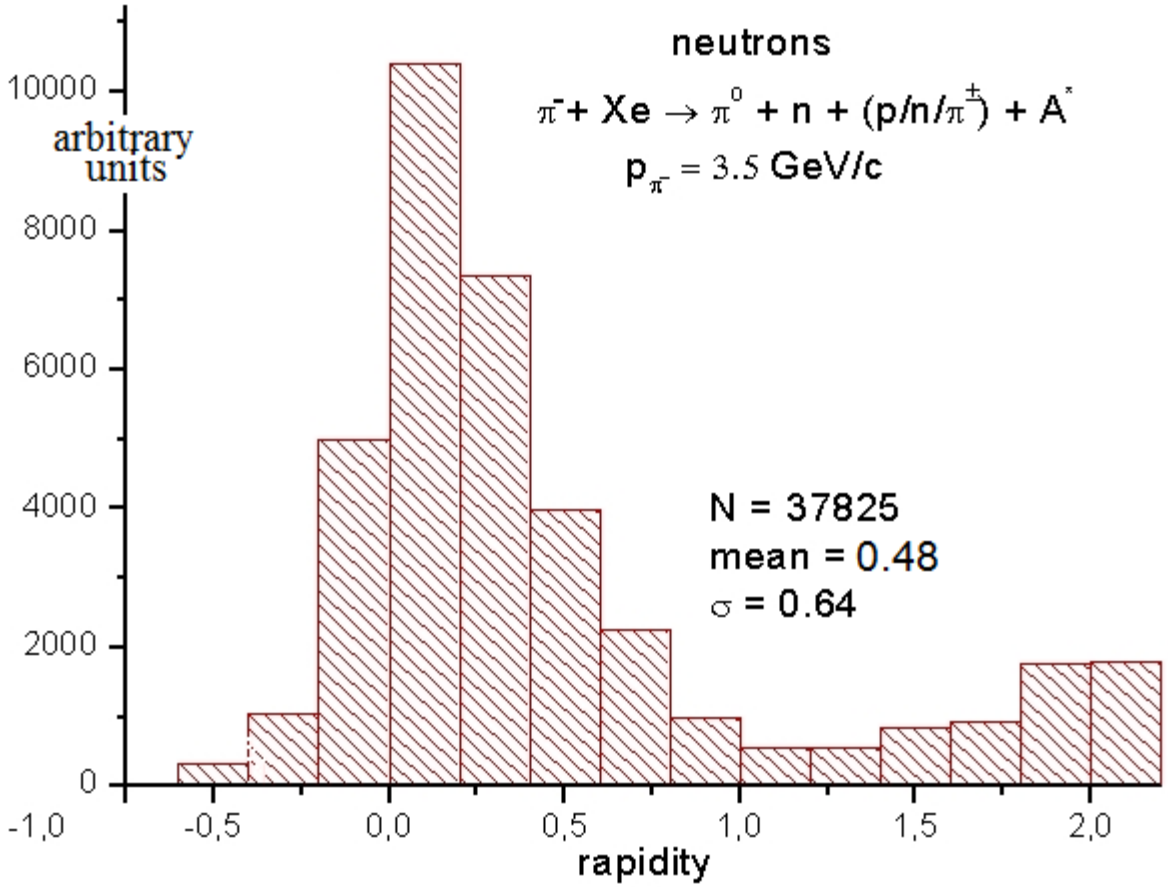

neutrons
$\pi^- + Xe \rightarrow \pi^0 + n + A^*$
$p_{\pi^-} = 3.5$ GeV/c
N = 17479
mean = 0.80
σ = 0.75
arbitrary units
2000
1500
1000
500
0
0
rapidity
2


$\pi^- + Xe \rightarrow \pi^0 + p + (p/n/\pi^{\pm}) + A^*$
$p_{\pi^-} = 3.5$ GeV/c
arbitrary units
N = 10228
mean = 5.3
σ = 1.6
impact paramter (fm)
0
200
400
600
800
1000
1400
1600
0
2
4
6
8


π^0 mesons
$\pi^- + Xe \rightarrow \pi^0 + p + (p/n/\pi^{\pm}) + A^*$
$p_{\pi^-} = 3.5$ GeV/c
N = 10228
mean = 0.84
σ = 1.16
arbitrary units
2000
1500
1000
500
0
-2,5 -2,0 -1,5 -1,0 -0,5 0,0 0,5 1,0 1,5 2,0 2,5 3,0 3,5 4,0
rapidity


protons
$\pi^- + Xe \rightarrow \pi^0 + p + (p/n/\pi^{\pm}) + A^*$
$p_{\pi^-} = 3.5$ GeV/c
N = 10228
mean = 0.46
σ = 0.56
arbitrary units
2500
2000
1500
1000
500
0
-0,5
0,0
0,5
1,0
1,5
2,0
rapidity


$\pi^- + Xe \rightarrow \pi^0 + n + (p/n/\pi^{\pm}) + A^*$
$p_{\pi^-} = 3.5$ GeV/c
arbitrary units
N = 37824
mean = 5.2
σ = 1.2
impact paramter (fm)
0
1000
2000
3000
4000
5000
6000
0
2
4
6
8


neutrons
$\pi^- + Xe \rightarrow \pi^0 + n + (p/n/\pi^{\pm}) + A^*$
$p_{\pi^-} = 3.5$ GeV/c
N = 37825
mean = 0.48
σ = 0.64
arbitrary units
10000
8000
6000
4000
2000
0
-1,0
-0,5
0,0
0,5
1,0
1,5
2,0
rapidity

π^- + Xe at 3.5 GeV/c

$\rightarrow \pi^0$ + p + A'

- impact parameter:
  b = 5.9 fm (5.8 fm)*/
  σ_b = 1.4 fm (1.5 fm)*/

$\rightarrow \pi^0$ + p+ (p/n/$\pi^{+/-}$) +A'

- impact parameter:
  b = 5.3 fm
  σ_b = 1.6 fm

$\rightarrow \pi^0$ + n + A'

impact parameter:
b = 5.7 fm
σ_b = 1.4 fm

$\rightarrow \pi^0$ + n+ (p/n/$\pi^{+/-}$) +A'

impact parameter:
b = 5.2 fm
σ_b = 1.2 fm

$$f(r_0) = \sigma_N \int_{-\sqrt{R^2-r_0^2}}^{\sqrt{R^2-r_0^2}} \rho\left(\sqrt{r_0^2+x^2}\right) \exp\left\{-\left[\sigma_N \int_{-\sqrt{R^2-r_0^2}}^{x} \rho(\sqrt{r_0^2+x^2})dx + \sum_i f_i(r_{0i})\right]\right\} dx$$

*/ π^+ + Xe $\rightarrow \pi^0$ + p + A' at 2.34 GeV/c

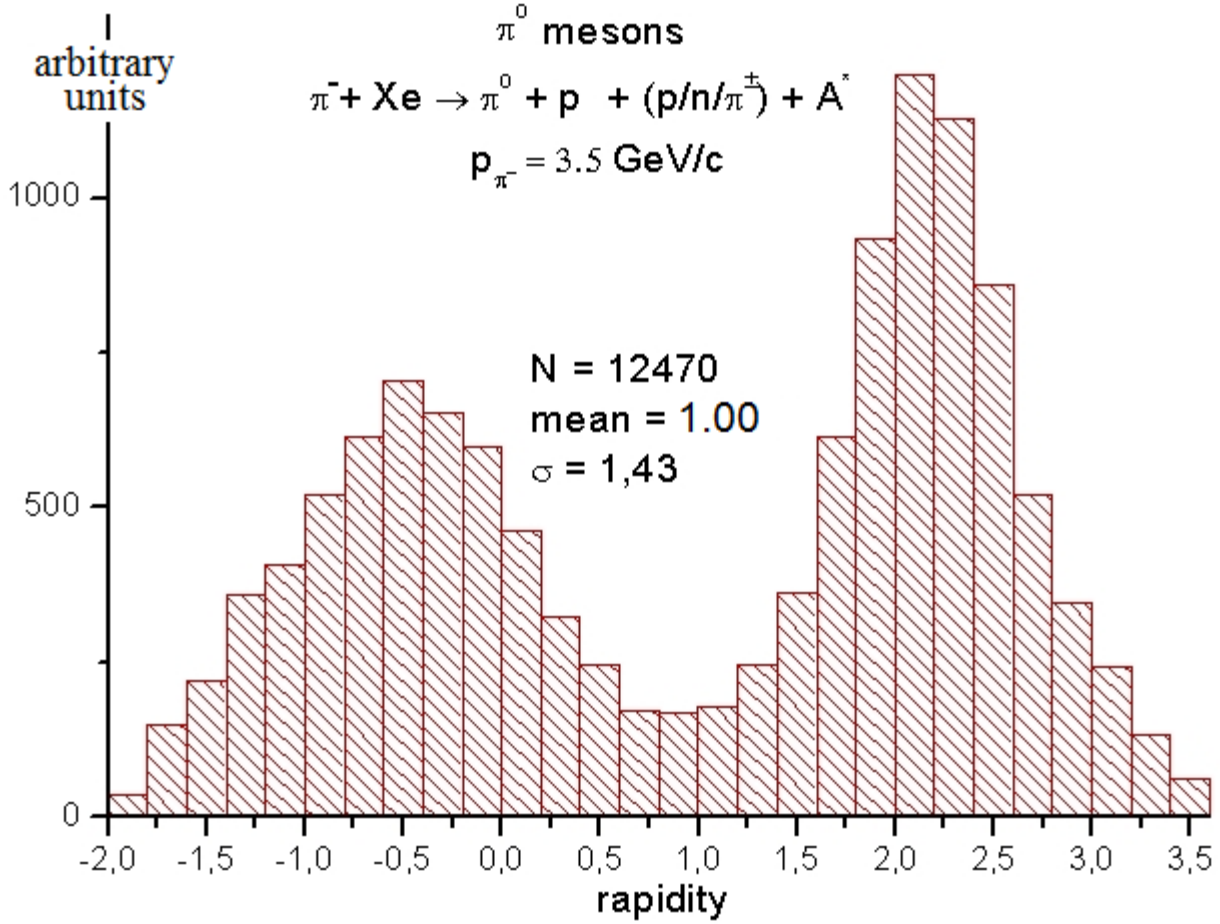
V. Summary and conclusion

1. **Comprehensive analysis of the correlation between multiplicity, rapidity and impact parameter has been done using JAM code by the example of the reaction $\pi^{+/-}$ + Xe $\rightarrow$ pions + protons + neutrons at 2.34, 3.5, 9 and 30 GeV/c.**
2. **The obtained results show that there exists the possibility to derive information about the impact paramter on the basis of the measured multiplicity of different kinds of emitted particles and their rapidity even at moderate energies.**
3. **The above mentioned task may be solved more succesfully if other scaling variables are used, too.**

THANK YOU

FOR

ATTENTION


π^0 mesons
$\pi^- + Xe \rightarrow \pi^0 + p + (p/n/\pi^{\pm}) + A^*$
$p_{\pi^-} = 3.5$ GeV/c
arbitrary units
1000
500
0
N = 12470
mean = 1.00
σ = 1,43
-2,0
-1,5
-1,0
-0,5
0,0
0,5
1,0
1,5
2,0
2,5
3,0
3,5
rapidity