# RELATIVISTIC INTERACTIONS FOR MESON-NUCLEON SYSTEMS IN THE CLOTHED-PARTICLE REPRESENTATION 

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## Some Recollections

> We will now prove the fundamental theorem: any operator $O$ may be expressed as a sum of products of creation and annihilation operators ...
> S. Weinberg
> Quantum Theory of Fields, Vol. I, 1995, p. 175 .

In accordance with the motto each of ten generators of the Poincaré group $\Pi$ may be expressed as a sum of products of creation and annihilation operators $a^{\dagger}(n)$ and $a(n)$ ( $n=1,2, \ldots$ ) for free particles, e.g., bosons and/or fermions.

In the framework of such a corpuscular picture Hamiltonian of a system of interacting mesons and nucleons can be written as

$$
H=\sum_{C=0}^{\infty} \sum_{A=0}^{\infty} H_{C A},
$$

$$
H_{C A}=£ H_{C A}\left(1^{\prime}, 2^{\prime}, \ldots, n_{C}^{\prime} ; 1,2, \ldots, n_{A}\right) a^{\dagger}\left(1^{\prime}\right) a^{\dagger}\left(2^{\prime}\right) \ldots a^{\dagger}\left(n_{C}^{\prime}\right) a\left(n_{A}\right) \ldots a(2) a(1),
$$

$C(A)$ - particle-creation (annihilation) number for operator substructure $H_{C A}$ and

$$
\begin{aligned}
& H_{C A}\left(1^{\prime}, 2^{\prime}, \ldots, C ; 1,2, \ldots, A\right)=\delta\left(\vec{p}_{1}^{\prime}+\vec{p}_{2}^{\prime}+\ldots+\vec{p}_{C}^{\prime}-\vec{p}_{1}-\vec{p}_{2}-\ldots-\vec{p}_{A}\right) \\
& \times h_{C A}\left(p_{1}^{\prime} \mu_{1}^{\prime} \xi_{1}^{\prime}, p_{2}^{\prime} \mu_{2}^{\prime} \xi_{2}^{\prime}, \ldots, p_{C}^{\prime} \mu_{C}^{\prime} \xi_{C}^{\prime} ; p_{1} \mu_{1} \xi_{1}, p_{2} \mu_{2} \xi_{2}, \ldots, p_{A} \mu_{A} \xi_{A}\right),
\end{aligned}
$$

$c$-number coefficients $h_{C A}$ do not contain! delta function.
"To free ourselves from any dependence on pre-existing field theories" (after S. Weinberg), boost operators $\vec{N}=\left(N^{1}, N^{2}, N^{3}\right)$

$$
\vec{N}=\sum_{C=0}^{\infty} \sum_{A=0}^{\infty} \vec{N}_{C A},
$$

$$
\vec{N}_{C A}=\mathcal{\sum} \vec{N}_{C A}\left(1^{\prime}, 2^{\prime}, \ldots, n_{C}^{\prime} ; 1,2, \ldots, n_{A}\right) a^{\dagger}\left(1^{\prime}\right) a^{\dagger}\left(2^{\prime}\right) \ldots a^{\dagger}\left(n_{C}^{\prime}\right) a\left(n_{A}\right) \ldots a(2) a(1)
$$

one of our purposes is to find some links between coefficients $H_{C A}$ and $\vec{N}_{C A}$, compatible with commutations

$$
\begin{gathered}
{\left[P_{i}, P_{j}\right]=0, \quad\left[J_{i}, J_{j}\right]=i \varepsilon_{i j k} J_{k}, \quad\left[J_{i}, P_{j}\right]=i \varepsilon_{i j k} P_{k},} \\
{[\vec{P}, H]=0, \quad[\vec{J}, H]=0, \quad\left[J_{i}, N_{j}\right]=i \varepsilon_{i j k} N_{k}, \quad\left[P_{i}, N_{j}\right]=i \delta_{i j} H,} \\
{[H, \vec{N}]=i \vec{P}, \quad\left[N_{i}, N_{j}\right]=-i \varepsilon_{i j k} J_{k},} \\
(i, j, k=1,2,3),
\end{gathered}
$$

$\vec{P}=\left(P^{1}, P^{2}, P^{3}\right)$ and $\vec{J}=\left(J^{1}, J^{2}, J^{3}\right)$ linear and angular momentum operators. For instant form of relativistic dynamics after Dirac only Hamiltonian and boost operators carry interactions,

$$
\begin{aligned}
& H=H_{F}+H_{l} \\
& \vec{N}=\vec{N}_{F}+\vec{N}_{l}
\end{aligned}
$$

while $\vec{P}=\vec{P}_{F}$ and $\vec{J}=\vec{J}_{F}$.

In turn,
with density

$$
H_{C A}=\int H_{C A}(\vec{x}) d \vec{x} \text { so } H=\int H(\vec{x}) d \vec{x}
$$

$$
H(\vec{x})=\sum_{C=0}^{\infty} \sum_{A=0}^{\infty} H_{C A}(\vec{x})
$$

For instance, in case with $C=A=2$,

$$
\begin{gathered}
H_{22}\left(1^{\prime}, 2^{\prime} ; 1,2\right)=\delta\left(\vec{p}_{1}^{\prime}+\vec{p}_{2}^{\prime}-\vec{p}_{1}-\vec{p}_{2}\right) h\left(1^{\prime} 2^{\prime} ; 12\right) \\
H_{22}(\vec{x})=\frac{1}{(2 \pi)^{3}} \sum \exp \left[-i\left(\vec{p}_{1}^{\prime}+\vec{p}_{2}^{\prime}-\vec{p}_{1}-\vec{p}_{2}\right) \vec{x}\right] h\left(1^{\prime} 2^{\prime} ; 12\right) a^{\dagger}\left(1^{\prime}\right) a^{\dagger}\left(2^{\prime}\right) a(2) a(1) .
\end{gathered}
$$

As usually $a(n)=a\left(\vec{p}_{n}, \mu_{n}, \xi_{n}\right)$. Further, transformation properties with respect to $\Pi$ in case of massive particle with spin $j$ :

$$
U_{F}(\Lambda, b) a^{\dagger}(p, \mu) U_{F}^{-1}(\Lambda, b)=e^{i \Lambda \rho b} D_{\mu^{\prime} \mu}^{(j)}(W(\Lambda, p)) a^{\dagger}\left(\Lambda p, \mu^{\prime}\right),
$$

$\forall \Lambda \in L_{+}$and arbitrary spacetime shifts $b=\left(b^{0}, \vec{b}\right)$,
with $D$-function whose argument is Wigner rotation $W(\Lambda, p), L_{+}$the homogeneous (proper) orthochronous Lorentz group, $(\Lambda, b) \rightarrow U_{F}(\Lambda, b)$ unitary irreducible representation of $\Pi$ in Hilbert space, e.g. hardronic states, for operators $a(p, \mu)=a(\vec{p}, \mu) \sqrt{p_{0}}$ that meet covariant commutation relations

$$
\begin{aligned}
{\left[a\left(p^{\prime}, \mu^{\prime}\right), a^{\dagger}(p, \mu)\right]_{ \pm} } & =p_{0} \delta\left(\vec{p}-\vec{p}^{\prime}\right) \delta_{\mu^{\prime} \mu} \\
{\left[a\left(p^{\prime}, \mu^{\prime}\right), a(p, \mu)\right]_{ \pm} } & =\left[a^{\dagger}\left(p^{\prime}, \mu^{\prime}\right), a^{\dagger}(p, \mu)\right]_{ \pm}=0 .
\end{aligned}
$$

Here $p_{0}=\sqrt{\vec{p}^{2}+m^{2}}$ is fourth component of 4-momentum $p=\left(p_{0}, \vec{p}\right)$.

Often one has to deal with field models where in Dirac (D) picture

$$
U_{F}(\Lambda, b) H_{l}(x) U_{F}^{-1}(\Lambda, b)=H_{l}(\Lambda x+b), \quad \forall x=(t, \vec{x}) .
$$

For interaction density

$$
H_{22}(x)=\frac{1}{(2 \pi)^{3}}\left\{\exp \left[i\left(p_{1}^{\prime}+p_{2}^{\prime}-p_{1}-p_{2}\right) x\right] \times h\left(1^{\prime} 2^{\prime} ; 12\right) a^{\dagger}\left(1^{\prime}\right) a^{\dagger}\left(2^{\prime}\right) a(2) a(1)\right.
$$

it means

$$
\begin{aligned}
D_{\eta_{1}^{\prime} \mu_{1}^{\prime}}^{\left(j_{1}^{\prime}\right)}\left(W\left(\Lambda, p_{1}^{\prime}\right)\right) D_{\eta_{2}^{\prime} \mu_{2}^{\prime}}^{\left(j_{2}^{\prime}\right)} & \left(W\left(\Lambda, p_{2}^{\prime}\right)\right) D_{\eta_{1} \mu_{1}}^{\left(j_{1}\right) *}\left(W\left(\Lambda, p_{1}\right)\right) D_{\eta_{2} \mu_{2}}^{\left(j_{2}\right) *}\left(W\left(\Lambda, p_{2}\right)\right) \\
& \times h\left(p_{1}^{\prime} \mu_{1}^{\prime}, p_{2}^{\prime} \mu_{2}^{\prime} ; p_{1} \mu_{1}, p_{2} \mu_{2}\right)=h\left(\Lambda p_{1}^{\prime} \eta_{1}^{\prime}, \Lambda p_{2}^{\prime} \eta_{2}^{\prime} ; \Lambda p_{1} \eta_{1}, \Lambda p_{2} \eta_{2}\right) .
\end{aligned}
$$

Of course, summations over all dummy labels are implied.
After these preliminaries we will show how one can build up interaction parts in Hamiltonian and boosts.

Recall that angular momentum $\vec{J}=\vec{J}_{F}=\vec{J}_{\pi}+\vec{J}_{\text {ferm }}$ with

$$
\vec{J}_{\pi}=\frac{i}{2} \int d \vec{k} \vec{k} \times\left(\frac{\partial a^{\dagger}(\vec{k})}{\partial \vec{k}} a(\vec{k})-a^{\dagger}(\vec{k}) \frac{\partial a(\vec{k})}{\partial \vec{k}}\right)
$$

and $\vec{J}_{\text {ferm }}=\vec{L}_{\text {ferm }}+\vec{S}_{\text {ferm }}$, where

$$
\begin{aligned}
\vec{L}_{\text {ferm }}= & \frac{i}{2} £ d \vec{p} \vec{p} \times\left(\frac{\partial b^{\dagger}(\vec{p} \mu)}{\partial \vec{p}} b(\vec{p} \mu)-b^{\dagger}(\vec{p} \mu) \frac{\partial b(\vec{p} \mu)}{\partial \vec{p}}\right. \\
& \left.+\frac{\partial d^{\dagger}(\vec{p} \mu)}{\partial \vec{p}} d(\vec{p} \mu)-d^{\dagger}(\vec{p} \mu) \frac{\partial d(\vec{p} \mu)}{\partial \vec{p}}\right), \\
& \vec{S}_{\text {ferm }}=\frac{1}{2}\left\{d \vec{p} \chi^{\dagger}\left(\mu^{\prime}\right) \vec{\sigma} \chi(\mu)\left(b^{\dagger}\left(\vec{p} \mu^{\prime}\right) b(\vec{p} \mu)-d^{\dagger}\left(\vec{p} \mu^{\prime}\right) d(\vec{p} \mu)\right),\right.
\end{aligned}
$$

boosts $\vec{N}_{F}=\vec{N}_{\pi}+\vec{N}_{\text {ferm }}$ with

$$
\vec{N}_{\pi}=\frac{i}{2} \int d \vec{k} \omega_{\vec{k}}\left(\frac{\partial a^{\dagger}(\vec{k})}{\partial \vec{k}} a(\vec{k})-a^{\dagger}(\vec{k}) \frac{\partial a(\vec{k})}{\partial \vec{k}}\right)
$$

and $\vec{N}_{\text {ferm }}=\vec{N}_{\text {ferm }}^{\text {orb }}+\vec{N}_{\text {ferm }}^{\text {spin }}$, where

$$
\begin{aligned}
& \vec{N}_{\text {ferm }}^{\text {reb }}= \frac{i}{2} \sum d \vec{p} E_{\vec{p}}\left(\frac{\partial b^{\dagger}(\vec{p} \mu)}{\partial \vec{p}} b(\vec{p} \mu)-b^{\dagger}(\vec{p} \mu) \frac{\partial b(\vec{p} \mu)}{\partial \vec{p}}\right. \\
&\left.+\frac{\partial d^{\dagger}(\vec{p} \mu)}{\partial \vec{p}} d(\vec{p} \mu)-d^{\dagger}(\vec{p} \mu) \frac{\partial d(\vec{p} \mu)}{\partial \vec{p}}\right), \\
& \vec{N}_{\text {ferm }}^{\text {spin }}=-\frac{1}{2} £ d \vec{p} \vec{p} \times \frac{\chi^{\dagger}(\mu) \vec{\sigma} \chi(\mu)}{E_{\vec{p}}+m}\left(b^{\dagger}(\vec{p} \mu) b(\vec{p} \mu)+d^{\dagger}(\vec{p} \mu) d(\vec{p} \mu)\right),
\end{aligned}
$$

$\omega_{\vec{k}}=\sqrt{\overrightarrow{k^{2}}+m_{\pi}^{2}}\left(E_{\vec{p}}=\sqrt{\vec{p}^{2}+m^{2}}\right)$ pion (nucleon) energy and $\chi(\mu)$ Pauli spinor.

## Clothed Particle Representation (CPR) of Hamiltonian and Other

 Generators of the Poincaré GroupAt this point, one can address the so-called Belinfante ansatz

$$
\vec{N}_{\text {oel }}=-\int \vec{x} H(\vec{x}) d \vec{x}
$$

which is helpful for a simultaneous blockdiagonalization of Hamiltonian and boost $[2,3]$, viz., both of them, being dependent on primary operators $\{\alpha\}$ (such as $a^{\dagger}(a)$, $b^{\dagger}(b)$ and $d^{\dagger}(d)$ for mesons and nucleons) in bare particle representation (BPR), are expressed through corresponding operators $\left\{\alpha_{c}\right\}$ for particle creation and annihilation in CPR via unitary clothing transformations (UCTs) $W(\alpha)=W\left(\alpha_{c}\right)$

$$
\alpha=W\left(\alpha_{c}\right) \alpha_{c} \boldsymbol{W}^{\dagger}\left(\alpha_{c}\right)
$$

A key point of clothing procedure in question is to remove so-called bad terms from Hamiltonian

$$
H \equiv H(\alpha)=H_{F}(\alpha)+H_{l}(\alpha)=W\left(\alpha_{c}\right) H\left(\alpha_{c}\right) W^{\dagger}\left(\alpha_{c}\right) \equiv K\left(\alpha_{c}\right)
$$

By definition, such terms prevent physical vacuum $|\Omega\rangle$ ( $H$ lowest eigenstate) and one-clothed-particle states $|n\rangle_{c}=a_{c}^{\dagger}(n)|\Omega\rangle$ to be $H$ eigenvectors for all $n$ included. Bad terms occur every time when any normally ordered product

$$
a^{\dagger}\left(1^{\prime}\right) a^{\dagger}\left(2^{\prime}\right) \ldots a^{\dagger}\left(n_{C}^{\prime}\right) a\left(n_{A}\right) \ldots a(2) a(1)
$$

of class [C.A] embodies, at least, one substructure $\in[k .0](k=1,2 \ldots)$ or/and [ $k .1]$ $(k=2,3, \ldots)$.

Respectively, let us write for boson-fermion system

$$
H_{l}(\alpha)=V(\alpha)+V_{\text {ren }}(\alpha)
$$

with primary (trial) interaction

$$
V(\alpha)=V_{b a d}+V_{\text {good }}
$$

"good" (e.g., $\in[k .2]$ ) as antithesis of "bad" while $V_{\text {ren }}(\alpha) \sim[1.1]+[0.2]+[2.0]$ "mass renormalization counterterms". Latter are important to ensure relativistic invariance (RI) in Dirac sense. In its turn, $V=\sum_{b} V_{b}$ comprises separate boson-fermion couplings $V_{b}$. In order to compare our calculations with those by Bonn group (Machleidt, Holinde, Elster) we have employed

$$
\begin{gathered}
V(\alpha)=V_{\mathrm{s}}+V_{\mathrm{ps}}+V_{\mathrm{v}} \\
V_{\mathrm{s}}=g_{\mathrm{s}} \int d \vec{x} \bar{\psi}(\vec{x}) \psi(\vec{x}) \varphi_{\mathrm{s}}(\vec{x}) \quad V_{\mathrm{ps}}=i g_{\mathrm{ps}} \int d \vec{x} \bar{\psi}(\vec{x}) \gamma_{5} \psi(\vec{x}) \varphi_{\mathrm{ps}}(\vec{x}) \\
V_{\mathrm{v}}=V_{\mathrm{v}}^{(1)}+V_{\mathrm{v}}^{(2)}, \quad V_{\mathrm{v}}^{(1)}=\int d \vec{x} H_{s c}(\vec{x}), \quad V_{\mathrm{v}}^{(2)}=\int d \vec{x} H_{\text {nonsc }}(\vec{x}) \\
H_{s c}(\vec{x})=g_{\mathrm{v}} \bar{\psi}(\vec{x}) \gamma_{\mu} \psi(\vec{x}) \varphi_{\mathrm{v}}^{\mu}(\vec{x})+\frac{f_{\mathrm{v}}}{4 m} \bar{\psi}(\vec{x}) \sigma_{\mu \nu} \psi(\vec{x}) \varphi_{\mathrm{v}}^{\mu \nu}(\vec{x}) \\
H_{\text {nonsc }}(\vec{x})=\frac{g_{\mathrm{v}}^{2}}{2 m_{\mathrm{v}}^{2}} \bar{\psi}(\vec{x}) \gamma_{0} \psi(\vec{x}) \bar{\psi}(\vec{x}) \gamma_{0} \psi(\vec{x})+\frac{f_{\mathrm{v}}^{2}}{4 m^{2}} \bar{\psi}(\vec{x}) \sigma_{0 i} \psi(\vec{x}) \bar{\psi}(\vec{x}) \sigma_{0 i} \psi(\vec{x})
\end{gathered}
$$

$\varphi_{\mathrm{v}}^{\mu \nu}(\vec{x})=\partial^{\mu} \varphi_{\mathrm{v}}^{\nu}(\vec{x})-\partial^{\nu} \varphi_{\mathrm{v}}^{\mu}(\vec{x})$ tensor of vector field in Schrödinger (S) picture.

Here we encounter scalar $H_{s c}$ and nonscalar $H_{\text {nonsc }}$ contributions to interaction densities of $\rho N N$ and $\omega N N$ couplings

$$
\begin{aligned}
U_{F}(\Lambda, a) H_{s c}(x) U_{F}^{-1}(\Lambda, a) & =H_{s c}(\Lambda x+a) \\
U_{F}(\Lambda, a) H_{\text {nonsc }}(x) U_{F}^{-1}(\Lambda, a) & \neq H_{\text {nonsc }}(\Lambda x+a)
\end{aligned}
$$

Therefore, in order to apply our approach to local field models with derivatives and/or spin $j \geq 1$ and also to their nonlocal extensions in framework of such a corpuscular picture we have developed clothing procedure $[2,3]$ removing from $V_{b a d}$ only its scalar part $V_{s c}$, if any. Clothing itself (cf. our talks at ISHEPP'02 and ISHEPP'04), as illustration for $\rho N N$ and $\omega N N$ couplings, is prompted by

$$
H(\alpha)=K\left(\alpha_{c}\right)=W\left(\alpha_{c}\right)\left[H_{F}\left(\alpha_{c}\right)+V_{\mathrm{v}}\left(\alpha_{c}\right)+V_{r e n}\left(\alpha_{c}\right)\right] W^{\dagger}\left(\alpha_{c}\right)
$$

or putting $W=\exp R$ with $R=-R^{\dagger}$ so

$$
\begin{aligned}
& K\left(\alpha_{c}\right)=H_{F}\left(\alpha_{c}\right)+V_{\mathrm{v}}^{(1)}\left(\alpha_{c}\right)+\left[R, H_{F}\right]+V_{\mathrm{v}}^{(2)}\left(\alpha_{c}\right) \\
& \quad+\left[R, V_{\mathrm{v}}^{(1)}\right]+\frac{1}{2}\left[R,\left[R, H_{F}\right]\right]+\left[R, V_{\mathrm{v}}^{(2)}\right]+\frac{1}{2}\left[R,\left[R, V_{\mathrm{v}}^{(1)}\right]+\ldots\right.
\end{aligned}
$$

and requiring $\left[R, H_{F}\right]=-V_{v}^{(1)}\left(^{*}\right)$ for the operator $R$ of interest to get

$$
H=K\left(\alpha_{c}\right)=K_{F}+K_{l}
$$

with a new free part $K_{F}=H_{F}\left(\alpha_{c}\right) \sim a_{c}^{\dagger} a_{c}$ and interaction

$$
K_{I}=\frac{1}{2}\left[R, V_{\mathrm{v}}^{(1)}\right]+V_{\mathrm{v}}^{(2)}+\frac{1}{3}\left[R,\left[R, V_{\mathrm{v}}^{(1)}\right]\right]+\ldots
$$

After a simple algebra we find

$$
\frac{1}{2}\left[R, V_{\mathrm{v}}^{(1)}\right](N N \rightarrow N N)=K_{\mathrm{v}}(N N \rightarrow N N)+K_{\text {cont }}(N N \rightarrow N N)
$$

Operator $K_{\text {cont }}(N N \rightarrow N N)$ may be associated with a contact interaction since it does not contain any propagators (details see in Refs. [6,7]). It has turned out that this operator cancels completely non-scalar operator $V^{(2)}$. In our opinion, such a cancellation, first discussed here, is a pleasant feature of the CPR.
Moreover, using property $V_{s c}(x)$ to be Lorentz scalar one can show that Lie algebra of $\Pi$ is satisfied with

$$
\vec{N}_{I}=\vec{N}_{B e l}+\vec{D} \equiv \int \vec{x} V_{\mathrm{v}}^{(1)}(\vec{x}) d \vec{x}+\vec{D}
$$

and get recursive formulae for finding contributions $\vec{D}^{(n)}$ to $\vec{D}=\sum_{n=2}^{\infty} \vec{D}^{(n)}$, label $(n)$ n'th order in coupling constants. It differs from expansion by Krueger and Gloeckle (1999).

In parallel, we have

$$
\vec{N}(\alpha)=\vec{B}\left(\alpha_{c}\right)=W\left(\alpha_{c}\right)\left\{\vec{N}_{F}(\alpha)+\vec{N}_{l}(\alpha)+\vec{N}_{r e n}(\alpha)\right\} W^{\dagger}\left(\alpha_{c}\right)
$$

with

$$
\vec{N}_{l}=-\int \vec{x} V_{v}(\vec{x}) d \vec{x}=-\int \vec{x}\left\{V_{v}^{(1)}(\vec{x})+V_{v}^{(2)}(\vec{x})\right\} d \vec{x}=\vec{N}_{l}^{(1)}+\vec{N}_{l}^{(2)}
$$

As before (see Refs. [2,3]) we find

$$
\left[R, \vec{N}_{F}\right]=-\vec{N}_{l}^{(1)},
$$

once operator meets condition (*) so boost generators in CPR acquire structure similar to $K\left(\alpha_{c}\right)$

$$
\vec{B}\left(\alpha_{c}\right)=\vec{B}_{F}+\vec{B}_{1} .
$$

Here $\vec{B}_{F}=\vec{N}_{F}\left(\alpha_{c}\right)$ the boost operator for noninteracting clothed particles (in our case fermions and vector mesons) and $\vec{B}_{\text {, }}$ includes the contributions induced by interactions between them

$$
\vec{B}_{l}=+\frac{1}{2}\left[R, \vec{N}_{l}^{(1)}\right]+\frac{1}{3}\left[R,\left[R, \vec{N}_{l}^{(1)}\right]\right]+\ldots
$$

## Relativistic Interactions in Meson-Nucleon Systems

Interaction operators

$$
\begin{aligned}
K_{I} \sim a_{c}^{\dagger} b_{c}^{\dagger} a_{c} b_{c}(\pi N \rightarrow \pi N)+b_{c}^{\dagger} b_{c}^{\dagger} b_{c} b_{c} & (N N \rightarrow N N)+d_{c}^{\dagger} d_{c}^{\dagger} d_{c} d_{c}(\bar{N} \bar{N} \rightarrow \bar{N} \bar{N}) \\
+b_{c}^{\dagger} b_{c}^{\dagger} b_{c}^{\dagger} b_{c} b_{c} b_{c}(N N N \rightarrow N N N)+\ldots & +\left[a_{c}^{\dagger} a_{c}^{\dagger} b_{c} d_{c}+H . c .\right](N \bar{N} \leftrightarrow 2 \pi)+\ldots \\
& +\left[a_{c}^{\dagger} b_{c}^{\dagger} b_{c}^{\dagger} b_{c} b_{c}+H . c .\right](N N \leftrightarrow \pi N N)+\ldots
\end{aligned}
$$

Pion-nucleon interaction operator

$$
\begin{aligned}
& K(\pi N \rightarrow \pi N)=\int d \vec{p}_{1} d \vec{p}_{2} d \vec{k}_{1} d \vec{k}_{2} V_{\pi N}\left(\vec{k}_{2}, \vec{p}_{2} ; \vec{k}_{1}, \vec{p}_{1}\right) a_{c}^{\dagger}\left(\vec{k}_{2}\right) b_{c}^{\dagger}\left(\vec{p}_{2}\right) a_{c}\left(\vec{k}_{1}\right) b_{c}\left(\vec{p}_{1}\right), \\
& V_{\pi N}\left(\vec{k}_{2}, \vec{p}_{2} ; \vec{k}_{1}, \vec{p}_{1}\right)=\frac{g^{2}}{2(2 \pi)^{3}} \frac{m}{\sqrt{\omega_{\vec{k}_{1}} \omega_{\vec{k}_{2}} E_{\vec{p}_{1}} E_{\vec{p}_{2}}}} \delta\left(\vec{p}_{1}+\vec{k}_{1}-\vec{p}_{2}-\vec{k}_{2}\right) \\
& \bar{u}\left(\vec{p}_{2}\right)\left\{\frac{1}{2}\left[\frac{1}{\hat{p}_{1}+\hat{k}_{1}+m}+\frac{1}{\hat{p}_{2}+\hat{k}_{2}+m}\right]\right. \\
& \left.\quad+\frac{1}{2}\left[\frac{1}{\hat{p}_{1}-\hat{k}_{2}+m}+\frac{1}{\hat{p}_{2}-\hat{k}_{1}+m}\right]\right\} u\left(\vec{p}_{1}\right)
\end{aligned}
$$

$\pi N$ quasipotential in momentum space is:

$$
\tilde{V}_{\pi N}\left(\vec{k}_{2}, \vec{p}_{2} ; \vec{k}_{1}, \vec{p}_{1}\right)=\left\langle a_{c}^{\dagger}\left(\vec{k}_{2}\right) b_{c}^{\dagger}\left(\vec{p}_{2}\right) \Omega\right| K(\pi N \rightarrow \pi N)\left|a_{c}^{\dagger}\left(\vec{k}_{1}\right) b_{c}^{\dagger}\left(\vec{p}_{1}\right) \Omega\right\rangle
$$



Figure 1: Different contributions to $\pi N$ quasipotential.

Graphs in Fig. 1 are topologically equivalent to well-known time-ordered Feynman diagrams. However, in Schrödinger picture used here, where all events are related to one and the same instant $t=0$, such an analogy could be misleading: line directions in Fig. 1 are given with the sole scope to discriminate between nucleon and antinucleon states.
Energy conservation is not assumed in constructing this and other quasipotentials. Indeed, coefficients in front of $a_{c}^{\dagger} b_{c}^{\dagger} a_{c} b_{c}$ generally do not fulfill on-energy-shell condition

$$
E_{\vec{p}_{1}}+\omega_{\vec{k}_{1}}=E_{\vec{p}_{2}}+\omega_{\vec{k}_{2}}
$$

In this connection, "left" four-vector $s_{1}$ is not necessarily equal to "right" Mandelstam vector $s_{2}=p_{2}+k_{2}$.

## Nucleon-nucleon interaction operator

After normal ordering of fermion operators we derive $N N \rightarrow N N$ interaction operator:

$$
\begin{gathered}
K_{N N}=\int d \vec{p}_{1} d \vec{p}_{2} d \vec{p}_{1}^{\prime} d \vec{p}_{2}^{\prime} V_{N N}\left(\vec{p}_{1}^{\prime}, \vec{p}_{2}^{\prime} ; \vec{p}_{1}, \vec{p}_{2}\right) b_{c}^{\dagger}\left(\vec{p}_{1}^{\prime}\right) b_{c}^{\dagger}\left(\vec{p}_{2}^{\prime}\right) b_{c}\left(\vec{p}_{1}\right) b_{c}\left(\vec{p}_{2}\right), \\
V_{N N}\left(\vec{p}_{1}^{\prime}, \vec{p}_{2}^{\prime} ; \vec{p}_{1}, \vec{p}_{2}\right)=-\frac{1}{2} \frac{g^{2}}{(2 \pi)^{3}} \frac{m^{2}}{\sqrt{E_{\vec{p}_{1}} E_{\vec{p}_{2}} E_{\vec{p}_{1}^{\prime}} E_{\vec{p}_{2}^{\prime}}^{\prime}}} \delta\left(\vec{p}_{1}^{\prime}+\vec{p}_{2}^{\prime}-\vec{p}_{1}-\vec{p}_{2}\right) \\
\times \bar{u}\left(\vec{p}_{1}^{\prime}\right) \gamma_{5} u\left(\vec{p}_{1}\right) \frac{1}{\left(p_{1}-p_{1}^{\prime}\right)^{2}-\mu^{2}} \bar{u}\left(\vec{p}_{2}^{\prime}\right) \gamma_{5} u\left(\vec{p}_{2}\right),
\end{gathered}
$$

Corresponding relativistic and properly symmetrized $N N$ interaction

$$
\tilde{V}_{N N}\left(\vec{p}_{1}^{\prime}, \vec{p}_{2}^{\prime} ; \vec{p}_{1}, \vec{p}_{2}\right)=\left\langle b_{c}^{\dagger}\left(\vec{p}_{1}^{\prime}\right) b_{c}^{\dagger}\left(\vec{p}_{2}^{\prime}\right) \Omega\right| K_{N N}\left|b_{c}^{\dagger}\left(\vec{p}_{1}\right) b_{c}^{\dagger}\left(\vec{p}_{2}\right) \Omega\right\rangle
$$

or through covariant (Feynman-like) "propagators",

$$
\begin{align*}
& \tilde{V}_{N N}\left(\vec{p}_{1}^{\prime}, \vec{p}_{2}^{\prime} ; \vec{p}_{1}, \vec{p}_{2}\right)=-\frac{1}{2} \frac{g^{2}}{(2 \pi)^{3}} \frac{m^{2}}{2 \sqrt{E_{\vec{p}_{1}} E_{\vec{p}_{2}} E_{\vec{p}_{1}^{\prime}} E_{\vec{p}_{2}^{\prime}}}} \delta\left(\vec{p}_{1}^{\prime}+\vec{p}_{2}^{\prime}-\vec{p}_{1}-\vec{p}_{2}\right) \\
& \times \bar{u}\left(\vec{p}_{1}^{\prime}\right) \gamma_{5} u\left(\vec{p}_{1}\right) \frac{1}{2}\left\{\frac{1}{\left(p_{1}-p_{1}^{\prime}\right)^{2}-\mu^{2}}\right. \\
& \left.\quad \quad+\frac{1}{\left(p_{2}-p_{2}^{\prime}\right)^{2}-\mu^{2}}\right\} \bar{u}\left(\vec{p}_{2}^{\prime}\right) \gamma_{5} u\left(\vec{p}_{2}\right)-(1 \leftrightarrow 2) . \tag{*}
\end{align*}
$$

Formula (*) determines $N N$ part of OBE interaction derived earlier via Okubo transformation method by Korchin, Shebeko [ Phys. At. Nucl. 56 (1993) 1663] (cf. Fuda, Zhang. Phys. Rev. C 51 (1995) 23 ) taking into account pion exchange and heavy-meson exchanges.
Distinctive feature of potential ( ${ }^{*}$ ) is the presence of covariant (Feynman-like) "propagator",

$$
\frac{1}{2}\left\{\frac{1}{\left(p_{1}-p_{1}^{\prime}\right)^{2}-\mu^{2}}+\frac{1}{\left(p_{2}-p_{2}^{\prime}\right)^{2}-\mu^{2}}\right\} .
$$

On the energy shell for $N N$ scattering, that is

$$
E_{i} \equiv E_{\vec{p}_{1}}+E_{\vec{p}_{2}}=E_{\vec{p}_{1}^{\prime}}+E_{\vec{p}_{2}^{\prime}} \equiv E_{f},
$$

this expression is converted into genuine Feynman propagator.

## $N N \leftrightarrow \pi N N$ transition operators

$$
\begin{array}{r}
K(N N \rightarrow \pi N N)=\int d \vec{p}_{1} d \vec{p}_{2} d \vec{p}_{1}^{\prime} d \vec{p}_{2}^{\prime} d \vec{k} V_{\pi N N}\left(\vec{p}_{1}^{\prime}, \vec{p}_{2}^{\prime}, \vec{k} ; \vec{p}_{1}, \vec{p}_{2}\right) \\
\\
a_{c}^{\dagger}(\vec{k}) b_{c}^{\dagger}\left(\vec{p}_{1}^{\prime}\right) b_{c}^{\dagger}\left(\vec{p}_{2}^{\prime}\right) b_{c}\left(\vec{p}_{1}\right) b_{c}\left(\vec{p}_{2}\right)
\end{array}
$$

$$
V_{\pi N N}\left(\vec{p}_{1}^{\prime}, \vec{p}_{2}^{\prime}, \vec{k} ; \vec{p}_{1}, \vec{p}_{2}\right)=V_{\pi N N}(\text { Feynman-like })+V_{\pi N N}(\text { off-energy-shell }),
$$

where

$$
\begin{aligned}
& V_{\pi N N}(\text { Feynman - like })=-i \frac{g^{3}}{(2 \pi)^{9 / 2}} \frac{m^{2} \delta\left(\vec{p}_{1}+\vec{p}_{2}-\vec{p}_{1}^{\prime}-\vec{p}_{2}^{\prime}-\vec{k}\right)}{\sqrt{2 \omega_{\vec{k}} E_{\vec{p}_{1}} E_{\vec{p}_{2}} E_{\vec{p}_{1}^{\prime}} E_{\vec{p}_{2}^{\prime}}}} \\
& \quad \times \frac{\bar{u}\left(\vec{p}_{2}^{\prime}\right) \gamma_{5} u\left(\vec{p}_{2}\right)}{\left(p_{2}-p_{2}^{\prime}\right)^{2}-\mu^{2}} \bar{u}\left(\vec{p}_{1}^{\prime}\right)\left[\frac{1}{\hat{p}_{1}^{\prime}+\hat{k}+m}+\frac{1}{\hat{p}_{1}-\hat{k}+m}\right] u\left(\vec{p}_{1}\right),
\end{aligned}
$$

Then we introduce quasipotential

$$
\tilde{V}_{\pi N N}\left(\vec{p}_{1}^{\prime}, \vec{p}_{2}^{\prime}, \vec{k} ; \vec{p}_{1}, \vec{p}_{2}\right)=\left\langle a_{c}^{\dagger}(\vec{k}) b_{c}^{\dagger}\left(\vec{p}_{1}^{\prime}\right) b_{c}^{\dagger}\left(\vec{p}_{2}^{\prime}\right) \Omega\right| K(N N \rightarrow \pi N N)\left|b_{c}^{\dagger}\left(\vec{p}_{1}\right) b_{c}^{\dagger}\left(\vec{p}_{2}\right) \Omega\right\rangle
$$

and draw respective graphs


Figure 2: Illustration of the "retarded" pion production mechanisms on the NN pair in the $g^{3}$-order.


Figure 3: Illustration of the "advanced" pion production mechanisms on the NN pair in the $g^{3}$-order.

## Three-Nucleon Forces

Normal ordering of fermion operators in $[R,[R,[R, V]]]$ leads to $N N N \rightarrow$ NNN interaction operator (antiparticle degrees of freedom are neglected),

$$
\begin{aligned}
K(3 N \rightarrow 3 N)=\int d \vec{p}_{1} d \vec{p}_{2} d \vec{p}_{3} d \vec{p}_{1}^{\prime} d \vec{p}_{2}^{\prime} d & \vec{p}_{3}^{\prime} V_{3 N}\left(\vec{p}_{1}^{\prime}, \vec{p}_{2}^{\prime}, \vec{p}_{3}^{\prime} ; \vec{p}_{1}, \vec{p}_{2}, \vec{p}_{3}\right) \\
& \times b_{c}^{\dagger}\left(\vec{p}_{1}^{\prime}\right) b_{c}^{\dagger}\left(\vec{p}_{2}^{\prime}\right) b_{c}^{\dagger}\left(\vec{p}_{3}^{\prime}\right) b_{c}\left(\vec{p}_{1}\right) b_{c}\left(\vec{p}_{2}\right) b_{c}\left(\vec{p}_{3}\right)
\end{aligned}
$$

$V_{3 N}\left(\vec{p}_{1}^{\prime}, \vec{p}_{2}^{\prime}, \vec{p}_{3}^{\prime} ; \vec{p}_{1}, \vec{p}_{2}, \vec{p}_{3}\right)$

$$
=-\frac{1}{8} \frac{g^{4} m^{4}}{(2 \pi)^{6}} \frac{\delta\left(\vec{p}_{1}^{\prime}+\vec{p}_{2}^{\prime}+\vec{p}_{3}^{\prime}-\vec{p}_{1}-\vec{p}_{2}-\vec{p}_{3}\right)}{\sqrt{E_{\vec{p}_{1}} E_{\vec{p}_{2}} E_{\vec{p}_{3}} E_{\vec{p}_{1}^{\prime}} E_{\vec{p}_{2}^{\prime}} E_{\vec{p}_{3}^{\prime}}}} D_{\vec{p}_{1}, \vec{p}_{2}, \vec{p}_{3}}^{\vec{p}_{3}^{\prime}, \vec{p}_{3}^{\prime}, \vec{p}_{3}^{\prime}} \frac{1}{E_{\vec{q}}} \bar{u}\left(\vec{p}_{1}^{\prime}\right) \gamma_{5} u\left(\vec{p}_{1}\right)
$$

$$
\times \bar{u}\left(\vec{p}_{2}^{\prime}\right) \frac{m-\hat{q}}{2 m} u\left(\vec{p}_{2}\right) \bar{u}\left(\vec{p}_{3}^{\prime}\right) \gamma_{5} u\left(\vec{p}_{3}\right), D_{\vec{p}_{1}, \vec{p}_{2}, \vec{p}_{3}}^{\vec{p}_{3}^{\prime}, \vec{p}_{3}^{\prime}} \overrightarrow{, \vec{x}}^{\prime}=\frac{E_{\vec{p}_{2}^{\prime}}-E_{\vec{q}}+E_{\vec{p}_{1}}-E_{\vec{p}_{1}^{\prime}}}{\left[\left(p_{1}-p_{1}^{\prime}\right)^{2}-\mu^{2}\right]\left[\left(p_{2}^{\prime}-q\right)^{2}-\mu^{2}\right]}
$$

$$
\times\left[\frac{3}{\left(p_{3}-p_{3}^{\prime}\right)^{2}-\mu^{2}}+\frac{1}{\left(p_{2}-q\right)^{2}-\mu^{2}}\right]+\frac{E_{\vec{p}_{2}}-E_{\vec{q}}+E_{\vec{p}_{3}^{\prime}}-E_{\vec{p}_{3}}}{\left[\left(p_{3}-p_{3}^{\prime}\right)^{2}-\mu^{2}\right]\left[\left(p_{2}-q\right)^{2}-\mu^{2}\right]}
$$

$$
\times\left[\frac{3}{\left(p_{1}-p_{1}^{\prime}\right)^{2}-\mu^{2}}+\frac{1}{\left(p_{2}^{\prime}-q\right)^{2}-\mu^{2}}\right], \quad \vec{q}=\vec{p}_{1}^{\prime}+\vec{p}_{2}^{\prime}-\vec{p}_{1}=\vec{p}_{2}+\vec{p}_{3}-\vec{p}_{3}^{\prime}
$$

In static limit for nucleons the quasipotential appears as a correction of nucleon-recoil order.

S Operator, Equivalence Theorem for S Matrix and Its Application to Elastic NN Scattering
By definition, with $H=H_{F}(\alpha)+H_{l}(\alpha)$

$$
S=\lim _{t_{2} \rightarrow+\infty} \lim _{t_{1} \rightarrow-\infty} e^{\imath H_{F} t_{2}} e^{-\imath H\left(t_{2}-t_{1}\right)} e^{-\imath H_{F} t_{1}}
$$

Let us introduce $S$ operator for decomposition $H=K\left(\alpha_{c}\right)=K_{F}\left(\alpha_{c}\right)+K_{l}\left(\alpha_{c}\right)$,

$$
S_{\text {cloth }}=\lim _{t_{2} \rightarrow+\infty} \lim _{t_{1} \rightarrow-\infty} e^{\imath K_{F} t_{2}} e^{-\imath K\left(t_{2}-t_{1}\right)} e^{-\imath K_{F} t_{1}}
$$

One can show that if $W_{D}(t)=\exp \left(i K_{F} t\right) W \exp \left(-i K_{F} t\right)$ meets condition
then

$$
\lim _{t \rightarrow \pm \infty} W_{D}(t)=1 \quad \text { or } \quad \lim _{t \rightarrow \pm \infty} R_{D}(t)=0
$$

$$
S_{\text {cloth }}=\lim _{t_{2} \rightarrow+\infty} \lim _{t_{1} \rightarrow-\infty} e^{\imath K_{F}\left(\alpha_{C}\right) t_{2}} e^{-\imath H\left(\alpha_{C}\right)\left(t_{2}-t_{1}\right)} e^{-\imath K_{F}\left(\alpha_{C}\right) t_{1}}
$$

Matrix elements of $S=S(\alpha)$ between bare states $\alpha^{\dagger} \ldots \Omega_{0}$ with $H_{F} \Omega_{0}=0$,

$$
\left\langle\alpha^{\dagger} \ldots \Omega_{0}\right| S(\alpha)\left|\alpha^{\dagger} \ldots \Omega_{0}\right\rangle
$$

and matrix elements of $S_{\text {cloth }}=S\left(\alpha_{c}\right)$ between clothed states $\alpha_{c}^{\dagger} \ldots \Omega$ with $K_{F} \Omega=0$,

$$
\left\langle\alpha_{c}^{\dagger} \ldots \Omega\right| S\left(\alpha_{c}\right)\left|\alpha_{c}^{\dagger} \ldots \Omega\right\rangle
$$

are equal to each other since $\alpha_{c}$-algebra with physical vacuum $\Omega$ is isomorphic to $\alpha$ -algebra with bare vacuum $\Omega_{0}$, i.e.,

$$
S_{f i} \equiv\langle f| S|i\rangle=\langle f ; c| S_{c l o t h}|i ; c\rangle
$$

## Application to Elastic NN Scattering

This result (ISHEPP'02, FB'03) has allowed us to reduce extremely complicated problem of describing $N N$ scattering in QFT to solution of integral equation

$$
\begin{aligned}
\left\langle 1^{\prime}, 2^{\prime}\right| T_{N N}(E+i 0)|1,2\rangle=\left\langle 1^{\prime}, 2^{\prime}\right| & K_{N N}|1,2\rangle \\
& +\left\langle 1^{\prime}, 2^{\prime}\right| K_{N N}\left(E+i 0-K_{F}\right)^{-1} T_{N N}(E+i 0)|1,2\rangle
\end{aligned}
$$

$|12\rangle=b_{c}^{\dagger} b_{c}^{\dagger}|\Omega\rangle$ any clothed two-nucleon state, once we will confine ourselves to approximation $K_{I}=K_{N N}$ or equation for $R$ - matrix

$$
\left\langle 1^{\prime} 2^{\prime}\right| \bar{R}(E)|12\rangle=\left\langle 1^{\prime} 2^{\prime}\right| \bar{K}_{N N}|12\rangle+\sum_{34}\left\langle 1^{\prime} 2^{\prime}\right| \bar{K}_{N N}|34\rangle \frac{\langle 34| \bar{R}(E)|12\rangle}{E-E_{3}-E_{4}}
$$

with $\bar{R}(E)=R(E) / 2\left(\bar{K}_{N N}=K_{N N} / 2\right)$, symbol $\sum_{34}$ implies the p.v. integration.
After angular-momentum decomposition in c.m.s

$$
\bar{R}_{L^{\prime} L}^{J S T}\left(p^{\prime}, p\right)=\bar{V}_{L^{\prime} L}^{J S T}\left(p^{\prime}, p\right)+\frac{1}{2} \sum_{L^{\prime \prime}} \mathrm{P} \int_{0}^{\infty} \frac{q^{2} d q}{E_{p}-E_{q}} \bar{V}_{L^{\prime} L^{\prime \prime}}^{J S T}\left(p^{\prime}, q\right) \bar{R}_{L^{\prime \prime} L}^{J S T}(q, p)
$$

$\bar{R}_{L^{\prime} L}^{J S T}\left(p^{\prime}, p\right) \equiv \bar{R}_{L^{\prime} L}^{J S T}\left(p^{\prime}, p ; 2 E_{p}\right)$
In our case such a decomposition means transition to matrix elements between states

$$
\begin{aligned}
\left|p J(L S) M_{J}\right\rangle=\sum\left(\left.\frac{1}{2} \mu_{1} \frac{1}{2} \mu_{2} \right\rvert\, S M_{S}\right)\left(L m_{L}\right. & \left.S M_{S} \mid J M_{J}\right) \\
& \times \int d \Omega_{\vec{p}} Y_{L m_{L}}(\hat{\vec{p}}) b_{c}^{\dagger}\left(\vec{p} \mu_{1}\right) b_{c}^{\dagger}\left(-\vec{p} \mu_{2}\right)|\Omega\rangle
\end{aligned}
$$

A careful exploration shows that our equation for $T$-matrix with cutoff functions

$$
F_{b}\left(p^{\prime}, p\right)=\left[\frac{\Lambda_{b}^{2}-m_{b}^{2}}{\Lambda_{b}^{2}-\left(p^{\prime}-p\right)^{2}}\right]^{n_{b}} \equiv F_{b}\left[\left(p^{\prime}-p\right)^{2}\right]
$$

has much common with equation by Bonn group in $J S T$-representation (in particular, for their Potential B). Nevertheless, one needs to keep in mind some distinctions, viz.,Potential B by Bonn group can be obtained from UCT quasipotentials with help of following transformations

- for boson propagators

$$
\left.\left[\left(p^{\prime}-p\right)^{2}-m_{b}^{2}\right]^{-1} \longrightarrow-\left[\vec{p}^{\prime}-\vec{p}\right)^{2}+m_{b}^{2}\right]^{-1}
$$

- for cutoff functions

$$
\left[\frac{\Lambda_{b}^{2}-m_{b}^{2}}{\Lambda_{b}^{2}-\left(p^{\prime}-p\right)^{2}}\right]^{n_{b}} \rightarrow\left[\frac{\Lambda_{b}^{2}-m_{b}^{2}}{\Lambda_{b}^{2}+\left(\vec{p}^{\prime}-\vec{p}\right)^{2}}\right]^{n_{b}}
$$

- omitting off-energy-shell correction in tensor-tensor term

$$
\frac{f_{\mathrm{v}}^{2}}{4 m^{2}}\left(E_{p^{\prime}}-E_{p}\right)^{2} \bar{u}\left(\vec{p}^{\prime}\right)\left[\gamma_{0} \gamma_{\nu}-g_{0 \nu}\right] u(\vec{p}) \bar{u}\left(-\vec{p}^{\prime}\right)\left[\gamma^{0} \gamma^{\nu}-g^{0 \nu}\right] u(-\vec{p}) \longrightarrow 0
$$

Table 1: The best-fit parameters for the two models. All masses are in $M e V$, and $n_{b}=2$ except for $n_{\rho}=n_{\omega}=4$.

| Meson |  | Potential B | UCT |
| :---: | :---: | :---: | :---: |
| $\pi$ | $g_{\pi}^{2} / 4 \pi$ | 14.4 | 14.574 |
|  | $\Lambda_{\pi}$ | 1700 | 2200 |
|  | $m_{\pi}$ | 138.03 | 138.03 |
| $\eta$ | $g_{\eta}^{2} / 4 \pi$ | 3 | 2.1 |
|  | $\Lambda_{\eta}$ | 1500 | 1200 |
|  | $m_{\eta}$ | 548.8 | 548.8 |
| $\rho$ | $g_{\rho}^{2} / 4 \pi$ | 0.9 | 1.3 |
|  | $\Lambda_{\rho}$ | 1850 | 1450 |
|  | $f_{\rho} / g_{\rho}$ | 6.1 | 5.953 |
|  | $m_{\rho}$ | 769 | 769 |
| $\omega$ | $g_{\omega}^{2} / 4 \pi$ | 24.5 | 25.325 |
|  | $\Lambda_{\omega}$ | 1850 | 2143.8 |
|  | $m_{\omega}$ | 782.6 | 782.6 |
| $\delta$ | $g_{\delta}^{2} / 4 \pi$ | 2.488 | 2.923 |
|  | $\Lambda_{\delta}$ | 2000 | 2092.2 |
|  | $m_{\delta}$ | 983 | 983 |
| $\sigma, T=0, T=1$ | $g_{\sigma}^{2} / 4 \pi$ | $18.3773,8.9437$ | $16.081,10.089$ |
|  | $\Lambda_{\sigma}$ | 2000,1900 | $2012.4,2200$ |
|  | $m_{\sigma}$ | 720,550 | $693.66,562.07$ |



Figure 4: Neutron-proton phase parameters plotted versus nucleon kinetic energy in lab. system. Solid curves calculated for Potential B. Dashed (dotted) - for UCT potential with Potential B (UCT) parameters from Table 1. The rhombs show original OBEP results.


Figure 5: Half-off-shell $R$-matrices at laboratory energy equal to $150 \mathrm{MeV}\left(p_{0}=265 \mathrm{MeV}\right)$. Other notations as in Fig.1.


Figure 6: Off-shell potentials with the momentum $p_{0}$ fixed as in Fig. 2. Other notations in Fig. 1.

## Clothing Procedure in the Theory of EM Interactions with Nuclei. Deuteron Properties

## The Deuteron Equation

Now, we consider a $K\left(\alpha_{c}\right)$ eigenstate from the $N N$ sector

$$
\left|\psi_{N N}\right\rangle=\sum_{\mu_{1} \mu_{2}} \int d \vec{p}_{1} d \vec{p}_{2} \psi_{N N}\left(\vec{p}_{1} \mu_{1}, \vec{p}_{2} \mu_{2}\right) b^{\dagger}\left(\vec{p}_{1} \mu_{1}\right) b^{\dagger}\left(\vec{p}_{2} \mu_{2}\right)|\Omega\rangle
$$

In the approximation $K_{l}=K_{l}^{(2)}$, the eigenvalue equation has the form

$$
\left[K_{F}+K_{N N}\right]\left|\psi_{N N}\right\rangle=E\left|\psi_{N N}\right\rangle
$$

In turn the deuteron state at rest can be written as the superposition

$$
\left|\psi_{d}^{M}\right\rangle=\sum_{l=0,2} \int_{0}^{\infty} d q q^{2}|q(11) 1 M\rangle \psi_{l}^{d}(q)
$$

with coefficients $\psi_{l}^{d}(q)=\left\langle q(11) 1 M \mid \psi_{N N}\right\rangle$ that satisfy the equations

$$
\psi_{l}^{d}(p)=\frac{1}{M_{d}-2 E_{\vec{p}}} \sum_{l^{\prime}} \int_{0}^{\infty} d q q^{2} \bar{V}_{l l^{\prime}}^{J=1, S=1, T=0}(p, q) \psi_{l^{\prime}}^{d}(q)
$$

where $M_{d}=2 m-\varepsilon_{d}$ deuteron mass, $\varepsilon_{d}$ deuteron binding energy.


Figure 7: Deuteron wave functions $\psi_{0}^{d}(q)=u(q)$ and $\psi_{2}^{d}(q)=w(q)$. Solid curves for Bonn Potential B. Dashed (dotted) - for UCT potential with Potential B (UCT) parameters from Table 1.

In case of the UCT potential after parameters fitting we have for the deuteron binding energy $\varepsilon_{d}=2.224 \mathrm{MeV}$ and for the D -state probability $P_{D}=5.494 \%$ vs Bonn values $\varepsilon_{d}=2.223 \mathrm{MeV}$ and $\left.P_{D}=4.986 \%\right)$.

## Deuteron Properties

In its most general form, the relativistic deuteron electromagnetic current can be written as

$$
\begin{aligned}
& \left\langle P^{\prime} M^{\prime}\right| J^{\mu}(0)|P M\rangle=-\left\{G_{1}\left(Q^{2}\right)\left[\xi_{M^{\prime}}^{*}\left(P^{\prime}\right) \cdot \xi_{M}(P)\right]\left(P^{\prime}+P\right)^{\mu}\right. \\
& +G_{2}\left(Q^{2}\right)\left[\xi_{M}(P)\left[\xi_{M^{\prime}}^{*}\left(P^{\prime}\right) \cdot q\right]-\xi_{M^{\prime}}^{*}\left(P^{\prime}\right)\left[\xi_{M}(P) \cdot q\right]\right] \\
& \left.\quad-G_{3}\left(Q^{2}\right) \frac{1}{2 m_{d}^{2}}\left[\xi_{M^{\prime}}^{*}\left(P^{\prime}\right) \cdot q\right]\left[\xi_{M}(P) \cdot q\right]\left(P^{\prime}+P\right)^{\mu}\right\}
\end{aligned}
$$

$\xi_{M}(P)\left(\xi_{M^{\prime}}\left(P^{\prime}\right)\right)$ - polarizations of incoming (outgoing) deuteron.

$$
G_{C}\left(Q^{2}\right)=G_{1}\left(Q^{2}\right)+\frac{2}{3} \eta G_{Q}\left(Q^{2}\right), \quad G_{M}\left(Q^{2}\right)=G_{2}\left(Q^{2}\right)
$$

$G_{Q}\left(Q^{2}\right)=G_{1}\left(Q^{2}\right)-G_{M}\left(Q^{2}\right)+(1+\eta) G_{3}\left(Q^{2}\right), \quad q=P^{\prime}-P, \quad Q^{2}=-q^{2}, \quad \eta=\frac{Q^{2}}{4 m_{d}^{2}}$
At $Q^{2}=0$, form factors $G_{C}, G_{M}$ and $G_{Q}$ give charge, magnetic and quadrupole moments of deuteron:

$$
Q_{C}(0)=1, \quad Q_{M}(0)=\frac{m_{d}}{m_{p}} \mu_{d}, \quad G_{Q}(0)=m_{d}^{2} Q_{d}
$$

For example, in case of deuteron magnetic moment we have

$$
\mu_{d} \sim \lim _{\eta \rightarrow 0} \frac{\left\langle P^{\prime} M^{\prime}=1\right| J^{x}(0)|P M=0\rangle}{\sqrt{\eta} \sqrt{1+\eta}}=\lim _{\eta \rightarrow 0} \frac{\left\langle P^{\prime} M^{\prime}=1\right| J^{x}(0)\left|P=\left(m_{d}, \overrightarrow{0}\right) M=0\right\rangle}{\sqrt{\eta} \sqrt{1+\eta}}
$$

Deuteron state in moving frame can be built up as

$$
\left|P^{\prime} M^{\prime}\right\rangle=e^{-i \vec{\beta}\left(P^{\prime}\right) \vec{B}}\left|\vec{O} M^{\prime}\right\rangle
$$

where boost operator

$$
\vec{B}=\vec{B}_{F}+\vec{B}_{l}
$$

contains interaction part and

$$
\vec{\beta}=\beta \vec{n}, \quad \vec{n}=\frac{\vec{v}}{v}, \quad \tanh \beta=v, \quad \vec{v}=\frac{\vec{P}^{\prime}}{m_{d}}
$$

Choosing $\vec{P}^{\prime}=(0,0, q)$ we have

$$
\mu_{d} \sim\left\langle\overrightarrow{0} M^{\prime}=1\right|\left(B_{F}^{z}+B_{l}^{z}\right) J^{x}(0)|\overrightarrow{0} M=0\rangle
$$

## Current Operator

For brevity, we omit any addressing to the Fock-Weyl criterion to satisfy the gauge independence principle, e.g., for reaction amplitude

$$
T(\gamma d \rightarrow p n)=\epsilon_{\mu}\langle p n ; \text { out }| J^{\mu}(0)|d\rangle
$$

and local analog of Siegert theorem based on transformation property of current density operator $J_{\mu}(x)$ with respect to Poincaré group (Shebeko Sov. J. Nucl. Phys. $90)$. For this illustration,

$$
J^{\mu}(0)=J_{N}^{\mu}(0)+J_{M}^{\mu}(0)
$$

where, for instance, $J_{N}^{\mu}(0)=\bar{\psi}(0) \frac{1+\tau_{3}}{2} \gamma^{\mu} \psi(0)$ and $J_{M}^{\mu}(0)=\left[\vec{\phi} \times \partial^{\mu} \vec{\phi}\right]_{3}$. In CPR

$$
J(0)=J_{\text {eff }}(0) \equiv W J_{c}(0) W^{\dagger}=J_{c}(0)+\left[R, J_{c}(0)\right]+\frac{1}{2}\left[R,\left[R, J_{c}(0)\right]\right]+\ldots
$$

$J_{c}(0)$ initial current in which "bare" operators are replaced by clothed ones. This decomposition involves one-body, two-body and more complicated effective currents if one uses terminology customary in the theory of meson exchange currents (MEC).

Following clothing procedure current operator $J^{\text {eff }}(0)$ can be written as

$$
\begin{aligned}
J_{e f f}^{\mu}(0)= & J_{N}^{\mu}(0)+J_{M E C}^{\mu}(0)+\cdots=\int d \vec{p}^{\prime} d \vec{p} \mathbf{F}_{N}^{\mu}\left(\vec{p}^{\prime}, \vec{p}\right) b_{c}^{\dagger}\left(\vec{p}^{\prime}\right) b_{c}(\vec{p}) \\
& +\int d \vec{p}_{1}^{\prime} d \vec{p}_{2}^{\prime} d \vec{p}_{1} d \vec{p}_{2} F_{M E C}^{\mu}\left(\vec{p}_{1}^{\prime}, \vec{p}_{2}^{\prime} ; \vec{p}_{1}, \vec{p}_{2}\right) b_{c}^{\dagger}\left(\vec{p}_{1}^{\prime}\right) b_{c}^{\dagger}\left(\vec{p}_{2}^{\prime}\right) b_{c}\left(\vec{p}_{1}\right) b_{c}\left(\vec{p}_{2}\right)+\cdots
\end{aligned}
$$

First term is contained nucleon form factors

$$
\begin{aligned}
& \left\langle\vec{q}^{\prime}, p[n]\right| J_{N}^{\mu}(0)|\vec{q}, p[n]\rangle=\frac{e}{(2 \pi)^{3}} \bar{u}\left(\vec{q}^{\prime}\right)\left\{F_{1}^{p[n]}\left[\left(q^{\prime}-q\right)^{2}\right] \gamma^{\mu}\right. \\
& \left.+\imath \sigma^{\mu \nu}\left(q^{\prime}-q\right)_{\nu} F_{2}^{p[n]}\left[\left(q^{\prime}-q\right)^{2}\right]\right\} u(\vec{q}),
\end{aligned}
$$

second - so-called interaction (or meson exchange) currents


## Conclusions and Prospects

- Starting from a total Hamiltonian for interacting meson and nucleon fields, we come to Hamiltonian and boost generator in CPR whose interaction parts consist of new relativistic interactions responsible for physical (not virtual) processes, particularly, in the system of bosons ( $\pi-, \eta-, \rho-, \omega-, \delta-$ and $\sigma$-mesons) and fermions (nucleons and antinucleons). The corresponding quasipotentials (these essentially nonlocal objects) for binary processes $N N \rightarrow N N, \bar{N} N \rightarrow \bar{N} N$, etc. are Hermitian and energy independent. It makes them attractive for various applications in nuclear physics. They embody the off-shell effects in a natural way without addressing to any off-shell extrapolations of the $S$-matrix for the $N N$ scattering.
- Using unitary equivalence of CPR to BPR, we have seen how in approximation $K_{l}=K_{l}^{(2)} N N$ scattering problem in QFT can be reduced to three -dimensional LS-type equation for the $T$-matrix in momentum space. The equation kernel is given by clothed two-nucleon interaction of class [2.2]. Such a conversion becomes possible owing to property of $K_{l}^{(2)}$ to leave two-nucleon sector and its separate subsectors to be invariant.
- Special attention has been paid to the elimination of auxiliary field components. We encounter such a necessity for interacting vector and fermion fields when in accordance with the canonical formalism the interaction Hamiltonian density embodies not only a scalar contribution but nonscalar terms too. It has proved (at least, for primary $\rho N$ and $\omega N$ couplings) that the UCT method allows us to remove such noncovariant terms directly in the Hamiltonian.
- Being concerned with constructing two-nucleon states from $\mathcal{H}$ and their angular-momentum decomposition we have not used the so-called separable ansatz, where every such state is a direct product of corresponding onenucleon (particle) states. The clothed two-nucleon partial waves have been built up as common eigenstates of the field total angular-momentum generator and its polarization (fermionic) part expressed through the clothed creation/destruction operators and their derivatives in momentum space.
- We have not tried to attain a global treatment of modern precision data. But a fair agreement with the earlier analysis by Bonn group and reasonable treatment of deuteron properties makes sure that our approach may be useful for a more advanced analysis. In the context, to have a more convincing argumentation one needs to do at least the following:

1) consider triple commutators $\left[R,\left[R,\left[R, V_{b}\right]\right]\right]$ to extract
two-boson-two-nucleon interaction operators of the same class [2.2] in fourth order in coupling constants.
2) extend our approach for describing the $N N$ scattering above pion production threshold.
As a whole, the persistent clouds of virtual particles are no longer explicitly contained in CPR, and their influence is included in properties of clothed particles (these quasiparticles of UCT method). In addition, we would like to stress that problem of the mass and vertex renormalizations is intimately interwoven with constructing the interactions between clothed nucleons. Renormalized quantities are calculated step by step in course of clothing procedure unlike some approaches, where they are introduced by "hands".
