Bethe-Salpeter Approach with the Covariant Separable Kernel for the Electrodisintegration of the Deuteron

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Plan

- Bethe-Salpeter formalism
- Description of the deuteron
- Electrodisintegration of the deuteron
- Conclusion

Formalism: Bethe-Salpeter equation

Bethe-Salpeter equation for scattered states:

$$T(p',p;P) = V(p',p;P) + \frac{\imath}{4\pi^3}\int d^4k\, V(p',k;P)\, S_2(k;P)\, T(k,p;P)$$

V - interaction kernel, T - scattering matrix

Two-particle Green function:

$$S_2^{-1}(k;P) = \left(\tfrac{1}{2}\, P\cdot\gamma + k\cdot\gamma - m\right)^{(1)}\left(\tfrac{1}{2}\, P\cdot\gamma - k\cdot\gamma - m\right)^{(2)}$$

Bethe-Salpeter equation for bound states:

$$\Phi^{JM}(p;P) = \frac{\imath}{(2\pi)^4} S_2(p;P)\int d^4k V(p,k;P)\Phi^{JM}(k;P)$$

Φ^{JM} - Bethe-Salpeter amplitude

$$\Phi^{JM}(p;P) = S_2(p;P)\Gamma^{JM}(p;P)$$

Γ^{JM} - vertex function

Formalism: Partial-wave decomposition

Partial-wave decomposition (in the np pair rest frame):

$$T_{\alpha\beta,\gamma\delta}(p',p;P_{(0)}) \qquad\qquad s = P_{(0)}^2$$

$$= \sum_{JMab} (\mathcal{Y}_{aM}(-\boldsymbol{p'})U_C)_{\alpha\beta} \otimes (U_C \mathcal{Y}^{\dagger}_{bM}(\boldsymbol{p}))_{\delta\gamma}\; T_{ab}(p'_0,|\boldsymbol{p'}|;p_0,|\boldsymbol{p}|;s)$$

Spin-angle functions:

$$\mathcal{Y}_{JM:LS\rho}(\boldsymbol{p})U_C$$

$$= \imath^L \sum_{m_L m_S m_1 m_2 \rho_1 \rho_2} C^{S_\rho \rho}_{\frac{1}{2}\rho_1 \frac{1}{2}\rho_2} C^{JM}_{Lm_L Sm_S} C^{Sm_S}_{\frac{1}{2}m_1 \frac{1}{2}m_2} Y_{Lm_L}(\boldsymbol{p}) u^{\rho_1}_{m_1}{}^{(1)}(\boldsymbol{p}) u^{\rho_2}_{m_2}{}^{(2)^T}(-\boldsymbol{p})$$

Normalization:

$$\int d\phi_{\mathbf{p}}\, d(\cos\theta_{\mathbf{p}})\, \mathrm{Sp}\left\{\mathcal{Y}^{\dagger}_{aM}(\boldsymbol{p})\mathcal{Y}_{a'M'}(\boldsymbol{p})\right\}$$

$$\equiv \int d\phi_{\mathbf{p}}\, d(\cos\theta_{\mathbf{p}}) (\mathcal{Y}^{\dagger}_{aM}(\boldsymbol{p}))_{\beta\alpha} (\mathcal{Y}_{a'M'}(\boldsymbol{p}))_{\alpha\beta} = \delta_{aa'}\delta_{MM'}$$

Bethe-Salpeter equation for partial-wave components:

$$T_{ab}(p'_0,|\boldsymbol{p'}|;p_0,|\boldsymbol{p}|;s) = V_{ab}(p'_0,|\boldsymbol{p'}|;p_0,|\boldsymbol{p}|;s)$$

$$+\frac{\imath}{4\pi^3}\sum_{cd}\int_{-\infty}^{+\infty} dk_0 \int_0^{\infty} \boldsymbol{k}^2 d|\boldsymbol{k}|\, V_{ac}(p'_0,|\boldsymbol{p'}|;k_0,|\boldsymbol{k}|;s) S_{cd}(k_0,|\boldsymbol{k}|;s)\, T_{db}(k_0,|\boldsymbol{k}|;p_0,|\boldsymbol{p}|;s)$$

Formalism: Separable ansatz

Interaction kernel:

$$V_{l'l}(p'_0, |\boldsymbol{p'}|; p_0, |\boldsymbol{p}|; s) = \sum_{i,j=1}^{N} \lambda_{ij}(s) g_i^{[l']}(p'_0, |\boldsymbol{p'}|) g_j^{[l]}(p_0, |\boldsymbol{p}|)$$

T **matrix:**

$$T_{l'l}(p'_0, |\boldsymbol{p'}|; p_0, |\boldsymbol{p}|; s) = \sum_{i,j=1}^{N} \tau_{ij}(s) g_i^{[l']}(p'_0, |\boldsymbol{p'}|) g_j^{[l]}(p_0, |\boldsymbol{p}|)$$

$$\tau_{ij}(s) = 1/(\lambda_{ij}^{-1}(s) + h_{ij}(s))$$

$$h_{ij}(s) = -\frac{\imath}{4\pi^3} \sum_l \int dk_0 \int \boldsymbol{k}^2 d|\boldsymbol{k}| \frac{g_i^{[l]}(k_0, |\boldsymbol{k}|) g_j^{[l]}(k_0, |\boldsymbol{k}|)}{(\sqrt{s}/2 - E_{\mathbf{k}} + \imath\epsilon)^2 - k_0^2},$$

$$E_{\mathbf{k}} = \sqrt{\boldsymbol{k}^2 + m^2}$$

Deuteron: Bethe-Salpeter equation for the deuteron

Bethe-Salpeter equation for the deuteron:

$$\Phi^{JM}(p;P) = \frac{\imath}{(2\pi)^4} S_2(p;P) \int d^4k V(p,k;P) \Phi^{JM}(k;P)$$

Radial part of the deuteron vertex function:

$$g_l(p) = \sum_{i,j=1}^{N} \lambda_{ij}(s) g_i^{[l]}(p_0, |\boldsymbol{p}|) c_j(s), \quad l \to {}^3S_1^+ - {}^3D_1^+$$

System of linear homogeneous equations:

$$c_i(s) - \sum_{k,j=1}^{N} h_{ik}(s) \lambda_{kj}(s) c_j(s) = 0$$

Normalization:

$$p_S + p_D = 1: \quad p_l = \frac{\imath}{2M_d(2\pi)^4} \int dp_0 \int \boldsymbol{p}^2 d|\boldsymbol{p}| \frac{(E_{\mathbf{p}} - M_d/2)[g_l(p_0, |\boldsymbol{p}|)]^2}{\left((M_d/2 - E_{\mathbf{p}} + \imath\epsilon)^2 - p_0^2\right)^2}$$

Deuteron: Interaction kernel

Rank-six kernel for $^3S_1^+$-$^3D_1^+$ state MY6:

$$g_1^{[S]}(p) = \frac{(p_{c1} - p_0^2 + \boldsymbol{p}^2)}{(p_0^2 - \boldsymbol{p}^2 - \beta_1^2)^2 + \alpha_1^4}$$

$$g_2^{[S]}(p) = \frac{(p_0^2 - \boldsymbol{p}^2)(p_{c2} - p_0^2 + \boldsymbol{p}^2)^2}{((p_0^2 - \boldsymbol{p}^2 - \beta_2^2)^2 + \alpha_2^4)^2}$$

$$g_3^{[S]}(p) = \frac{(p_0^2 - \boldsymbol{p}^2)^3(p_{c3} - p_0^2 + \boldsymbol{p}^2)^2}{((p_0^2 - \boldsymbol{p}^2 - \beta_3^2)^2 + \alpha_3^4)^3}$$

$$g_4^{[D]}(p) = \frac{-(p_0^2 - \boldsymbol{p}^2)(p_{c4} - p_0^2 + \boldsymbol{p}^2)^2}{((p_0^2 - \boldsymbol{p}^2 - \beta_{41}^2)^2 + \alpha_{41}^4)((p_0^2 - \boldsymbol{p}^2 - \beta_{42}^2)^2 + \alpha_{42}^4)}$$

$$g_5^{[D]}(p) = \frac{-(p_0^2 - \boldsymbol{p}^2)}{(p_0^2 - \boldsymbol{p}^2 - \beta_5^2)^2 + \alpha_5^4}$$

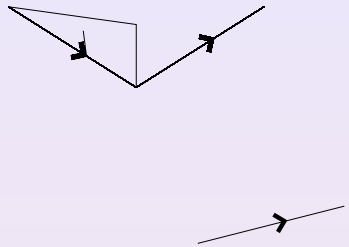
$$g_6^{[D]}(p) = \frac{(p_0^2 - \boldsymbol{p}^2)^4(p_{c6} - p_0^2 + \boldsymbol{p}^2)}{((p_0^2 - \boldsymbol{p}^2 - \beta_{61}^2)^2 + \alpha_{61}^4)^2((p_0^2 - \boldsymbol{p}^2 - \beta_{62}^2)^2 + \alpha_{62}^4)}$$

$$g_4^{[S]} = g_5^{[S]} = g_6^{[S]} = 0 = g_1^{[D]} = g_2^{[D]} = g_3^{[D]} = 0$$

S.G. Bondarenko, V.V. Burov, W.-Y.Pauchy Hwang, E.P. Rogochaya, Nucl. Phys. **A**, doi:10.1016/j.nuclphysa.2010.08.007, 17 pp. [arXiv:1002.0487[nucl-th]]

Electrodisintegration of the deuteron: Impulse approximation

One-photon approximation:



Cross section: JLab experiment

Cross section (for unpolarized particles):

$$\frac{d^3\sigma}{dQ^2 d|\boldsymbol{p}_n| d\Omega_n} = \frac{\sigma_{\rm Mott}\pi \boldsymbol{p}_n^2}{(2\pi)^3 2M_d E_e E'_e} \times$$
$$\times \left[l^0_{00}W_{00} + l^0_{++}(W_{++}+W_{--}) + l^0_{+-}\cos 2\phi\ 2{\rm Re}W_{+-} - l^0_{+-}\sin 2\phi\ 2{\rm Im}W_{+-}\right.$$
$$\left. -l^0_{0+}\cos\phi\ 2{\rm Re}(W_{0+}-W_{0-}) - l^0_{0+}\sin\phi\ 2{\rm Im}(W_{0+}+W_{0-})\right].$$

$\sigma_{\rm Mott} = (\alpha\cos\frac{\theta_e}{2}/2E_e\sin^2\frac{\theta_e}{2})^2$, $\alpha = e^2/(4\pi)$ - fine structure constant

Lepton tensor:

$$l^0_{00} = \frac{Q^2}{\boldsymbol{q}^2}, \quad l^0_{0+} = \frac{Q}{|\boldsymbol{q}|\sqrt{2}}\sqrt{\frac{Q^2}{\boldsymbol{q}^2}+\tan^2\frac{\theta_e}{2}},$$
$$l^0_{++} = \tan^2\frac{\theta_e}{2} + \frac{Q^2}{2\boldsymbol{q}^2}, \quad l^0_{+-} = -\frac{Q^2}{2\boldsymbol{q}^2}$$

Hadron tensor:

$$W_{\mu\nu} = \frac{1}{3}\sum_{s_d s_n s_p} < np: SM_S|j_\mu|d:1M >< np: SM_S|j_\nu|d:1M >^*$$

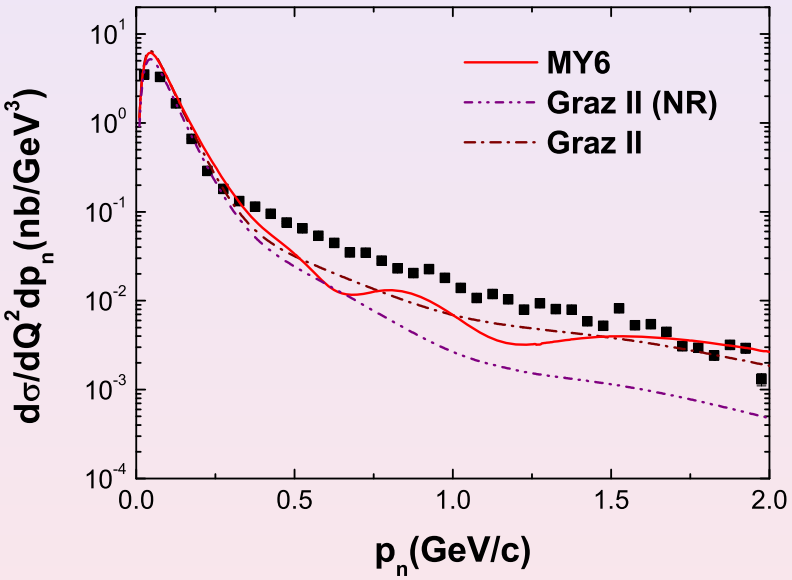
K.S. Egiyan, Phys. Rev. Lett. **98**, 262502 (2007):

$$\frac{d^2\sigma}{dQ^2 d|\boldsymbol{p}_n|} = \int\limits_{0^\circ}^{360^\circ} d\phi \int\limits_{20^\circ}^{160^\circ} d\theta_n \frac{d^3\sigma}{dQ^2 d|\boldsymbol{p}_n| d\Omega_n}$$

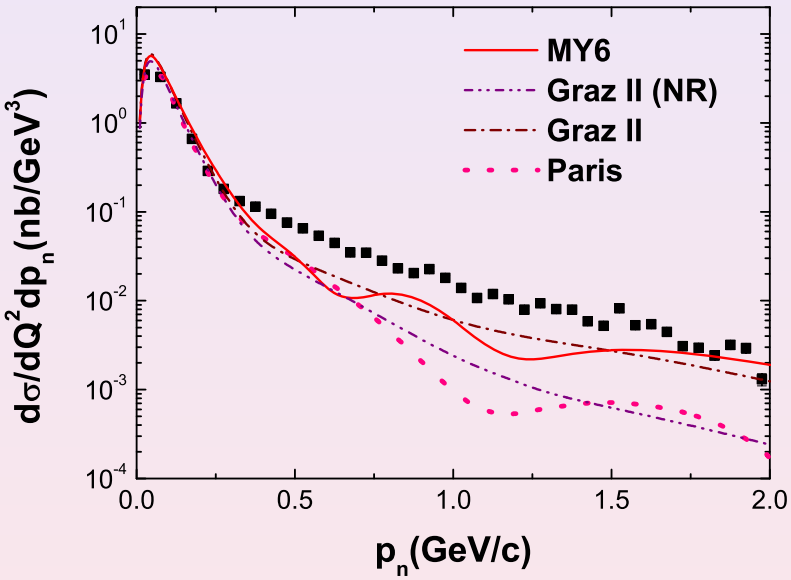
Cross section: Comparison with experiment, $Q^2 = 2\text{GeV}^2$

$$Q^2 = 2 \pm 0.25\,\text{GeV}^2$$

dipole FF



modified dipole FF



Experimental data were taken from K.Sh. Egiyan *et al.*, CLAS Note No. 2006-025, http://www1.jlab.org/ul/Physics/Hall-B/clas/
Graz II (NR): L. Mathelitsch, W. Plessas, W. Schweiger, Phys. Rev. **C26**, 65 (1982)
Graz II: G. Rupp, J.A. Tjon, Phys. Rev. **C41**, 472 (1990)
Paris: M. Lacombe *et al.*, Phys. Rev. **C21**, 861 (1980)

Cross section: Comparison with experiment, $Q^2 = 2\text{GeV}^2$

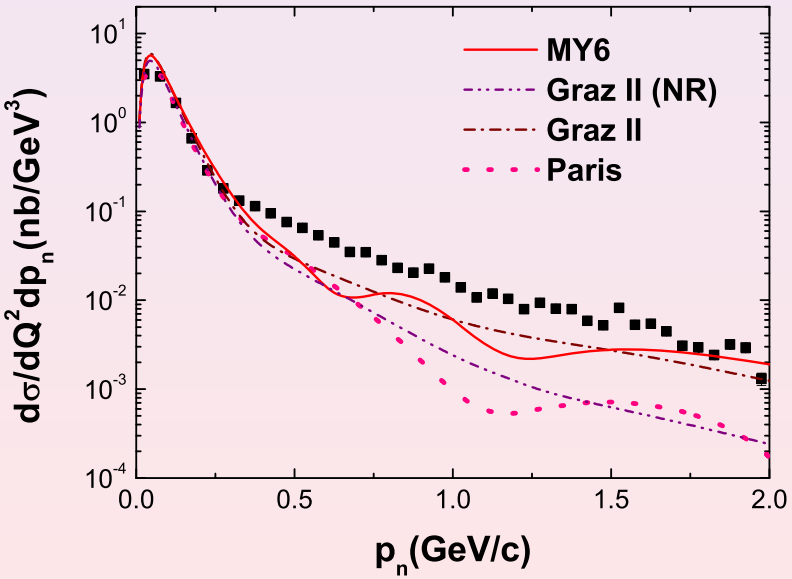
S.G. Bondarenko, V.V. Burov, M. Beyer, S.M. Dorkin, Phys. Rev. **C58**, 3143 (1998)

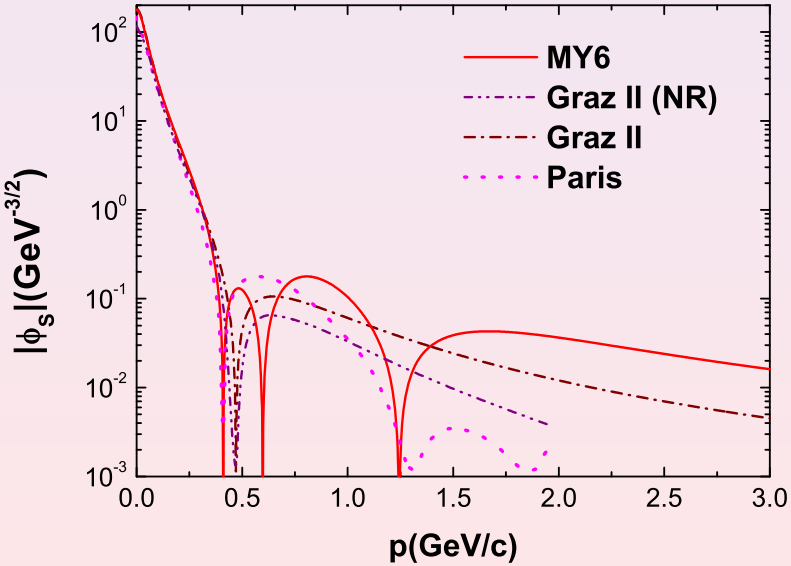
wave function for ${}^3S_1^+$ partial-wave state

$$\phi_l(p_0, |\boldsymbol{p}|) = \frac{g_l(p_0, |\boldsymbol{p}|)}{(M_d/2 - E_{\mathbf{p}} + \imath\epsilon)^2 - p_0^2}$$

at $p_0 = M_d/2 - E_{\mathbf{p}}$

cross section

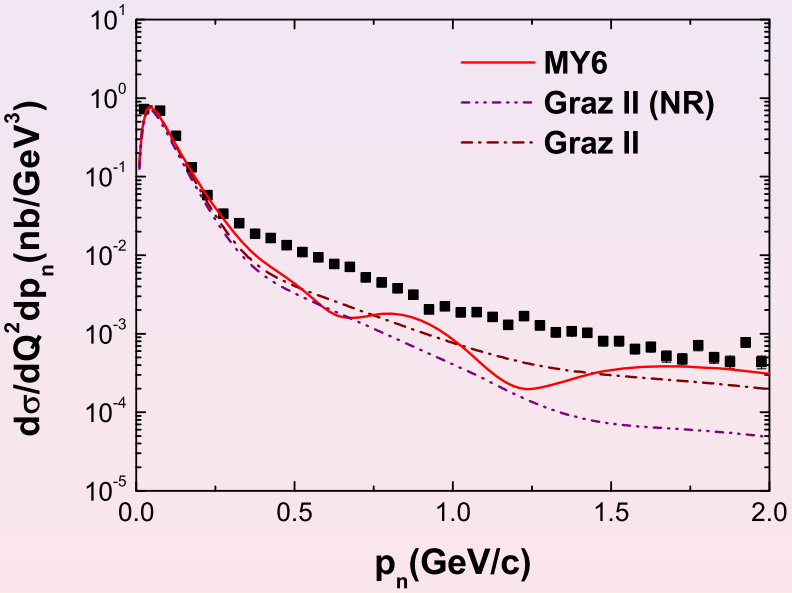




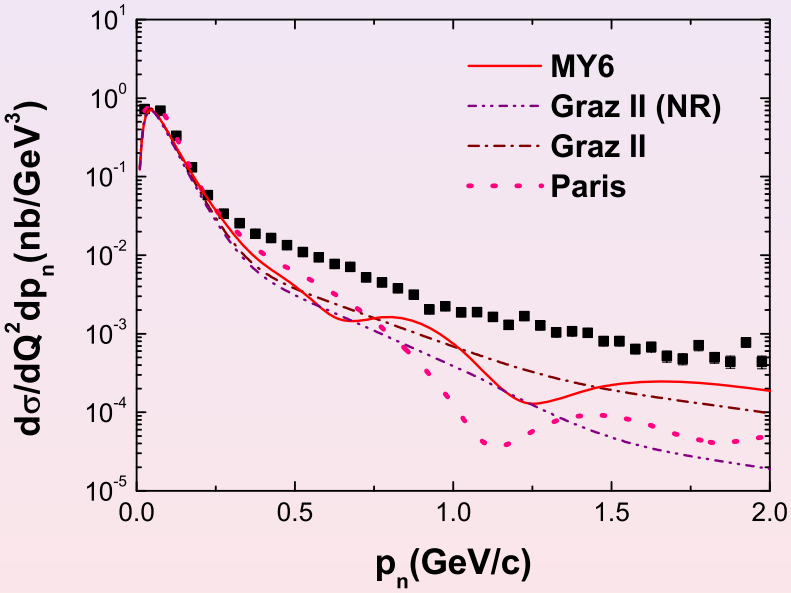
Cross section: Comparison with experiment, $Q^2 = 3\text{GeV}^2$

$$Q^2 = 3 \pm 0.5\,\text{GeV}^2$$

dipole FF



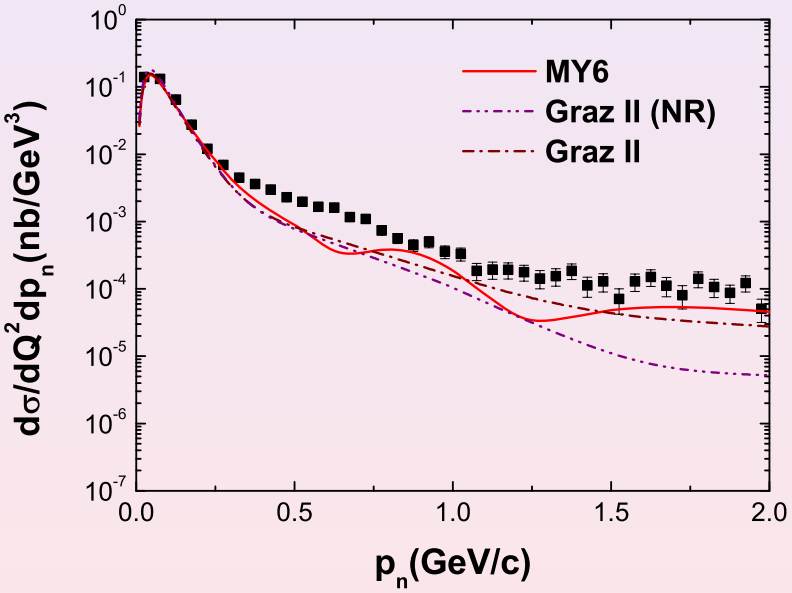
modified dipole FF



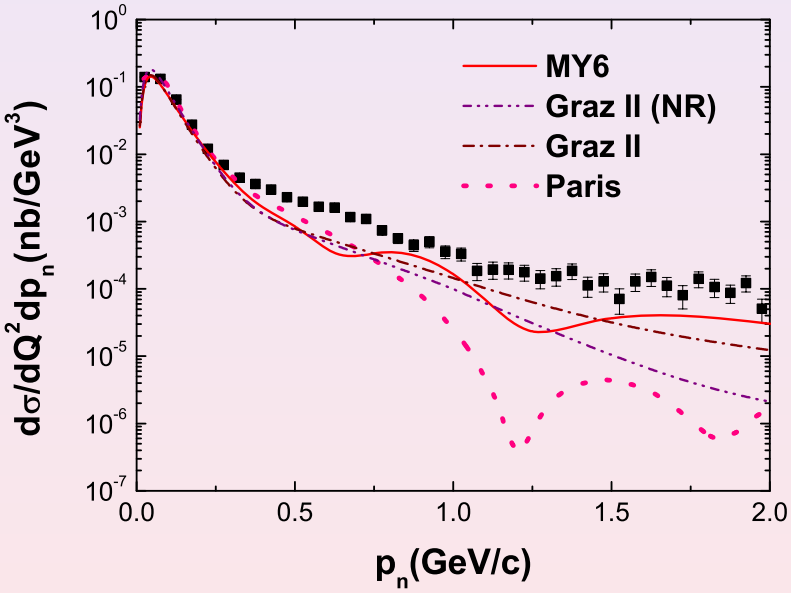
Cross section: Comparison with experiment, $Q^2 = 4\text{GeV}^2$

$$Q^2 = 4 \pm 0.5\,\text{GeV}^2$$

dipole FF



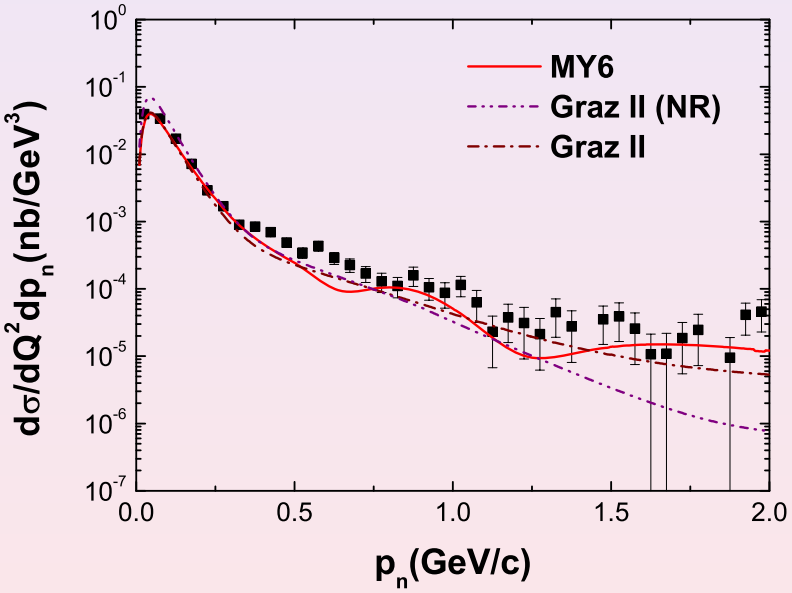
modified dipole FF



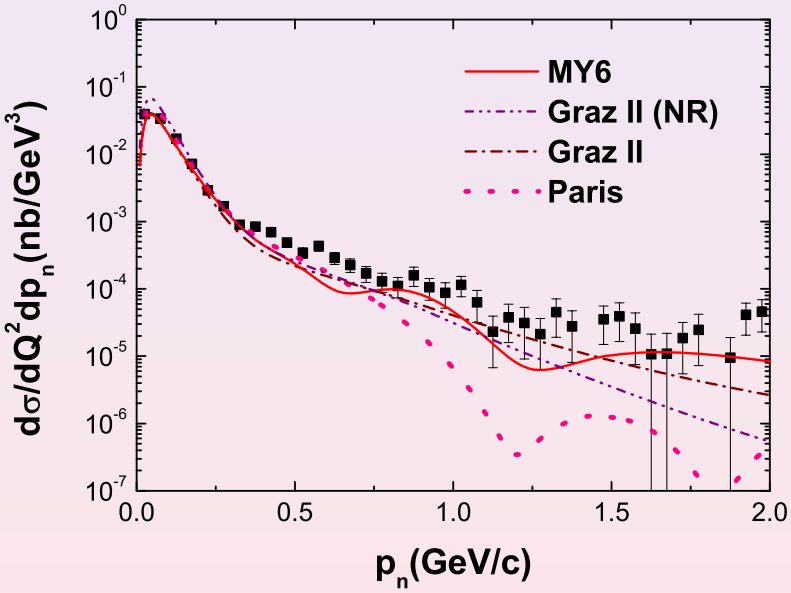
Cross section: Comparison with experiment, $Q^2 = 5\text{GeV}^2$

$$Q^2 = 5 \pm 0.5\,\text{GeV}^2$$

dipole FF



modified dipole FF



Conclusion

- The Bethe-Salpeter approximation with a covariant separable interaction kernel for the description of the nucleon-nucleon interactions was presented.
- Four different models of NN interactions and two models of nucleon electromagnetic form factors were considered for the description of the deuteron electrodisintegration at high momentum transfer.
- The considerable influence of relativistic effects with the increase of the momentum transfer and the outgoing neutron momentum was demonstrated.
- The investigation showed slight dependence of the calculated cross section on the used model for nucleon form factors. This conclusion should be tested in further calculations when final state interaction (possible only with MY6) and pair currents effects will be taken into account.