Possible effect of mixed phase and deconfinement upon spin correlations in the $\Lambda\bar{\Lambda}$ pairs generated in relativistic heavy-ion collisions

V.L.Lyuboshitz, Valery V. Lyuboshitz

(JINR, Dubna)

Introduction

- Spin correlations for the $\Lambda\Lambda$ and $\Lambda\overline{\Lambda}$ pairs, generated in one act of collision of hadrons or nuclei , and respective angular correlations at the joint registration of hadronic decays of two hyperons give the important information about the character of multiple processes.

- The advantage of $\Lambda\Lambda$ and $\Lambda\overline{\Lambda}$ systems is due to the fact that the *P*-odd decays $\Lambda \to p + \pi^-$ and $\overline{\Lambda} \to \overline{p} + \pi^+$ serve as effective analyzers of the spin states of Λ and $\overline{\Lambda}$ particles.

1. General structure of the spin density matrix of the pairs of ΛΛ and $\Lambda\bar{\Lambda}$

- The spin density matrix of the $\Lambda\Lambda$ and $\Lambda\bar{\Lambda}$ pairs, just as the spin density matrix of two spin-½ particles in general, can be presented in the following form :

$$\hat{\rho}^{(1,2)} = \frac{1}{4}\left[\ \hat{I}^{(1)} \otimes \hat{I}^{(2)} + (\hat{\sigma}^{(1)}\mathbf{P}_1) \otimes \hat{I}^{(2)} + \hat{I}^{(1)} \otimes (\hat{\sigma}^{(2)}\mathbf{P}_2) + \sum_{i=1}^{3}\sum_{k=1}^{3} T_{ik}\ \hat{\sigma}_i^{(1)} \otimes \hat{\sigma}_k^{(2)}\right]$$

in doing so, $tr_{(1,2)}\,\hat{\rho}^{(1,2)} = 1$.

Here $\hat{I}$ is the two-row unit matrix, $\hat{\sigma} = (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)$ is the vector Pauli operator (x, y, $z \to$ 1, 2, 3),

$\mathbf{P}_1$ and $\mathbf{P}_2$ are the polarization vectors of the first and second particle ($\mathbf{P}_1 = \langle\hat{\sigma}^{(1)}\rangle$, $\mathbf{P}_2 = \langle\hat{\sigma}^{(2)}\rangle$), $T_{ik} = \langle\, \hat{\sigma}_i^{(1)} \otimes \hat{\sigma}_k^{(2)}\rangle$ are the correlation tensor components . In the general case $T_{ik} \neq P_{1i}\, P_{2k}$. The tensor with components $C_{ik} = T_{ik} - P_{1i}\, P_{2k}$ describes the spin correlations of two particles .

2. Spin correlations and angular correlations at joint registration of decays of two Λ particles into the channel $\Lambda \to p + \pi^-$

- Any decay with the space parity nonconservation may serve as an analyzer of spin state of the unstable particle .

The normalized angular distribution at the decay $\Lambda \to p + \pi^-$ takes the form:

$$\frac{dw(\mathbf{n})}{d\Omega_{\mathbf{n}}} = \frac{1}{4\pi}(1 + \alpha_\Lambda \mathbf{P}_\Lambda \mathbf{n}) \ .$$

Here $\mathbf{P}_\Lambda$ is the polarization vector of the Λ particle, $\mathbf{n}$ is the unit vector along the direction of proton momentum in the rest frame of the Λ particle, α_Λ is the coefficient of *P*-odd angular asymmetry ($\alpha_\Lambda = 0.642$).

The decay $\Lambda \to p + \pi^-$ selects the projections of spin of the Λ particle onto the direction of proton momentum; the analyzing power equals $\xi = \alpha_\Lambda \mathbf{n}$.

- Now let us consider the double angular distribution of flight directions for protons formed in the decays of two Λ particles into the channel $\Lambda \rightarrow p + \pi^-$, normalized by unity (the analyzing powers are $\xi_1 = \alpha_\Lambda \mathbf{n}_1$, $\xi_2 = \alpha_\Lambda \mathbf{n}_2$) . It is described by the following formula :

$$\frac{d^2 w(\mathbf{n}_1, \mathbf{n}_2)}{d\Omega_{\mathbf{n}_1} d\Omega_{\mathbf{n}_2}} = \frac{1}{16\pi^2}\left[1 + \alpha_\Lambda \mathbf{P}_1 \mathbf{n}_1 + \alpha_\Lambda \mathbf{P}_2 \mathbf{n}_2 + \alpha_\Lambda^2 \sum_{i=1}^{3}\sum_{k=1}^{3} T_{ik}\, n_{1i}\, n_{2k}\right]$$

where $\mathbf{P}_1$ and $\mathbf{P}_2$ are polarization vectors of the first and second Λ particle, T_{ik} are the correlation tensor components, $\mathbf{n}_1$ and $\mathbf{n}_2$ are unit vectors in the respective rest frames of the first and second Λ particle, defined in the common (unified) coordinate axes of the c.m. frame of the pair

($i, k = \{1, 2, 3\} = \{x, y, z\}$) .

The polarization parameters can be determined from the angular distribution of decay products by the method of moments -- as a result of averaging combinations of trigonometric functions of angles of proton flight over the double angular distribution :

$$P_{1i} = \frac{3}{\alpha_\Lambda} \langle n_{1i} \rangle , \quad P_{2k} = \frac{3}{\alpha_\Lambda} \langle n_{2k} \rangle , \quad T_{ik} = \frac{9}{\alpha_\Lambda^2} \langle n_{1i} \, n_{2k} \rangle$$

Here

$$\langle \, ... \, \rangle \equiv \int (\, ... \,) \left(\frac{d^2 w(\mathbf{n}_1, \mathbf{n}_2)}{d\Omega_{\mathbf{n}_1} d\Omega_{\mathbf{n}_2}} \right) d\Omega_{\mathbf{n}_1} d\Omega_{\mathbf{n}_2} \quad ;$$

$n_{1x} = \sin\theta_1 \cos\varphi_1$; $n_{1y} = \sin\theta_1 \sin\varphi_1$; $n_{1z} = \cos\theta_1$;
$n_{2x} = \sin\theta_2 \cos\varphi_2$; $n_{2y} = \sin\theta_2 \cos\varphi_2$; $n_{2z} = \cos\theta_2$,

where θ_1 and φ_1 , θ_2 and φ_2 are the polar and azimuthal angles of emission of protons in the rest frames of the first and second Λ particle, respectively – with respect to the unified system of coordinate axes of c.m. frame of pair;

$d\Omega_{\mathbf{n}_1} = \sin\theta_1 \, d\theta_1 \, d\varphi_1$ and $d\Omega_{\mathbf{n}_2} = \sin\theta_2 \, d\theta_2 \, d\varphi_2$
are the elements of solid angles of proton emission .

- The angular correlation, integrated over all angles except the angle θ between the vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ and described by the formula :

$$dw(\cos\theta) = \frac{1}{2}\left(1 + \frac{1}{3}\alpha_{\Lambda}^{2}\, T\cos\theta\right)\sin\theta\, d\theta\ ,$$

is determined only by the "trace" of the correlation tensor $T = T_{11} + T_{22} + T_{33} = W_t - 3W_s$, and it does not depend on the polarization vectors (single-particle states may be unpolarized).

So, finally we have :

$$dw(\cos\theta) = \frac{1}{2}\left(1 - \alpha_{\Lambda}^{2}\,(W_s - \frac{W_t}{3})\,\cos\theta\right)\sin\theta\, d\theta\ ,$$

W_s and W_t are relative fractions of the singlet state and triplet states, respectively .

3. Correlations at the joint registration of the decays $\Lambda \to p + \pi^-$ and $\overline{\Lambda} \to \overline{p} + \pi^+$

- Due to *CP* invariance, the coefficients of *P*-odd angular asymmetry for the decays $\Lambda \to p + \pi^-$ and $\overline{\Lambda} \to \overline{p} + \pi^+$ have equal absolute values and opposite signs: $\alpha_{\overline{\Lambda}} = -\alpha_{\Lambda} = -0.642$.
  The double angular distribution for this case is as follows :

$$\frac{d^2 w(\mathbf{n}_1,\mathbf{n}_2)}{d\Omega_{\mathbf{n}_1}\, d\Omega_{\mathbf{n}_2}} = \frac{1}{16\,\pi^2}\left[1 + \alpha_{\Lambda} \mathbf{P}_{\Lambda}\, \mathbf{n}_1 - \alpha_{\Lambda} \mathbf{P}_{\overline{\Lambda}}\, \mathbf{n}_2 - \alpha_{\Lambda}^2 \sum_{i=1}^{3}\sum_{k=1}^{3} T_{ik}\, n_{1i}\, n_{2k} \right]$$

  (here $-\alpha_{\Lambda} = +\alpha_{\overline{\Lambda}}$ and $-\alpha_{\Lambda}^2 = +\alpha_{\Lambda}\alpha_{\overline{\Lambda}}$) .

- Thus, the angular correlation between the proton and antiproton momenta in the rest frames of the Λ and $\overline{\Lambda}$ particles is described by the expression :

$$dw(\cos\theta) = \frac{1}{2}\left(1 - \frac{1}{3}\alpha_{\Lambda}^2\, T \cos\theta \right) \sin\theta\, d\theta = \frac{1}{2}\left(1 + \alpha_{\Lambda}^2\, (W_s - \frac{W_t}{3})\, \cos\theta \right) \sin\theta\, d\theta ,$$

- where θ is the angle between the proton and antiproton momenta .

4. Spin correlations at the generation of $\Lambda\bar{\Lambda}$ pairs in multiple processes

- Further we will use the model of one-particle sources, which is the most adequate one in the case of collisions of relativistic ions .
- Spin and angular correlations at the decays of two Λ particles, being identical particles, with taking into account Fermi statistics and final-state interaction , were considered previously .
- In the present report we are interested in spin correlations in the decays of $\Lambda\bar{\Lambda}$ pairs . In the framework of the model of independent one-particle sources, spin correlations in the $\Lambda\bar{\Lambda}$ system arise only on account of the difference between the interaction in the final triplet state ($S = 1$) and the interaction in the final singlet state. At small relative momenta, the s -wave interaction plays the dominant role as before, but, contrary to the case of identical particles ($\Lambda\Lambda$) , in the case of non-identical particles ($\Lambda\bar{\Lambda}$) the total spin may take both the values $S = 1$ and $S = 0$ at the orbital momentum $L = 0$. In doing so, the interference effect, connected with quantum statistics, is absent .

.

If the sources emit unpolarized particles, then, in the case under consideration, the correlation function describing momentum-energy correlations has the following structure (in the c.m. frame of the $\Lambda\bar{\Lambda}$ pair) :

$$R(\mathbf{k},\mathbf{v}) = 1 + \frac{3}{4} B_t^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v}) + \frac{1}{4} B_s^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v})$$

Here $B_t^{(\Lambda\bar{\Lambda})}$ and $B_s^{(\Lambda\bar{\Lambda})}$ -- contributions of interaction of Λ and $\bar{\Lambda}$ in the final triplet (singlet) state, which are expressed through the amplitudes of scattering of non-identical particles Λ and $\bar{\Lambda}$, and depend on space-time dimensions of the generation region of $\Lambda\bar{\Lambda}$ -pair.

R.Lednicky, V.L.Lyuboshitz (1982) // Yad. Fiz. 35, 1316

k – momentum of $\bar{\Lambda}$ in the c.m. frame of the pair, **v** – velocity of the pair

The spin density matrix of the $\Lambda\bar{\Lambda}$ pair is given by the formula :

$$\hat{\rho}^{(\Lambda\bar{\Lambda})} = \hat{I}^{(1)} \otimes \hat{I}^{(2)} + \frac{B_t^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v}) - B_s^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v})}{4R(\mathbf{k},\mathbf{v})} \hat{\boldsymbol{\sigma}}^{(1)} \otimes \hat{\boldsymbol{\sigma}}^{(2)} \quad ,$$

and the components of the correlation tensor are as follows:

$$T_{ik} = \frac{B_t^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v}) - B_s^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v})}{4 + 3B_t^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v}) + B_s^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v})}\ \delta_{ik}$$

- At sufficiently large values of k, one should expect that :

$$B_s^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v}) = 0, \quad B_t^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v}) = 0 \quad .$$

In this case the angular correlations in the decays $\Lambda \rightarrow p + \pi^-$ and $\bar{\Lambda} \rightarrow p + \pi^+$, connected with the final-state interaction, are absent :

$$T_{ik} = 0, \qquad T = 0\ .$$

5. Angular correlations in the decays $\Lambda \to p + \pi^-$ and $\bar{\Lambda} \to \bar{p} + \pi^+$ and the "mixed phase"

- Thus, at sufficiently large relative momenta (for $k >> m_\pi$) one should expect that the angular correlations in the decays $\Lambda \to p + \pi^-$ and $\bar{\Lambda} \to \bar{p} + \pi^+$, in the framework of the model of one-particle sources are absent. In this case two-particle (and multiparticle) sources may be, in principle, the cause of the spin correlations. Such a situation may arise, if at the considered energy the dynamical trajectory of the system passes through the so-called "mixed phase"; then the two-particle sources, consisting of the free quark and antiquark , start playing a noticeable role . For example, the process $s\bar{s} \to \Lambda\bar{\Lambda}$ may be discussed .

  The *CP* parity of the fermion-antifermion pair is $CP = (-1)^{S+1}$.

- In the case of one-gluon exchange, *CP* = 1, and then *S* = 1, i.e. the $\Lambda\bar{\Lambda}$ pair is generated in the triplet state; in doing so, the "trace" of the correlation tensor $T = 1$.

- Even if the frames of one-gluon exchange are overstepped, the quarks s and $\bar{s}$, being ultrarelativistic, interact in the triplet state (S = 1) . In so doing, the primary CP parity CP = 1, and, due to the CP parity conservation, the $\Lambda\bar{\Lambda}$ pair is also produced in the triplet state. Let us denote the contribution of two-quark sources by x . Then at large relative momenta $T = x > 0$.

- Apart from the two-quark sources, there are also two-gluon sources being able to play a comparable role. Analogously with the annihilation process $\gamma\gamma \to e^+e^-$, in this case the "trace" of the correlation tensor is described by the formula (the process $gg \to \Lambda\bar{\Lambda}$ is implied) :

$$T = 1 - \frac{4(1-\beta^2)}{1 + 2\beta^2 \sin^2\theta - \beta^4 - \beta^4 \sin^4\theta} ,$$

where β is the velocity of Λ (and $\overline{\Lambda}$) in the c.m. frame of the $\Lambda\overline{\Lambda}$ pair, θ is the angle between the momenta of one of the gluons and Λ in the c.m. frame . At small β ($\beta << 1$) the $\Lambda\overline{\Lambda}$ pair is produced in the singlet state (total spin $S = 0$, $T = -3$), whereas at $\beta \approx 1$ – in the triplet state ($S = 1$, $T = 1$) . Let us remark that at ultrarelativistic velocities β (i.e. at extremely large relative momenta of Λ and $\overline{\Lambda}$) both the two-quark and two-gluon mechanisms lead to the triplet state of the $\Lambda\overline{\Lambda}$ pair ($T = 1$) .

V.L.Lyuboshitz, V.V.Lyuboshitz (2010) // Yad. Fiz. **73** (5), 836
[Phys. At. Nucl. **73** (5), 805]

6. Summary

- It is advisable to investigate the spin correlations of $\Lambda\Lambda$ and $\Lambda\overline{\Lambda}$ pairs produced in relativistic heavy ion collisions.

- The spin correlations are studied by the method of angular correlations – method of moments.

In the general case, the appearance of angular correlations in the decays $\Lambda \to p + \pi^-$ and $\overline{\Lambda} \to \overline{p} + \pi^+$ with the nonzero values of the “trace” of the correlation tensor T at large relative momenta of the Λ and $\overline{\Lambda}$ particles may testify to the passage of the system through the “mixed phase” .

Thank you !