Heavy-light quark systems in the instanton vacuum

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Outline

Current experiments

- *B*- and *D*-mesons experiments with unprecedented integrated luminosities:
  Belle, BaBar and CDF collaborations.
- Neutrino-production of open and hidden charm in neutrino-hadron processes:
  K2K , MiniBoone , NuTeV and Minerva collaborations.

Theory status

- Pre-QCD quantum-mechanical potential models with undefined phenomenological constants. The relation of them to QCD is quite obscure.
- An advanced version – NRQCD (Bodwin et al94). But here light-heavy quarks interactions is done in a phenomenological way.
- HQET (Isgur,Wise89) treats the heavy mesons using the pQCD methods but does not take into account nonperturbative effects.
- Phenomenological chiral lagrangian for heavy and light mesons taking into account the chiral and heavy quark symmetries of QCD (reviews Wise93, Casalbuoni et al97).

Application of the instanton vacuum model to a heavy-light quark systems

We propose to study the physics of heavy-light quark systems in the instanton vacuum model:

- light quark contribution to the properties of the heavy quarks;
- the couplings at the phenomenological chiral lagrangian for heavy and light mesons.

There is a tool for such a work.

$S\chi$SB in QCD instanton vacuum

- Correct description of the spontaneous breaking of the chiral symmetry ($S\chi$SB), which is responsible for properties of most hadrons and nuclei.
- $S\chi$SB is due to the delocalization of single-instanton quark zero modes in the instanton medium.
- Only two parameters:
  - average instanton size $\rho \sim 0.3\,\mathrm{fm}$,
  - average inter-instanton distance $R \sim 1\,\mathrm{fm}$,

  – suggested phenomenologically (Shuryak1981),
  – derived variationally from $\Lambda_{\overline{\mathrm{MS}}}$ (Diakonov,Petrov1983)
  – confirmed by lattice measurements (Negele et al1998, DeGrand et al2001, Faccioli et al2003, Bowman etal2004).
- The model provided a consistent description of the light quark physics (Diakonov et al, Goeke et al, Musakhanov et al).

QCD instantons

Instantons –classical solutions of the equations of motion in Euclidean space. In singular gauge (Belavin *et.al.*, 1975):

$$A^{I,a}_{\mu}(x) = \frac{2\rho^2 \bar{\eta}^{\nu}_{\mu a}(x-z)_{\nu}}{(x-z)^2[\rho^2+(x-z)^2]}\,.$$

For the antiinstanton just change the t'Hooft symbol $\bar{\eta} \to \eta$.

- The solutions are (anti)self-dual, *i.e.* $G^a_{\mu\nu} = \pm \tilde{G}^a_{\mu\nu}$.
- The topological charge $Q = \frac{1}{32\pi^2}\int d^4x\, G^a_{\mu\nu}\tilde{G}^a_{\mu\nu} = +1$ for instantons and -1 for antiinstantons.
- The action on both instantons and antiinstantons $S_I = \frac{8\pi^2}{g^2} \Rightarrow$ the amplitude of tunneling $\sim \exp(-S_I)$ with $|\Delta N_W| = 1$,
  $N_W = \frac{1}{24\pi^2}\int d^3x \epsilon_{ijk} \left\langle \left(U^\dagger \partial_i U\right)\left(U^\dagger \partial_j U\right)\left(U^\dagger \partial_k U\right)\right\rangle$.
- Number of collective coordinates for each instanton:

$$4\,(centre)\ +\ 1\,(size)\ +\ (4N_c{-}5)\,(orientations)\ =\ 4N_c$$

Dependence on N_{CS}

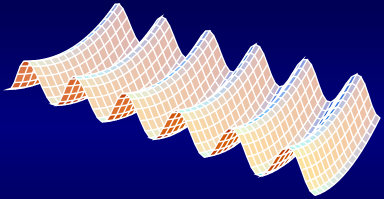

Figure: Vacuum gluon energy vs Chern-Simons number
$N_{CS} = \int d^3x\, K_0 = \frac{1}{16\pi^2} \int d^3x\, \epsilon^{ijk} \left(A_i^a \partial_j A_k^a + \frac{1}{3} \epsilon^{abc} A_i^a A_j^b A_k^c \right),$
$N_{CS} \Rightarrow N_{CS} + N_W.$

Instanton ensemble

- Sum ansatz $A = \sum_I A^I + \sum_{\bar{I}} A^{\bar{I}}$ for dilute gas approximation. Allows analytical evaluation, even with quarks.
- Example of exact multiinstanton solution (self-duality):

$$A_\mu^a = \bar{\eta}_{a\mu\nu}\partial_\nu \ln\left(1 + \sum_i \frac{\rho_i^2}{(x - z_i)^2}\right)$$

- Instanton-antiinstanton interactions: Ratio ansatz, Streamline ansatz. *Sum ansatz gives too strong repulsion for $R \leq \rho$.*
- Partition function–only numerically (lattice).

Parameters of instanton ensemble

- Size distribution $D(\rho)$ and average value $\bar{\rho}$
- Density of instantons (or average interinstanton distance $\bar{R}$)
- Results:
  - Lattice estimate: $\bar{R} \approx 0.89\, fm$, $\bar{\rho} \approx 0.36\, fm$,
  - Phenomenological estimate: $\bar{R} \approx 1\, fm$, $\bar{\rho} \approx 0.33\, fm$,
  - Our estimate (with account of $1/N_c$ corrections): $\bar{R} \approx 0.76\, fm$, $\bar{\rho} \approx 0.32\, fm$, correspond $F_{\pi,m=0} = 88 MeV, \langle \bar{q}q \rangle_{m=0} = -(255 MeV)^3$

  Thus within $10 - 15\%$ uncertainty different approaches give similar estimates
- Packing parameter $\underline{\pi^2 (\frac{\bar{\rho}}{\bar{R}})^4 \sim 0.1 - 0.3}$

  $\Rightarrow$ Independent averaging over instanton positions and orientations.

QCD vacuum on the lattice

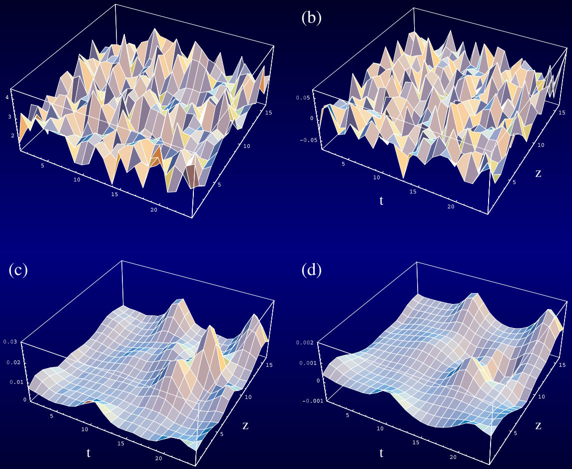

Figure: Action and topological charge densities in different configurations on the lattice.

Light quarks in the instanton background

Basic assumptions (Diakonov *et.al.*, 1986-2006):

- Sum ansatz as background. Quarks $\Rightarrow$ quenched approximation.
- Zero-mode approximation

$$S(x,y) \approx \frac{|\Phi_0(x,\zeta)\rangle\,\langle\Phi_0(y,\zeta)|}{im} + \frac{1}{i\hat{\partial}}, \quad (i\hat{\partial} + g\hat{A})\Phi_0(x,\zeta) = 0,$$

  collective coordinates ζ: a instanton position z and color orientation U.
- The number of colors $N_c \to \infty$, LO over N_c is kept.
- The width of the size distribution is suppressed as $1/N_c$

are working well at $m \Rightarrow 0$ but wrong beyond the chiral limit.

Zero mode vs. Chiral Symmetry

Extension of zero-mode approximation beyond the chiral limit:

$$S_i = S_0 - S_0 \hat{p} \frac{|\Phi_{0i}\rangle\langle\Phi_{0i}|}{\langle\Phi_{0i}|\hat{p} S_0 \hat{p}|\Phi_{0i}\rangle} \hat{p} S_0, \quad S_0 = \frac{1}{\hat{p} + im},$$

$$S_i|\Phi_{0i}\rangle = \frac{1}{im}|\Phi_{0i}\rangle, \quad \langle\Phi_{0i}|S_i = \langle\Phi_{0i}|\frac{1}{im}.$$

Sum-up of multi-scattering series $\Rightarrow$ full light quark propagator:

$$S - S_0 = -S_0 \sum_{i,j} \hat{p}|\Phi_{0i}\rangle \left\langle \Phi_{0i} \left| \left(\frac{1}{B(m)} \right) \right| \Phi_{0j} \right\rangle \langle\Phi_{0j}|\hat{p} S_0,$$

$$B(m) = \hat{p} S_0 \hat{p}$$

Low-frequency part of the light quark determinant with the quark sources

We have to calculate $\mathrm{Det}(\hat{P}+im)e^{(-\xi^+ S\xi)}$:

- $\ln \mathrm{Det}(\hat{P}+im) = -i\mathrm{Tr}\int_m^{M_{PV}} dm' \,\tilde{(}S(m)-S_0(m'))$.
- $\mathrm{Det}(\hat{P}+im) = \mathrm{Det}_{\mathrm{high}}\cdot\mathrm{Det}_{\mathrm{low}}$.
- $\mathrm{Det}_{\mathrm{high}}$ is accounted Dirac eigenvalues from M_1 to the Pauli–Villars mass M_{PV}.
- $\mathrm{Det}_{\mathrm{low}}$ is accounted eigenvalues less than M_1.

We get for each flavor $\mathrm{Det}_{\mathrm{low}}\exp\left(-\xi^+ S\xi\right) =$

$$= \det B(m)\exp\left(-(\xi^+ S_0\xi) + \sum_{i,j}\xi_i^+\left(\frac{1}{B(m)}\right)_{ij}\xi_j\right)$$

$\xi_i^+ = \xi^+ S_0\hat{p}|\Phi_{0i}>$, $\xi_j =<\Phi_{0j}|\hat{p}S_0\xi$.

Fermionized representation

The tricks:

- Grassmanian variables representation of $\det B(m)$,
- introducing of fermionic fields $\psi^\dagger, \psi$,
- changing the order of the integrations.

provides finally $\det B(m) \exp(-\xi^+ S \xi) =$

$$= \int \prod_f D\psi_f D\psi_f^\dagger \exp \int \left(\psi_f^\dagger (\hat{p} + i m_f) \psi_f + \psi_f^\dagger \xi_f + \xi_f^+ \psi_f \right)$$

$$\times \prod_f \left\{ \prod_{+}^{N_+} V_{+,f}[\psi^\dagger, \psi] \prod_{-}^{N_-} V_{-,f}[\psi^\dagger, \psi] \right\} ,$$

where $V_{\pm,f}[\psi^\dagger, \psi] =$

$$= i \int dx \left(\psi_f^\dagger(x)\, \hat{p} \Phi_{\pm,0}(x; \zeta_\pm) \right) \int dy \left(\Phi_{\pm,0}^\dagger(y; \zeta_\pm) (\hat{p}\, \psi_f(y) \right) .$$

Partition function

Averaging over instantons collective coordinates $\Rightarrow$ partition function $Z[\xi_f, \xi_f^+] =$

$$= \int \prod_f D\psi_f D\psi_f^\dagger \exp \int \left(\psi_f^\dagger (\hat{p} + i m_f) \psi_f + \psi_f^\dagger \xi_f + \xi_f^+ \psi_f \right)$$

$$\times \int D\zeta \prod_f \left\{ V_{+,f}^{N_+}[\psi^\dagger, \psi] V_{-,f}^{N_-}[\psi^\dagger, \psi] \right\} ,$$

Small packing parameter provided here independent averaging:

$$\int d\zeta_\pm \prod_f V_{\pm,f}[\psi^\dagger, \psi] = \prod_f \bar{V}_{\pm,f}[\psi^\dagger, \psi]$$

Partition function at $N_f = 1$ and 2

At $N_f = 1$ and $N_\pm = N/2$

$$Z[\xi, \xi^+] = \exp\left[-\xi^+ \frac{1}{\hat{p} + i(m + M(p))}\xi + \mathrm{Tr}\ln\left(\hat{p} + i(m + M(p))\right) + N\ln\frac{N/2}{\lambda} - N\right]$$

$N = \mathrm{Tr}\frac{iM(p)}{\hat{p}+i(m+M(p))}$, $\quad M(p) = \frac{\lambda}{N_c}(2\pi\rho F(p))^2$

At $N_f = 2$, $N_\pm = N/2$ and saddle-point approximation

$$Z[\xi_f, \xi_f^+] = \exp\left[-\sum_f \xi_f^+ \left(\hat{p} + im_f + iM_f(p)\right)^{-1}\xi_f + N\ln\frac{N/2}{\lambda} - N - \frac{1}{2}V\sigma^2 + \mathrm{Tr}\ln\frac{\hat{p} + im + iM(p)}{\hat{p} + im}\right]$$

$\lambda, \sigma,$ $M(p) = \lambda^{0.5}(2g)^{-1}(2\pi\rho)^2 F^2(p)\sigma$ from the Eqs.
$N = 0.5V\sigma^2 = 0.5\mathrm{Tr}\, iM(p)(\hat{p} + im + iM(p))^{-1}$

Heavy quarks in the instanton background

(Infinitely) heavy quark propagator (Wilson line) $S_H =$

$$= \frac{1}{Z}\int \prod_f D\psi_f D\psi_f^\dagger \prod_{\pm}^{N_\pm} \bar{V}_{f,\pm}[\psi^\dagger,\psi] e^{\int \psi_f^\dagger(\hat{p}+im_f)\psi_f} w[\psi,\psi^\dagger]$$

$$w[\psi,\psi^\dagger] = \left\{\prod_{f,\pm}^{N_\pm} \bar{V}_{f,\pm}[\psi^\dagger,\psi]\right\}^{-1} \int D\zeta \left\{\prod_{f,\pm}^{N_\pm} V_{f,\pm}[\psi^\dagger,\psi]\right\}$$

$$\times \frac{1}{\theta^{-1} - \sum_i a_i},\ w_\pm = \frac{1}{\theta^{-1} - a_\pm},\ < t|\theta|t' >= \theta(t-t'),$$

$$< t|\theta^{-1}|t' >= -\frac{d}{dt}\delta(t-t'), a_i(t) = iA_{i,\mu}(x(t))\frac{d}{dt}x_\mu(t)$$

Heavy quark propagator at $N_f = 1$

Extension of DPP89 solution (planar graphs) is $w^{-1}[\psi,\psi^\dagger] =$

$$= \theta^{-1} - \frac{N}{2}\sum_{\pm}\frac{1}{\bar{V}_\pm[\psi^\dagger,\psi]}\Delta_{H,\pm}[\psi^\dagger,\psi] + O(N^2/V^2),$$

$$\Delta_{H,\pm}[\psi^\dagger,\psi] = \int d\zeta_\pm\; V_\pm[\psi^\dagger,\psi]\theta^{-1}(w_\pm-\theta)\theta^{-1}.$$

Then at $N_f = 1$

$$S_H = \frac{1}{\theta^{-1} - \lambda\sum_\pm \Delta_{H,\pm}[\frac{\delta}{\delta\xi},\frac{\delta}{\delta\xi^+}]}\exp\left[-\xi^+\left(\hat{p} + iM(p)\right)^{-1}\xi\right]|_{\xi=\xi^+}$$

DPP89 solution is reproduced at the approximation:

$$S_H \approx \frac{1}{\theta^{-1} - \lambda\sum_\pm \Delta_{H,\pm}[\frac{\delta}{\delta\xi},\frac{\delta}{\delta\xi^+}]\exp\left[-\xi^+\left(\hat{p} + iM(p)\right)^{-1}\xi\right]|_{\xi=\xi^+}}$$

At any N_f and in saddle-point approximation no an essential difference with $N_f = 1$.

Heavy–light quarks interactions from instantons at $N_f = 1$

From S_H^{-1} the heavy–light quarks interaction term as

$$-\lambda \sum_{\pm} Q^\dagger \Delta_{H,\pm}[\psi^\dagger, \psi] Q = -i\lambda \sum_{\pm} \int d^4 z_\pm \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4}$$
$$\times e^{i(k_2-k_1)z_\pm} (2\pi\rho)^2 F(k_1) F(k_2) \frac{1}{N_c^2} \psi^+(k_1) \frac{1 \pm \gamma_5}{2} \psi(k_2)$$
$$\times Q^+ \mathrm{tr}_c \left(\theta^{-1}(w_\pm - \theta)\theta^{-1}\right) Q + \frac{1}{32(N_c^2-1)} \psi^+(k_1)(\gamma_\mu\gamma_\nu$$
$$\times \frac{1 \pm \gamma_5}{2}) \lambda^i \psi(k_2) \mathrm{tr}_c(\tau_\mu^\mp \tau_\nu^\pm \lambda^j) Q^+ \mathrm{tr}_c \left(\theta^{-1}(w_\pm - \theta)\theta^{-1} \lambda^j\right) \lambda^i Q$$

Here $Q, Q^\dagger$ are heavy quark fields.
At any N_f the interaction term will have 2 heavy and $2N_f$ light quark legs. The actual structure is defined by the color orientation integration like here.

Comments on heavy–light quarks interactions from instantons

- Instantons generate nonlocal quark-quark interactions. Range of the nonlocality $\sim \rho \approx 0.3\, fm$.
- At $N_f = 2$ case among varieties of terms there is a term with 2 heavy quarks 2 light quarks and pion legs like: $qQ \Rightarrow q'Q'\pi$.

Heavy quark–antiquark system correlator, $N_f = 1$

The correlator $C(L_1, L_2) =$

$$= \frac{1}{Z}\int D\psi D\psi^\dagger \prod_{\pm}^{N_\pm} \bar{V}_\pm[\psi^\dagger,\psi] \exp\int\left(\psi^\dagger(\hat{p + im})\psi\right) W[\psi,\psi^\dagger]$$

$$< T|W[\psi,\psi^\dagger]|0 >= \left(\prod_{\pm}^{N_\pm} \bar{V}_\pm[\psi^\dagger,\psi]\right)^{-1} \int D\zeta \prod_{\pm}^{N_\pm} V_\pm[\psi^\dagger,\psi]$$

$$\times < T|\frac{1}{\theta^{-1} - \sum_i a_i^{(1)}}|0 >< 0|\frac{1}{\theta^{-1} - \sum_i a_i^{(2)}}|T > .$$

is a Wilson loop along the rectangular contour $L \times r$. The sides $L_1 = (0, T), L_2 = (T, 0)$ are parallel to x_4 axes and separated by the distance r. The $a^{(1)}, a^{(2)}$ are the projections of the instantons onto the lines L_1, L_2.

Heavy quark–antiquark system correlator, $N_f = 1$

The extension of DPP89 solution is $W^{-1}[\psi,\psi^\dagger] =$

$$= w_1^{-1}[\psi,\psi^\dagger] \times w_2^{-1,T}[\psi,\psi^\dagger] - \frac{N}{2}\sum_{\pm} \bar{V}_\pm^{-1}[\psi^\dagger,\psi]\int d\zeta_\pm$$

$$\times V_\pm[\psi^\dagger,\psi]\theta^{-1}\left(w_\pm^{(1)} - \theta\right)\theta^{-1}(\times)\left(\theta^{-1}\left(w_\pm^{(2)} - \theta\right)\theta^{-1}\right)^T$$

where, superscript T means the transposition, $(\times)$ – tensor product and

$$w^{(1,2)^{-1}}[\psi,\psi^\dagger] = \theta^{-1} - \frac{N}{2}\sum_{\pm}\frac{1}{\bar{V}_\pm[\psi^\dagger,\psi]}\Delta_{H,\pm}^{(1,2)}[\psi^\dagger,\psi] + O(\frac{N^2}{V^2}).$$

Heavy quark–antiquark potential V_{lq}, generated by light quarks, $N_f = 1$

Explicitly the integration of the first term in $W^{-1}[\psi, \psi^\dagger]$ over $\psi, \psi^\dagger$ leads to $V_{lq} =$

$$= \left(\lambda \sum_{\pm} \Delta^{(1)}_{H,\pm}[\frac{\delta}{\delta \xi_1}, \frac{\delta}{\delta \xi_1^+}]\right) (\times) \left(\lambda \sum_{\pm} \Delta^{(2)}_{H,\pm}[\frac{\delta}{\delta \xi_2}, \frac{\delta}{\delta \xi_2^+}]\right)^T$$
$$\times \exp\left[-\xi_2^+ \left(\hat{p} + iM(p)\right)^{-1} \xi_1 - \xi_1^+ \left(\hat{p} + iM(p)\right)^{-1} \xi_2\right]_{\xi=\xi^+=0}$$

V_{lq} represent the heavy quark -antiquark interaction potential, generated by light quark-antiquark exchange.

Comment on heavy quark–antiquark potential V_{lq}

- The range of heavy quark–antiquark potential V_{lq}, generated by light quarks exchange between heavy quarks, is controlled by dynamical light quark mass $M \sim 0.35$ GeV.

Discussion

It was developed the QCD instanton vacuum based framework which is naturally lead to the consistent treatment of light quark physics and now is applied to heavy quarks too. Within this one we find:

- QCD vacuum instantons lead to the light quark interactions, responsible for the $S\chi$SB and the most important properties of light hadrons and nuclei.
- In the presence of heavy quarks instantons generate also their interactions with light quarks.
- Such an interactions give a contributions to the heavy quark and heavy quark–antiquark system properties.
- There is a consistent way to estimate the couplings in the phenomenological chiral lagrangian for heavy and light mesons, accounting $S\chi$SB and heavy quark symmetries.