Spin-tensor Effects at Channelling and a Potential for Measuring Quadrupole Moments of Unstable Nuclei

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Overview

Quadrupole moments of unstable nuclei can be measured by optical methods

Restriction for the lifetime of nuclei:

$$t > 10^{-6}\ \mathrm{s}$$

see **Yu. P. Gangrsky, Soros Educ. J. 6,** *N 8, 93 (2000).*

Another method:

Spin rotation and oscillation of nuclei *(I≥1)* at channeling in straight and bent crystals

Second-order spin interactions transform vector and tensor polarizations each to other

straight crystals

V. G. Baryshevsky and A. A. Sokolsky, Pis'ma v Zs. Tekh. Fiz. 6, *N 23, 1419 (1980).*

We consider the spin dynamics in bent crystals because their use allows to avoid hitting quasi-channeled and dechanneled nuclei into a polarimeter

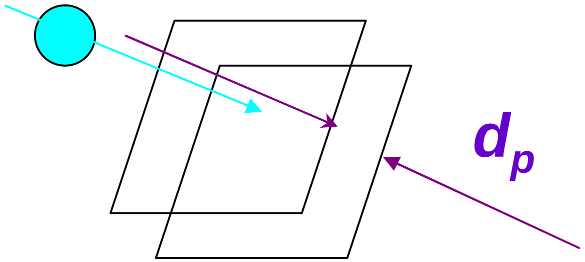
We suppose that the use of planar channeling in crystals permits to measure quadrupole moments of unstable nuclei with the lifetime of

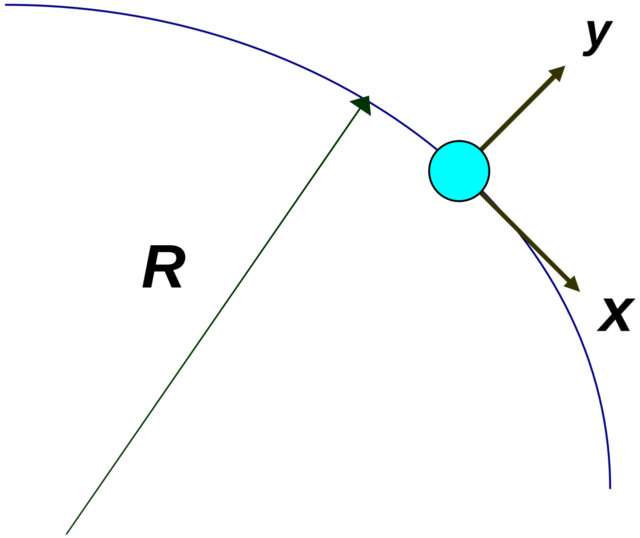
$$t \geq 10^{-7} \text{ s}$$

Basic equations

For positive-charged nuclei, the harmonic potential can be used:

$$\Phi(x) = \frac{ax^2}{2}, \quad a = \frac{8U_0}{d_p^2}$$





Nonrelativistic nuclei

Operator of quadrupole nomentum tensor:

$$Q_{ij} = \frac{3Q}{2I(2I-1)}\left[I_i I_j + I_j I_i - \frac{2}{3} I(I+1) d_{ij} \right]$$

spin

Interaction energy:

$$V_q = \frac{1}{6} Q_{xx} \frac{\partial^2 \Phi(x)}{\partial x^2} = \frac{a}{6} Q_{xx}$$

Its spin-dependent part:

$$V = \frac{aQ}{2I(2I-1)} I_x^2$$

Resulting expression for the Hamiltonian:

$$H = H_0 + I_z (w_a)_z + \frac{aQ}{2I(2I-1)} I_x^2$$

Polarization vector and polarizatiion tensor:

$$\mathbf{P} = \frac{\langle \mathbf{I} \rangle}{I}, \quad P_{ij} = \frac{3\left\langle I_i I_j + I_j I_i \right\rangle - 2I(I+1) d_{ij}}{2I(2I-1)}$$

g-2 frequency:

$$(w_a)_z = \frac{a(g^2-1)^{3/2}}{g^2 R} \left(\frac{g-2}{2} - \frac{1}{g^2-1} \right)$$

We use the matrix Hamiltonian for determining the evolution of the spin wave function:

$$i\frac{d\Psi}{dt} = \mathrm{H}\Psi, \qquad \Psi = \begin{pmatrix} C_1(t) \\ C_0(t) \\ C_{-1}(t) \end{pmatrix}, \qquad \mathrm{H}_{ij} = < i \mid H \mid j > .$$

Initial matrix Hamiltonian:

$$\mathrm{H} = \begin{pmatrix} E_0 + w_0 + \mathrm{A} & 0 & \mathrm{A} \\ 0 & E_0 + 2\mathrm{A} & 0 \\ \mathrm{A} & 0 & E_0 - w_0 + \mathrm{A} \end{pmatrix},$$

$$w_0 \equiv (w_a)_z, \qquad \mathrm{A} = \frac{aQ}{4I(2I-1)}$$

Spin dynamics of nuclei

The polarization vector and tensor:

$$P_i = \frac{\langle S_i \rangle}{S}, \quad P_{ij} = \frac{3\langle S_i S_j + S_j S_i \rangle - 2S(S+1)\delta_{ij}}{2S(2S-1)},$$

The spin matrices:

$$S_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_y = \frac{i}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$S_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

Connection between the polarization vector and tensor and the spin amplitudes:

$$P_\rho = \frac{1}{\sqrt{2}}(C_1C_0^* + C_1^*C_0 + C_0C_{-1}^* + C_0^*C_{-1}),$$

$$P_\phi = \frac{i}{\sqrt{2}}(C_1C_0^* - C_1^*C_0 + C_0C_{-1}^* - C_0^*C_{-1}),$$

$$P_z = (C_1C_1^* - C_{-1}C_{-1}^*),$$

$$P_{\rho\rho} = \frac{3}{2}(C_1C_{-1}^* + C_1^*C_{-1} + C_0C_0^*) - \frac{1}{2},$$

$$P_{\phi\phi} = -\frac{3}{2}(C_1C_{-1}^* + C_1^*C_{-1} - C_0C_0^*) - \frac{1}{2},$$

$$P_{\rho\phi} = i\frac{3}{2}(C_1C_{-1}^* - C_1^*C_{-1}).$$

When the direction of the initial tensor polarization is defined by the spherical angles ***θ*** and ***ψ***,

$$\mathbf{P}(0) = 0, \quad P_{xx}(0) = 1 - 3\sin^2 q \cos^2 y \ ,$$

$$P_{yy}(0) = 1 - 3\sin^2 q \sin^2 y \ , \quad P_{zz}(0) = 1 - 3\cos^2 q \, ,$$

$$P_{xy}(0) = -\frac{3}{2}\sin^2 q \sin 2y \ , \quad P_{xz}(0) = -\frac{3}{2}\sin 2q \cos y \ ,$$

$$P_{yz}(0) = -\frac{3}{2}\sin 2q \sin y \ .$$

Initial tensor polarization is the most favorable

Final polarization:

$$P_x(t) = \sin 2q \left\{ -\left[\cos(w't)\sin y + \frac{w_0}{w'}\sin(w't)\cos y \right]\sin(\mathrm{A}t) + \frac{\mathrm{A}}{w'}\sin(w't)\cos(\mathrm{A}t)\sin y \right\},$$

$$P_y(t) = \sin 2q \left\{ \left[\cos(w't)\cos y - \frac{w_0}{w'}\sin(w't)\sin y \right]\sin(\mathrm{A}t) + \frac{\mathrm{A}}{w'}\sin(w't)\cos(\mathrm{A}t)\cos y \right\},$$

$$P_z(t) = -\frac{2A}{w'}\sin^2 q \sin(w't) \times \left[\cos(w't)\sin(2y) + \frac{w_0}{w'}\sin(w't)\cos(2y) \right].$$

When the direction of the initial vector polarization is defined by the spherical angles *θ* and *ψ*,

$$\boldsymbol{P}(0) = \sin\theta\cos\psi\,\boldsymbol{e}_\rho + \sin\theta\sin\psi\,\boldsymbol{e}_\phi + \cos\theta\,\boldsymbol{e}_z$$

$$P_{\rho\rho} = \tfrac{1}{2}\left[3\sin^2(\theta)\cos^2(\psi) - 1\right],$$

$$P_{\phi\phi} = \tfrac{1}{2}\left[3\sin^2(\theta)\sin^2(\psi) - 1\right], \quad P_{\rho\phi} = \tfrac{3}{4}\sin^2(\theta)\sin(2\psi)$$

Final vertical polarization:

$$P_z(t) = \left[1 - \frac{2A^2}{w'^2}\sin^2(w't)\right]\cos q$$

$$+\frac{A}{w'}\sin^2 q\,\sin(w't)\left[\cos(w't)\sin(2y) + \frac{w_0}{w'}\sin(w't)\cos(2y)\right].$$

For straight crystals, the formulas derived coincide with those obtained by Baryshevsky and Sokolsky

Vertical polarization is caused by the quadrupole moment. For the initial tensor polarization, the spin-tensor interaction is the only reason of the final vector polarization

The effect of appearance of a vector polarization of a tensor-polarized nuclear beam can be discovered!

Conclusions

charge density

$$a = \frac{\partial^2 \Phi(x)}{\partial x^2} \sim r(x)$$

a **is very different for channeled, quasi-channeled and dechanneled nuclei**

The use of bent crystals allows to avoid hitting quasi-channeled and dechanneled nuclei into a polarimeter. Excluding such nuclei significantly decrease a systematical error.

velocity of nuclei $v \sim 10^7$ m/s, $l \sim 1$ m $\Rightarrow$ $t \geq 10^{-7}$ s!

Quadrupole moments of unstable nuclei up to 10^{-29} m^2 (0.1 b) can be measured

Summary

- **Spin dynamics of polarized beams in bent crystals caused by electric quadrupole moments of nuclei is calculated by matrix method**
- **The results agree with those by Baryshevsky and Sokolsky**
- **Channeling of polarized beams in bent crystals can be successfully used for measuring quadrupole moments of unstable nuclei with**

$$t \geq 10^{-7}\ \mathrm{s}$$

- **The effect of appearance of a vector polarization of a tensor-polarized nuclear beam can be discovered**
- **Quadrupole moments of unstable nuclei up to 10^{-29} m^2 (0.1 b) can be measured**

Thank you for attention