Asymptotics in Relativistic Nuclear Physics

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Between previous and present Baldin Seminars Professor V.Burov was performed **60 years** (14.09.2009). V.Burov is one of the pupils of academician A.M.Baldin and is a permanent organizer Baldin Seminars on length of the long time. V.Burov is well known as large scientist in the field of Relativistic Nuclear Physics.

I offer to congratulate Prof. Burov V.V. with this event!

$$\mathbf{I + II \rightarrow 1 + 2 + 3 + \ldots}$$

$$b_{ik} = -(u_i - u_k)^2$$

$$u_i = p_i / m_i$$
$$u_k = p_k / m_k$$

$$i, k = \mathrm{I}, \mathrm{II}, 1, 2, 3, \ldots.$$

A.M.Baldin and L.A.Didenko. Fortschr. Phys. 38 (1990) 4, 261-332.

Classification of relativistic nuclear collisions on b_{ik}

$$b_{ik} \sim 10^{-2}$$

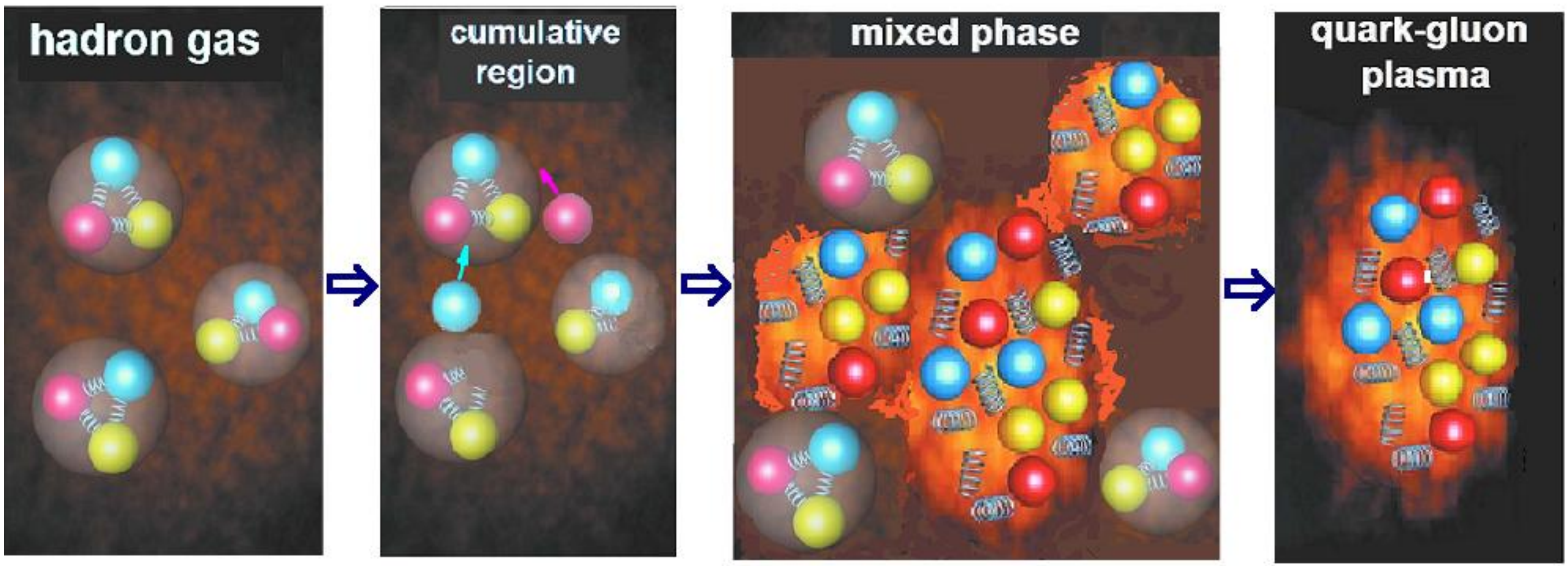
classic nuclear physics

$$0.1 \leq b_{ik} < 1$$

intermediate domain

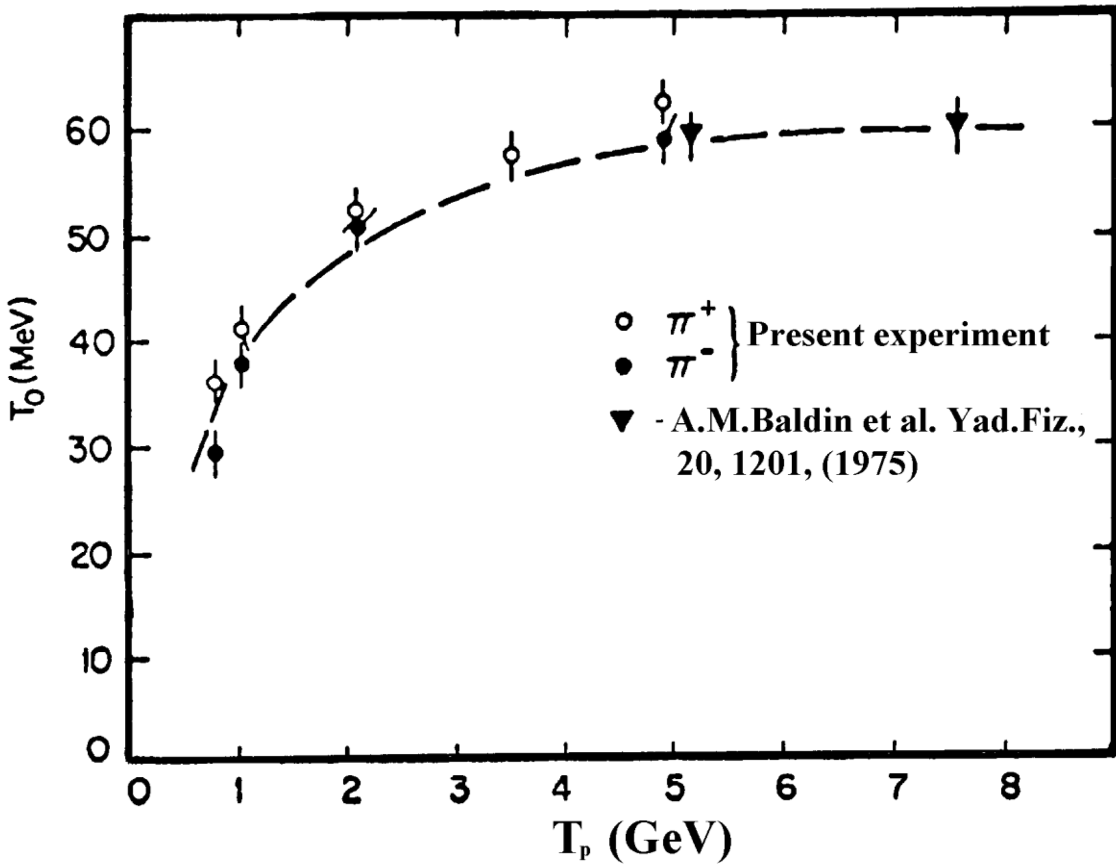
$$b_{ik} >> 1$$

nuclei should be considered as quark-gluon systems



$b_{ik} \approx 0.01$ $b_{ik} \approx 5$ $b_{ik} > 5$ $b_{ik} >> 1$

L.S.Schroeder et al. Phys. Rev. Lett., v.43, No.24, 1979, p.1787



Dependence of T_0 parameter for pion at 180° for p-Cu collisions on the energy of incident proton T_p. Pion cross-section parameterized by the form $E \cdot d\sigma/dp^3 = C \cdot \exp(-T/T_0)$, where T is the pion laboratory kinetic energy

Correlation Depletion Principle (CDP)

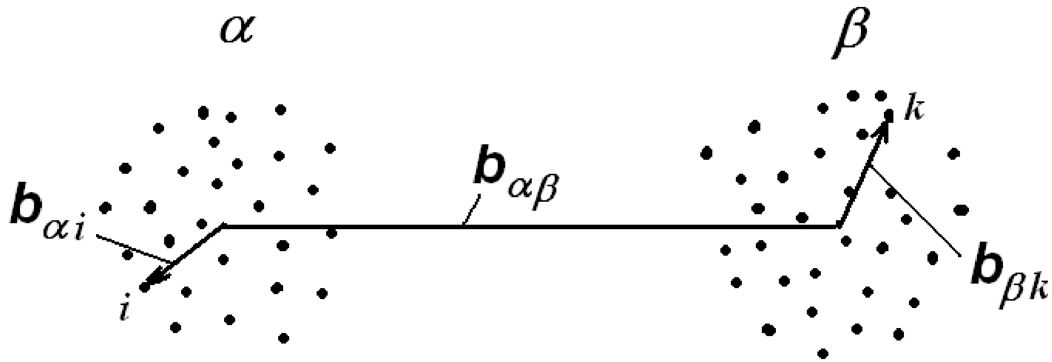
CDP was offered by N.N.Bogolyubov in statistical physics as universal property of the probability distributions for particle location in ordinary space-time (*r, t*).

The principle is based on the idea that the correlation between largely spaced parts of a macroscopic system practically vanishes and the distribution is factorized.

CDP in Relativistic Nuclear Physics was suggested by A.M.Baldin in four-velocity space.

Due to complementary of r_{ik} and b_{ik} this principle is quite opposite to the Bogolyubov CDP.

The Bogolyubov CDP is valid for $|r_i - r_k|^2 \longrightarrow \infty$
while the Baldin CDP is fulfilled for $|r_i - r_k|^2 \longrightarrow 0$ (or $b_{ik} \longrightarrow \infty$)
according to the asymptotic freedom.



$$W\,\Big|_{b_{\alpha\beta}\to\infty} \longrightarrow W_\alpha \cdot W_\beta$$

I + II → 1 + …

$\alpha = I\,, \quad \beta = II$

$d^2\sigma\,/db_{II\,1}dx_1 \;\to\; F_I \cdot F_{II}\,(b_{II\,1},\, N_1)$

$b_{II\,1} = (u_{II} - u_1)^2$

N_1 – *cumulative number*

V_α - middle point of system

u_i - four-velocity of particle *i* in the system

u_k - four-velocity of particle *k* in the system

$$b_{\alpha i} = -(V_\alpha - u_i)^2$$
$$b_{\alpha k} = -(V_\alpha - u_k)^2$$

Isolated system $\longrightarrow$ $b_{\alpha i} \ll b_{\alpha k}$

Example of the Isolated system: W_α and W_β (F_I and F_{II})

The important example of the Isolated system → JETS

Invariant definition of hadron jets

The jet is a claster of hadrons with small relative b_{ik} .

The jet axis (a unit four-vector):

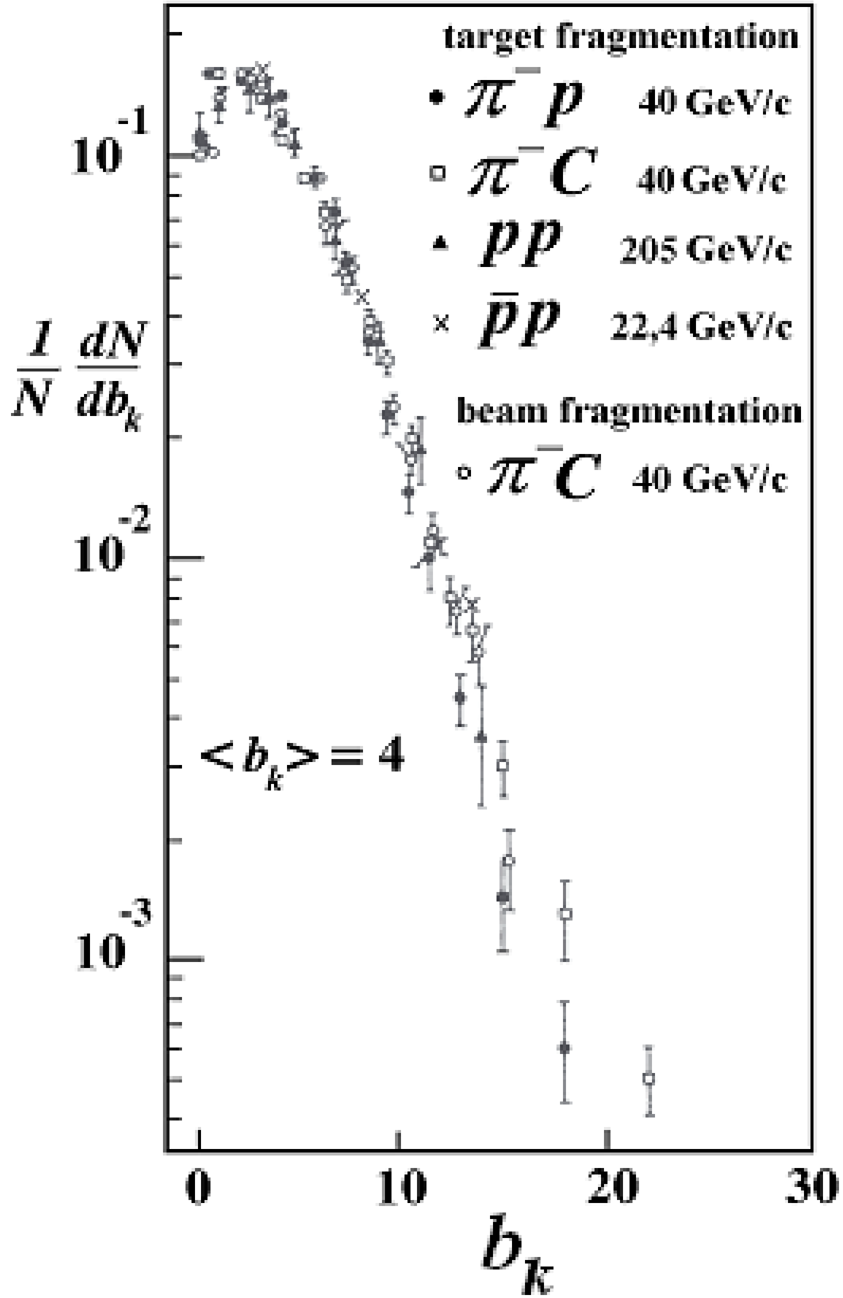
$$V = \sum_i (u_i / \sqrt{(\Sigma u_i)^2}$$

$$V_o^2 - \vec{V}^2 = 1$$

The summation is performed over all the particles belonging to a selected particle group (claster).

It is possible to determine the squared four-velocity with respect to the axis:

$$b_k = - (V - u_k)^2$$


target fragmentation
π⁻p 40 GeV/c
π⁻C 40 GeV/c
pp 205 GeV/c
p̄p 22,4 GeV/c
beam fragmentation
π⁻C 40 GeV/c
<b_k> = 4
1/N dN/db_k
b_k
0
10
20
30
10^-1
10^-2
10^-3

• Distribution of π – mesons on b_k in jets of hadron-hadron and hadron-nucleus collisions at high energies are UNIVERSAL. They do not depend either on the collision energy, or on the type of fragmenting system.

• This universality of properties of four-dimensional hadron jets indicates that the hadronisation of quark systems is defined by the dynamics of interaction of a color charge with QCD vacuum.

For jets : $<b_k> \sim 1$

For hadrons: $<b_k> \ll 1$

$$A_2 = \min [-\sum_{K}(V_\alpha - u_k)^2 - \sum_{i}(\mathrm{V}_\beta - u_i)^2],$$

$$A_3 = \min [-\sum_{K}(V_\alpha - u_k)^2 - \sum_{i}(\mathrm{V}_\beta - u_i)^2 - \sum_{j}(V_\gamma - u_j)^2],$$

……………………..

etc.

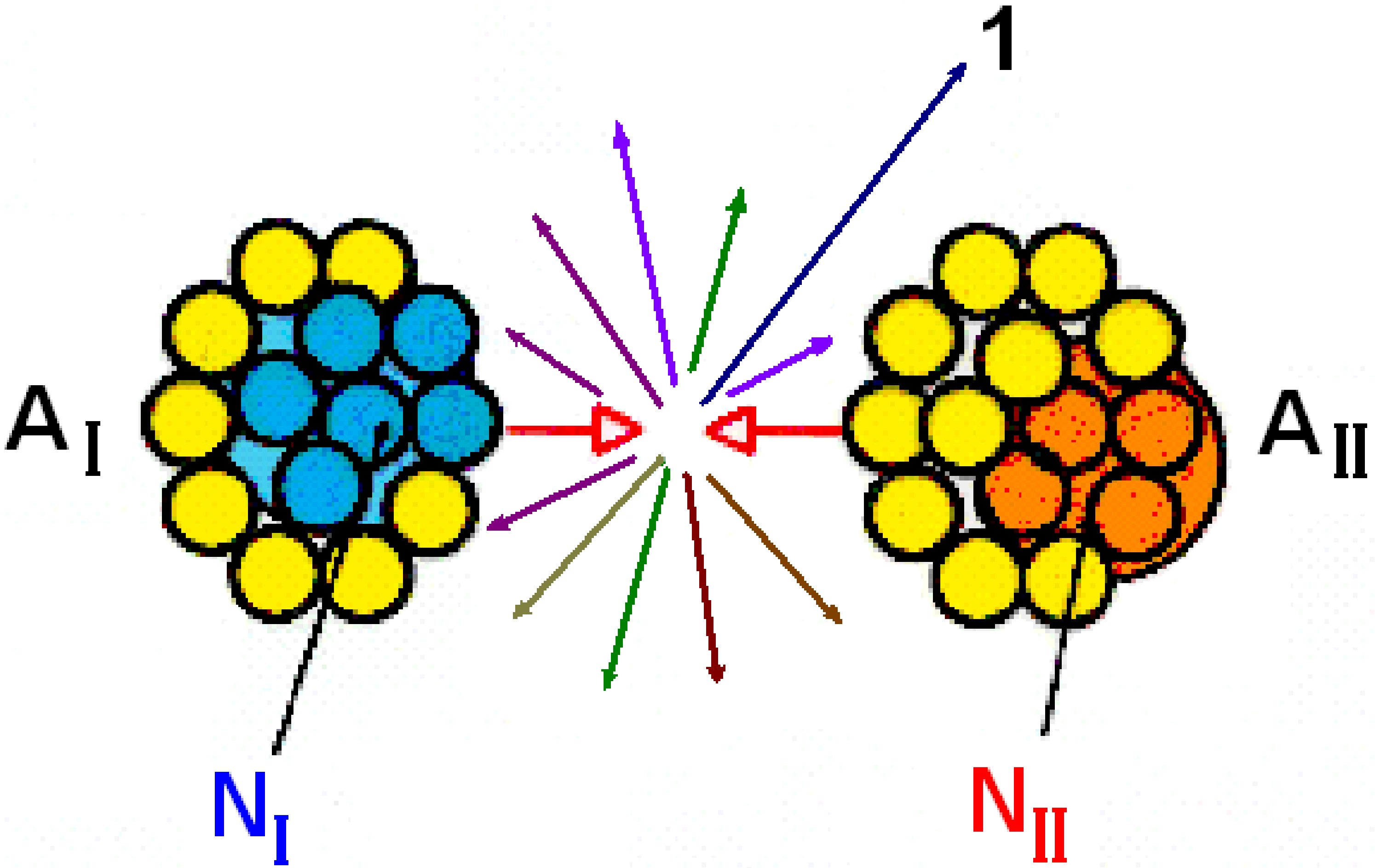
Distribution of jet axes is well described QCD as result of the production of quarks and gluons (i.e. color charges).

In other words $\rightarrow$ studying of jets is studying of deformation of vacuum by a color charge, studying of the deconfinment.

Automodelity Principle

Automodelity solutions means reduction of number of arguments of researched function due to importance only some combinations of independent variables → **similarity parameters**.

Automodelity ***W***, i.e. the assumption of existence universal asymptotic in the relativistic nuclear collisions (nuclear, intermediate and a quark-gluon) → HYPOTHESIS demanding an experimental substantiation.


1
A I
A II
N I
N II

$$\Pi = min[{}^{1}/_{2}\sqrt{(u_I N_I + u_{II} N_{II})^2}]$$

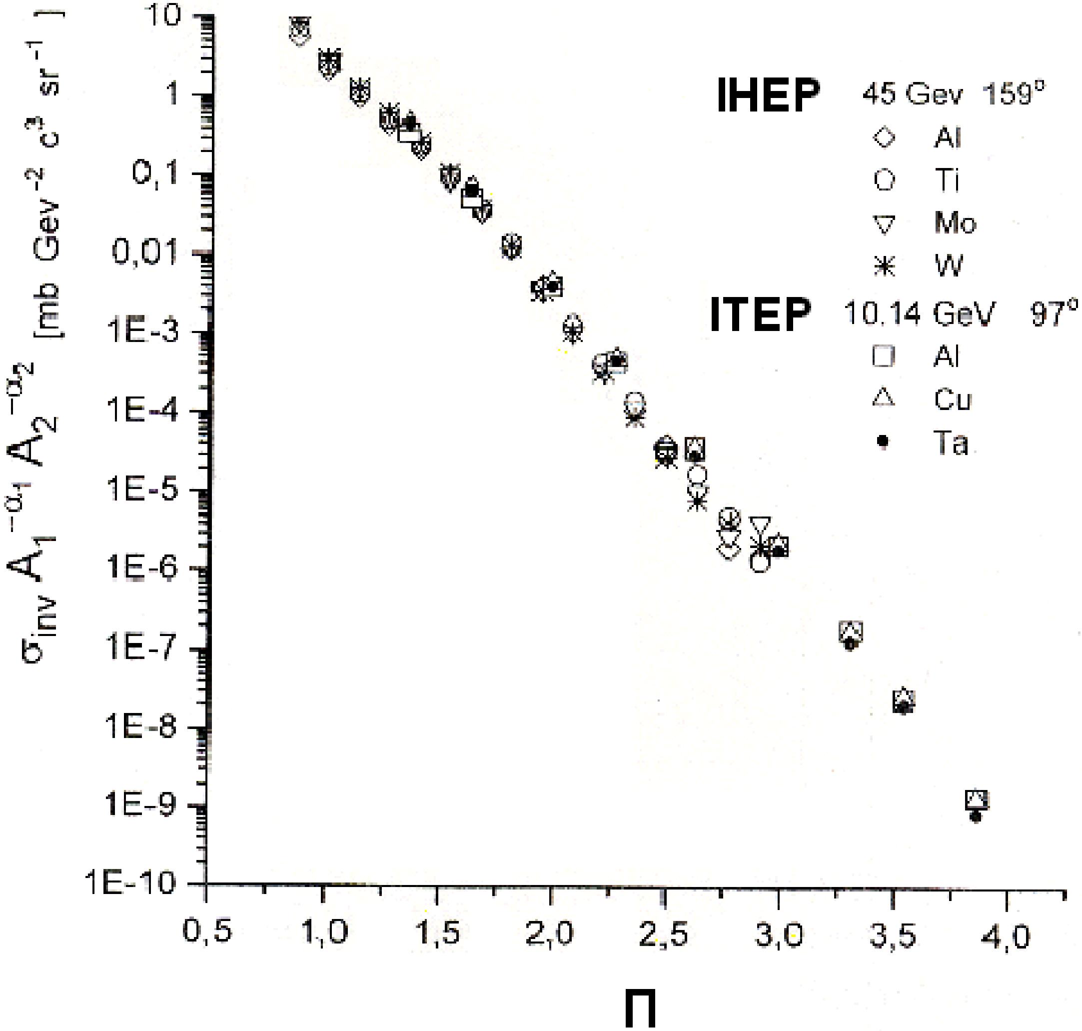
$$E \cdot \frac{d^3\sigma}{d\vec{p}} = C_1 \cdot A_I^{\alpha(N_I)} \cdot A_{II}^{\alpha(N_{II})} \cdot \exp\left(-\frac{\Pi}{C_2}\right)$$

$$\alpha(N_I) = \frac{1}{3} + \frac{N_I}{3}$$

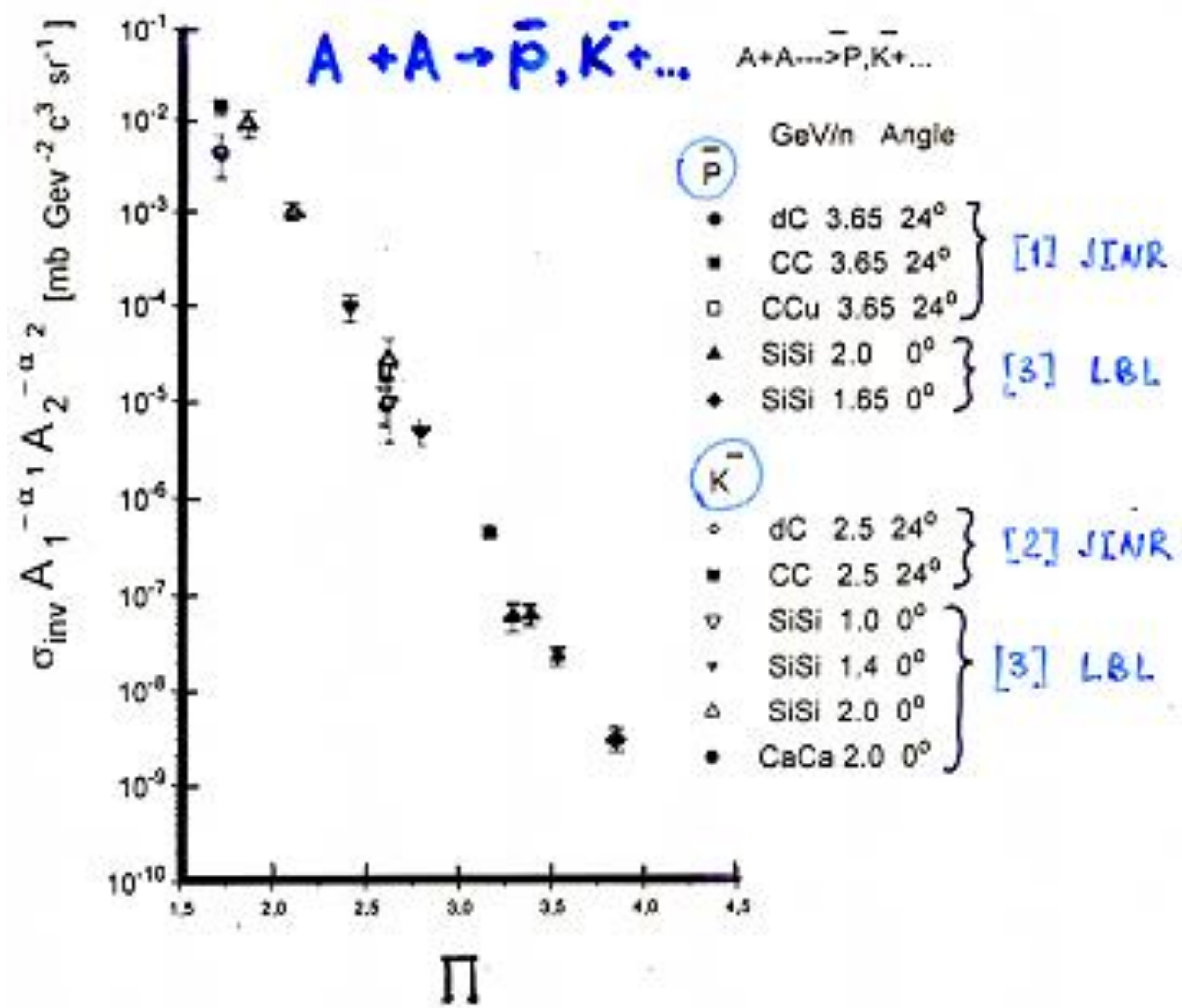
$$\alpha(N_{II}) = \frac{1}{3} + \frac{N_{II}}{3}$$

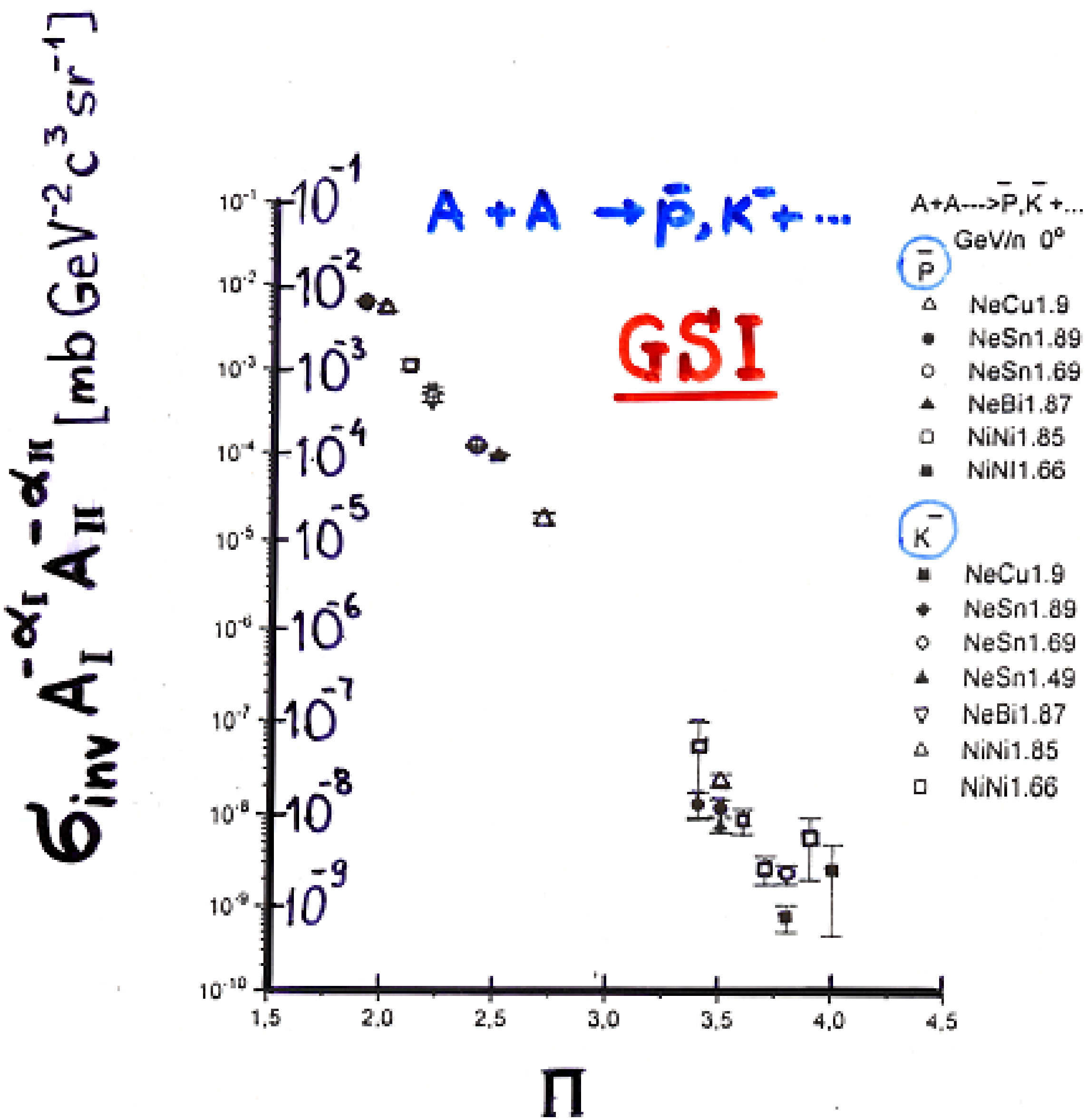
$$C_1 = 1.9 \cdot 10^4 \ mb \cdot GeV^{-2} \cdot c^3 \cdot sr^{-1}$$

$$C_2 = 0.125 \pm 0.002$$



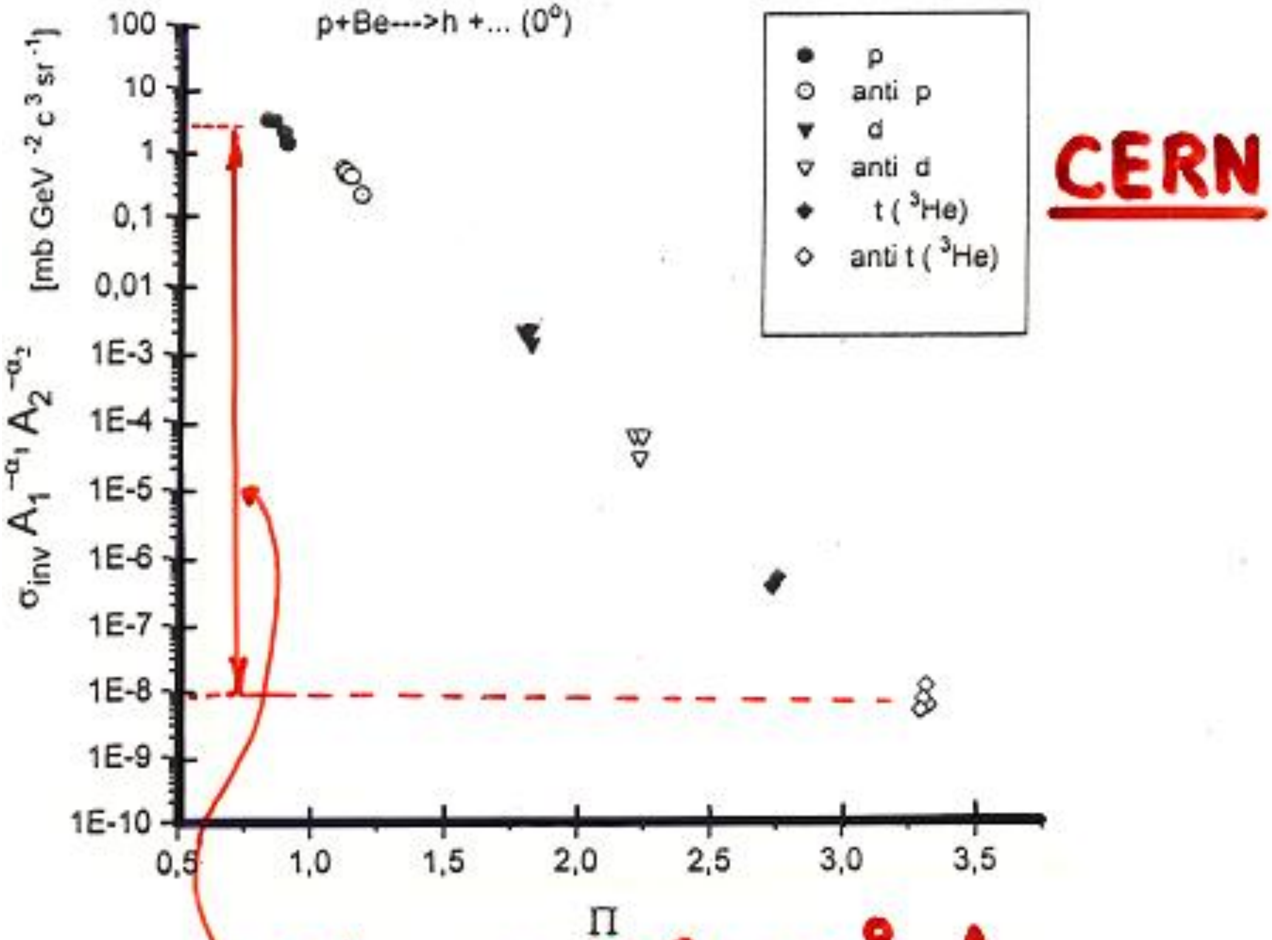
A. M. Балдин, A. A. Балдин. Phys.of Element. Part. and Atomic Nuclei, V.29, 1998.


A + A → p̄, K̄ + ...
A+A--->P̄,K̄+...
$\sigma_{inv} A_1^{-\alpha_1} A_2^{-\alpha_2}$ [mb Gev^{-2} c^3 sr^{-1}]
Π
GeV/n Angle
P̄
dC 3.65 24°
CC 3.65 24°
CCu 3.65 24°
[1] JINR
SiSi 2.0 0°
SiSi 1.65 0°
[3] LBL
K̄
dC 2.5 24°
CC 2.5 24°
[2] JINR
SiSi 1.0 0°
SiSi 1.4 0°
SiSi 2.0 0°
CaCa 2.0 0°
[3] LBL



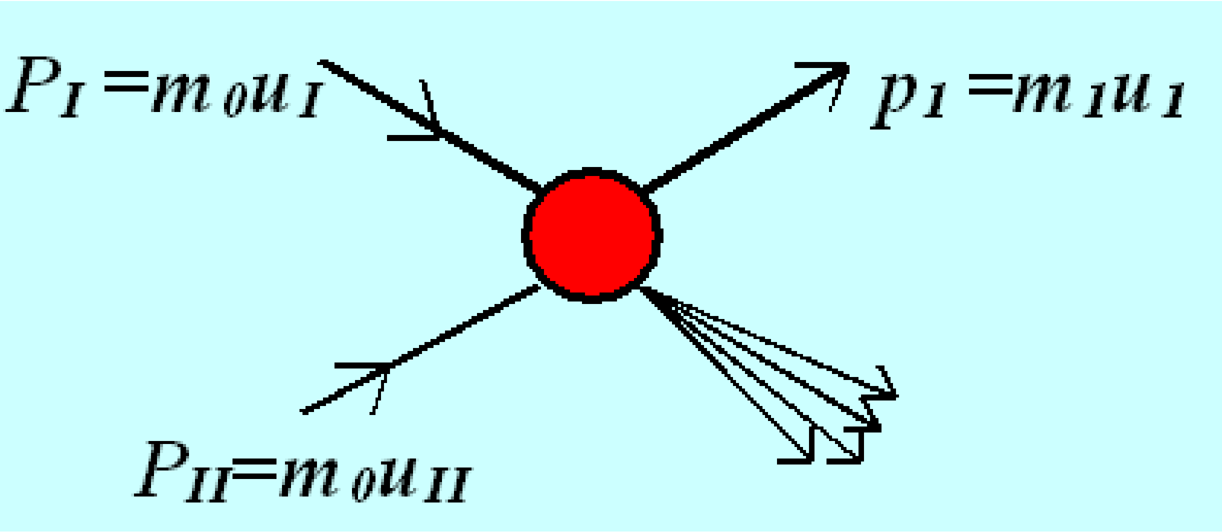
A + A → p̄, K⁻ + ...
GSI
A+A--->P̄,K̄ +...
GeV/n 0°
P̄
NeCu1.9
NeSn1.89
NeSn1.69
NeBi1.87
NiNi1.85
NiNi1.66
K̄
NeCu1.9
NeSn1.89
NeSn1.69
NeSn1.49
NeBi1.87
NiNi1.85
NiNi1.66
Π

p + Be → h + ... (200 GeV)



$\sigma_{inv} \to 10^0 \div 10^8$!

$$\mathbf{I + II \rightarrow 1 + \ldots}$$



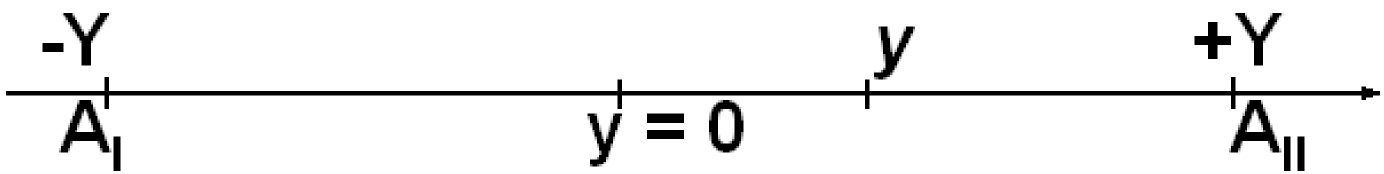
$$(N_I P_I + N_{II} P_{II} - p_1)^2 = (N_I m_0 + N_{II} m_0 + \Delta)^2$$

Δ is the mass of the particle providing conservation of the barion number, strangeness and other quantum numbers

$$\Pi^{min} \Rightarrow d\Pi/dN_I = 0; \;\; d\Pi/dN_{II} = 0$$

In the central rapidity region ($y = 0$)

$$(u_1 u_I) = (u_1 u_{II})$$



$$N_I = N_{II} =$$

$$N = [1 + \sqrt{1 + (\Phi_\delta/\Phi^2)}]\Phi,$$

where

$$\Phi = (1/m_0)[m_T chY + \Delta](½\, sh^2 Y)$$

$$\Phi_\delta = (\Delta^2 - m_1^2)(4m_0^{\,2}\, sh^2 Y)$$

and

$$\Pi^{min} = N \cdot chY$$

Baldin A.M., Malakhov A.I. JINR Rapid Communications, No.1(87)-98, 1998, pp.5-12.

$$\Pi_{min} = \min\left[1/2\sqrt{(u_I N_I + u_{II} N_{II})^2}\right]$$
$$(N_I m_0 u_I + N_{II} u_{II} m_0 - m_1 u_1)^2 = (N_I m_0 + N_{II} m_0 + \Delta)^2$$

$$N_I \cdot N_{II} - \Phi_I N_I - \Phi_{II} N_{II} = \Phi_\delta ,$$
$$\Phi_I = [(m_1/m_0)\cdot(u_I u_1) + \Delta/m_0]/[(u_I u_{II}) - 1]$$
$$\Phi_{II} = [(m_1/m_0)\cdot(u_{II} u_1) + \Delta/m_0]/[(u_I u_{II}) - 1]$$
$$\Phi_\delta = (\Delta^2 - m_1^2)/[2m_0^2((u_I u_{II}) - 1)].$$

$$[(N_I/\Phi_{II}) - 1]\cdot[(N_{II}/\Phi_I) - 1] = 1 + [\Phi_\delta/(\Phi_I \cdot \Phi_{II})]$$

$$d\Pi/dN_I = 0 , \quad d\Pi/dN_{II} = 0.$$

$$F_I = [(N_I/\Phi_{II}) - 1] , \qquad F_{II} = [(N_{II}/\Phi_I) - 1].$$
$$F_I \cdot F_{II} = 1 + \Phi_\delta/(\Phi_I \cdot \Phi_{II}) = \alpha$$

$$d\Pi/dF_I = 0 , \quad d\Pi/dF_{II} = 0.$$

$$4\Pi^2 = N_I^2 + N_{II}^2 + 2N_I \cdot N_{II} \cdot (u_I u_{II})$$
$$4\Pi^2 = (F_I + 1)^2 \Phi_{II}^2 + (F_{II}+1)^2 \Phi_I^2 + 2\Phi_I \cdot \Phi_{II} \cdot (F_I + 1)\cdot(F_{II}+1)\cdot(u_I u_{II})$$
$$F_{II} = \alpha/F_I \qquad d(4\Pi^2)/dF_I = 0$$

$$F_I^4 + F_I^3[1 + (u_I u_{II})/z] - (\alpha/z)\cdot F_I \cdot[(u_I u_{II}) + (1/z)] - \alpha^2/z^2 = 0$$

$z = \Phi_{II}/\Phi_I$

$I \leftrightarrow II$: $\quad z \to (1/z); \quad F_I \to (\alpha/F_{II}).$

$$F_{II}^4 + F_{II}^3[1+(u_I u_{II})z] - z\cdot\alpha\cdot F_{II}\cdot[z+(u_I u_{II})] - \alpha^2 z^2 = 0.$$

In the central rapidity region $(u_I u_I) = (u_I u_{II}) \to \quad z = 1 \to$

$$F_I = F_{II},\ \Phi_I = \Phi_{II} = \Phi$$
$$F_I = F_{II},\ (N_I/\Phi - 1) = (N_{II}/\Phi - 1),\ N_I = N_{II} = N$$
$$F^2 = \alpha ,\ F_I = F_{II} = \sqrt{\alpha} = \sqrt{1 + (\Phi_\delta/\Phi^2)}$$

$$N_I = N_{II} = N = (1+F)\cdot\Phi = [1 + \sqrt{1 + (\Phi_\delta/\Phi^2)}]\cdot\Phi$$

Baldin A.M., Malakhov A.I.
JINR Rapid Communications,
No.1(87)-98, 1998, pp.5-12.

Asymptotics

$$E \to \infty$$

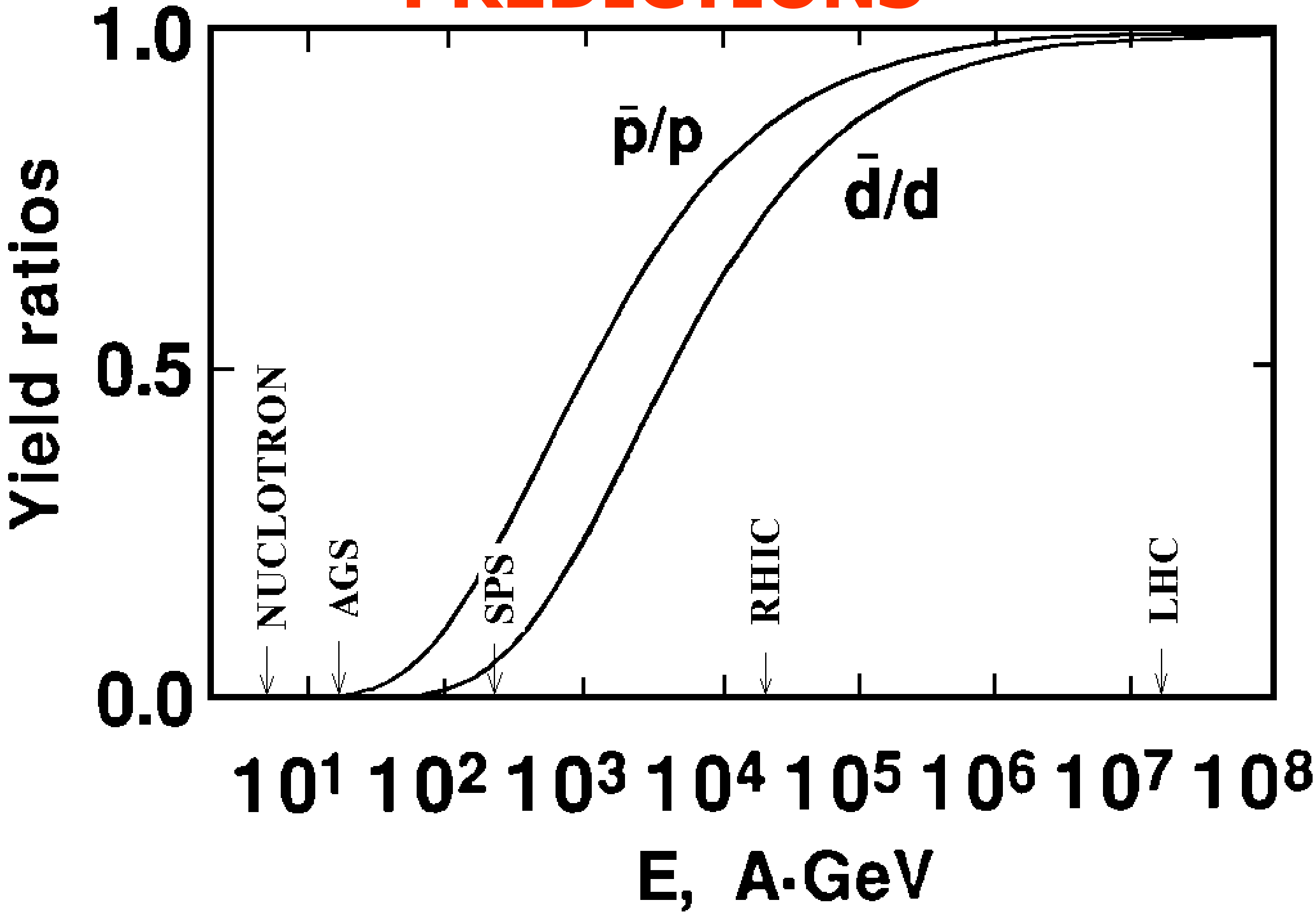
$$\Pi_\infty^{min} = (m_T/2m_0)[1 + \sqrt{1+(\Delta^2 - m_1^2)/m_T^2}]$$

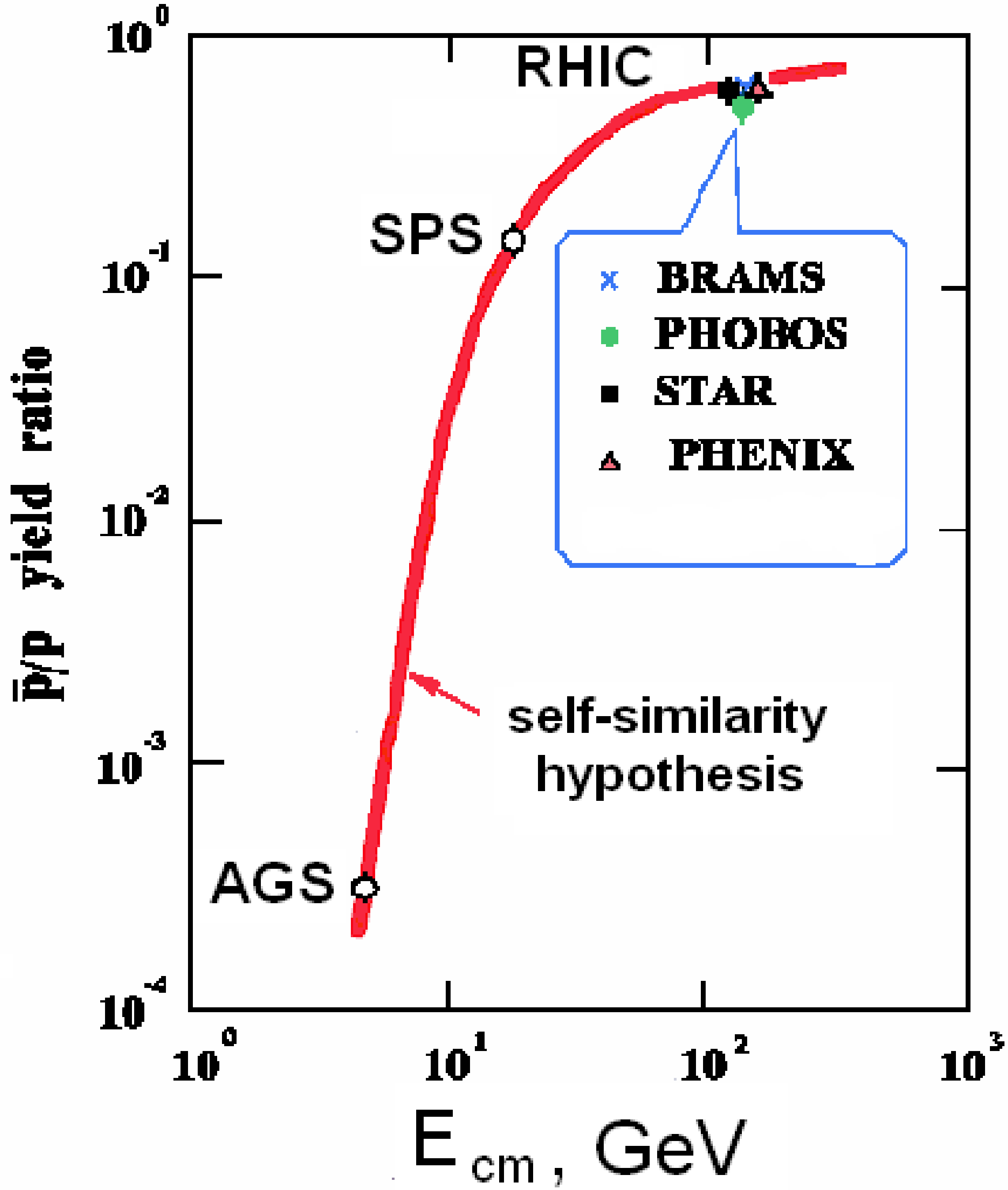
$$N_\infty \to 0$$

The analytical representation for Π leads to the following conclusions:

- There is the limiting value of Π at high energies
- The ratio of particle to antiparticle and nucleus to antinucleus production cross-section goes to the unit while energy rising
- The effective number of nucleons involved in the reaction decreases with increasing collision energy
- Probability of observation of antinuclei and fragments in the central rapidity region is small
- Strange particles yield increases with increasing collision energy

PREDICTIONS





10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$\bar{p}/p$ yield ratio
RHIC
SPS
AGS
BRAMS
PHOBOS
STAR
PHENIX
self-similarity
hypothesis
10^0
10^1
10^2
10^3
E_{cm}, GeV

It is possible see that parameters of the CBM and MPD and energy region of FAIR and NICA are very well corresponded for investigations of the initial stage of asymptotic regime

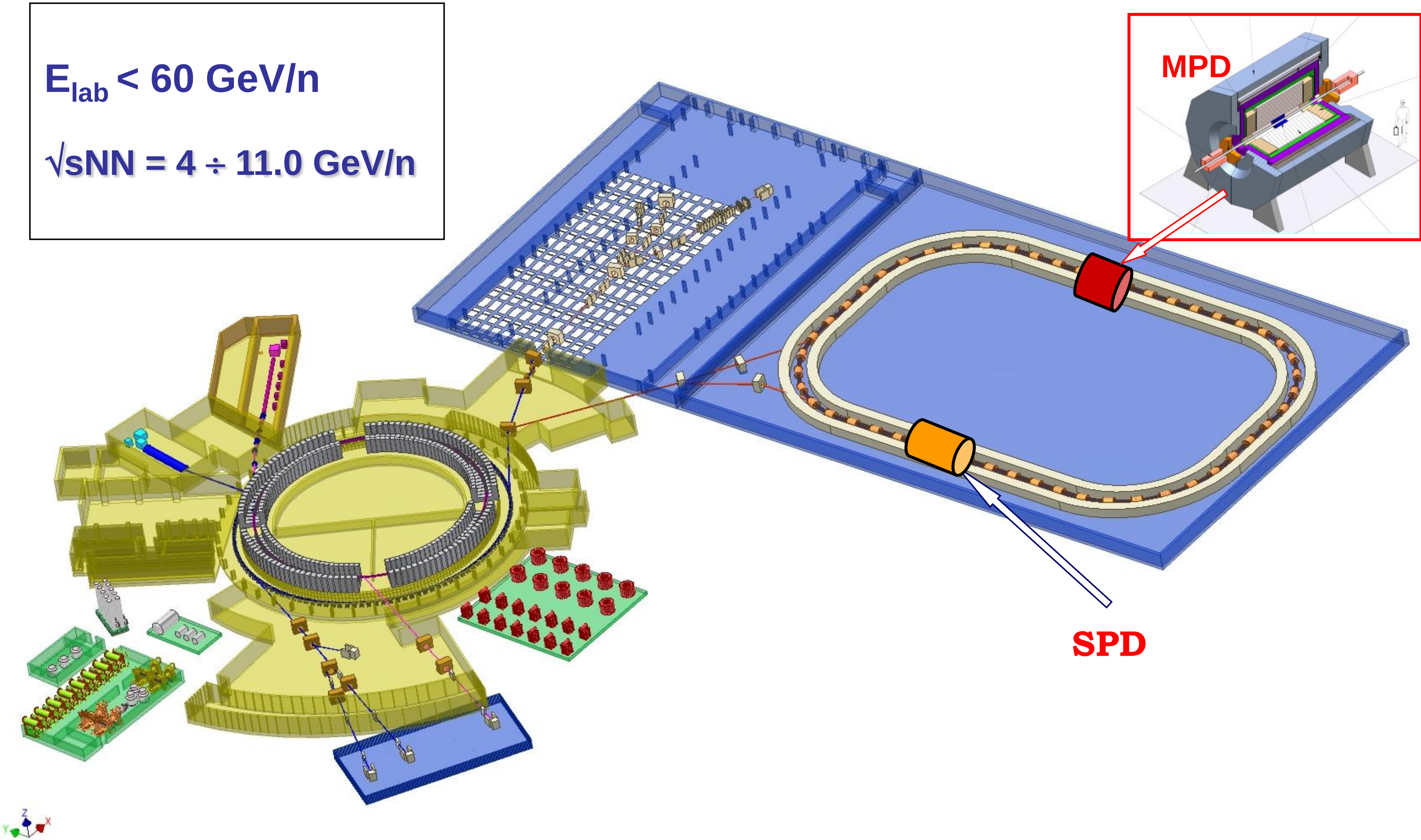
FAIR

Facility for Antiproton and Ion Research

NICA
Nuclotron-based Ion Collider fAcility

E_{lab} < 60 GeV/n

$\sqrt{s_{NN}}$ = 4 ÷ 11.0 GeV/n

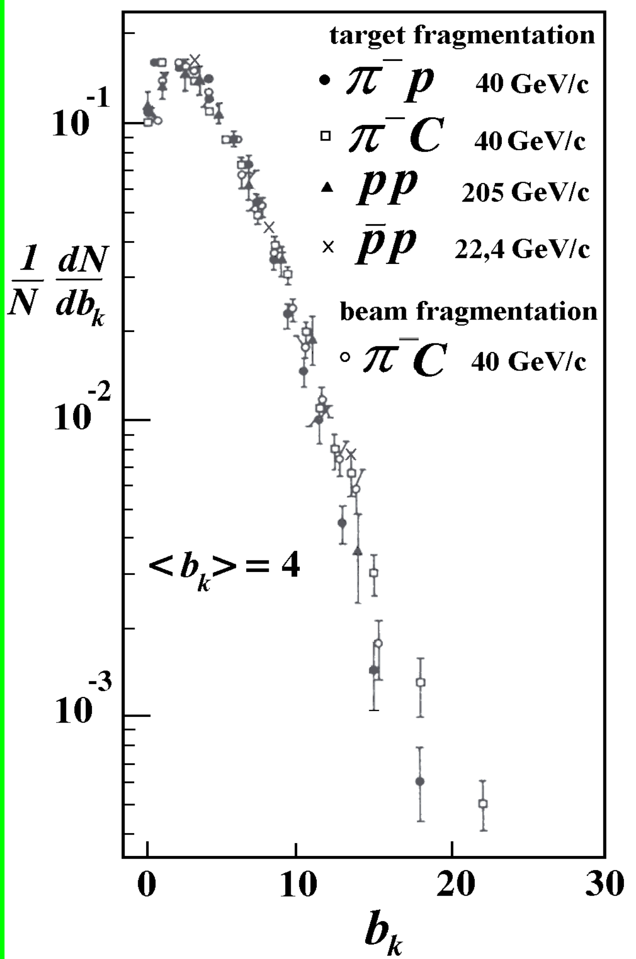


Dark energy

Astronomical observations testify that today (and in recent times) the Universe extends with acceleration: rate of expansion grows in time. In this sense also it is possible to speak about antigravitation: the usual gravitational attraction would slow down scattering galaxies, and in our universe, it turns out, on the contrary.

One of candidates for a role of dark energy - vacuum. The density of energy of vacuum does not change at expansion of the Universe, and it means negative pressure of vacuum.

Change of energy at change of volume is defined by pressure, $\Delta E = -p\Delta V$. At expansion of the Universe energy of vacuum grows together with volume (the density of energy is constant) that is possible, only if pressure of vacuum negatively.



- **Distributions of π - mesons on b_k in jets of hadron-hadron and hadron-nucleus collisions at high energies are UNIVERSAL. They do not depend either on the collision energy, or on the type of fragmenting system (p, π, p^-, C, ...)**

- **$<b_k>$ = 4 characterizes the average four-velocity of π mesons with respect to the jet axis in the fragmentation of various quark objects**

- **The discovered UNIVERSALITY of the properties of four-dimensional hadron jets indicates that the hadronisation of quark systems is defined by the dynamics of interaction of a color charge with QCD vacuum**

$$\Delta E = -p\Delta V$$

$$I + II \rightarrow 1 + 2 + \ldots$$

$$b_{I\,II} = -(u_I - u_{II})^2 = -(1 - 2u_I u_{II} - 1) =$$

$$= 2(u_I u_{II} - 1) = 2(E_I/m_I - 1)$$

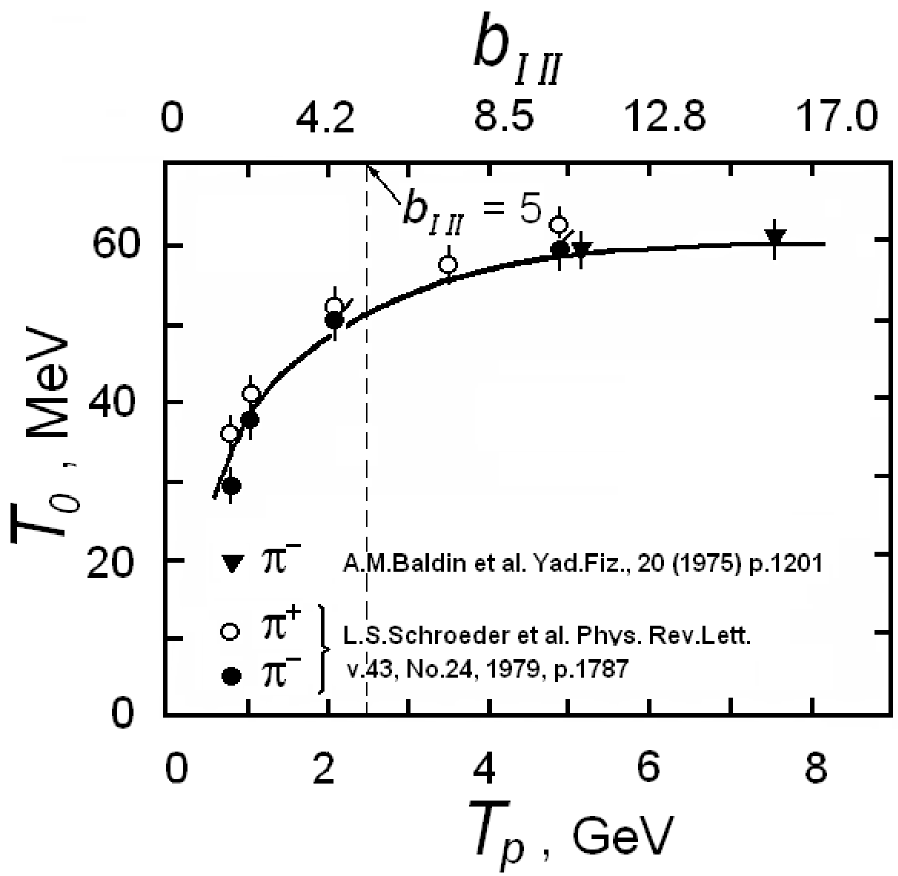
$$\Delta b_{I\,II} = 2\Delta E_I/m_I \longrightarrow \Delta E_I = (m_I/2)\Delta b_{I\,II} = -p\Delta V$$

$$p = -(m_I/2)\Delta b_{I\,II}/\Delta V)$$

$$V = \sigma\, 2r_0 A^{1/3} \longrightarrow \Delta V = \Delta\sigma\, 2r_0 A^{1/3}$$

$$p = -[m_I/4\, r_0 A^{1/3}]\Delta b_{I\,II}/\Delta\sigma$$

$$p = -[m_I / 4\, r_0 A^{1/3}]\Delta b_{I\,II} / \Delta\sigma$$



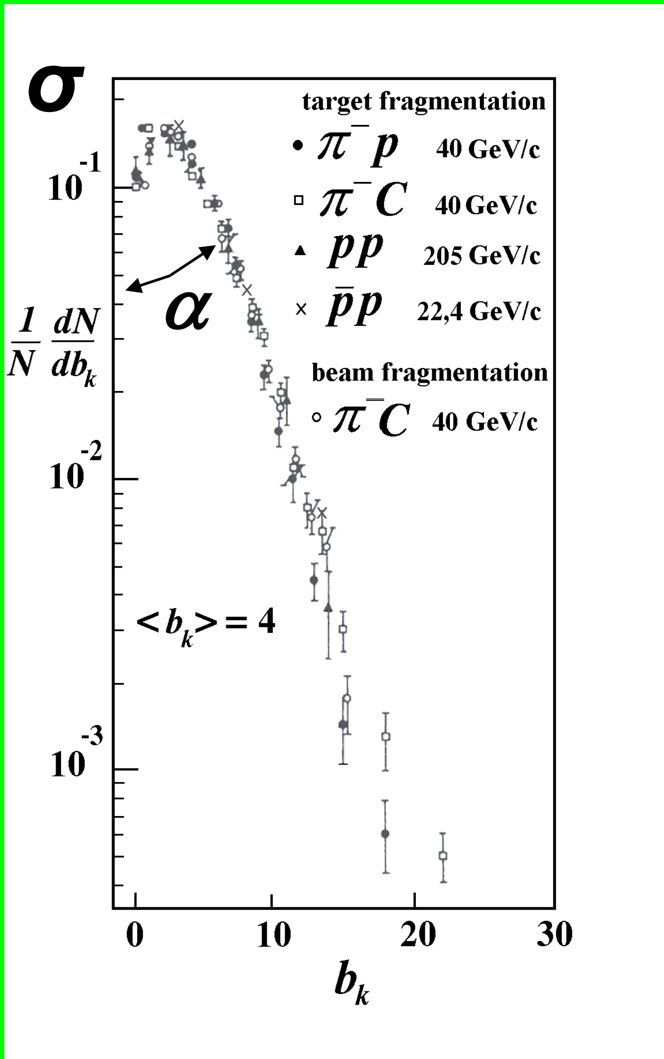
$\Delta b_{I\,II} > 0$

$\Delta\sigma > 0$

$p < 0$

$\sigma \sim exp\ (-T_\pi / T_0)$

Dependence of T_0 parameter for pion at 180º for $p+Cu$ collisions on the energy of incident proton T_p. Pion cross-section parameterized by the form $E\cdot d^3\sigma/dp^3 = C\cdot exp(-T/T_0)$, where T is the pion laboratory kinetic energy



$$\Delta E = -p\Delta V$$

$$\Delta E \sim \Delta b_k$$

$$\Delta V \sim \Delta \sigma$$

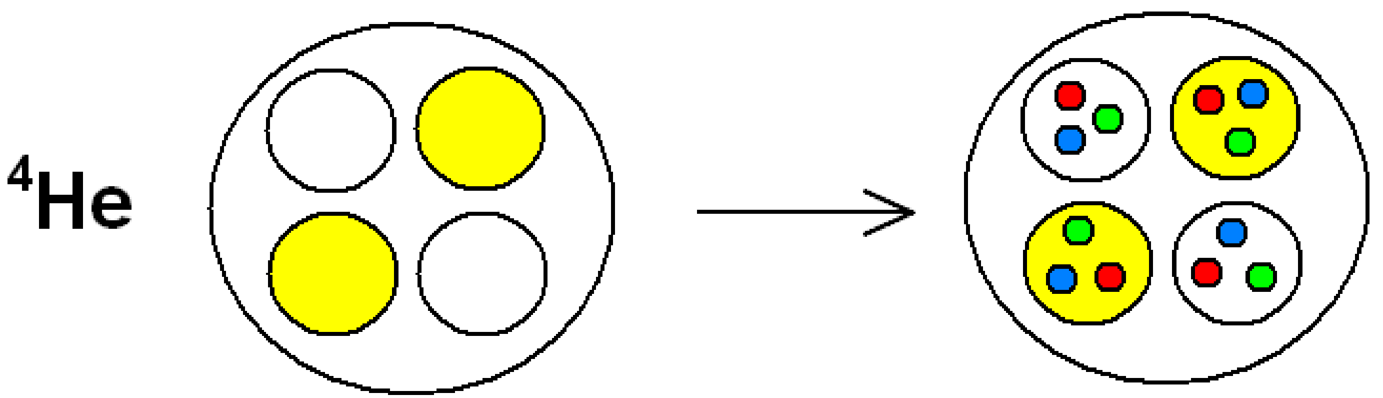
$$P = -\Delta E/\Delta V \sim \Delta b_k / \Delta b_k = tg\ \alpha$$

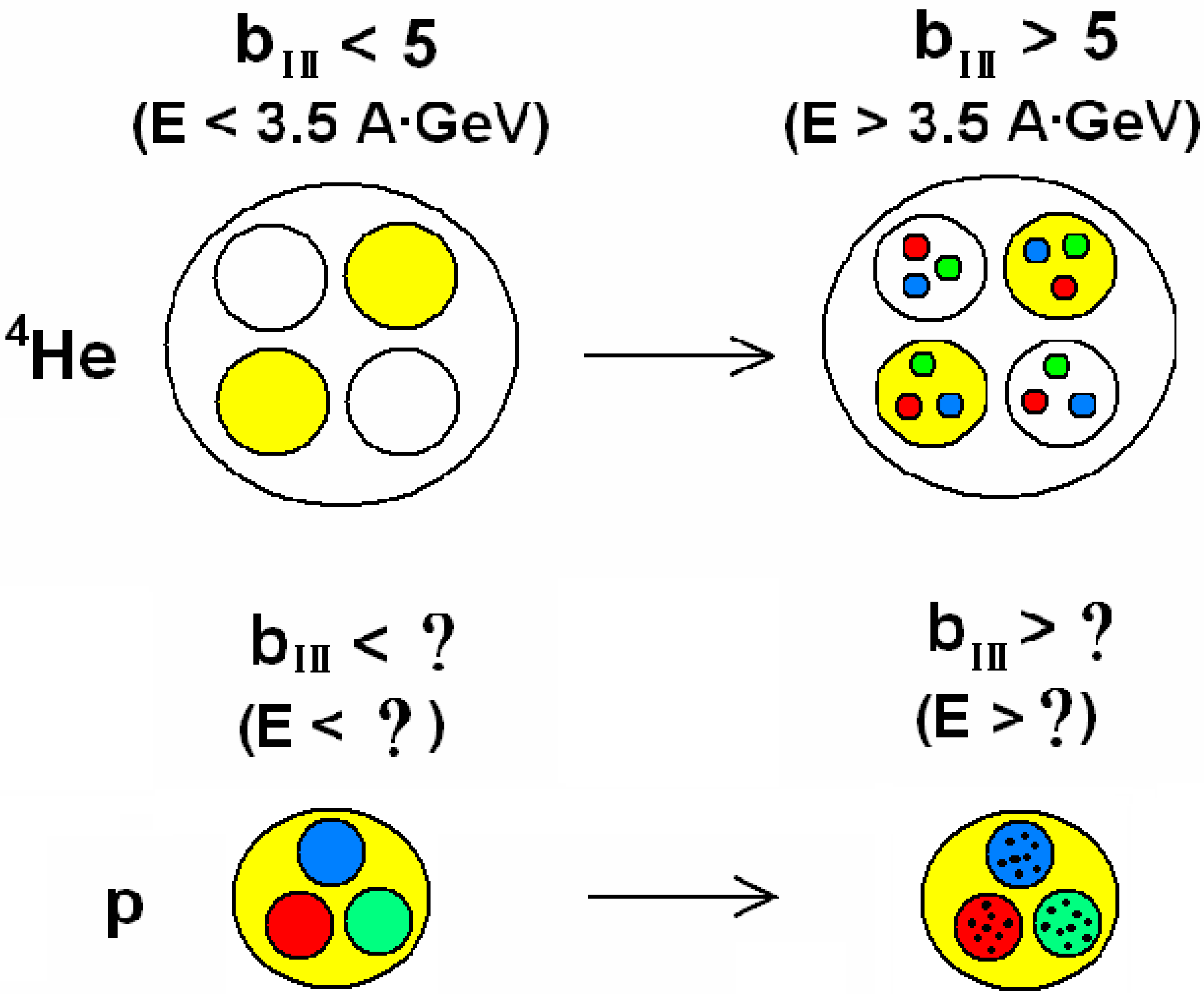
$$P \sim tg\ \alpha$$

Quark Structure

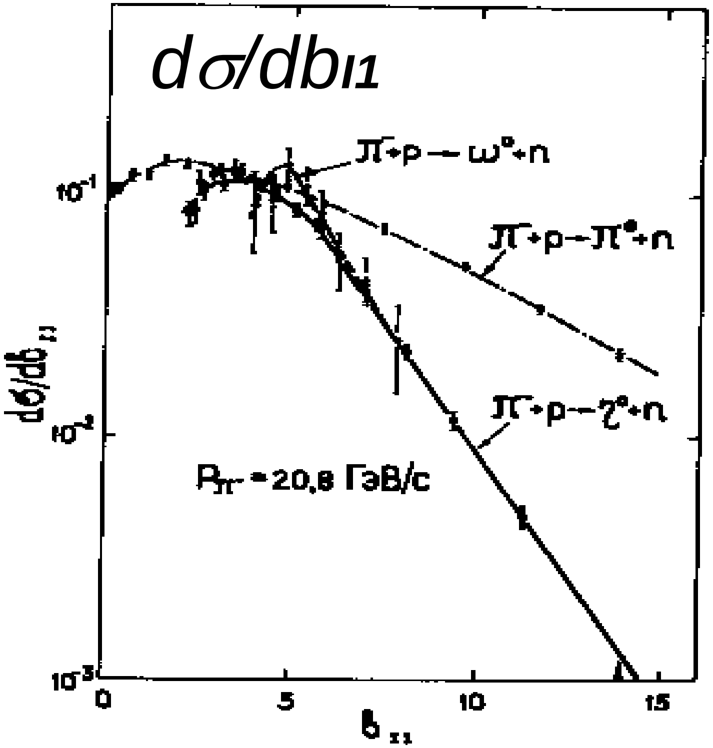
A.Malakhov. Relativistic Nuclear Physics: from Handres of MeV to TeV. 8th Intern. Workshop. Dubna, May 23-28, 2005. E1,2-2006-30, Dubna (2006) pp.44-46.

$b_{III} < 5$ (E < 3.5 A·GeV) $b_{III} > 5$ (E > 3.5 A·GeV)

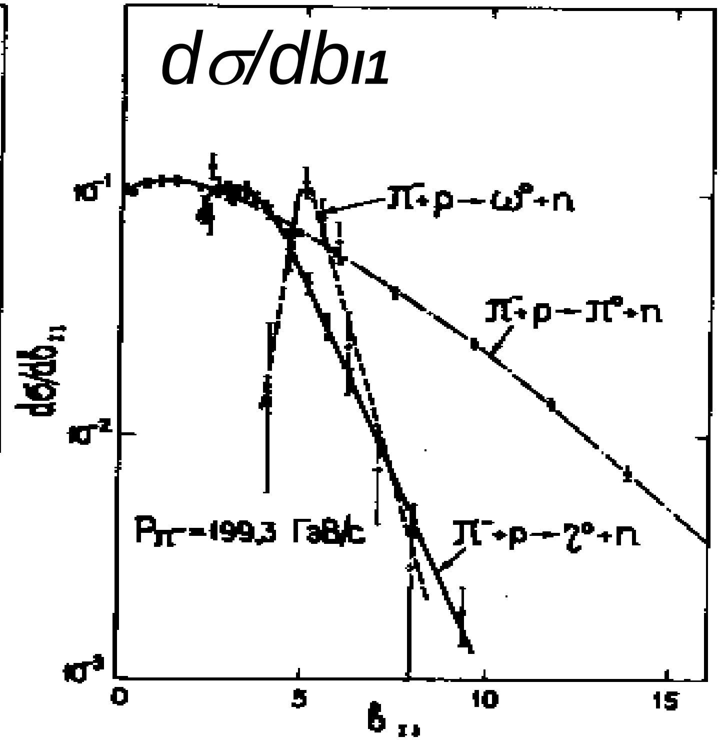




$b_{III} < 5$
(E < 3.5 A·GeV)
$b_{III} > 5$
(E > 3.5 A·GeV)
^4He
$b_{III} < ?$
(E < ?)
$b_{III} > ?$
(E > ?)
p



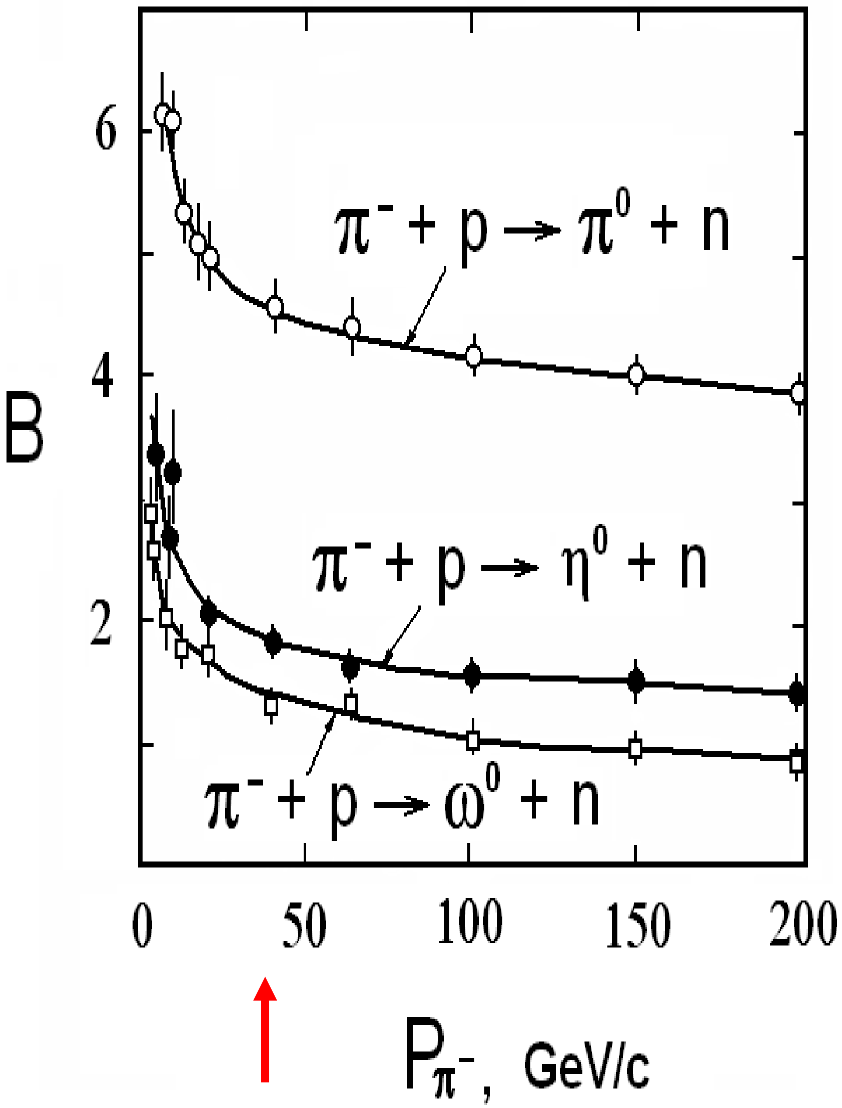
b I1



b I1

$$d\sigma/db_{I1} = A\cdot \exp(-b_{I1}/B)$$

A.I.Malakhov, G.L.Melkumov. JINR Rapid Commun., No.19-86 (1986) pp.32-39).



As it possible see in figure asymptotic regime is beginning with pion momentum $\boldsymbol{p_\pi}$ **~ 25 GeV** which is correspond $\boldsymbol{b_{I\,II}} \sim 380$. Thus we can propose that at $\boldsymbol{b_{I\,II}} \geq 380$ beginning manifestation of the internal structure of quarks and in this region possible to study internal structure of quarks.

$$b_{I\,II} = -\,(u_\pi - u_p)^2 = -\,(1 - 2u_\pi u_p - 1) =$$

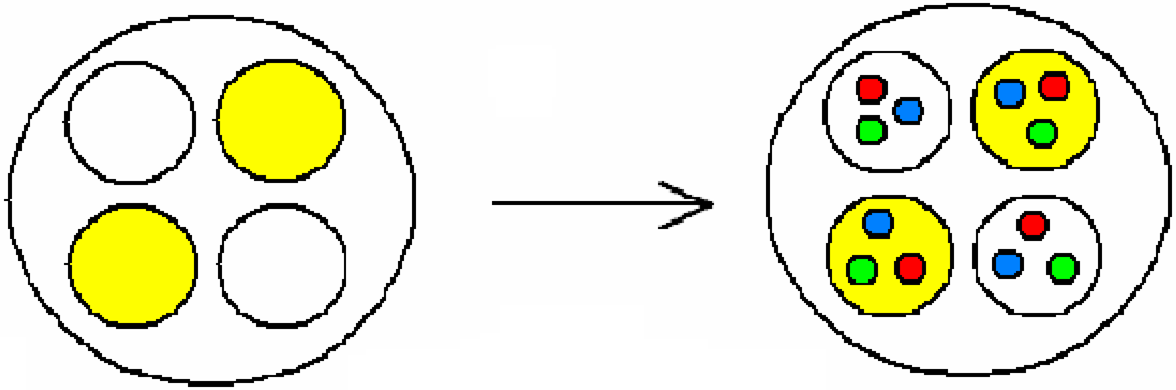
$$= 2(u_\pi u_p - 1) = 2(E_\pi/m_\pi - 1) = 2T_\pi / m_\pi \cong$$

$$\cong 2 \cdot 25/0.130 \cong 380.$$

$b_{III} < 5$ $(E < 3.5 \cdot A\ \mathrm{GeV})$

$b_{III} > 5$ $(E > 3.5 \cdot A\ \mathrm{GeV})$

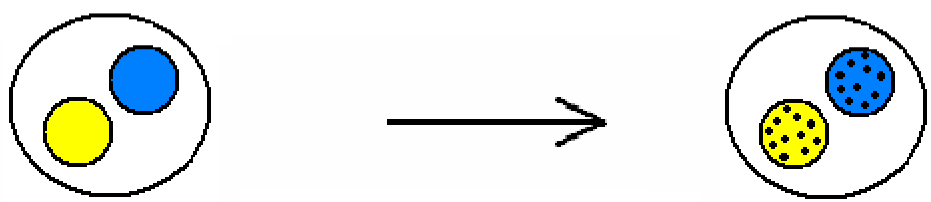
^4He

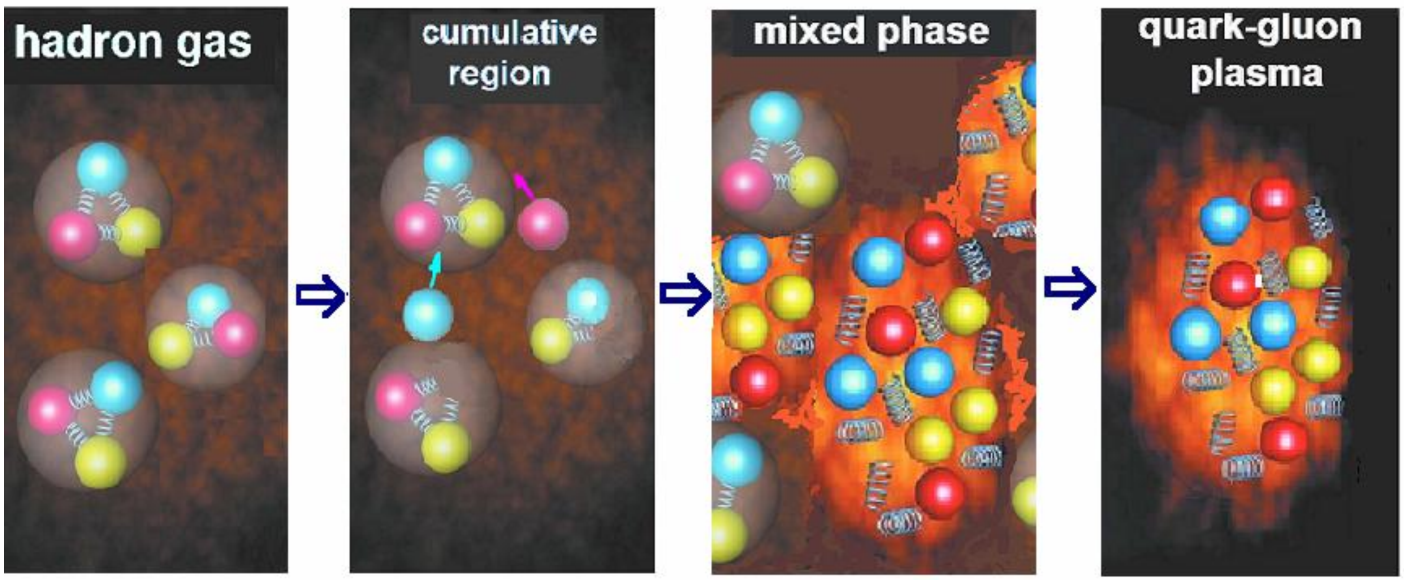


$b_{III} < 380$ $(E < 25 \cdot A\ \mathrm{GeV})$

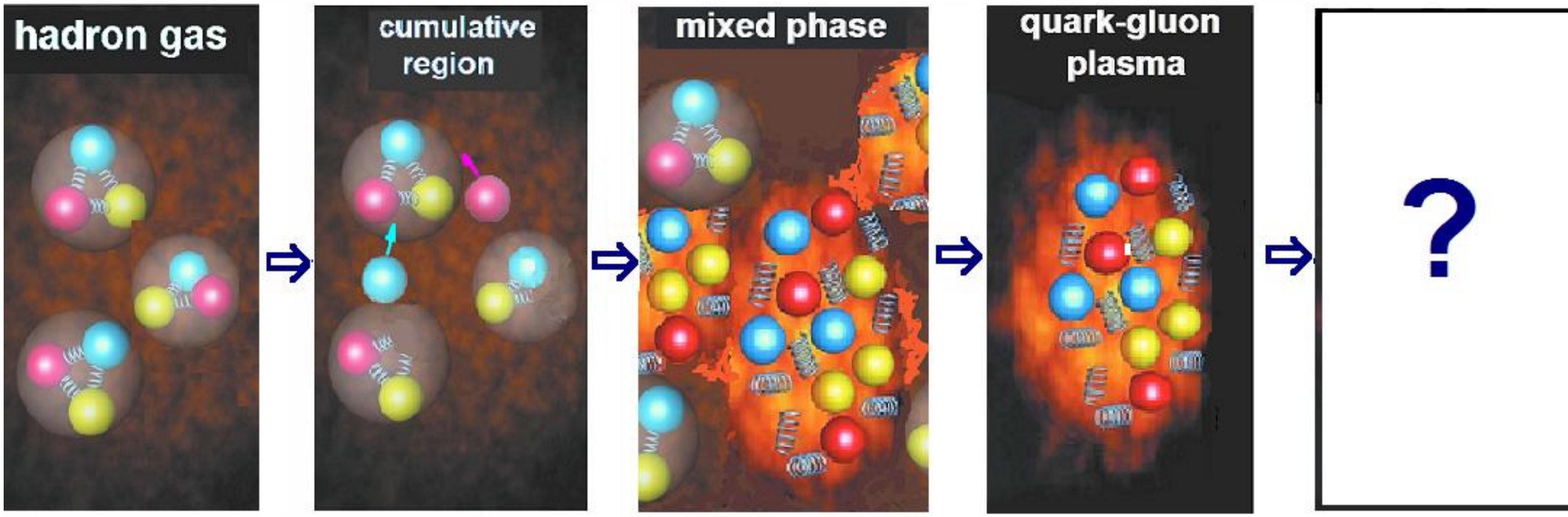
$b_{III} > 380$ $(E > 25 \cdot A\ \mathrm{GeV})$

π

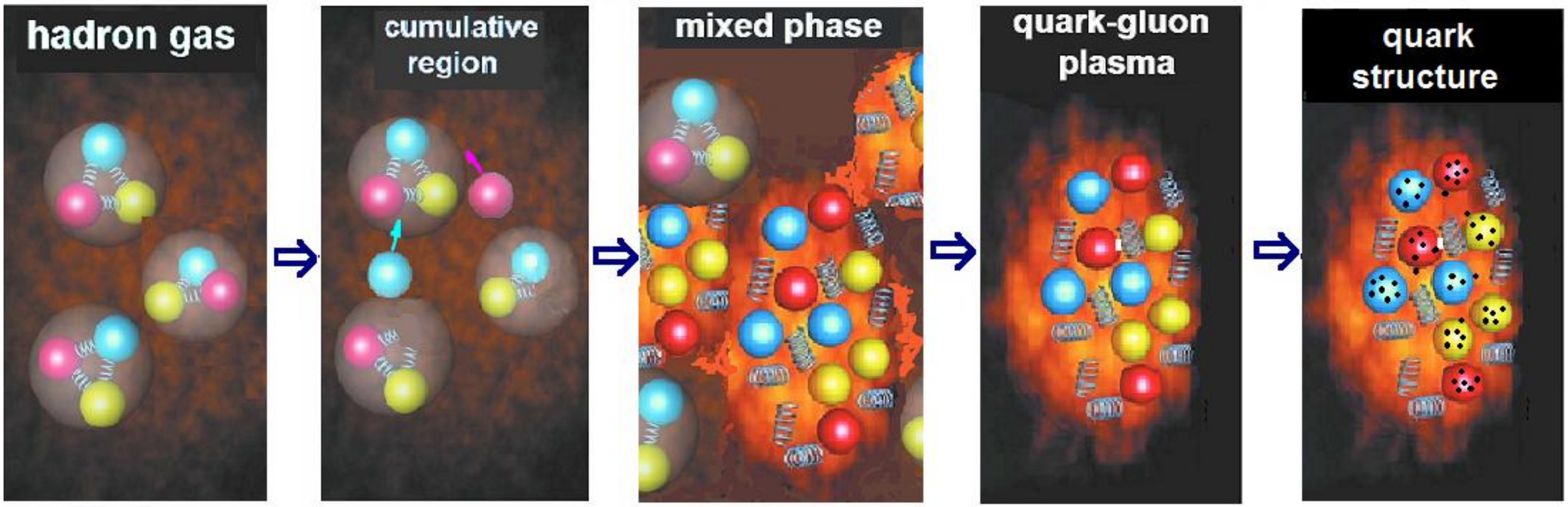




$b_{ik} \approx 0.01$ $\quad$ $b_{ik} \approx 5$ $\quad$ $b_{ik} > 5$ $\quad$ $b_{ik} >> 1$



$b_{ik} \approx 0.01$ $b_{ik} \approx 5$ $b_{ik} > 5$ $b_{ik} >> 1$



$b_{ik} \approx 0.01$ $b_{ik} \approx 5$ $b_{ik} > 5$ $b_{ik} >> 1$ $b_{ik} > 400$

Conclusions

- The FAIR and NICA energy region allows to investigate in details transition and asymptotic areas of nuclear interactions
- Experiments at these accelerators allow to check up validity of a Automodelity Principle (Self-similarity Hypothesis) and Correlation Depletion Principle for nuclear interactions
- Probably also studying of the exotic phenomena: a dark energy (QCD vacuum) and quark structure

Thank you !

$$b_k = -(V - u_k)^2 = -(\overset{=1}{V^2} - 2Vu_k + \overset{=1}{u_k^2}) = 2(Vu_k - 1) =$$

$$= 2\left(\frac{Vp_k}{m_k} - 1\right) = 2\left(\frac{E_k}{m_k} - \frac{\vec{V}\vec{P}_k}{m_k} \quad 1\right) = 2\frac{E_k}{m_k} - 2\frac{\vec{V}\vec{P}_k}{m_k} - 2 \, .$$

$$b_k - 2\frac{E_k}{m_k} + 2 = -2\frac{\vec{V}\vec{P}_k}{m_k}$$

$$\left(b_k - 2\frac{E_k}{m_k} + 2\right)^2 = 4\,\frac{\overset{=1}{\vec{V}^2}\vec{P}_k^2}{m_k} = 4\,\frac{p_k^2}{m_k^2}$$

$$b_k - 2\frac{E_k}{m_k} + 2 = \pm 2\frac{p_k}{m_k}$$

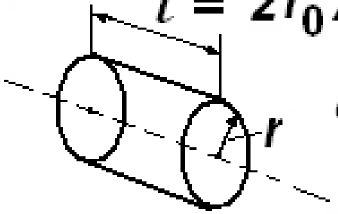
$$\boxed{b_k = 2\frac{p_k}{m_k} + 2\frac{E_k}{m_k} - 2 = \frac{2}{m_k}(\pm p_k + E_k - m_k)}$$

$$\boxed{p = -\frac{\Delta E}{\Delta V}}$$

$$V = \sigma \cdot l = \sigma \cdot 2 r_0 A^{\frac{1}{3}}$$

$$\Delta V = \Delta\sigma \cdot 2 r_0 A^{\frac{1}{3}}$$

$$l = 2 r_0 A^{\frac{1}{3}}$$



$$\sigma = \pi r^2$$

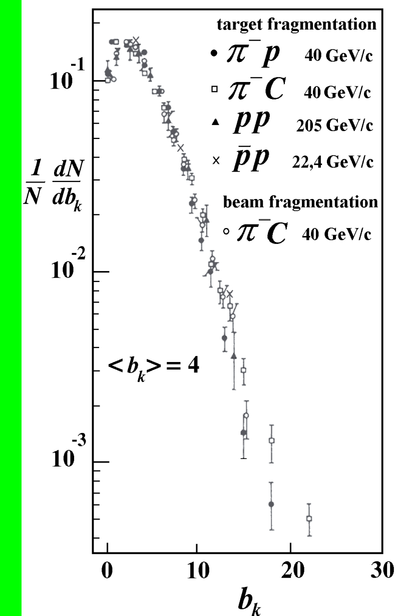
$$\boxed{b_k = 2\frac{p_k}{m_k} + 2\frac{E_k}{m_k} - 2 = \frac{2}{m_k}(\pm p_k + E_k - m_k)}$$

$$\Delta b_k = \frac{2}{m_k}(\pm \Delta p_k + \Delta E_k) = \frac{2}{m_k}\left[\pm \Delta (E_k^2 + m_k^2)^{\frac{1}{2}} + \Delta E_k\right] =$$

$$= \frac{2}{m_k}\left[\pm \frac{1}{2}(E_k^2 - m_k^2)^{-\frac{1}{2}} 2 E_k \Delta E_k + \Delta E_k\right] = \frac{2}{m_k}\left(\pm \frac{E_k}{p_k} + 1\right)\Delta E_k$$

$$\boxed{\Delta E_k = \frac{m_k}{2\left(1 \pm \frac{E_k}{p_k}\right)}\Delta b_k = \frac{m_k}{2\left(1 \pm \frac{1}{\beta_k}\right)}\Delta b_k}$$

$$p = -\frac{\Delta E}{\Delta V} = -\frac{\left[\dfrac{m_k}{2\left(1 \pm \dfrac{1}{\beta_k}\right)}\right]\Delta b_k}{2 r_0 A^{\frac{1}{3}} \cdot \Delta\sigma} =$$



$$= -\frac{m_k}{4 r_0 A^{\frac{1}{3}} \underbrace{\left(1 \pm \frac{1}{\beta_k}\right)}_{\approx 1}} \frac{\Delta b_k}{\Delta\sigma} \approx \begin{cases} -\dfrac{1}{8}\dfrac{m_k}{r_0 A^{\frac{1}{3}}}\dfrac{\Delta b_k}{\Delta\sigma} \\ +2.25\dfrac{m_k}{r_0 A^{\frac{1}{3}}}\dfrac{\Delta b_k}{\Delta\sigma}, \end{cases} \qquad \beta_k \approx 0{,}9$$

$$b_k = -(V - u_k)^2 = -(\overset{=1}{V^2} - 2Vu_k + \overset{=1}{u_k^2}) = 2(Vu_k - 1) =$$

$$= 2\left(\frac{VP_k}{m} - 1\right) = 2\left(\frac{E_k}{m_k} - \frac{\vec{V}\vec{P}_k}{m_k} - 1\right) = 2\frac{E_k}{m_k} - 2\frac{\vec{V}\vec{P}_k}{m_k} - 2 .$$

$$= 2\left(\frac{E_k}{m_k} - \frac{p_k\, Cos\theta_k}{m_k} - 1\right) = \frac{2}{m_k}\left(E_k - p_k\, Cos\theta_k - 1\right)$$

$$b_k = \frac{2}{m_k}\,(E_k - p_k\,Cos\Theta_k - 1)$$

$$\Delta b_k = \frac{2}{m_k}(\Delta E_k - \Delta p_k\,Cos\Theta_k + p_k\,Sin\Theta_k\,\Delta\Theta_k) \approx$$

$$\approx \frac{2}{m_k}(\Delta E_k - \Delta p_k) = \frac{2}{m_k}(\Delta E_k - \frac{E_k}{p_k}\Delta E_k) = \frac{2\,\Delta E_k}{m_k}(1 - \frac{E_k}{p_k}) =$$

$$= \frac{2\,\Delta E_k}{m_k}(1 - \frac{1}{\beta_k})$$

$$\boxed{\Delta E_k = \frac{m_k}{2\,(1 - \frac{1}{\beta_k})}\,\Delta b_k}$$

$$p = -\frac{\Delta E}{\Delta V} = -\frac{\left[\frac{m_k}{2\,(1-\frac{1}{\beta_k})}\right]\Delta b_k}{2\,r_0\,A^{\frac{1}{3}}\cdot\Delta\sigma} =$$

$$\beta_k \approx 0{,}9$$

$$= -\frac{m_k}{4\,r_0\,A^{\frac{1}{3}}\,(1-\frac{1}{\beta_k})}\,\frac{\Delta b_k}{\Delta\sigma} \simeq 9\,\frac{m_k}{4\,r_0\,A^{\frac{1}{3}}}\,\frac{\Delta b_k}{\Delta\sigma} = 2.25\,\frac{0.14}{r_0\,A^{\frac{1}{3}}}\,\frac{\Delta b_k}{\Delta\sigma} =$$

$$\frac{0.315}{r_0 A^{\frac{1}{3}}}\,\frac{\Delta b_k}{\Delta\sigma} = \frac{0.315}{1.4\cdot 10^{-13}\cdot 2.27}\,\frac{\Delta b_k}{\Delta\sigma} = 10^{12}\,\frac{\Delta b_k}{\Delta\sigma}\left[\frac{\mathrm{GeV}}{cm}\right]$$

$$r_0 = 1.4\cdot 10^{-13}\ cm$$

$$A^{\frac{1}{3}} = 12^{\frac{1}{3}} = 2.27$$