Three jet production in peripheral antiproton (proton)-proton coliisions

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We consider creation of some hadronic state by two reggeized gluons emitted by colliding hadrons (protons, antiprotons), accompanied by two jets resulting from interaction of initial hadrons with colored reggeized gluons. Differential cross sections of creation of a single gluon, quark-anti quark pair and pair of real gluons presented. Describing the creation of jets in fragmentation regions we use some anzatz, based on gauge invariance, which relate the probability of jet production with generalized parton distribution and in particular with gluon density in proton. Differential distribution are presented.

For completeness we put the similar distributions for QED processes of lepton pair creation, production of pseudo-scalar and scalar particles through the two virtual photon mechanism in high energy protons collisions. The relevant cross sections in spite of smallness of coupling constant are enhanced by Wezsaecker-Williams factors compared with jet production ones.

I. INTRODUCTION

In the early seventies of last century the processes of creation of some set of particles where intensively studied [1]. For the case when the lepton pair created outside the fragmentation regions of protons the cross section of process (see Fig. 1 a)

$$p(p_1) + p(\bar{p})(p_2) \to p(p_1') + p(\bar{p})(p_2') + \mu^+(q_+) + \mu^-(q_-), \quad (1)$$

have a form (phase volume is defined in Appendix A)

$$d\sigma^{p\bar{p}\to q\bar{q}p\bar{p}} = \frac{2\alpha^4}{\pi}\frac{d^2q_1 d^2q_2 d^2k_1 dx}{\pi^3}\frac{d\beta_1}{\beta_1} \times \frac{\vec{q}_1^2\vec{k}^2}{(\vec{q}_1^2 + M^2\beta_1^2)^2(\vec{q}_2^2 + M^2\alpha^2)^2} \cdot F; \quad (2)$$

where M is proton mass,

$$s\alpha = \frac{-c}{\beta_1 x(1-x)},\ 0 < x = \frac{\beta_2}{\beta_1} < 1;\ \beta_1 << 1, \quad (3)$$

$$c = m^2 + \vec{q}_2^{\,2} + \vec{q}_1^{\,2} x + 2\vec{q}_1\vec{q}_2 x, \quad (4)$$

$$c_1 = m^2 + (\vec{k}_2 - \vec{q}_2)^2 + \vec{q}_1^{\,2} x + 2\vec{q}_1(\vec{q}_2 - \vec{k}_2)x; \quad (5)$$

$$\vec{q}_1^{\,2}\vec{q}_2^{\,2} F = \frac{\vec{q}_2^{\,2}\vec{q}_1^{\,2}}{cc_1} - \frac{x\bar{x}}{c^2 c_1^2}[(\vec{q}_1^{\,2} + 2\vec{q}_1\vec{q}_2)(\vec{q}_2^{\,2} - 2\vec{k}_2\vec{q}_2) + 2(\vec{q}_2\vec{q}_1)(m^2 + \vec{k}_2^{\,2})]^2. \quad (6)$$

Here m is lepton mass, $x_1 = 1 - \beta_1 \approx 1$, $-\vec{q}_1$ is the energy fraction of the scattered proton and it's momentum, transversal to the initial proton direction $\vec{p}_1$ (center of mass of initial particles implied). $1 + \alpha \approx 1$, $\vec{q}_2$-the similar quantities for the scattered proton(anti-proton). $x\beta_1 + \frac{\mu^2+\vec{k}_1^2}{s\beta_1 x}$, $-\vec{k}_1$ and $(1-x)\beta_1 + \frac{\mu^2+\vec{k}_2^2}{s\beta_1(1-x)}$, $\vec{k}_2 = \vec{q}_1 - \vec{q}_2 - \vec{k}_1$-corresponding quantities for negative and positive charged leptons from the pair created, m

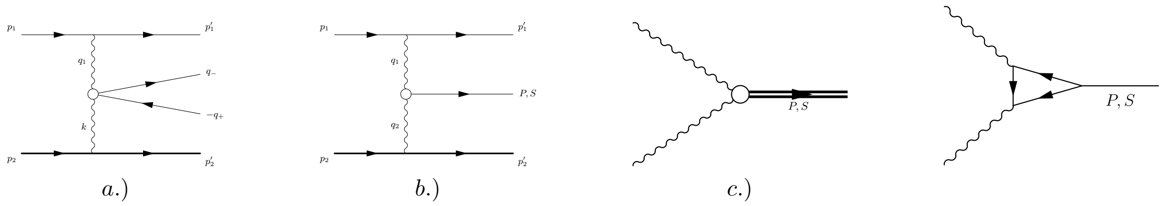

Figure 1: Feynman diagram

is the mass of the created particle. For two-photon processes with creation of pseudo-scalar and scalar particle we use the corresponding sub-process $\gamma(q_1,\mu)+\gamma(q_2,\nu) \rightarrow P(S)$ (see Fig. 1,b,c) with matrix elements described in terms of triangle Feynman loop diagrams with quarks as an internal fermions:

$$M^{\gamma\gamma P} = \frac{2\alpha N_P g_p}{\pi m_q}(q_1 e_1 q_2 e_2) I_P, (q_1 e_1 q_2 e_2) = \epsilon^{\alpha\beta\gamma\sigma} q_{1\alpha} e_{1\beta} q_{2\gamma} e_{2\sigma};$$

$$M^{\gamma\gamma S} = \frac{2\alpha N_S g_S}{\pi m_q}[q_1 q_2)(e_1 e_2) - (e_1 q_2)(e_2 q_1)] I_S, \quad (7)$$

where $e_{1,2}(q_{1,2})$ - the polarization vectors of photons, $N_{P,S}$ - color factors

$$N_P = N_c(\frac{4}{9} - \frac{1}{9}) = 1;$$
$$N_S = N_c(\frac{4}{9} + \frac{1}{9}) = \frac{5}{3}. \quad (8)$$

Performing the loop momentum integration we obtain

$$I_{P,S} = \int\limits_0^1 dx \int\limits_0^1 \frac{ydy}{d_{P,S}}(1, 1 - 4y^2x(1-x)),$$

$$d_{P,S} = 1 - y^2x(1-x)\frac{M_{P,S}^2}{m_q^2} -$$
$$y(1-y)\left[x\frac{q_1^2}{m_q^2} + (1-x)\frac{q_2^2}{m_q^2}\right], \quad (9)$$

$M_{P,S}, m_q$-masses of produced particles and quark mass. We can use the Goldberger-Treiman relation $g_P/m_q = 1/F_\pi$, with $F_\pi = 93MeV$ is decay constant of charged pion; and the similar relation $g_S/m_q = 1/F_\sigma, F_\sigma \sim F_\pi$.

When inserting these matrix elements to the matrix element of process $2 \to 3$ the combination is used $M^{\gamma\gamma F}(e_1 \to p_1, e_2 \to p_2)/s = m^{\gamma\gamma F}$, we obtain

$$m^{\gamma\gamma P} = \frac{\alpha N_P}{\pi F_\pi}[\vec{q}_1, \vec{q}_2]_z I_P;$$
$$m^{\gamma\gamma S} = \frac{\alpha N_S}{\pi F_\sigma}(\vec{q}_1, \vec{q}_2) I_S, \qquad (10)$$

where we consider the four-momenta of virtual photons to be essentially transversal two- component euclidean vectors $\vec{p}_1 \vec{q}_{1,2} = 0; \quad q_{1,2}^2 = -\vec{q}_{1,2}^{\,2} < 0$.

Cross sections of processes of single meson production in the pionization region are

$$d\sigma^{pp\to ppP} = \frac{2\alpha^4}{\pi}\frac{d\beta_1}{\beta_1}dN_1 dN_2 C_P \sin^2\theta;$$
$$d\sigma^{pp\to ppS} = \frac{2\alpha^4}{\pi}\frac{d\beta_1}{\beta_1}dN_1 dN_2 C_S \cos^2\theta, \quad (11)$$

with θ - azimuthal angle between two-dimensional vectors $\vec{q}_1, \vec{q}_2$,

$$C_P = \left|\frac{N_P}{F_\pi} I_P\right|^2; \quad C_S = \left|\frac{N_S}{F_\sigma} I_S\right|^2; \quad (12)$$

and Weizsaecker-Williams (WW) enhanced factors

$$dN_1 = \frac{\vec{q}_1^{\,2} d^2\vec{q}_1}{(\vec{q}_1^{\,2} + m^2\beta_1^2)^2},$$
$$dN_2 = \frac{\vec{q}_2^{\,2} d^2\vec{q}_2}{(\vec{q}_2^{\,2} + m^2\alpha_2^2)^2},$$
$$s\alpha_2\beta_1 = M_{P,S}^2 + (\vec{q}_1 + \vec{q}_2)^2. \quad (13)$$

We use the expression of the squared 4-vectors of momenta transferred to lepton pair:

$$q_1^2 \approx -(\vec{q}_1^{\,2} + m_p^2 \beta_1^2);$$
$$q_2^2 \approx -(\vec{q}_2^{\,2} + m_p^2 \alpha_2^2), m_p = m. \qquad (14)$$

These factors being integrated, produce the "large logarithmic" factors

$$\int_0^{Q^2} dN_1 = \ln \frac{q^2}{m^2 \beta_1^2} - 1, m^2 >> Q^2 >> s. (15)$$

Performing the numerical integrations Vermasseren formula for matrix element of conversion of two (virtual) photons to real lepton pair [2]

$$\gamma(e_1, q_1) + \gamma(e_2, q_2) \to \mu^-(q_-) + \mu^+(q_+) \quad (16)$$

was found

$$\bar{u}(q_-)\left\{\hat{e}_1\frac{\hat{k}_1+m}{k_1^2-m^2}\hat{e}_2+\hat{e}_2\frac{\hat{k}_2+m}{k_2^2-m^2}\hat{e}_1\right\}v(q_+)=\frac{(q_1e_1\alpha\mu)(q_2e_2\beta\mu)}{(k_1^2-m^2)(k_2^2-m^2)}\bar{u}(q_1)\times\left[\gamma^\beta\hat{k_1}\gamma^\alpha+\gamma^\alpha\hat{k_2}\gamma^\beta\right]v(q_+),\quad(17)$$

which reveal the explicit property of gauge invariance.

Considering the processes with creation of single gluon and pairs of gluons and quark-anti-quark pairs, the exchange by the reggeized gluons between protons becomes relevant. Two phenomena compared with the QED case appears. First -the absence of WW factors. This statement follows from the gauge invariance of vertex describing the conversion of proton to jet after emission of a reggeized gluon

$$q_1^\mu < j_1|J_\mu^a|p(p_1) >= (\alpha_1 p_2 + q_{1\perp})^\mu J_\mu^a = 0.(18)$$

Quantity $N_a = p_2^\mu J_\mu^a/s$ which enters in matrix element of the whole process being squared and summed on final states of the jet with fixed invariant mass square 9definition of phase volume is given in Appendix A):

$$\int \sum N^a N^{a_1 *} d\gamma_1 = |\frac{(\vec{q}_1 \vec{J}^a)(\vec{q}_1 \vec{J}^{a_1})}{s\alpha_1}|^2 d\gamma_1 = \frac{\vec{q}_1^2}{M_1^2}\delta^{aa_1}\Phi_1(\vec{q}_1^2) dM_1^2 F(\beta_1),$$

$$\Phi_1(\vec{q}_1^2) = \frac{M_1^2}{(M_1^2 + \vec{q}_1^2)^2}, \quad (19)$$

with M_1^2 is the invariant mass squared of the jet produced by proton and

$$F(x) = xg(x), \quad (20)$$

is the gluon density into proton $F(0) \neq 0$. Here we use some interpretation of the General Parton Distribution (GPD) hypothesis [7].

We will see below that using the gauge invariance of matrix element of subprocess $RR \to F$, (R denotes the reggeized gluon) an additional factor $\vec{q}_1^2 \vec{q}_2^2$ in matrix element squared appears. So the factor $1/(q_1^2 q_2^2)^2$ is canceled. So the effect of WW enhancement disappears in processes $pp \to 3jet$.

The second effect is the appearance of gluon reggeization factor R in expression for the cross section

$$R = \left(\frac{S_1}{s_0}\right)^{2(\alpha_g(q_1^2)-1)} \left(\frac{S_2}{s_0}\right)^{2(\alpha_g(q_2^2)-1)}, \quad (21)$$

where $\alpha_g(q_1^2) = 1 - \alpha_s \vec{q}_1^2/(\pi q_0^2)$- is the Regge trajectory of gluon, $q_0^2 \sim 1Gev^2$ $\alpha_g(0) = 0$. The partial invariant mass squared are defined as

$$S_1 = (p_1 - q_2)^2 \approx -s\alpha_2; S_2 = (q_1 + p_2)^2 \approx s\beta_1,$$

$$S_1 S_2 = s(M_F^2 + (\vec{q}_1 - \vec{q}_2)^2). \quad (22)$$

Scale factor $s_0 \approx 1GeV^2$ is in principle one of fitting parameters.

Effect of gluon reggeization $R(\vec{q}^2)$ is illustrated in Fig.6.

II. PROCESS $pp \to jjg$

Vertex function which describe interaction of two reggeized gluon with ordinary gluon (see Fig. 2a)

$$R(-, q_1, a) + R(+, -q_2, b) \to g(\mu, q_2 - q_1, c), \quad (23)$$

have a form [5]

$$m^{RR\to g} = 2g f_{abc} C_\mu e^{c\mu} \quad (24)$$

with f_{abc}-structure constants of color $SU(N)$ group, $e^{c\mu}$ is the polarization vector of a real gluon and vector C_μ have a form

$$C_\mu = (q_1 + q_2)_\mu - (q_2^- + \frac{\vec{q}_2^2}{q_1^+})(n^+)_\mu - (q_1^+ + \frac{\vec{q}_1^2}{q_2^-})(n^-)_\mu. \quad (25)$$

Note that vector C obey the gauge condition $C_\mu(q_2 - q_1)^\mu = 0$. Using the on mass shell condition of gluon $s_1 = (q_2 - q_1)^2 = q_2^- q_1^+ - (\vec{q}_2 - \vec{q}_1)^2$ and $q_1 = q_1^+(n^-)/2 + q_{1\perp}$, $q_2 = q_2^-(n^+)/2 + q_{2\perp}$ we obtain

$$|C_\mu^2| = \frac{4\vec{q}_1^2\vec{q}_2^2}{(\vec{q}_2 - \vec{q}_1)^2}. \quad (26)$$

Keeping in mind the further conversion of gluon to the gluon jet with invariant mass squared M^2, we obtain the cross section of process $pp \rightarrow$

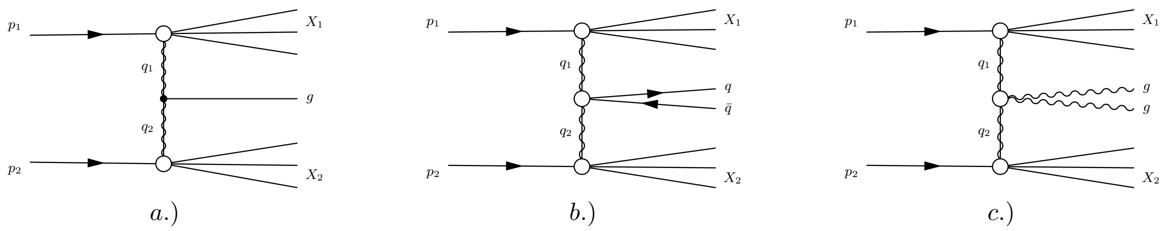

Figure 2: Feynman diagram

$j_1 j_2 j_g$ the expression given in Appendix B with

$$\Phi^g = \frac{16N(N^2-1)}{M^2 + (\vec{q}_2 - \vec{q}_1)^2}. \qquad (27)$$

The quantity

$$I^g = \frac{M_1 M_2}{16N(N^2-1)} \int \frac{d^2\vec{q}_1 d^2\vec{q}_2}{\pi^2} \Phi_1(\vec{q}_1^2)\Phi_2(\vec{q}_2^2)\Phi^g,$$

$$\Phi_i(\vec{q}_i^2) = \frac{M_i^2}{(\vec{q}_i^2 + M_i^2)^2}, \quad (28)$$

for some values M_1^2, M_2^2, M^2 is presented in Fig. 3.

III. PROCESS $pp \to jjgg$

The quantity Φ^{gg} entering the cross section (see Appendix) have a form

$$\Phi^{gg} = \frac{1}{\vec{q}_1^{\,2} \vec{q}_2^2} \sum |M^{RRPP}|^2, \qquad (29)$$

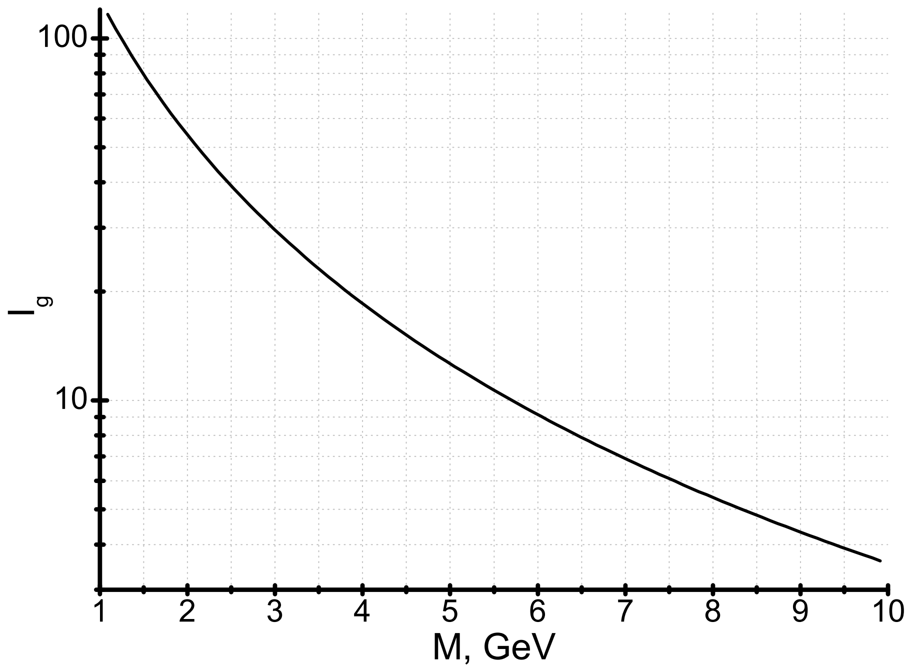

Figure 3: Value I_g as a function of produced gluon jet mass M in case of $M_1 = M_2 = 1$ GeV.

with M^{RRPP} is the effective vertex describing the conversion of two reggeized gluons to two real gluons (see Fig. 2 b)

$$R(-, c, q_1) + R(+, d, -q_2) \rightarrow g(\nu_1, d_1, k_1) + g(\nu_2, d_2, k_2). \quad (30)$$

It was obtained [3–6]

$$\sum |M^{RRPP}|^2 = G_1(a^{\nu_1\nu_2}(k_1,k_2))^2 +$$
$$G_2\Omega_{\sigma\sigma'}(k_1)\Omega_{\rho\rho'}(k_2)a^{\sigma\rho}(k_1,k_2)a^{\rho'\sigma'}(k_2,k_1) +$$
$$(k_1 \leftrightarrow k_2), \quad (31)$$

with

$$G_1 = (f_{d_1d_2r}f_{cdr})^2 = N^2(N^2-1);$$
$$G_2 = f_{d_1d_2r}f_{cdr}f_{d_2cr}f_{d_1dr} = -\frac{1}{2}N^2(N^2-1), (32)$$

projection operators

$$\Omega_{\sigma\sigma'}(k) = -g^{\perp}_{\sigma\sigma'} - \frac{2}{\vec{k}^2}k_{\sigma\perp}k_{\sigma'\perp}, \qquad (33)$$

and

$$a^{\nu_1\nu_2}(k_1,k_2) = 4\Big[\frac{1}{t}q_\perp^{\nu_1}q_\perp^{\nu_2} - \frac{1}{\chi}q_\perp^{\nu_1}(k_1 - \frac{x}{\bar{x}}k_2)^{\nu_2} +$$
$$\frac{1}{\chi}q_\perp^{\nu_2}(k_2 - \frac{\bar{y}}{y}k_2)^{\nu_1} - \frac{x\vec{q}_2^{\,2}}{\chi\vec{k}_1^2}k_1^{\nu_1}k_1^{\nu_2} - \frac{\bar{y}\vec{q}_1^{\,2}}{\chi\vec{k}_2^2}k_2^{\nu_1}k_2^{\nu_2} -$$
$$\frac{1}{\chi}(1 + \frac{tx}{\bar{x}\vec{k}_1^2})k_1^{\nu_1}k_2^{\nu_2} + \frac{1}{\chi}k_1^{\nu_1}k_2^{\nu_2} - 2Dg_\perp^{\nu_1\nu_2}\Big], (34)$$

with

$$D = 1 + \frac{t}{\chi} + \frac{\bar{x}\vec{k}_1^2}{tx} + \frac{1}{\chi}[\frac{\bar{x}}{x}\vec{k}_1^2 - \frac{x}{\bar{x}}\vec{k}_2^2] + \frac{\vec{q}_1^2}{\chi}\bar{y} + \frac{\vec{q}_2^2}{\chi}x. \quad (35)$$

We use here the notations

$$k_i = \alpha_i p_2 + \beta_i p_1 + q_{i\perp}, s\alpha_i\beta_i = \vec{k}_i^2;$$
$$q = q_1 - k_1 = q_2 + k_2; t = q^2; \chi = (k_1 + k_2)^2;$$
$$x = \frac{\beta_1}{\beta_1 + \beta_2}, y = \frac{\alpha_1}{\alpha_1 + \alpha_2}. \quad (36)$$

Using the different (but equivalent) forms for t, χ

$$t = -(\vec{q}_1 - \vec{k}_1)^2 - \frac{\bar{x}}{x}\vec{k}_1^2; \chi = \frac{1}{x\bar{x}}(\bar{x}\vec{k}_1 - x\vec{k}_2)^2;$$
$$t = -(\vec{q}_2 + \vec{k}_2)^2 - \frac{y}{\bar{y}}\vec{k}_2^2; \chi = \frac{1}{y\bar{y}}(\bar{y}\vec{k}_1 - y\vec{k}_2)^2 \quad (37)$$

one can be convinced that the gauge conditions

$$D|\vec{q}_1 \to 0 = D|\vec{q}_2 \to 0 = 0;$$
$$a^{\nu_1\nu_2}(k_1, k_2)|\vec{q}_1 \to 0 = 0 \quad (38)$$

are fulfilled. Cross section of process $pp \to j_1 j_2 gg$ is given in Appendix, with

$$\Phi^{gg} = \frac{1}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}} \sum |M^{RRPP}|^2. \qquad (39)$$

Due to gauge properties of $a^{\nu_1\nu_2}$, the quantity Φ^{gg} is finite at $\vec{q}_1, \vec{q}_2 \to 0$ which provide the convergence of the quantity

$$I^{gg} = M_1^2 M_2^2 \int \frac{d^2\vec{q}_1 d^2\vec{q}_2}{\pi^2} \Phi_1(\vec{q}_1^{\,2})\Phi_2(\vec{q}_2^{\,2})\Phi^{gg}. \qquad (40)$$

This quantity is presented in Fig. 4 for some values of M_1^1, M_2^2 and gluon jets characteristics.

IV. PROCESS $pp \to jjq\bar{q}$

Matrix element of subprocess of conversion of two reggeized gluons to the quark-anti-quark pair (see Fig. 2c)

$$R(-, a, q_1) + R(+, b, -q_2) \to q(k_1) + \bar{q}(k_2), \qquad (41)$$

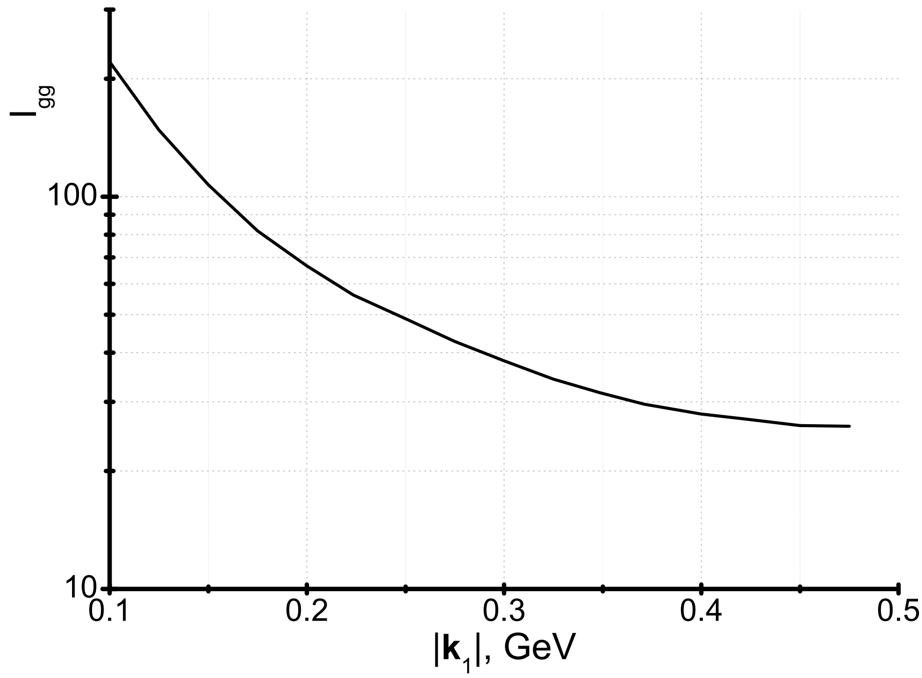

Figure 4: Value I_{gg} as a function of transverse momentum modulus $|\vec{k}_1|$ of one of the gluon in the produced gluon pair in case of $M_1 = M_2 = 1$ GeV, $x = 0.2$ and $y = 0.3$.

is described by two different mechanisms: direct interaction and production of gluon with the subsequent it's conversion to the quark pair

$$M^{q\bar{q}} = \bar{u}(k_1)[At_a t_b - Bt_b t_a]v(k_2), \qquad (42)$$

with t_a-generator of color $SU(N)$ group in fermion representation,

$$[t_a, t_b] = i f_{abc} t_c, a, b, c = 1, 2, ..., N^2 - 1,$$
$$Tr t_a = 0, Tr I = N; Tr t_a t_b = \frac{1}{2}\delta_{ab},$$
$$\sum (t_a^2)^2 = I\frac{N^2-1}{2N}; \sum Tr t_a t_b t_a t_b = -\frac{N^2-1}{4N};$$
$$\sum Tr t_a t_a t_b t_b = \frac{(N^2-1)^2}{4N}, \quad (43)$$

and [6]

$$A = \gamma^- \frac{\hat{q}_1 - \hat{k}_1 - m}{(q_1 - k_1)^2 - m^2}\gamma^+ - \frac{2}{q^2}\hat{C};$$
$$B = \gamma^+ \frac{\hat{q}_1 - \hat{k}_2 - m}{(q_1 - k_2)^2 - m^2}\gamma^- - \frac{2}{q^2}\hat{C},$$
$$q = k_1 + k_2, \quad (44)$$

with m-quark mass and 4-vector C_μ describing the conversion of two reggeized gluons to the ordinary gluon was given above. The gauge properties of $M^{q\bar{q}}$, i.e. turning it to zero in limit $\vec{q}_1 \to 0$

as well as in the limit $\vec{q}_1 \to 0$ can be seen using the expressions (we use Dirac equations for quarks)

$$A = \frac{1}{-\vec{q}_1^{\,2} - k_1^- q^+}(\gamma^- \hat{q}_1^\perp - 2k_1^-)\gamma_+ - \frac{2}{(k_1+k_2)^2}\hat{C};$$
$$A = \frac{1}{-\vec{q}_2^{\,2} - k_2^+ q^-}\gamma^-(\hat{q}_2^\perp \gamma_- + 2k_2^+) - \frac{2}{(k_1+k_2)^2}\hat{C} \qquad (45)$$

and

$$B = \frac{1}{-\vec{q}_2^{\,2} - k_1^+ q^+}(\gamma^+ \hat{q}_2^\perp + 2k_1^+)\gamma_- - \frac{2}{(k_1+k_2)^2}\hat{C};$$
$$B = \frac{1}{-\vec{q}_2^{\,2} - k_1^+ q^-}\gamma^+(\hat{q}_1^\perp \gamma_- - 2k_2^-) - \frac{2}{(k_1+k_2)^2}\hat{C}; \qquad (46)$$

$$\hat{C} = 2\hat{q}_1^\perp - \frac{\vec{q}_1^{\,2}}{q_2^-}\gamma^- + \frac{1}{q_1^+}[q^2 + \vec{q}_1^{\,2} - 2\vec{q}_1\vec{q}_2]\gamma^+;$$
$$\hat{C} = 2\hat{q}_2^\perp - \frac{\vec{q}_2^{\,2}}{q_1^+}\gamma^+ + \frac{1}{q_2^-}[q^2 + \vec{q}_2^{\,2} - 2\vec{q}_1\vec{q}_2]\gamma^- \qquad (47)$$

These properties provide convergence of the relevant integrals on $\vec{q}_{1,2}$. We obtain

$$\vec{q}_1^{\,2}\vec{q}_2^{\,2}\Phi^{q\bar{q}} = 4[N_1(S_A + S_B) - 2N_2 S_{AB}], \quad (48)$$

with

$$S_A = \frac{1}{4} Sp(\hat{k}_1 + m)A(\hat{k}_2 - m)\tilde{A};$$
$$S_B = \frac{1}{4} Sp(\hat{k}_1 + m)B(\hat{k}_2 - m)\tilde{B};$$
$$S_{AB} = \frac{1}{4} Sp(\hat{k}_1 + m)A(\hat{k}_2 - m)\tilde{B}. \quad (49)$$

Note that the value $\Phi^{q\bar{q}}$ is finite in both limits $\vec{q}_1 \to 0$ and $\vec{q}_2 \to 0$. The expression for cross section is given in Appendix with $\Phi^{q\bar{q}}$ given above.

Result of numerical integration of the quantity

$$I^{q\bar{q}} = \int \frac{d^2\vec{q}_1 d^2\vec{q}_2}{\pi^2} \Phi_1(\vec{q}_1^{\,2})\Phi_2(\vec{q}_2^{\,2})\Phi^{q\bar{q}} \quad (50)$$

is presented in Fig.5.

V. DISCUSSION

In paper [8] consideration similar to ones given in section IV was done to investigate heavy quark production. But, unfortunately the effect of Regge factor R was not taken into account.

Open charm and b quark production was considered in papers [9]. Regge-factors as well was not taken into account. But it seems to be rather important since intersept of quark Regge trajectory at zero transfer momentum less than unity $\alpha_q(0) = 1/2$.

Effect of reggeization can be approximated as

$$R \approx \left(\frac{sM_F^2}{s_0^2}\right)^{-2\alpha_s\vec{q}^2/(\pi q_0^2)}. \qquad (51)$$

For typical values $s_0 \sim M_F^2 \sim q_0^2 \approx 1GeV^2$ this factor is presented in Fig.6.

Anzatz for fragmentation function of protons used above is in agreement with the commonly used [7] in terms of unintegrated gluon distribution

$$F(x) = xg(x) = \int\limits_0^\infty \frac{d\vec{k}^2}{\vec{k}^2}\theta(\vec{q}^2 - \vec{k}^2)\mathcal{F}(x, \vec{k}) \to$$

$$F(x) \int \frac{M_1^2 dM_1^2}{(M_1^2 + \vec{q}_1^2)^2} \frac{d^2 q_1}{\pi} \int \frac{M_2^2 dM_2^2}{(M_2^2 + \vec{q}_2^2)^2} \frac{d^2 q_2}{\pi}. \quad (52)$$

Besides it provide the correct $\vec{q}^2$ dependence of proton inelastic form factors.

We underline that in processes with final state $F = q\bar{q}; gg$ two particles with open color as well must be converted to jets. We imply that these jets belong to the same boost factor $d\beta_1/\beta_1$. So there is no rapidity gap between these jets.

It is interesting to generalize the QED result for process of Higgs boson production through two reggeized gluons mechanism. As well as color trace of triangle quark Feynman diagram give $Trt_a t_b = \delta_{ab}/2$, the color structures of protons jets become to be connected. A a result and additional factor compared with QED case $(3/2)^2(N^2-1)$. Keeping in mind that the main contribution arise from top quark in fermion loop and that the mass of the intermediate state with two top quarks exceed Higgs boson mass $M_H < 2M_t$ we can estimate the corresponding integral on Feynman parameters as

$$I_S \to I_H \approx \int_0^1 dx \int_0^1 y(1-4y^2x(1-x)) = \frac{1}{3}. \tag{53}$$

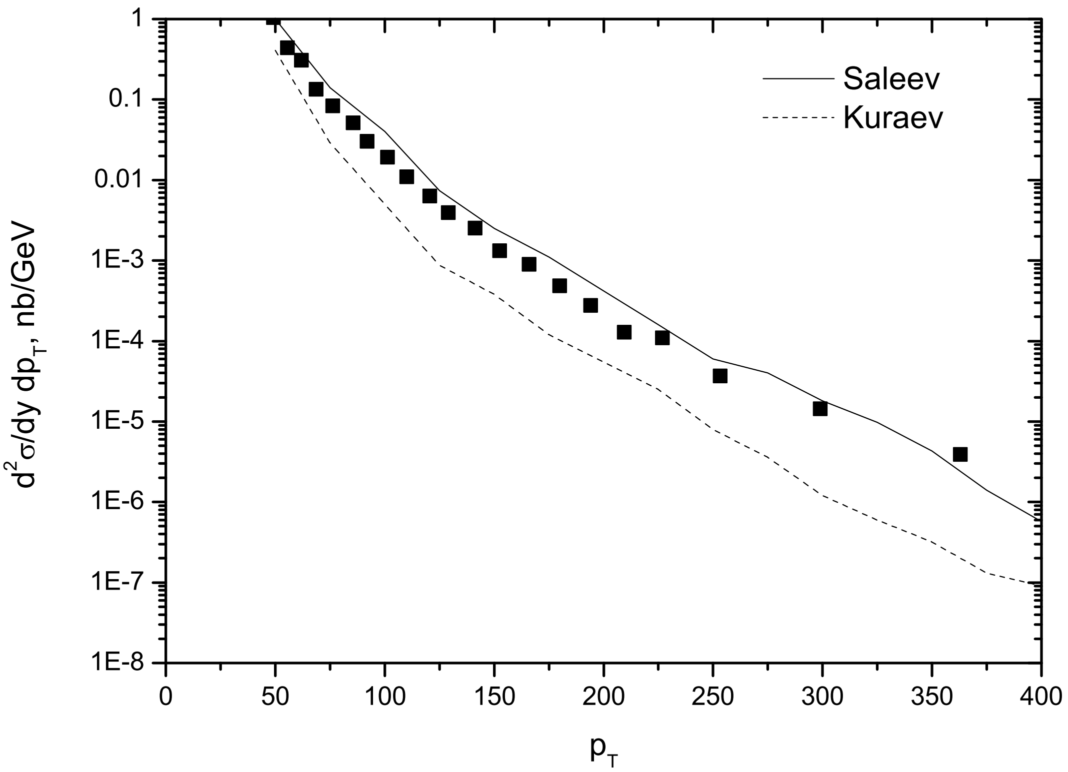

Figure 5: The cross section of dependence from p_T

As a result we have

$$d\sigma^{pp\to jjH} = \sigma_H(N^2-1)R\frac{d\beta_1}{\beta_1},$$
$$\sigma_H = \frac{\alpha_s^4}{2\pi}(\frac{g_{tth}}{M_t})^2. \qquad (54)$$

Estimation in frames Standard Model leads $g_{ttH} \sim \sqrt{2\sqrt{2}G_F}m_t \sim 1$. The quantity σ_H is rather large $\sigma_H \sim 6\mu b$. Gluon reggeization factor R can reduce the Higgs meson production cross section in a middle by one order of magnitude. This factor is presented in Fig. 4.

For process of a single gluonic jet production $pp \to j_p j_p j_g$ the transverse momentum of the created gluon must be of order of momenta tranferred to the nucleons $\vec{q_1} = \vec{q_2} + \vec{q_g}$, so the quantity of order of invariant mass of jets created by the nucleons $|\vec{q_1}| \sim= |\vec{q_2}| \sim |\vec{q_g}| \sim 1GeV/c$. For the case of large values of momenta $|\vec{q}|$ the reggeization factor suppression take place $R \sim (s_i/s_0)^{-(\alpha_s/\pi)(\vec{q}_i^2/q_0^2)} << 1$, $q_0^2 \sim 1Gev^2/c^2$, $\vec{q}_i^2/q_0^2 >> 1$.

The importance of reggeization factor $F(z) = \int\limits_0^\infty \frac{dx}{(x+1)^2} z^{-\lambda x}$ is illustrated in Fig.3, for $\lambda = -2\frac{\alpha_s}{\pi} \approx -0.2$, $z = \frac{s}{s_0}$.

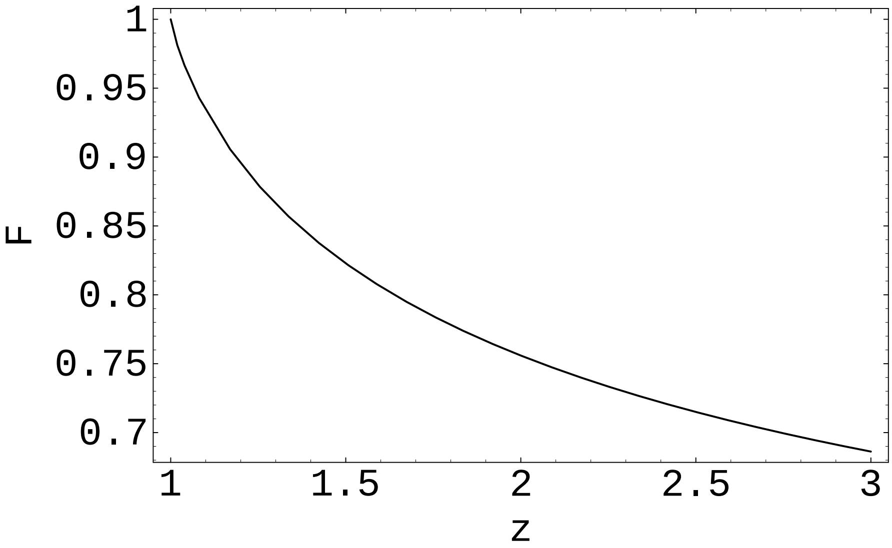

Figure 6: Reggeization factor

VI. APPENDIX A. SUDAKOV'S PARAMETRIZATION. PHASE VOLUME

Here we use Sudakov parametrization of four-vectors of the problem:

$$q_i = \alpha_i \tilde{p}_2 + \beta_i \tilde{p}_1 + q_{i\perp}, i = 1, 2. \qquad (55)$$

with light-like 4-vectors $\tilde{p}_i$ builded from p_i.

First we rearrange the phase volume of process $pp \to ppF$ with some state F in terms of Sudakov variables. Starting from the standard expression

$$d\Gamma^{2\to 2F} = (2\pi)^4 \cdot \delta^4(p_1 + p_2 - p_1' - p_2' - \sum_F q_i)\frac{d^3p_1'}{2\varepsilon_1'(2\pi)^3}\frac{d^3p_2'}{2\varepsilon_2'(2\pi)^3}\prod_F \frac{d^3q_i}{2\varepsilon_i(2\pi)^3}. \quad (56)$$

Introducing two auxiliary variables and two relevant δ functions

$$\int d^4q_1 d^4q_2 \delta^4(p_1 - q_1 - p_1')\delta^4(q_2 + p_2 - p_2') = 1, \quad (57)$$

using the relation

$$\frac{d^3q_i}{2\varepsilon_i} = d^4q_i\delta(q_i^2 - m_i^2), \quad (58)$$

and besides

$$q_i = \alpha_i p_2 + \beta_i p_1 + q_{i\perp}, q_i^2 = s\alpha_i\beta_i - \vec{q}_i^{\,2},$$

$$p_1^2 = p_2^2 = 0; q_{i\perp}p_1 = q_{i\perp}p_2 = 0; q_{i\perp}^2 = -\vec{q}_i^{\,2} < 0,$$

$$d^4q_i = \frac{s}{2}d\alpha d\beta d^2\vec{q}_i, \quad (59)$$

and using the δ^4 functions to perform the integration on the momenta of the scattered protons

$p'_i, p'_2,$

$$\frac{1}{(2\pi)^6}\frac{ds\alpha_1}{2}d\beta_1\frac{ds\beta_2}{2}d\alpha_2\pi^2\frac{d^2\vec{q}_1}{\pi}\frac{d^2\vec{q}_2}{\pi}\delta(-s\alpha_1-\vec{q}_1^{\,2})\delta(s\beta_2-\vec{q}_2^{\,2})d\Gamma_F, \quad (60)$$

we put it as

$$d\Gamma^{2\to 2F}=\frac{\pi^2}{4s(2\pi)^6}\frac{d\beta_1}{\beta_1}dM^2\frac{d^2\vec{q}_1}{\pi}\frac{d^2\vec{q}_1}{\pi}d\Gamma_F,$$

$$d\Gamma_F=(2\pi)^4\delta^4(q_1-q_2-\sum_F q_i)\prod_F\frac{d^3q_i}{2\varepsilon_i(2\pi)^3}, \quad (61)$$

with $M^2=(q_1-q_2)^2=-s\alpha_2\beta_1-(\vec{q}_1-\vec{q}_2)^2$-invariant mass squared of the state F.

For the phase volume of $3jet$ production we have

$$d\Gamma^{pp\to j_1j_2F}=(2\pi)^{-6}\frac{\pi^2}{4s}\frac{d\beta_1}{\beta_1}dM_1^2d\gamma_1dM_2^2d\gamma_2\times\frac{d^2\vec{q}_1}{\pi}\frac{d^2\vec{q}_2}{\pi}d\Gamma_FdM^2, \quad (62)$$

with

$$d\gamma_1 = (2\pi)^3\delta^4(p_1 - q_1 - \sum_{j1} q_i)\prod_{j1}\frac{d^3q_i}{2\varepsilon_i(2\pi)^3};$$

$$d\gamma_2 = (2\pi)^3\delta^4(p_2 + q_2 - \sum_{j2} q_i)\prod_{j2}\frac{d^3q_i}{2\varepsilon_i(2\pi)^3}. \quad (63)$$

For the case of subprocess $\gamma(q_1)+\gamma(-q_2) \to p(k)$ with $k^2 = M^2$ we have

$$\int dM^2 d\Gamma_1 = \int dM^2 \frac{1}{(2\pi)^3} d^4k\delta(k^2 - M^2)$$
$$(2\pi)^4\delta^4(q_1 - q_2 - k) = 2\pi. \qquad (64)$$

Consider now the subprocess $\gamma(q_1) + \gamma(-q_2) \to a(k_1)+b(k_2)$, $(q_1-q_2)^2 = (k_1+k_2)^2 = M^2$, $k_1^2 = m_1^2, k_2^2 = m_2^2$. Using Sudakov representation

$$q_1 = \beta_1 + q_{1\perp}, q_2 = \alpha_2 + q_{2\perp},$$
$$k_i = \alpha^{(i)}p_2 + \beta^{(i)}p_1 + k_{i\perp}, \qquad (65)$$

we have

$$\int dM^2 d\Gamma_2 = \int dM^2 \frac{(2\pi)^4}{(2\pi)^6} d^4k_1 \delta(k_1^2 - m_1^2)$$
$$d^4k_2 \delta(k_2^2 - m_2^2)\delta^4(q_1 - q_2 - k). \quad (66)$$

Using $dM^2 = d(s\alpha_2\beta_1$, and introduce the notations

$$x = \beta^{(1)}/\beta_1, 1 - x = x = \bar{x} = \beta^{(2)}/\beta_1;$$
$$y = \alpha^{(1)}/(-\alpha_2), 1 - y = \alpha^{(2)}/(-\alpha_2),$$

we can write down this quantity in two equivalent forms

$$\int dM^2 d\Gamma_2 = \int \frac{d^2\vec{k}_1 dx}{2(2\pi)^2 x\bar{x}}; 0 < x < 1;$$
$$\int dM^2 d\Gamma_2 = \int \frac{d^2\vec{k}_1 dy}{2(2\pi)^2 y\bar{y}}; 0 < y < 1. \quad (67)$$

We put here the typical invariants

$$2q_1k_1 = \frac{\vec{k}_1^2 + m_1^2}{x} - 2\vec{q}_1\vec{k}_1;$$
$$2q_2k_1 = \frac{\vec{k}_1^2 + m_1^2}{y} - 2\vec{q}_2\vec{k}_1. \quad (68)$$

VII. APPENDIX B. MATRIX ELEMENT AND CROSS SECTION OF PERIPHERICAL PROCESSES

Main contribution to the matrix element of processes $2 \to 3$ in peripherical kinematics

$$s = 2p_1p_2 >> |q_1^2| \sim |q_2^2| \sim M_1^2 \sim M_2^2 \sim M^2, \quad (69)$$

arise from Feynman diagrams with photon (gluons) state in the the scattering channel. It can be written in factorized form. For the case of QED processes

$$P(p_1) + p(p_2) \to p(p_1') + p(p_2') + F(q_1, q_2, X_F)$$

we have

$$M^{2\to 2F} = \frac{(4\pi\alpha)^2}{q_1^2 q_2^2} < p(p_1')|J_\mu|p(p_1) >$$
$$< p(p_2')|J_\nu|p(p_2 > g^{\mu\mu_1} g^{\nu\nu_1} m_{\mu_1\nu_1}(q_1, q_2; F). \quad (70)$$

Using the gauge invariance we can choice the Gribov's form of presentation of the photon Green function

$$g^{\mu\mu_1} = g_{\perp}^{\mu\mu_1} + \frac{2}{s}[p_2^{\mu} p_1^{\mu_1} + p_2^{\mu_1} p_1^{\mu}]. \qquad (71)$$

Omitting the terms leading to the contribution of order m^2/s compared to ones with the contribution to the cross section of order of unity one can replace

$$g^{\mu\mu_1} \to \frac{2}{s} p_2^{\mu} p_1^{\mu_1}; g^{\nu\nu_1} \to \frac{2}{s} p_1^{\mu} p_2^{\nu_1}. \qquad (72)$$

So matrix element of QED process have a form

$$M^{QED} = \frac{(4\pi\alpha)^2}{q_1^2 q_2^2} \left(\frac{2}{s}\right)^2 sN_1 sN_2 \frac{s}{4} m^{+-}(q_1, q_2, F), \qquad (73)$$

with

$$N_1 = \frac{1}{s}\bar{u}(p_1')\hat{p}_2 u(p_1); N_2 = \frac{1}{s}\bar{u}(p_2')\hat{p}_1 u(p_2). \qquad (74)$$

and

$$m^{+-} = n_\mu^+ n_\nu^- m^{\mu\nu}, n_\mu^+ = \frac{2p_2}{\sqrt{s}}; n_\mu^- = \frac{2p_1}{\sqrt{s}}. \qquad (75)$$

and $m^{\mu\nu}$ is the matrix element of subprocess. Summed on spin states matrix element square is

$$\sum |M^{QED}|^2 = 4\frac{(4\pi\alpha)^4}{(q_1^2 q_2^2)^2} s^2 \sum |m^{+-}|^2. \quad (76)$$

For the case of 3 jets production we obtain

$$M^{QCD} = \frac{(4\pi\alpha_s)^2}{q_1^2 q_2^2} s N^a N^b m_{ab}^{+-}, \quad (77)$$

with quantities $N^{a,b}$ specified above.

Cross section of process $pp \to j_1 j_2 F$ is defined as

$$d\sigma = \frac{1}{8s} \sum |M^{QCD}|^2 d\Gamma_{3j}.$$

We use the anzatz for fragmentation region of proton with momentum p_1:

$$\int d\gamma_1 N_a^1 N_{a_1}^{1*} = \frac{\vec{q}_1^{\,2}}{M_1^2} \delta_{aa_1} \Phi_1 F(\beta_1),$$

$$\Phi_1 = \frac{M_1^2}{(M_1^2 + \vec{q}_1^{\,2})^2}, \quad (78)$$

and similar expression for proton with momentum p_2.

Cross section of process of single gluon creation accompanied by two jets $pp \to jjg$can be written in form

$$d\sigma^{jjg} = \frac{\alpha_s^3}{16M_1^2M_2^2}\Phi_1 dM_1^2\Phi_2 dM_2^2\Phi^g \times \frac{d\beta_1}{\beta_1}\frac{d^2\vec{q}_1}{\pi}\frac{d^2\vec{q}_2}{\pi}F(\beta_1)F(\alpha_2)R, \qquad (79)$$

with

$$\Phi^g = \sum \frac{(f_{abc}C_\mu)^2}{\vec{q}_1^2\vec{q}_2^2}. \qquad (80)$$

Explicit expression of Φ^g is given above.

Cross section of production of pair of colored particles a_1, a_2 (gg or $q\bar{q}$) has a form

$$d\sigma^{jja_1a_2} = \frac{\alpha_s^4}{8M_1^2M_2^2}\Phi^{a_1a_2}\Phi_1 dM_1^2\Phi_2 dM_2^2 \times \frac{d\beta_1}{\beta_1}\frac{d^2\vec{q}_1}{\pi}\frac{d^2\vec{q}_2}{\pi}F(\beta_1)F(\alpha_2)RdM^2d\Gamma_2. \quad (81)$$

Explicit expressions for $\Phi^{a_1 a_2}$ are given above for the cases $q\bar{q}$ and gg.

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[1] V. N Baier, V. S. Fadin, V. A. Khoze and V. M. Kuraev, Phys. Rep. v 78 N3 (1981),,293. V. M. Budnev, I. F. Ginzburg, G. V. Meledin and V. G. Serbo, Phys. Rep. v. 15c(1975), 181.

[2] J. A. M. Vermasseren, Nucl. Phys. B 229(1983), 347.

[3] L. N. Lipatov, Nucl. Phys. B 542(1995),369

[4] V. S. Fadin, M. I. Kotski and L. N. Lipatov, hep-ph/9704267v2(1997).

[5] E. N. Antonov, I. O. Cherednikov, E. A. Kuraev, L. N. Lipatov, Nucl. Phys. B 721(2005), 111.

[6] L. Lipatov and V. Fadin, Yad. Fiz. v 50 (1989),1141.; Nucl. Phys. B477 (1996),767.

[7] M. A. Kimber, A. D. Martin and M. G. Ryskin, Phys. Rev. D **63**, 114027 (2001) [arXiv:hep-ph/0101348].

[8] P. Hagler, R. Kirschner, A. Schafer, L. Szymanowski and O. Teryaev, Phys. Rev. D **62**, 071502 (2000) [arXiv:hep-ph/0002077].

[9] B. A, Kniehl, A. V. Shipilova and V. A. Saleev, Phys. Rev. D 79, 034007 (2009); hep-ph/1003.0346v2 (2010).