Two body and three body field theoretical equations with and without quark-gluon degrees of freedom

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Short list of related publications:

[I] *A. I. MACHAVARIANI, A. J. BUCHMANN, AMAND FAESSLER and G. A. EMELYANENKO* Annals of Physics **253**(1997)149.

[II] *A. I. MACHAVARIANI/ and/ AMAND/ FAESSLER* Nuclear Physics **A646** (2002) 231.

[III] *A. I. MACHAVARIANI* Phys. Letters **B540**(2002) 81.

[IV] *A. I. MACHAVARIANI and AMAND FAESSLER;* Annals of Phys.**409**(2004)p.49-92.

[V] *A. I. MACHAVARIANI and AMAND FAESSLER;* Phys. Review **C72** (2005) 024002.

[VI] *A. I. MACHAVARIANI and AMAND FAESSLER;* Journal of Physics G: Nucl. Part. Phys. **37** (2010) 075004.

OUR NUMERICAL APPLICATIONS:

Description of the experimental data of the NN, πN and $\pi N - \gamma\pi N$ scattering reactions in the Δ resonance region.

CONTENT & Intention:

(I) Are the 4D Bethe-Salpeter equations more general and consistent than the time ordered from the beginning 3D field-theoretical equations?

(A) Nonphysical degrees of freedom in the 4D Bethe-Salpeter equations

(B) INPUT and OUTPUT in the 4D and 3D field-theoretical equations

(C) 3D time-ordered field-theoretical equations as the non avoidable intermediate reduction of the 4D Bethe-Salpeter equation.

(D) Why are the 3D time-ordered field-theoretical equations more convenient by numerical solutions?

(II) 4D and 3D field theoretical equations with and without quarks.

The most compact (convenient) 3D time-ordered field-theoretical equations with quarks.

(III) The three-body 3D time-ordered field-theoretical equations.

(A) Construction of the COMPLETE set of the 3-body forces.

(B) Relativistic field-theoretical (with creation and annihilation) generalization of the Faddeev equations.

(C) Principal non linearity of the 3D and 4D field theoretical equations

Why do we need the field-theoretical approach:

Construction of the relativistic (or non-relativistic) potentials in the 3D Schrödinger-type equations i.e. a relativistic field-theoretical generalization of the usual non-relativistic collision theory.

$$(H_O + V)|\Psi >= E|\Psi >$$

or

$$T(E) = V + V(E - H_O - V + i\epsilon)^{-1}T(E); \quad T(E) \equiv V|\Psi >$$

via the vertex functions. The non-relativistic limit of all relativistic field-theoretical equations reproduce the same Schrödinger equation.

A. Quasipotential 3D reductions of the Bethe-Salpeter equation, (Logunov-Tavkhelidze, Blancenbecler-Sugar etc.)

B. Relativistic Hamiltonian approach within Old perturbation theory (Kadyshevsky, Karmanov, etc),

C. Old perturbation theory or spectral decomposition over the asymptotic states (Chew-Low equations, our approach)

Our approach presents a linearization of the generalized unitarity condition.

This condition is also a matrix representation of the Bogoljubov-Medvedev-Polivaniv causality principle.

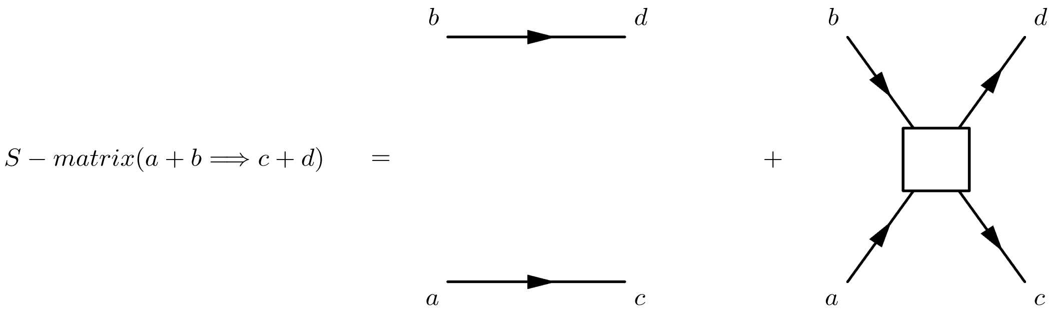


Figure 1:
$\mathcal{S}_{a+b\leftarrow c+d} = \delta(\mathbf{p_a} - \mathbf{p_c})\delta(\mathbf{p_b} - \mathbf{p_d}) + (2\pi)^4 i\delta^{(4)}(p_a + p_b - p_c - p_d)\mathcal{A}_{a+b\leftarrow c+d}.$

Sought amplitude:

$$\mathcal{A}_{a+b\leftarrow c+d} =< out; \mathbf{p}_a|\overline{u}(\mathbf{p_b}J(0)|\mathbf{p}_c\mathbf{p}_d; in >$$

$$\mathcal{A}_{a+b\leftarrow c+d} =< 0|b_{\mathbf{p}_a}(out)\overline{u}(\mathbf{p_b})J(0)b^+_{\mathbf{p}_c}(in)b^+_{\mathbf{p}_d}(in)|0 >$$

Nucleon Source $\qquad J(x) = (i\gamma_\mu \frac{\partial}{\partial x_\mu} - m_N)\Psi(x)$

$$b_{\mathbf{p}}(out(in)) = \int d^3x e^{ipx}\overline{u}(\mathbf{p})\gamma_o\Psi_{out(in)}(x)$$

INTRODUCING: $\quad b_{\mathbf{p}}(x_o) = \int d^3x e^{ipx}\overline{u}(\mathbf{p})\gamma_o\Psi(x)$
USING:

$$b^+_{\mathbf{p}}(out) - b^+_{\mathbf{p}}(in) = \int dx_o \frac{\partial}{\partial x_o} b_{\mathbf{p}}(x_o) = \int d^4x \overline{J}(x) e^{-ipx} u(\mathbf{p})$$

$$\mathcal{A}_{a+b\leftarrow c+d} = \overline{u}(\mathbf{p_b} \int d^4 x e^{-ip_c x} < out; \mathbf{p}_a | \mathsf{T}(J_b(0)\overline{J}_c(x)) | \mathbf{p}_d; in > u(\mathbf{p_c}$$

SUBSTITUTION: $\Sigma_n \, |n; in >< in; n| = \hat{\mathbf{1}}$

FINAL EQUATION:

$$\mathcal{A}_{a+b\leftarrow c+d} = \mathcal{W}_{a+b\to c+d} + \sum_{g+h} \mathcal{A}^{+}_{a+b\leftarrow g+h} \frac{\delta^{(3)}(\mathbf{p_a} + \mathbf{p_b} - \mathbf{p_g} - \mathbf{p_h})}{p_a^o + p_b^o - p_g^o - p_h^o + i\epsilon} \mathcal{A}_{g+h\leftarrow c+d}$$

have the form of the generalised unitarity condition for the Lippmann-Schwinger-type equations: $T = V + T^{+} G_o T$

This condition is also the matrix representation of the Bogoljubov-Medvedev-Polivaniv causality principle.

OUR RESULT:

$$\Longleftrightarrow T_{a+b\leftarrow c+d}(E) = U_{a+b\leftarrow c+d}(E) + \sum_{g+h} U_{a+b\leftarrow g+h}(E) \frac{1}{E - E_{g+h} + i\epsilon} T_{g+h\leftarrow c-}$$

$$T(E = E_{a+b} = E_{c+d}) = \mathcal{A}_{a+b\to c+d}; \quad U_{a+b\to c+d}(E = E_{a+b}) = \mathcal{W}_{a+b\to c+d}$$

Final relativistic Lippmann-Schwinger-type equation for the reaction $a + b \leftarrow c + d$

$$T(\mathbf{p}', \mathbf{p}, E) = V(\mathbf{p}', \mathbf{p}, E) + \int \frac{d^3\mathbf{q}}{E + i\epsilon - E(\mathbf{q})} V(\mathbf{p}', \mathbf{q}, E) T(\mathbf{q}, \mathbf{p}, E)$$

$$E(\mathbf{q}) = \sqrt{m_a^2 + \mathbf{q}^2} + \sqrt{m_b^2 + \mathbf{q}^2}$$

$NN \rightarrow N'N'$ Reaction

$$V = [equal\!-\!time commutator] + [on\, mass\!-\!shell\, meson\, exchange$$

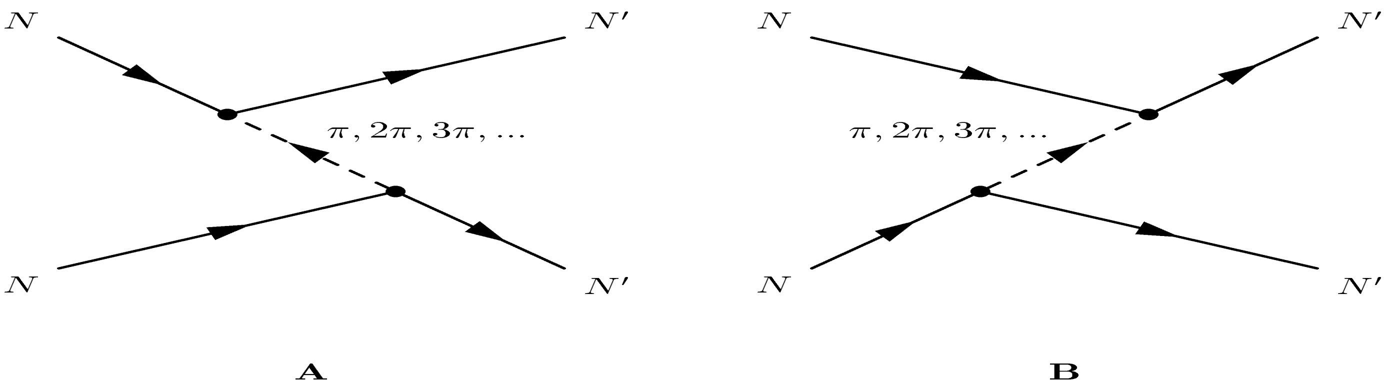
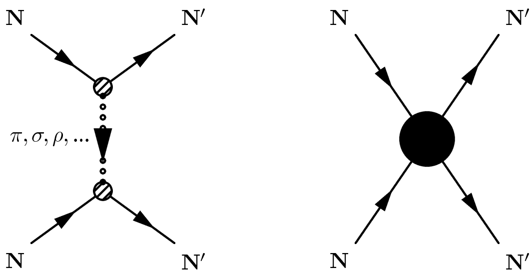


Figure 2: *Time-ordered part of the NN potential with ON MASS SHELL intermediate pions. The full circles denote the vertex functions the one off-mass shell nucleon.*



*The NN potential $\overline{u}(\mathbf{p_b} < out; \mathbf{p}'|\{\mathcal{J}(0), b^+_{\mathbf{p}}(0)\}|\mathbf{p}; in >$. For the 3-point renormalizable meson $-$ NN Lagrangian ($\mathcal{L} = g_\pi \overline{\Psi}\gamma_5 \Psi \Phi_\pi$) arise ONE OFF MASS-SHELL $\pi, \sigma, \rho, \omega, \ldots$ meson exchange-diagram (**A**). The non-renormalizable Lagrangian ($\mathcal{L} = g_\pi / m_\pi \overline{\Psi}\gamma_\mu\gamma_5\Psi\partial^\mu\Phi_\pi$) generate the four nucleon overlapping (contact) term diagram (**B**) without intermediate propagators. The shaded circle corresponds to the vertex function $< \mathbf{p'}_N | j_{meson}(0) | \mathbf{p}_N >$ with off-mass shell meson.*

INPUT & OUTPUT

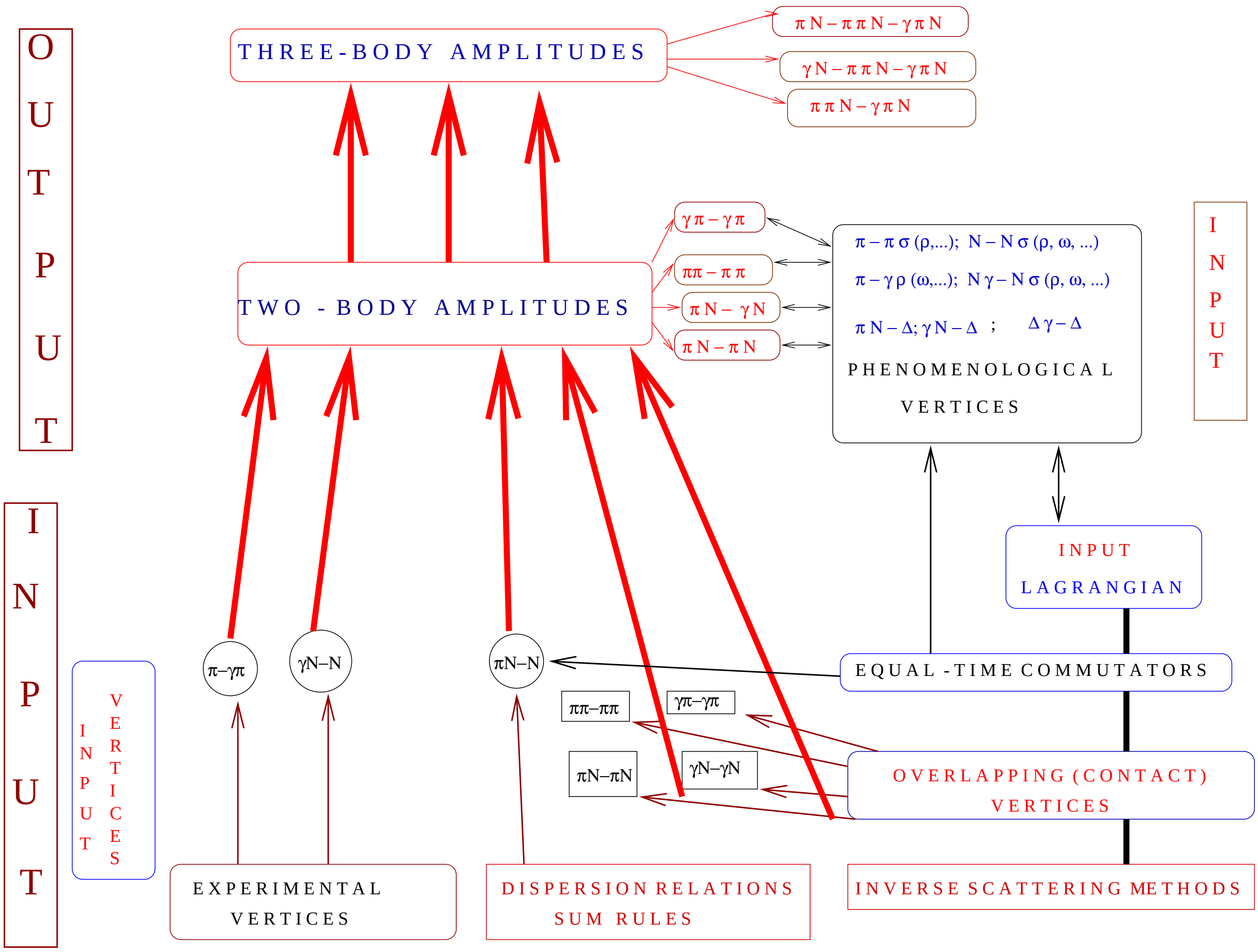


Figure 3: Every kind of input vertex functions contains two on-mass shell particle states and one can construct these vertices directly from experimental data, or using the dispersion relations and other inverse scattering methods.

INPUT 4D Bethe-Salpeter equation: 3-variable vertex.
OUTPUT for binary reactions: 2-variable on shell ammplitudes.

Nonphysical degrees of freedom in the 4D Bethe-Salpeter equations

Relative time-coordinate (A. Klein, Logunov-Tavkelidze

3D time-ordered field-theoretical equations as the non avoidable intermediate reduction of the 4D Bethe-Salpeter equation.

(I) on shell 4D amplitude of Bethe-Salpeter and $\mathcal{A}_{a+b\leftarrow c+d} =< out; \mathbf{p}_a|\overline{u}(\mathbf{p_b}J(0)|\mathbf{p}_c\mathbf{p}_d; in >$ coincides.

(II) One can reproduce the 3D time-ordered equation $t = V + VG_oT$ from the Bethe-Salpeter equation $\mathcal{T} = w + w\mathcal{G}_{\wr}\mathcal{T}$.

$v = w + w\Big[\mathcal{G}_{\prime} - G_o\Big]v$, where $\mathcal{T}^{3D} = T$

A special sum of the Bethe-Salpeter potentials reproduce 3D time-ordered potential v.

v is ANALYTICALLY defined via equal-time commutators and on mass shell particle exchange amplitudes.

FORMULATION WITH QUARKS:

HAAG-NISHIJIMA-ZIMMERMANN (1958) and HUANG-WELDON (1975) approach of the composite particle in quantum field theory.

$$\Psi_{\mathbf{p_N}}(Y) = \int d^4r_3 d^4r_{1,2}\widetilde{\chi}^{\dagger}_{\mathbf{p_N}}(Y=0, r_{1,2}.r_3)\mathsf{T}\Big(q_1(y_1)q_2(y_2)q_3(y_3)\Big),$$

where Y**,** $r_{1,2}$ **and** r_3 **are the Jacobi coordinates** $y_1 = Y - \eta_3 r_3 + \eta_2 r_{1,2}$**,** $y_2 = Y - \eta_3 r_3 - \eta_1 r_{1,2}$**,** $y_3 = Y + \eta_{1,2} r_3$

$$\chi_{\mathbf{p_N}}(y_1, y_2, y_3) =< 0|\mathsf{T}\Big(q_i(y_1)q_j(y_2)q_k(y_3)|\mathbf{p}_N, s_N, i_N; in >$$

$$\mathcal{B}_{\mathbf{p_N}}(x^o) = \int d^3\mathbf{x} \exp{(ip_N x)}\overline{u}(\mathbf{p}_N)\gamma_o\Psi_{\mathbf{p_N}}(x)$$

$$\mathcal{B}_{in(out)}(\mathbf{p}_N) = \lim_{x^o\to-\infty(+\infty)} \mathcal{B}_{\mathbf{p_N}}(x^o),$$

$$\Big\{\mathcal{B}^+_{in(out)}(\mathbf{p}'), \mathcal{B}_{in(out)}(\mathbf{p})\Big\} = (2\pi)^3\frac{p^o_N}{m_N}\delta(\mathbf{p}' - \mathbf{p});$$

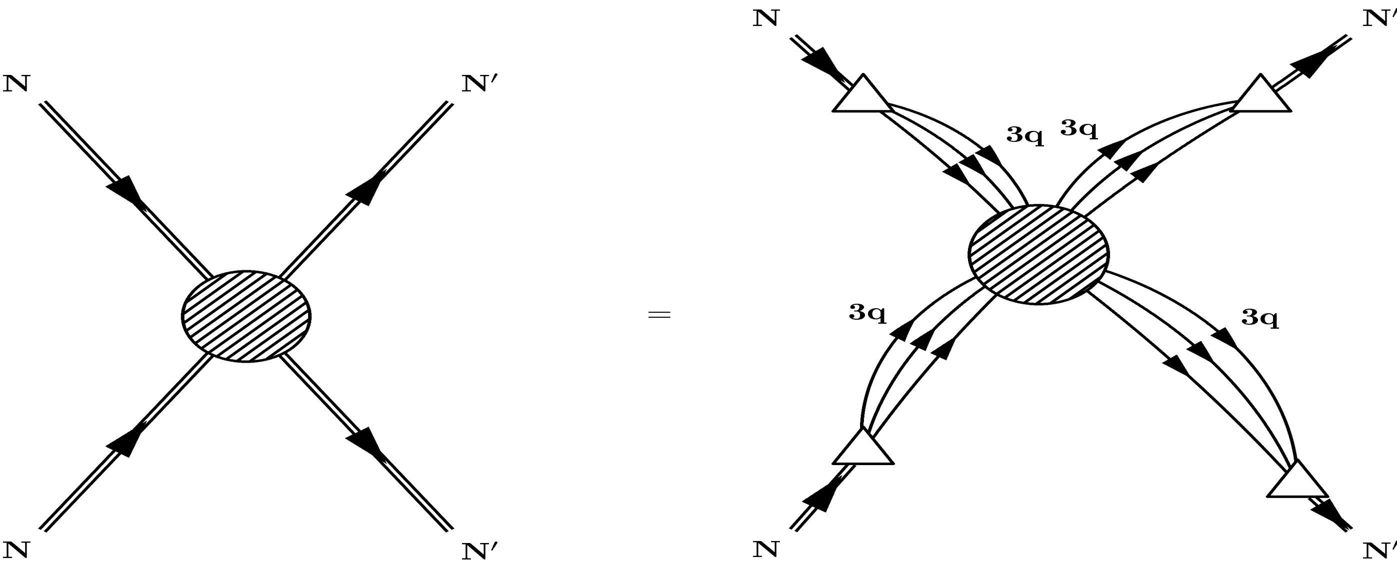


Figure 4: *The NN amplitude with quarks*

$$\mathcal{A}_{N'+N'\leftarrow N+N} =< 0|B_{out}(\mathbf{p}_{N'})\overline{u}(\mathbf{p}_{\mathbf{N'}})J(0)B^{+}_{in}(\mathbf{p}_{N})B^{+}_{in}(\mathbf{p}_{N})|0 >$$

The completeness condition $\sum_n |n; in >< in; n| = \hat{\mathbf{1}}$, $\mathcal{S}$-matrix $\mathcal{S}_{mn} =< out; m|n; in >$, $\mathcal{S}$-matrix reduction formulas, the form of the sought amplitudes $\mathcal{A}_{N'+N'\leftarrow N+N}$, generalized unitarity condition in the quantum field theory with and without quarks are the same in the field theoretical approach with quarks.

Consequently, the form of the considered 3D equation $T = v + vG_oT$ and the form of the potential v remain be the same.

RESULT: the intermediate quark propagation does not contribute in the unutarity condition of the hadron-hadron scattering reactions.

$NN \rightarrow N'N'$ potential with quarks

$V = [equal{-}time commutator] + [on\, mass{-}shell\, meson\, exchange$

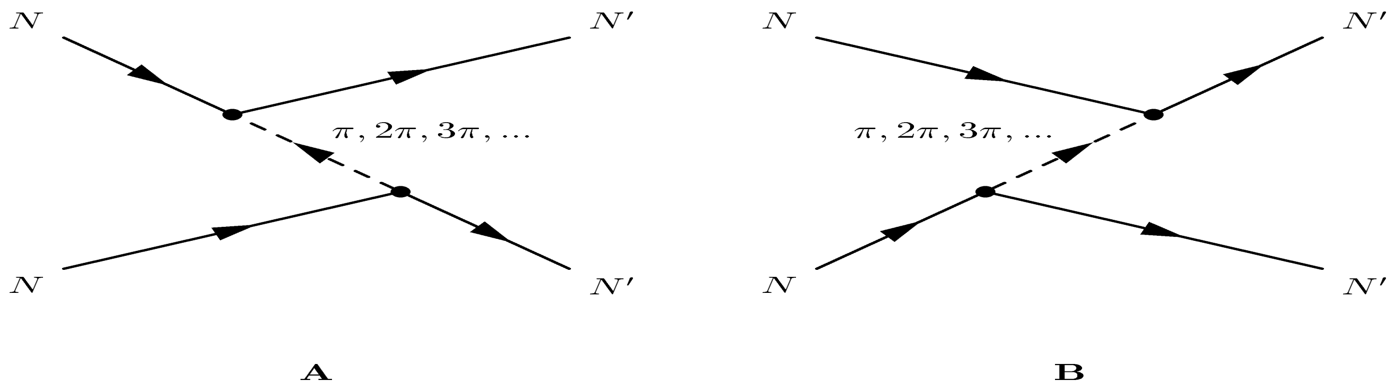
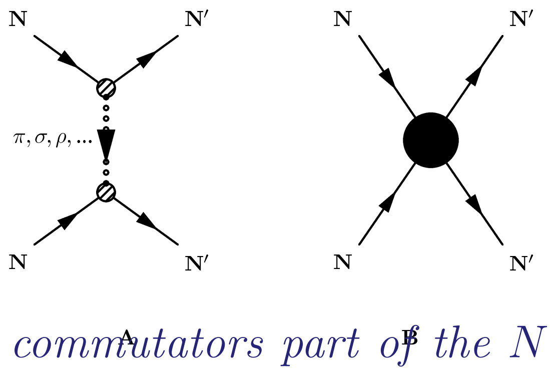


Figure 5: *Time-ordered part of the NN potential with ON MASS SHELL particle exchange remains be the same.*

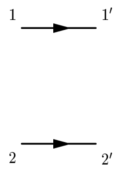


*The equal-time commutators part of the NN potential consists of the ONE OFF MASS-SHELL $\pi, \sigma, \rho, \omega, ...$ meson exchange-part (**A**). and the overlapping (contact) part which is constructed through the quark-gluon exchange diagrams (**B**).*

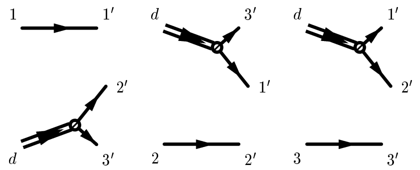
Thus instead of the contact terms from the non-renormalizable Lagrangian ($\mathcal{L} = g_\pi/m_\pi \overline{\Psi}\gamma_\mu\gamma_5\Psi\partial^\mu\Phi_\pi$) in the local quantum field theory in the formulation with quarks appears contact terms from the quark-gluon exchange.

II.THREE-BODY EQUATIONS for 3 fermions (3e,Nd-NNN)

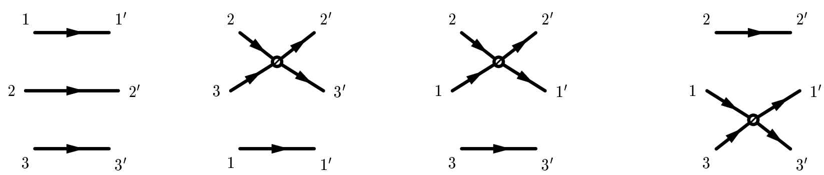
$\alpha = 1', 2', 3', f'd'$ **and** $\beta = 1, 2, 3, fd$



A



B



C

$$S_{\alpha,\beta} =< out; \alpha|\beta; in >=< out; \tilde{\alpha}|b_{\mathbf{p_a}}(in)|\tilde{\beta}; in > -(2\pi)^4 i\delta^{(4)}(P_\alpha - P_\beta)\mathcal{T}_{\alpha,\beta}$$

$$\alpha = a + \tilde{\alpha}; \quad \beta = b + \tilde{\beta}$$

$p_a = (\sqrt{m_a^2 + \mathbf{p}_a^2}, \mathbf{p}_a) \equiv (E_{\mathbf{p_a}}, \mathbf{p}_a)$**.**

In the S-matrix approach one can separate from the beginning the connected and the disconnected parts of amplitudes and one can write independent equations for these connected and disconnected parts

$$\mathcal{M}_{\alpha\beta} =< out; \tilde{\alpha}|J_{\mathbf{p_a}}(0)|\beta; in >= \mathcal{M}_{\alpha\beta}^{dc} + \mathcal{M}_{\alpha\beta}^{con}$$

Equation for the Heisenberg fermion field operators

$J_{\mathbf{p_a}}(x) = Z_a^{-1/2}\overline{u}(\mathbf{p_a})(i\gamma_\mu\partial_x^\mu - m_a)\psi_a(x)$

Using S-matrix reduction formulas, in the same way as for the 2-body case we get

$$\mathcal{M}^{con}_{\alpha\beta} =< out; \tilde{\alpha}| b^{+}_{\mathbf{p_b}}(out) J_{\mathbf{p_a}}(0)|\tilde{\beta}; in >^{con}$$

$$- < out; \tilde{\alpha}|\{J_{\mathbf{p_a}}(0), b^{+}_{\mathbf{p_b}}(0)\}|\tilde{\beta}; in > +i\int d^4x e^{-ip_b x} < out; \tilde{\alpha}|T(J_{\mathbf{p_a}}(0)\overline{J}_{\mathbf{p_b}}(x))|\tilde{\beta}; in >^{con},$$

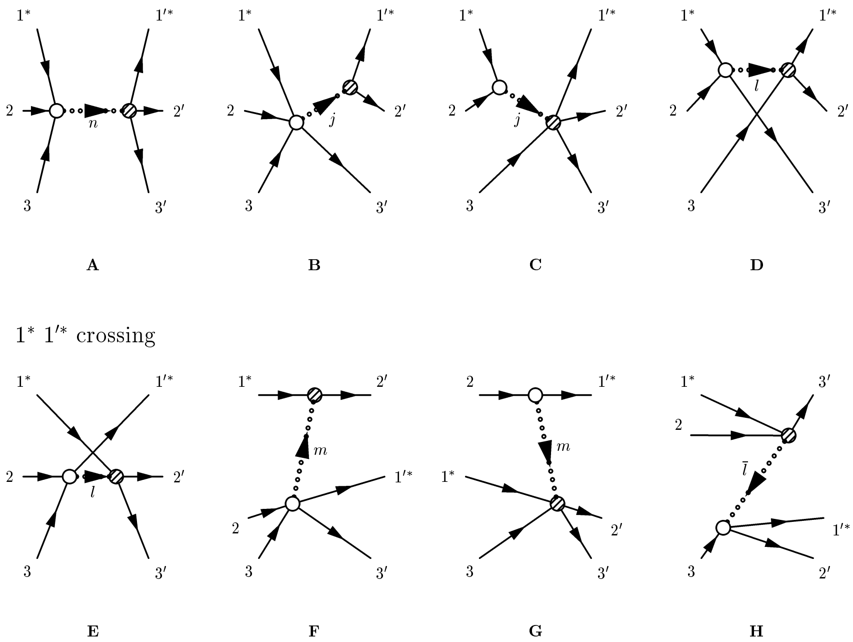
where

$$b^{+}_{\mathbf{p}_b}(x_0) = Z_b^{-1/2}\int d^3x e^{-ip_b x}\overline{u}(\mathbf{p_b})\gamma_o\psi_b(x)$$

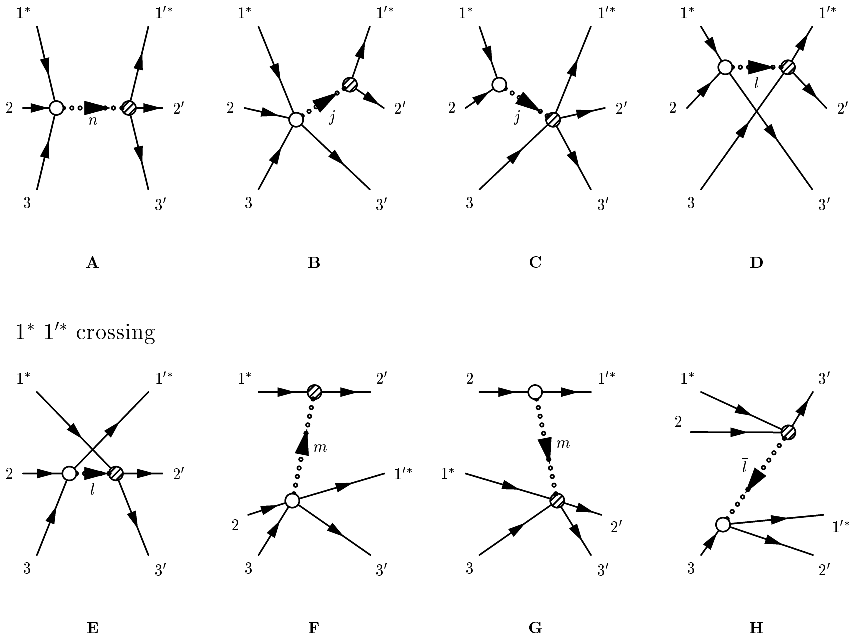
and for the disconnected part of the 3-body amplitude we have independent set of equations

$$\mathcal{M}^{dc}_{\alpha\beta}(E_\beta) = V^{dc}_{\alpha\beta} + \sum_\gamma \mathcal{M}^{dc}_{\alpha\beta}(E_\gamma)\frac{1}{E_\beta - E_\gamma + i\epsilon}\mathcal{M}^{dc\ *}_{\beta,\gamma},$$

Substituting $\sum_n |n; in >< in; n| = \hat{\mathbf{1}}$ **we obtain for the connected part of amplitude**

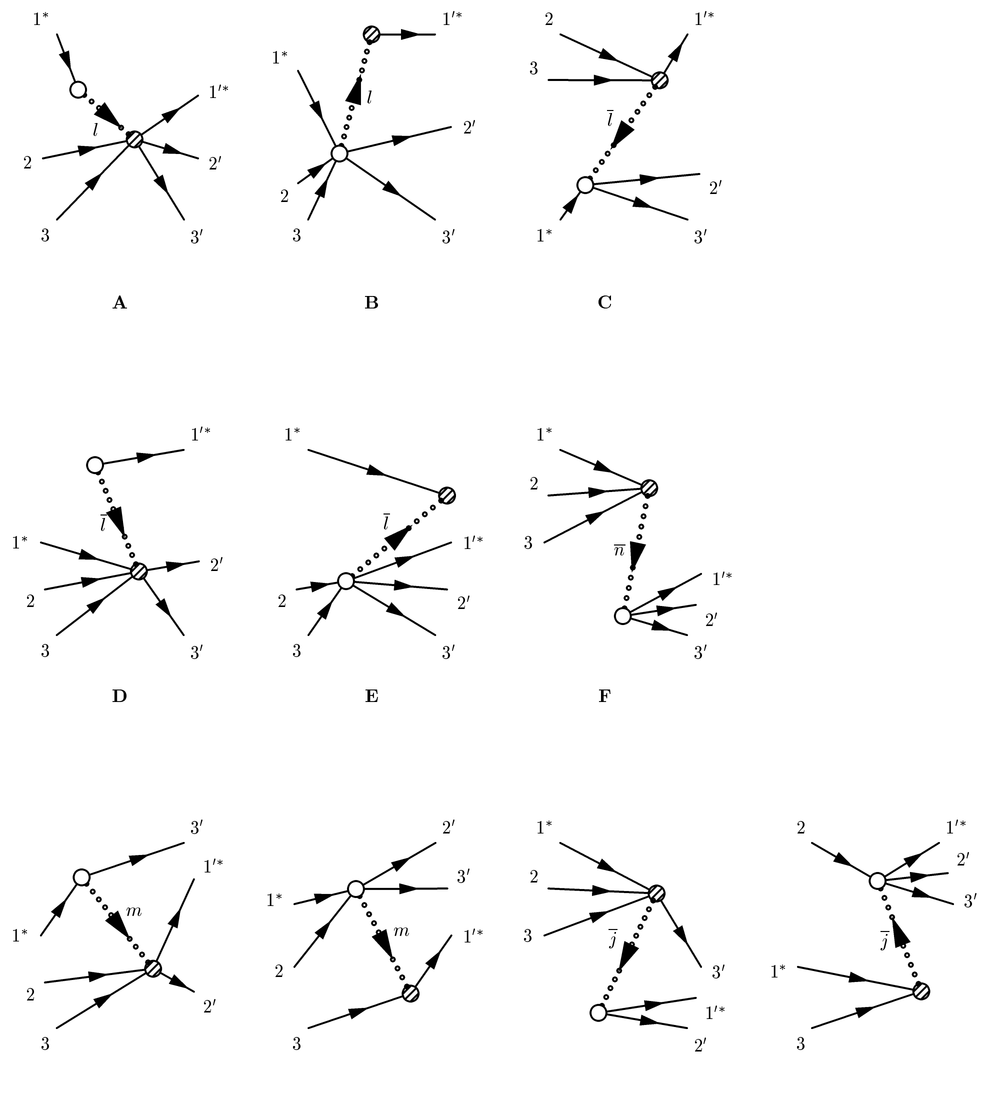


$$\mathcal{M}^{con}_{\alpha\beta} = W_{\alpha\beta} + (2\pi)^3\sum_\gamma \mathcal{M}^{con}_{\alpha\beta}\frac{\delta^{(3)}(\mathbf{p}_b + \mathbf{P}_{\tilde{\beta}} - \mathbf{P}_\gamma)}{E_{\mathbf{p_b}} + \ P^o_{\tilde{\beta}} - P^o_\gamma + i\epsilon}\mathcal{M}^{con*}_{\beta\gamma}$$



$$
\begin{aligned}
W_{\alpha\beta} = & - < out;\tilde{\alpha}|\{J_{\mathbf{p_a}}(0), b^{+}_{\mathbf{p_b}}(0)\}|\tilde{\beta}; in > \\
& +(2\pi)^3 \sum_{n=1''2''3''b'',f''d'',...} < out;\tilde{\alpha}|J_{\mathbf{p_a}}(0)|n;in > \frac{\delta^{(3)}(\mathbf{p}_b+\mathbf{P}_{\tilde{\beta}}-\mathbf{P}_n)}{E_{\mathbf{p_b}}+P^o_{\tilde{\beta}}-P^o_n+i\epsilon} < in;n|\overline{J}_{\mathbf{p_b}}(0)|\tilde{\beta};in \\
& -(2\pi)^3 \sum_{l=f,fb,...} < out;\tilde{\alpha}|\overline{J}_{\mathbf{p_b}}(0)|l;in > \frac{\delta^{(3)}(-\mathbf{p}_b+\mathbf{P}_{\tilde{\alpha}}-\mathbf{P_l})}{-E_{\mathbf{p_b}}+P^o_{\tilde{\alpha}}-P^o_l} < in;l|J_{\mathbf{p_a}}(0)|\tilde{\beta};in >,
\end{aligned} \tag{3.8}
$$

where $n = 1''2''3''b'', f''d'', ...$ denotes the four body states with the intermediate $b'' = \gamma''$ for $3e$ and $b'' = \pi'', \sigma'', ...$ for NNN or Nd.



Diagrams obtained after the two particle $(2, 3)$ and $(2', 3')$ transposition from the s-channel diagram in Fig.2A and Fig.2E

The advantage of this spectral decomposition formula is that it does not need the splitting into four pieces $V_{\alpha\beta}=\Sigma_{i=1}^{3}V^{i}_{\alpha\beta}+V^{c}_{\alpha\beta}$ in order to taken into account the disconnected parts in the perturbation series. Besides this equation are free from the double-counting problem which appear due to special disconnected diagrams.

Using the linearisation procedure of such equations, one get the equivalent Lippmann-Schwinger-type equations

$$T_{\alpha,fd}(E_{fd})=U_{\alpha,fd}(E_{fd})+\sum_{\gamma}U_{\alpha\gamma}(E_{fd})\frac{1}{E_{fd}-E_{\gamma}+i\epsilon}T_{\gamma,fd}(E_{fd}),$$

where $E_{fd}=E_d+E_f$, $\alpha,\gamma=3f,fd$ and $U_{\alpha\gamma}(E)$ is unambiguously determined from the connected part of potentials $w_{\alpha\gamma}$.

The solution of the three-body equations (i.e the $123\rightarrow 1''2''3'$ and $123\rightarrow 3'd''$ transition amplitudes) participate in the $w^{c}_{\alpha\gamma}$ potential in the diagrams 3B and 3C. One can rid the three-fermion potential of such type nonlinearities after introduction of a new amplitudes $f_{\alpha,\beta}=F_{\alpha,\beta}+A_{\alpha,\beta}$, where the choice of $A_{\alpha,\beta}$ is conditioned by cancellation of the terms in Fig. 2B and in Fig. 2C which have the form fg_oA^{+} and Ag_of^{+}. Afterwards we get the linear Lippmann-Schwinger-type equation for $F_{\alpha,\beta}$ amplitudes with the disconnected terms. Thus this linearisation procedure shows connections between the nonlinear Low-type equations and the Faddeev-type equations for the three-fermion scattering problems.

Thus we have obtained the spectral decomposition formulas (or off shell unitarity conditions) for the three-body amplitudes in the standard quantum field theory.

$$\mathcal{M}^{con}_{\alpha\beta}=W_{\alpha\beta}+(2\pi)^3\sum_{\gamma}\mathcal{M}^{con}_{\alpha\beta}\frac{\delta^{(3)}(\mathbf{p}_b+\mathbf{P}_{\widetilde{\beta}}-\mathbf{P}_{\gamma})}{E_{\mathbf{p_b}}+\ P^{o}_{\widetilde{\beta}}-P^{o}_{\gamma}+i\epsilon}\mathcal{M}^{con*}_{\beta\gamma}$$

and corresponding 3-body Lippmann-Schwinger-type equation

$$T_{\alpha,fd}(E_{fd})=U_{\alpha,fd}(E_{fd})+\sum_{\gamma}U_{\alpha\gamma}(E_{fd})\frac{1}{E_{fd}-E_{\gamma}+i\epsilon}T_{\gamma,fd}(E_{fd}),$$

CONCLUSION:

[I.] From the beginning 3D time-ordered equation are necessary intermediate step between the 4D Bethe-Salpeter equations and the real 3D observables ($\mathcal{S}$-matrix, amplitudes and corresponding cross sections, unitarity conditions in the Fock space, causality etc are given in 3D form).

Therefore one can not suppose, that 4D equations contain MORE information as the 3D equations in the old perturbation theory.

[II.] The considered 3D equations allows to avoid the principal ambiguities and approximations of other relativistic field theoretical formulations:

(A) These equations require as INPUT one-variable phenomenological theoretical vertices.

(B) These equations are free from the 3D ambiguities of the quasiopotential reductions of the Bethe-Salpeter equations.

(C) Suggested equations have the same compact form within formulations with and without quark degrees of freedom

(D) Suggested equations are convenient for the numerical calculations because they have form of the well known Schrödinger or Lippmann-Schwinger equations with well known analytical properties and the solution procedure.

[III.] The suggested three-body equations satisfy au-

tomatically unitarity condition and they are free from the double-counting problems.

(A) Suggested three-body equations presents the general recipe of construction of the complete set of the three-body forces.

(B) These equation have the most compact and convenient form.