Relativistic structure of nuclei

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Issues in quantum field theory description of bound states

- Final and initial states are not asymptotically free.

Solution: Mandelstam reduction technique.

Reformulation for multiparticle systems is needed

- Perturbation theory does not work – infinite number of terms have to be considered.

Solution: Non-perturbative kernels of the Bethe-Salpiter equation

Axiomatic approach, connection with observables or numerical modeling on the lattice are needed.

- Bound particles are separated by a four-dimensional interval.

Bound nucleons are in different times.

Arthur Eddington: «Yesterday electron and tomorrow proton cannot bound an hydrogen atom today»

Existing solutions for the relative time issue

Quasi-potential approaches: the relative time is an artifact of the relativistic theory

- Approaches with the spectator on the mass-shell: Gross, Kadyshevskiy
- Approaches with the relative time set to zero
- Approaches with the relative time set to zero and the energy and momentum conservation law preserved by the ghost particles: Karmanov

All of the approaches require additional model-dependent dynamical degrees of freedom

3D limit of a theory

$$S = \int d^4x \mathcal{L}(\phi(x), \partial_\mu \phi(x))$$

To get the Lagrange function

$$L = \int d^3x \mathcal{L}(\phi(x), \partial_\mu \phi(x))$$

one has to choose a 3D hypersurface in the 4D space

The ways $d^4x \longrightarrow d^3x dt$ define different Hamiltonian dynamical systems

The transformation $d^4x \longrightarrow d^3xdt$ induces smoth bundle on the 4D manifold

Definition of a smoth bundle

1) bundle space E $\qquad d^3xdt$

2) base of the bundle M $\qquad dt$

3) Projection $p : E \longrightarrow M$

4) layers F $\qquad d^3x$

5) transformation group G for the layers F $\qquad G(1,3)$

6) structure of the bundle:

M covered by $\{U_\alpha\}$ where the direct product is introduced:

$$\phi_\alpha : F \times U_\alpha \longrightarrow p^{-1}(U_\alpha) \qquad p\phi_\alpha(y,x) = x$$

Time - evolution in the bundle space - time

Smooth transformation from layer to layer: Bundle space with connectedness

$\{F_t\}$ - set of spaces, members of which depend of the parameter t

$E = \mathrm{U} F_t$ - is the bundle space

Each path $\gamma(s), a \leq s \leq b$ in M corresponds transition of the layer *F* along the line from the point *a* to the point *b*.

$$\phi_\gamma : F_{t_0} \to F_{t_1} \quad t_0 = \gamma(a) \qquad t_1 = \gamma(b)$$

General definition: connectedness is a distribution, which defines for each point *y* of the space *E* a direction orthogonal to *F* in the point *y*. Locally in the base coordinates *x* the connectedness can be defined the following differential form:

$$A = A_\mu dx^\mu$$

This transition can be considered as time evolution of a dynamical system

Instant form of dynamics

E — orthogonal bundle with space-like layers

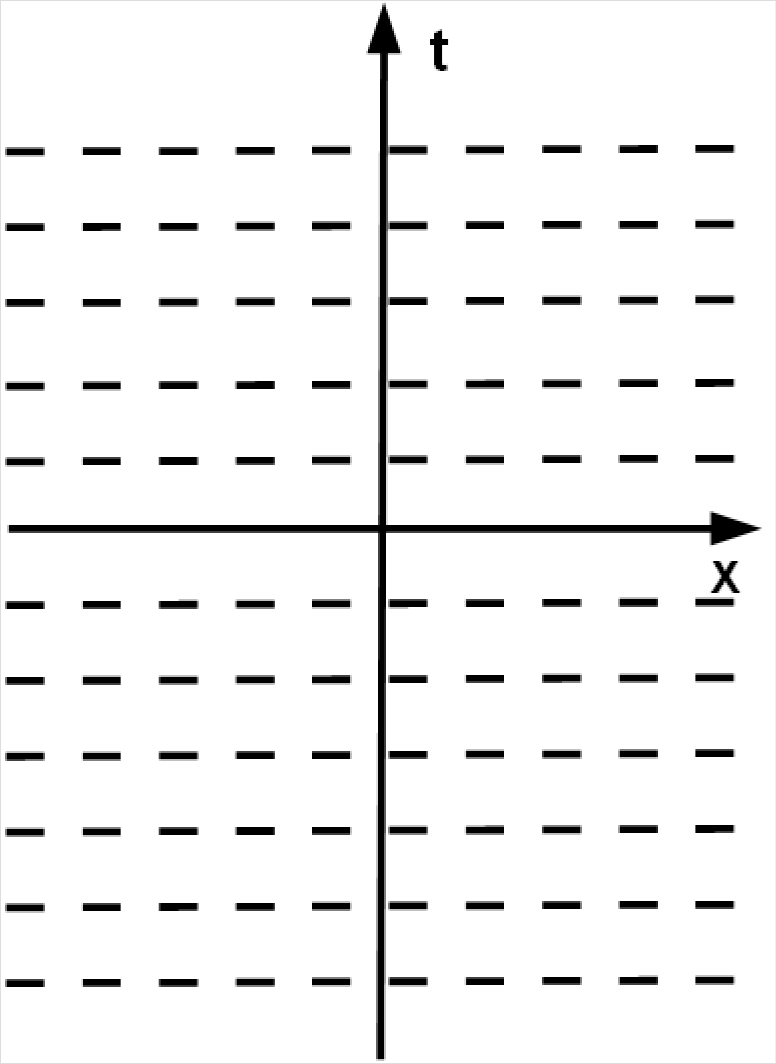
G — Galilean transformations

$$G = G(1,3)$$

layer — nonrelativistic 3D space

$$ct = Const$$

The space that we usually imagine ourselves



Point form of dynamics

G — 4D rotations,
Lorentz group

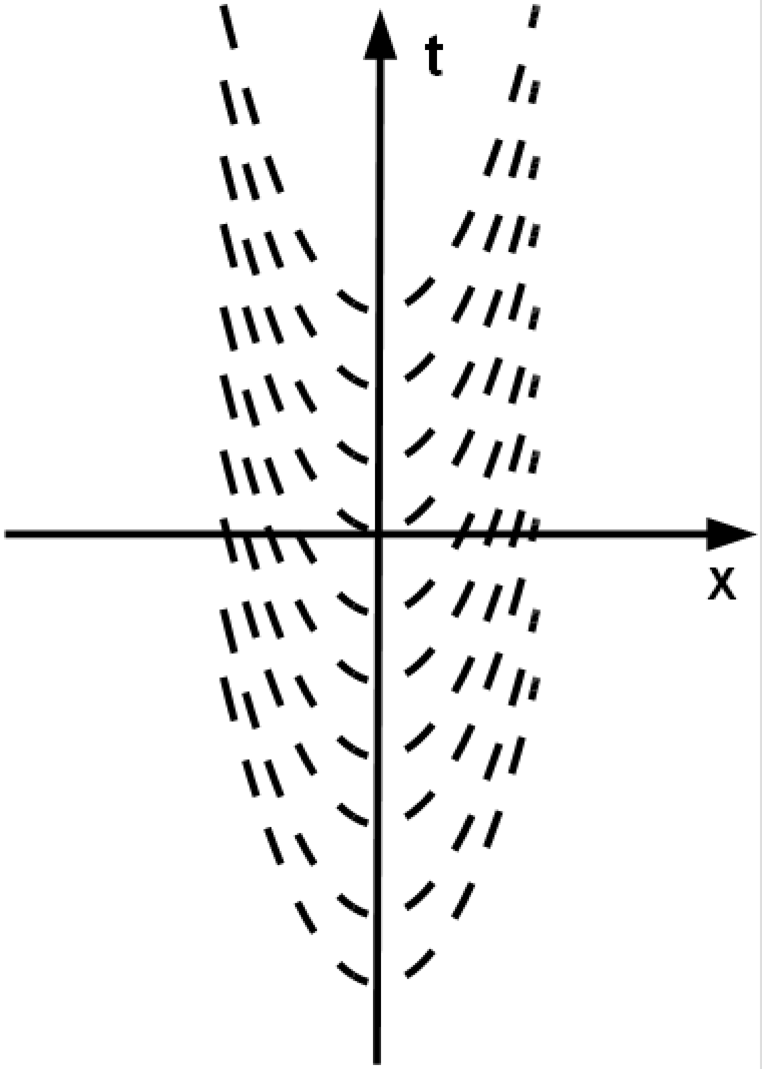
$$G = SL(2, C)$$

Layer — 4D sphere with a zero radius

$$c^2 t^2 - x^2 = 0$$

3D space with synchronized clocks

This is the space with equal time events



Light cone dynamics

G — Symmetry transformations of the light cone, spinor transformations

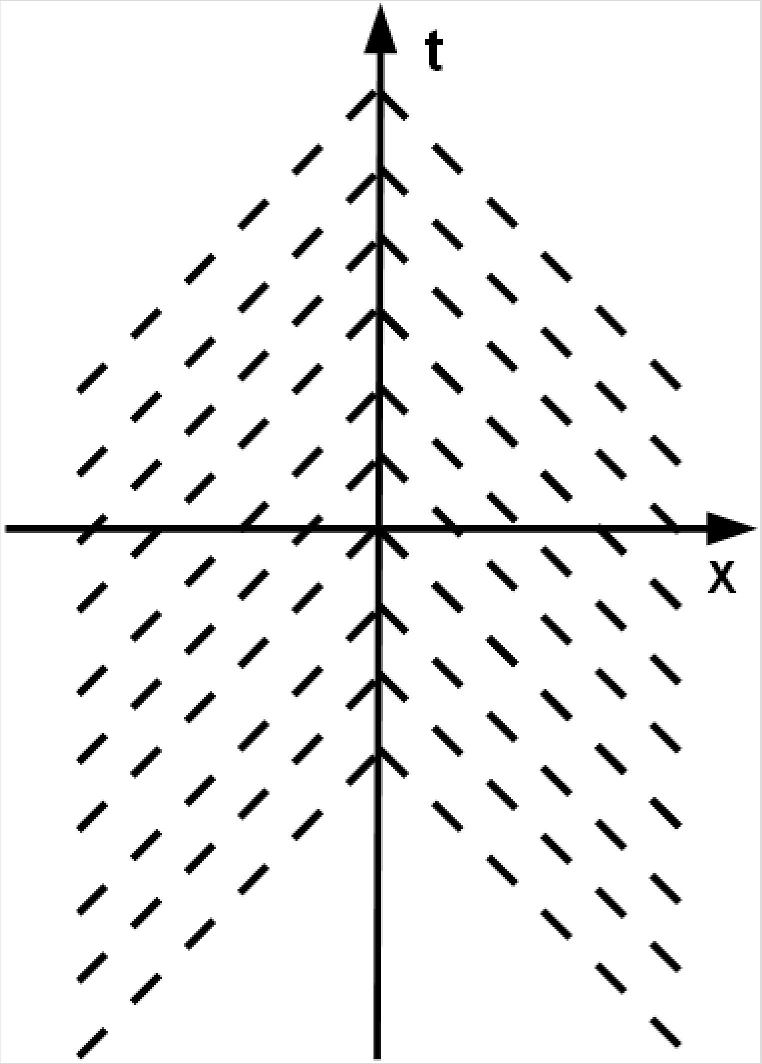
$$G = SU(2)$$

Layer — light cone

$$c|t| - |x| = 0$$

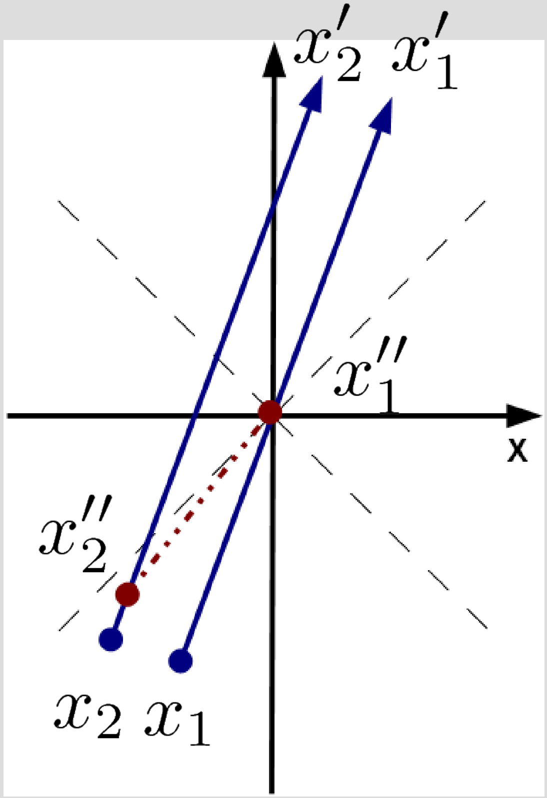
Layers — Celestial sphere, which is the world that we really see by our eyes

Universe observations are made in this system

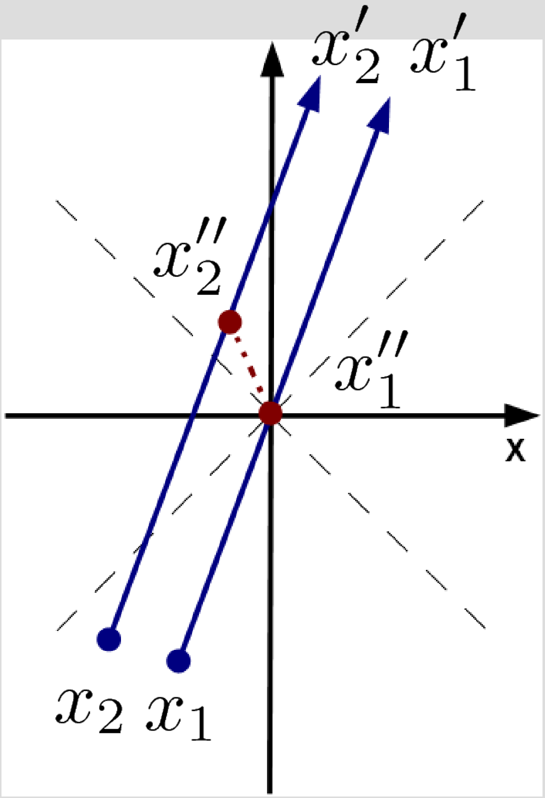


Interacting particles

Retarded interaction



Advanced interaction



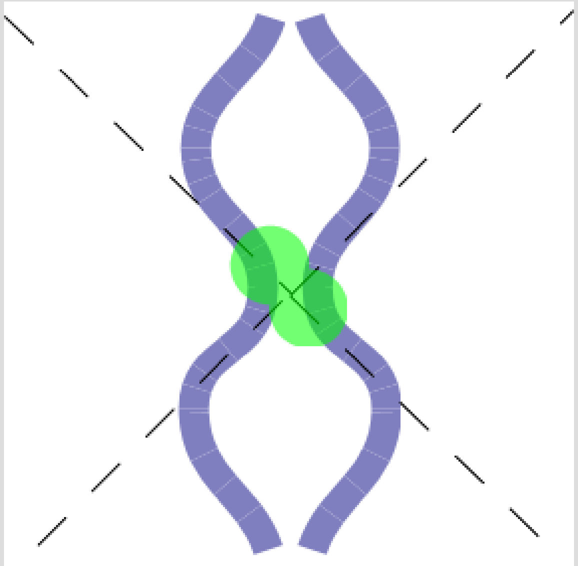
$$(\hat{\partial}_{x_1} - m) \otimes (\hat{\partial}_{x_2} - m) G(x_1, x_1'; x_2, x_2') =$$

$$\delta(x_1' - x_1)\delta(x_2' - x_2) + \int d^4x_1'' d^4x_2'' \bar{G}(x_1, x_1''; x_2, x_2'') G(x_1'', x_1'; x_2'', x_2')$$

Interacting particles: bound state

2 particles — 1 partcile: $p + n \longrightarrow d + \gamma$

Bound state has compact internal space that define its structure

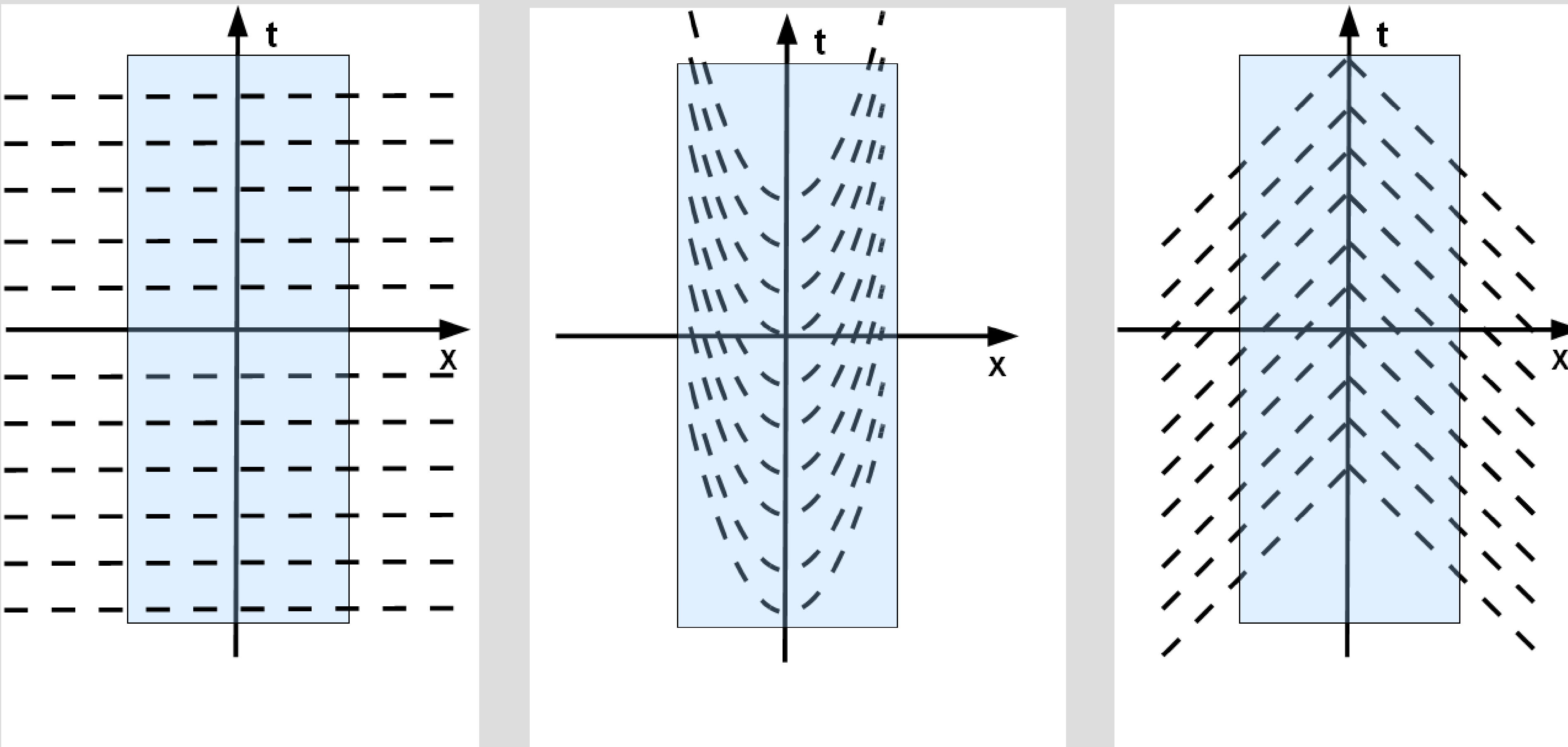


If the deuteron is situated in the time point t, then its constituents cannot be in past or future.

Bound nucleons are at space-like distance

Space-time bundle of bound states: compact layers

Using different bundles we get different structure of the layer



Using different forms of dynamics we get different particle structure.

Space-time Fermi smearing of the bound nucleon

Uncertainty relation establishes bounds for the Fermi smearing of the bound nucleons:

$$\Delta x \propto 1/\Delta p$$

$$\tau \propto 1/(E_{\mathrm{N}} - \Delta_{\mathrm{N}}) - 1/E_{\mathrm{N}}$$

Δ_{N} - separation energy of the bound nucleon

$$\tau \leq \frac{\Delta_{\mathrm{N}}}{E_{\mathrm{N}}(E_{\mathrm{N}} - \Delta_{\mathrm{N}})}$$

Kinematic distortion of the bound nucleon structure that cannot be avoided.

Bound states and off-shell particles

2D base manifold that cannot be reduced, choice of the path leads to different dynamics of the system

$$(\hat{\partial}_{x_1} - m) \otimes (\hat{\partial}_{x_2} - m) G(x_1, x_1'; x_2, x_2') =$$

$$\delta(x_1' - x_1)\delta(x_2' - x_2) + \int d^4x_1'' d^4x_2'' \bar{G}(x_1, x_1''; x_2, x_2'') G(x_1'', x_1'; x_2'', x_2')$$

$\bar{G}$ is defined by the symmetry properties of the layers in the bundle space

- space-time symmetries
- symmetries of the particles

⇒ Symmetry properties of the interaction kernel

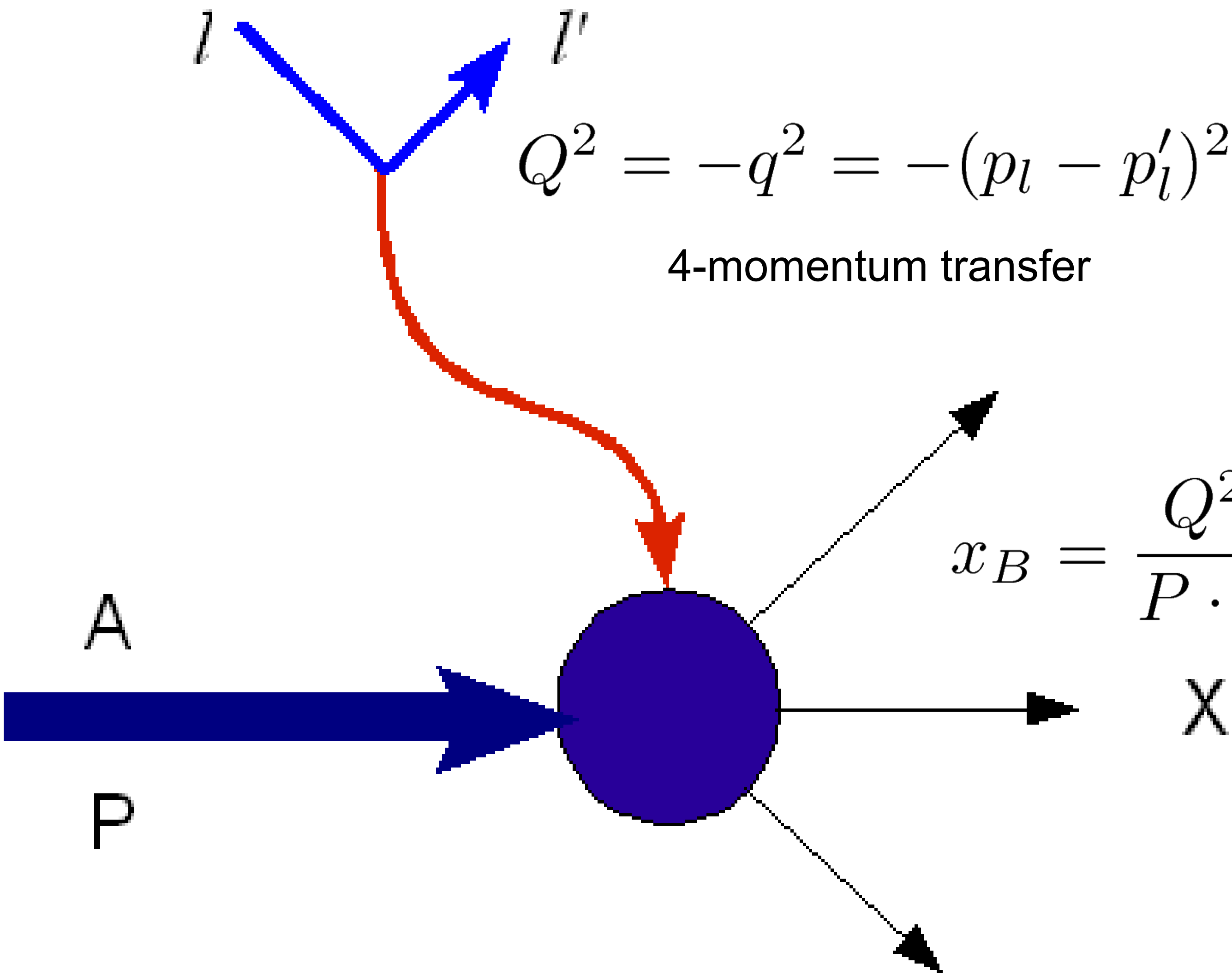
Issue: We cannot choose a hyper-surface in this case

Solution: Use of the additional conditions according the observation procedure and symmetry properties

Deep inelastic scattering off nuclei

Sensitive to the valence quark distribution

$$d\sigma \propto \frac{\alpha^2}{q^4} L^{\mu\nu}(k,k')W_{\mu\nu}(P, p_l' - p_l)$$

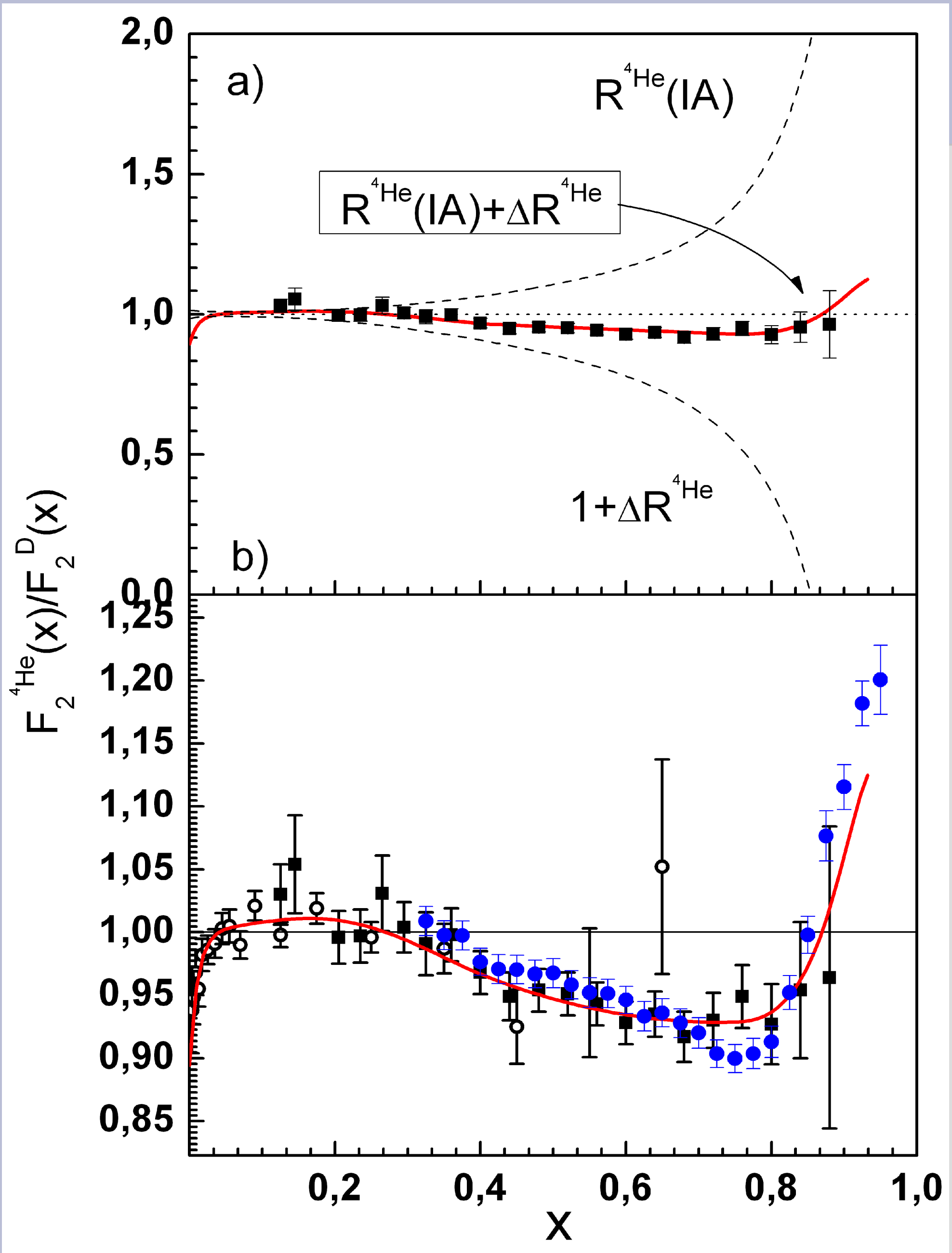


$$Q^2 = -q^2 = -(p_l - p_l')^2$$

4-momentum transfer

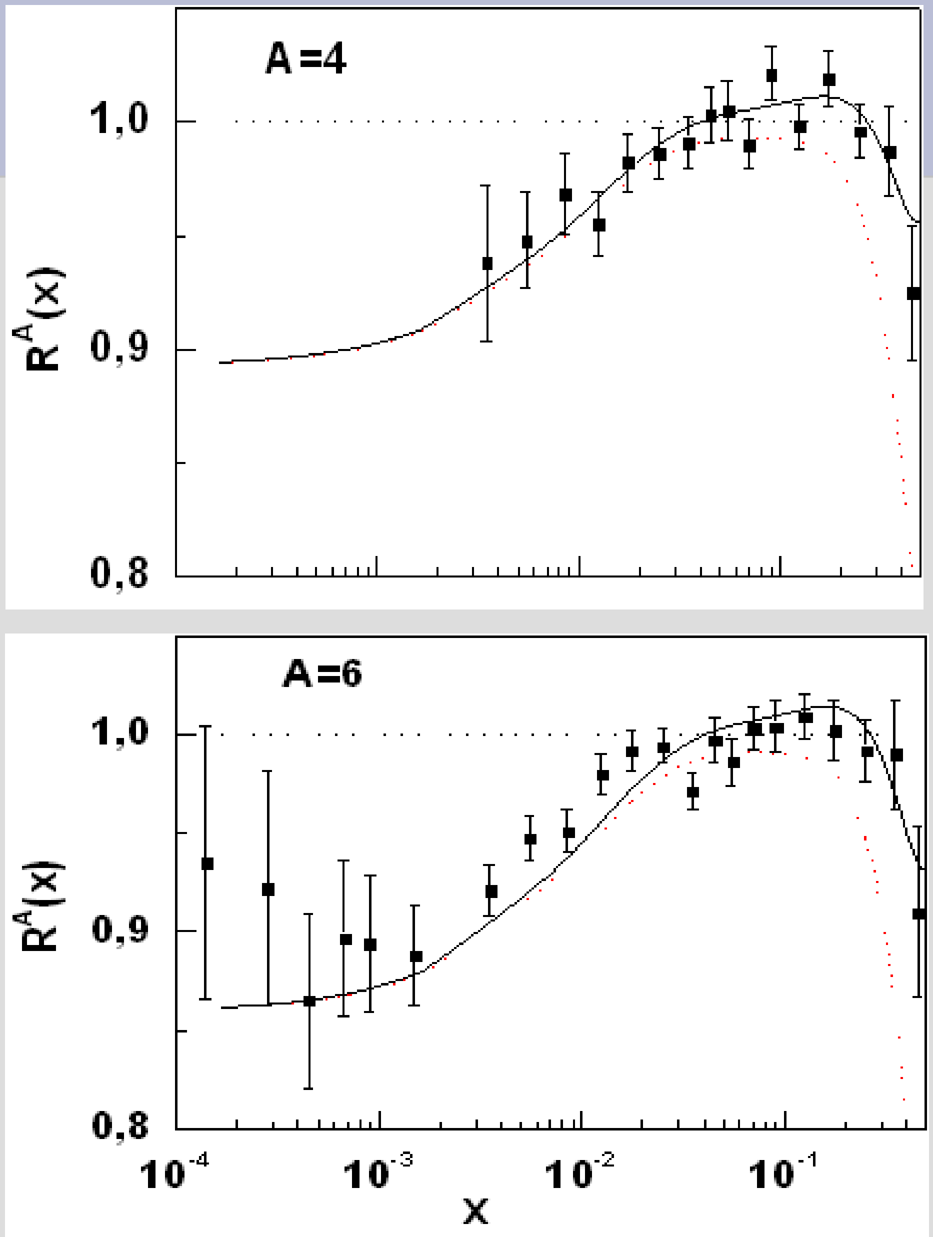
$$x_B = \frac{Q^2}{P \cdot q} = \frac{Q^2}{M_A(q_0 - q_3)}$$

DIS on the helium



$$\frac{dW^{N}_{\mu\nu}(p,q)}{dp_0} = \frac{dx_N}{dp_0}\left(\frac{1}{x_N}\frac{dF_2^N(x_N,Q^2)}{dx_N} - \frac{F_2^N(x_N,Q^2)}{x_N^2}\right)$$

Shadowing effect in the DIS



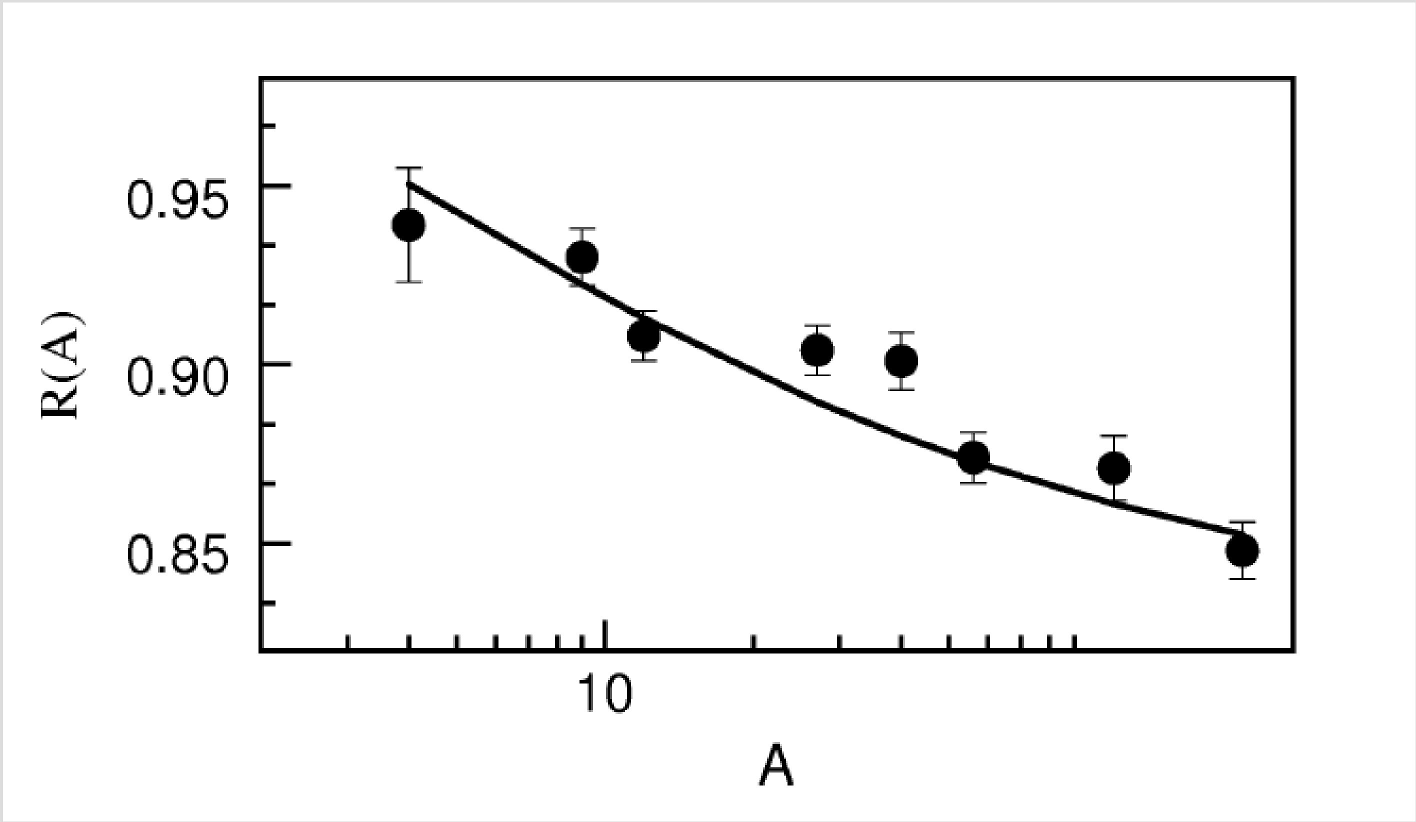
The red dashed curve on the picture:

$$R^A \simeq 1 - \frac{\langle \Delta_A^N \rangle}{m} \frac{1}{F_2^{\rm N}(x)} \frac{dF_2^{\rm N}(x)}{dx}$$

Full calculation — solid curve

A- dependence of the EMC-effect

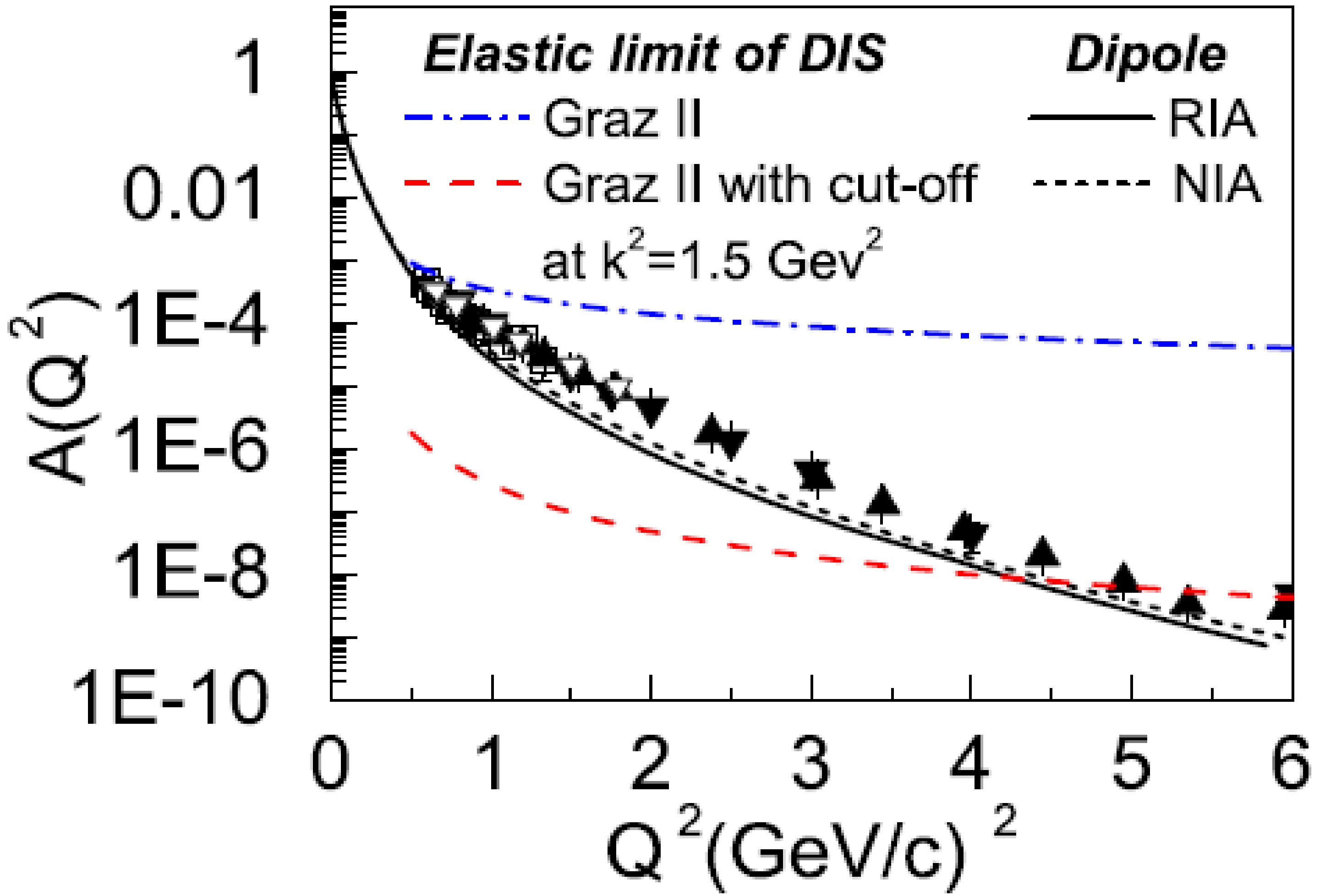
A-dependence is define by the ratio of surface and internal nucleons



$$M_A - E_{A-1} - E_N = - < T > + \epsilon$$

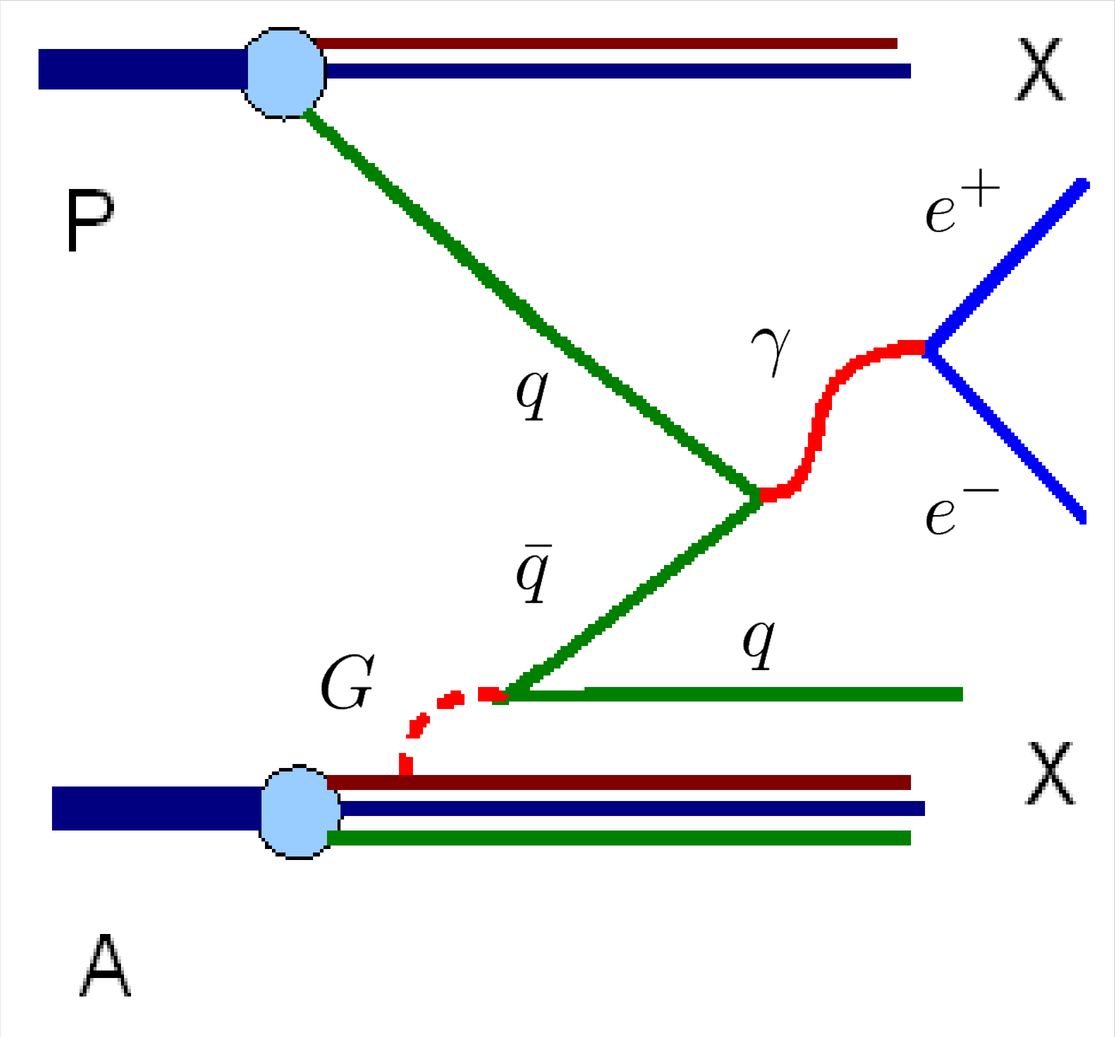
Surface nucleons: $< T_{surf} > \ll < T_{int} >$

Elastic limit of the DIS off deuteron: Information about interaction kernel



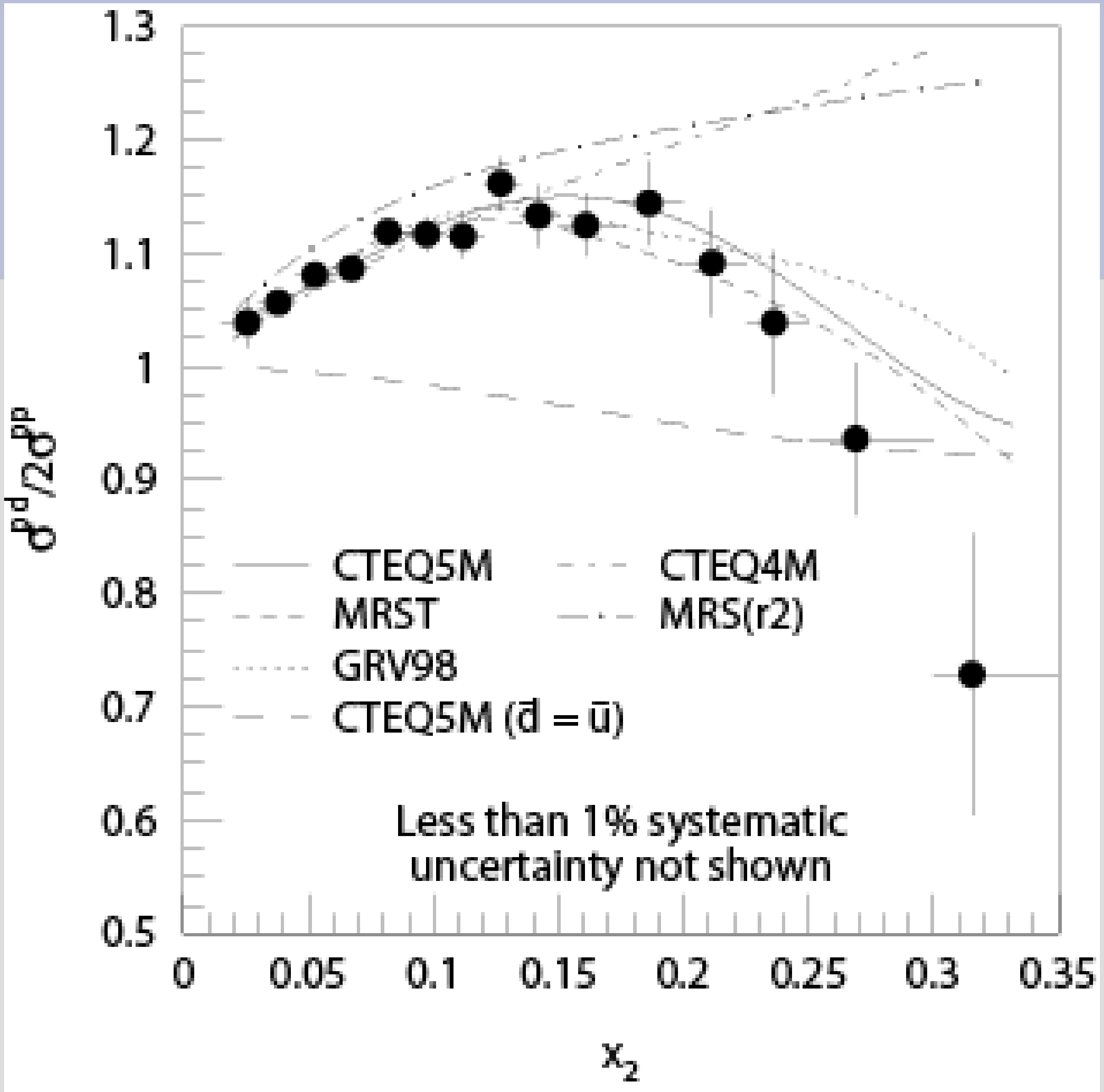
Drell-Yan pairs production

Sensitive to the vacuum structure of the nucleon



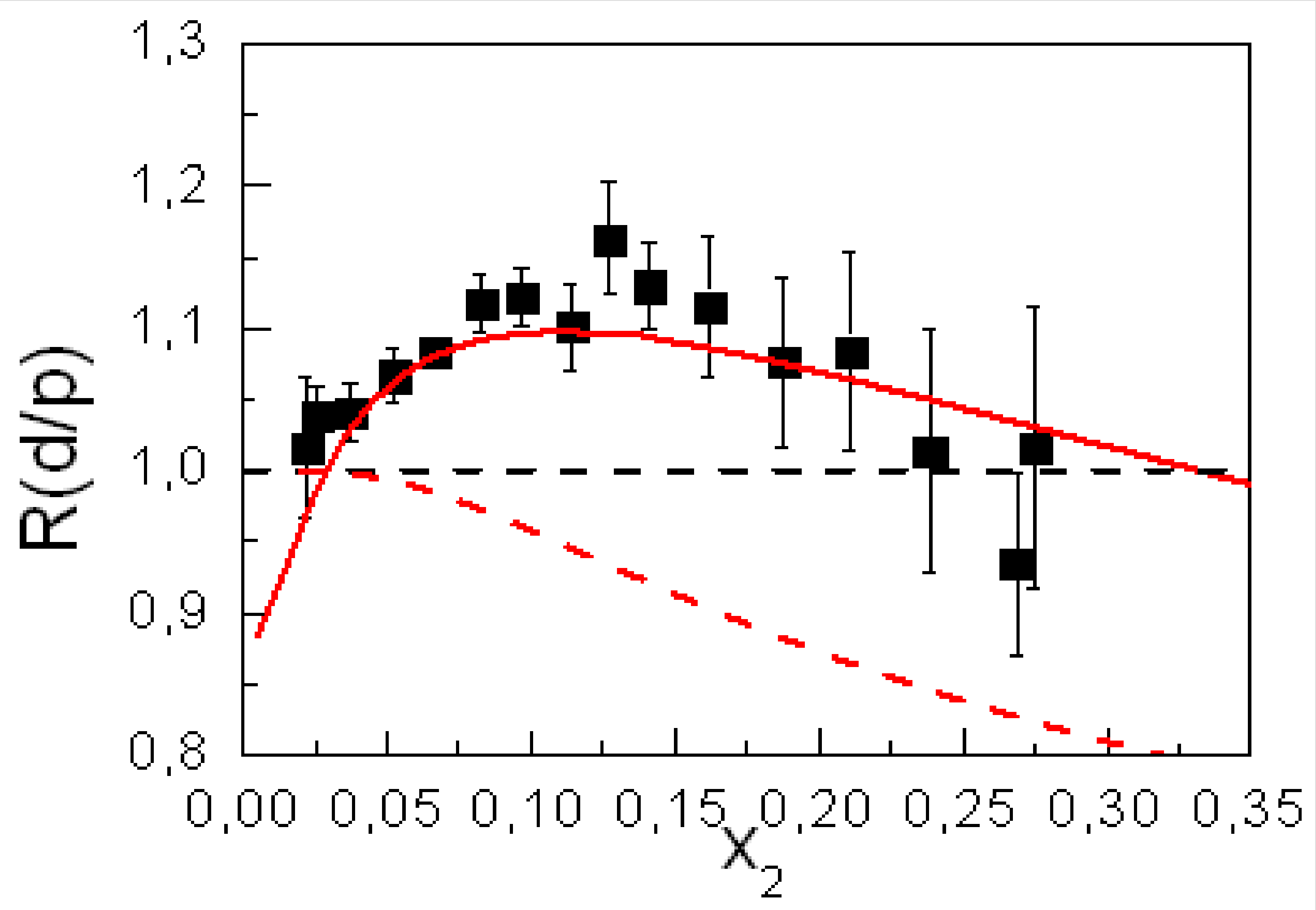
FNAL E866 data 2001

Deviations from R=1 are considered as evidences of vacuum flavor asymmetry



Effects of 4D Fermi smearing can change the conclusion

Ratio of the Drell-Yan pair production with symmetrical vacuum



Conclusion

- 3D projection does not give full description of the relativistic dynamics of the bound states.
- Interaction particles are smeared in time, magnitude of the effect is defined by the uncertainty relation.
- One of the observable effects of the time-smearing is the nucleon structure distortion in nuclei.