MATHEMATICAL PRINCIPLES UNIFYING GENERAL RELATIVITY AND QUANTUM MECHANICS

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Abstract

Subject of present announcement is unification of General Relativity and Quantum Mechanics into an unified full theory, which will be named here as General Quantum Mechanics. To have objective nature and absolutely rigid structure the theory should be absolutely free of any outer conditions, especially of observers. The main goal (and result) of General Quantum Mechanics is creation of the new conceptual basis that provides full understanding of the physics of the so-called elementary particles and cosmology. in the framework of unique structure.

Our intention here is to exhibit the key points of GR and QM unification. It is widely accepted that this is the most actual problem of modern physics. Unification is fundamental problem because in the another case we will not be able to finish building of Babylon Tower.

1 General Principle

From the principle of general covariance and uncertainty principle (notion of trajectory disappears in quantum theory of particles) it follows that coordinates have no physical meaning both in GR and QM and this is a very important common basis of the unification. From this it follows that only General Principle of GQM reads: Physical Space is smooth manifold because this structure does not distinguish intrinsically between different coordinate systems (the principle of general covariance is naturally included into this notion. (framed manifold and manifold) Framed manifold emerges when the connection between physical space and physical fields is established.

Smooth manifold consists of topological manifold and differential structure defined on it. It is known that a topological manifold always admits

differential structure if and only if its dimension is not larger than four. We conclude that dimension of physical space can be equal 2,3 or 4.

The conceptual structure of GQM includes the following foundational notions: concept of really geometrical quantity i.e., quantity connected with the geometrical structure of physical space; new field concept of time as the cornerstone of any dynamical theory; concept of geometrical internal symmetry which comes in after the introduction of really geometrical quantities to make these quantities variable.

2 Concept of Really Geometrical Quantity

The guide principle of geometrization and unification reads: the geometrical structure of physical space (points, curves, congruences of curves, families of curves) determines a very restricted set of really geometrical quantities (fundamental fields) and along with that a geometrical internal symmetry that make these quantities variable. The set of geometrical quantities that are connected with the geometrical structure of the physical space is very restricted since it is defined by the natural functionals and systems of ordinary differential equations.

Let us enumerate really geometrical quantities:
1)the positive definite Riemann metric g_{ij};
2) the linear (affine) connection P^i_{jk};
3) the scalar and covariant vector fields, antisymmetric covariant tensor fields. For our goals it is useful to show the last set of fields as 2^n-tuple $(n = 2, 3, 4)$

$$\mathbf{A} = (a, a_i, a_{ij}, \cdots a_{ijk\ldots l}).$$

This is very small zoo of the really geometrical quantities should be sufficient for understanding of everything.

List of physical fields: 1) Gravitational field with potential g_{ij};
2) General electromagnetic field with potential P^i_{jk}; 3) Spinning field with potential

$$\mathbf{A} = (a, a_i, a_{ij}, \cdots a_{ijk\ldots l}).$$

By definition, the geometrical internal transformations come in after the introduction of really geometrical quantities to make them variable. The general role of this symmetry can be outlined as follows: **geometrical internal symmetry makes the really geometrical quantities variable putting in correspondence a given quantity some new quantity. To**

be "observable" it should be broken and being broken it leaves a trace in the form of differential equations for these quantities that describe all effects connected with internal symmetry.

Spin symmetry is geometrical internal symmetry that describes such phenomena as spin. Being broken, spin symmetry designs equations of spinstatic and spindynamics. Spin symmetry group is general linear group $GL(2^n, \mathbf{R})$ that act in the space of the spinning fields. It can be shown that this group has a structure of the direct product of two groups of the same structure

$$GL(2^n, \mathbf{R}) = S \times \tilde{S}.$$

This result express in the abstract form that spinning field has internal angular momentum and magnetic momentum.

3 Concept of Time

In geometry there is no motion, which is tightly connected with the concept of time. Since, in general, coordinates have no physical sense, time should be first introduced into the theory as really geometrical quantity. This is first observation. Further, the quantum-mechanical causality shows clearly that all dynamical laws of undisturbed systems have the following form: **the rate of change with time of certain quantity equals the result of action of some operator on this quantity. The rate of change with time is the operator of evolution which defines causality in the field theory. In general, the coordinate independent definition of the rate of change with time of field quantity cannot be given without a new field concept of time.** The idea that **the time is a scalar field** was put forward suggesting, by way of justification, a self-consistent theory of fields which does not depend on outer conditions. In GQM the properties of time and physical space are not defined by the properties of devices and by the methods of measurements. Let me remind that GQM should be free from any external conditions.

The temporal field with respect to the coordinate system u^1, u^2, u^3, u^4 is denoted by $f(u) = f(u^1, u^2, u^3, u^4)$. The gradient of the temporal field (the stream of time) is the vector field $\mathbf{t}$ with the components $t^i = (\nabla f)^i = g^{ij}\frac{\partial f}{\partial u^j} = g^{ij}\partial_j f = g^{ij}t_j$, where g^{ij} are the contravariant components of the Riemann metric (1). The rate of change with time of some quantity is the Lie derivative with respect to the stream of time $\mathbf{t}$ and the symbol $D_{\mathbf{t}}$ denotes

this operation. For the rate of change with time of the temporal field itself we get $D_{\mathbf{t}} f = t^i \partial_i f = g^{ij} \partial_i f \, \partial_j f$. The temporal field obeys the fundamental equation :

$$D_{\mathbf{t}} f = (\nabla f)^2 = g^{ij} \frac{\partial f}{\partial u^j} \frac{\partial f}{\partial u^j} = 1. \qquad (1)$$

which means that time flows equably. The other possible operator of evolution has the form of covariant derivative in the direction of the stream of time. This operator is denoted by $\nabla_{\mathbf{t}} = t^i \nabla_i$, where ∇_i is a covariant derivative with respect to the connection that belongs to the Riemann metric g_{ij}.

Stream of time defines fundamental discrete symmetry, the symmetry of the right and left or bilateral symmetry. This symmetry is the fundamental realization of the concept of geometrical internal symmetry. A pair of vector fields $\mathbf{v}$ and $\overline{\mathbf{v}}$ has bilateral symmetry with respect to the stream of time if the sum of these fields is collinear to the gradient of a temporal field and their difference is orthogonal to it, $\overline{\mathbf{v}} + \mathbf{v} = \lambda \mathbf{t}, \quad (\overline{\mathbf{v}}, \mathbf{t}) = (\mathbf{v}, \mathbf{t})$, where $(\mathbf{v}, \mathbf{w}) = g_{ij} v^i w^j = v^i w_i$ is a scalar product. In components, we have $\overline{v}^i = 2(\mathbf{v}, \mathbf{t})\, t^i - v^i = (2 t^i t_j - \delta^i_j) v^j$. Thus, the bilateral symmetry may be represented as a linear transformation $\overline{v}^i = R^i_j v^j$, where $R^i_j = 2 t^i t_j - \delta^i_j$. This transformation is natural to call reflection. For the metric we get $\overline{g}_{ij} = g_{kl} R^k_i R^l_j = g_{ij}$.

The bilateral symmetry defines the causal structure and gives the straightforward method of introduction of temporal field into the Lagrangian of physical fields.

4 Static and Dynamics

From the new concept of time it follows directly that there are phenomena outside the time (in this case a temporal field is simply absent and system in question is at absolute rest). In GQM this take the form of two divisions: Statics and Dynamics. Statics defines the laws of the initial states of dynamical processes and, hence, initial states by itself. Only in this case GQM can be considered as absolutely closed structure free of any outer conditions. The connection between Statics and Dynamics is very simple. We consider the 3- dimensional physical space in the Statics as an initial space cross section of 4-dimensional physical space in the Dynamics. Due to this the problem of initial singularity disappears. The Big Bang is simply transition from a timeless state to a dynamical state (as a release of the potential

energy of a timeless system). The inverse process is also possible.

5 Some Results of Unification

1) It is shown that spinstatics and spindynamics consist the essence of the physics of elementary particles.

2) It is discovered that in the Universe there is two causal structure. One of them is defined by the general solution of the equation of temporal field. The another solution and new causal structure emerges as special solution of this equation. The second solution represents the essence of the so-called strong interactions.

3) The role of the gravitational field in the physics of elementary particles is recognized since the bipolar structure of the spin symmetry (spin) cannot be discovered without gravitational field.

4) The problem of dark energy is solved. Dark energy is simply the energy of the gravitational field. It is shown that in the case of the Universe filled only gravitational energy the potential energy of the gravitational field is strictly positive and, hence, in this case we have repulsion that is observed.

5) The Problem of Dark matter is solved. The Dark matter is heavy light that interact only gravitationally. Heavy photons is quants of general electromagnetic field. Massless photon represent singlet state of general electromagnetic field.

6) The problem of electric charge is solved. The operator of electric charge is found. This opens only real possibility to understand the electron-muon puzzle.

7) The problem of confinement can be treated as existence of second causal structure.

8) The most general form of the law of energy conservation is discovered.

9) The equations of the gravitational field, the spinning field, the general

electromagnetic field are derived.

The observer himself can use GQM as a unique instrument to learn more objective information about the physical phenomena itself, and to find new creative approaches and innovative possibilities for his own goals. The calculations and any activity can be fruitful only on the reliable basis of understanding that GQM provides in full measure.

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