THE RELATIVISTIC ELECTROMAGNETIC STRUCTURE OF THE NUCLEON

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Problem

1. *Composite quark model.*
2. *Polarization behavior* $F_e(Q^2)/F_m(Q^2)$.
3. *Construction of conserving current.*

These problems will be considered in the framework of the instant form of the reletivistic quantum mechanics. This approach was developed for two body systems by Krutov A.F. and Troitsky V.E. and gives good description of the deuteron and different two-quark systems.[1]

[1] A.F. Krutov V.E. Troitsky, Phys.Rev.C, 65,04501(2002).

Instant form of dynamics

Full set of commuting operators:

$$\hat{M}_I, \hat{J}^2, \hat{J}_3, \hat{\mathbf{P}}\ . \tag{1}$$

In the instant form of dynamics $\hat{J}^2$, $\hat{J}_3$, $\hat{\vec{P}}$ operators are coincided with non-interaction system operators, therefore their diagonalization is reduced to appropriate basis choice.

$$\hat{M}_I = \hat{M}_0 + \hat{V}\ , \tag{2}$$

where $\hat{M}_0$ - non-interactions system invariant mass operator , $\hat{V}$ - interaction operator.

Conditions limiting mass operator:

$$\hat{M}_I = \hat{M}_I^+, \hat{M}_I > 0; \quad (3)$$

$$[\hat{\vec{P}}, \hat{V}] = [\hat{\vec{J}}, \hat{V}] = [\hat{\vec{\nabla}}_P, \hat{V}] = 0. \quad (4)$$

$$\hat{M}_I|\psi(1,2,3)\rangle \equiv (\hat{M}_0 + \hat{V})|\psi(1,2,3)\rangle = \lambda|\psi(1,2,3)\rangle. \quad (5)$$

where λ - eigenvalue of mass operator.

Three body basis

$$|\vec{p}_1, m_1; \vec{P}_{23}, \sqrt{s_{23}}, m_{J_{23}}, L_{23}, S_{23}, J_{23}\rangle =$$
$$= |\vec{p}_1, m_1;\rangle \otimes |\vec{P}_{23}, \sqrt{s_{23}}, m_{J_{23}}, L_{23}, S_{23}, J_{23}\rangle \ , \qquad (6)$$

where

$$|\vec{P}_{23}, \sqrt{s_{23}}, m_{J_{23}}, L_{23}, S_{23}, J_{23}\rangle =$$
$$= \sum \int d\hat{k}_{23} |\vec{p}_2, m_2; \vec{p}_3, m_3\rangle Y_{L_{23} m_{L_{23}}}(\hat{k}_{23})$$
$$\langle \frac{1}{2}\frac{1}{2}\tilde{m}_2\tilde{m}_3|S_{23} m_{S_{23}}\rangle\langle S_{23} L_{23} m_{S_{23}} m_{L_{23}}|J_{23} m_{J_{23}}\rangle$$
$$D^{\frac{1}{2}}_{m_2' m_2}(P_{23} p_2) D^{\frac{1}{2}}_{m_3' m_3}(P_{23} p_3) \ . \qquad (7)$$

Three body basis

Normalization.

$$\langle \vec{p'}_1, m'_1 | \vec{p}_1, m_1 \rangle = 2p_{10}\delta(\vec{p}_1 - \vec{p}\,'_1)\delta_{m_1 m'_1} \ , \quad (8)$$

$$\langle \vec{P'}_{23}, \sqrt{s'_{23}}, m'_{J_{23}}, L'_{23}, S'_{23}, J'_{23} | \vec{P}_{23}, \sqrt{s_{23}}, m_{J_{23}}, L_{23}, S_{23}, J_{23} \rangle =$$
$$= N^2 2P^0_{23}\delta(\vec{P}_{23} - \vec{P}'_{23})\delta(\sqrt{s_{23}} - \sqrt{s'_{23}})\delta_{m_{J_{23}} m'_{J_{23}}}\delta_{S_{23} S'_{23}}\delta_{L_{23} L'_{23}}\delta_{J_{23} J'_{23}}, \quad (9)$$

$$|\vec{P}, m_J; \sqrt{s}, \sqrt{s_{23}}, L, S, J, (L, S, J)_{23}\rangle \; . \qquad (10)$$

$$\langle\vec{P}', m'_J; \sqrt{s'}, \sqrt{s'_{23}}, L', S', J', (L', S', J')_2|\vec{P}, m_J; \sqrt{s}, \sqrt{s_{23}}, L, S, J, (L, S, J)_2$$

$$= N_1^2 \frac{2P_0}{kk_{23}} \delta(\vec{P} - \vec{P}')\delta(\sqrt{s} - \sqrt{s'})\delta(\sqrt{s_{23}} - \sqrt{s'_{23}})$$

$$\delta_{m_J m'_J}\delta_{S'S}\delta_{L'L}\delta_{L'_{23}L_{23}}\delta_{S'_{23}S_{23}}\delta_{J'J}\delta_{J'_{23}J_{23}} \; . \qquad (11)$$

$$|\vec{P}, m_J; \sqrt{s}, \sqrt{s_{23}}, L, S, J, (L, S, J)_{23}\rangle =$$

$$= \sum |\vec{p}_1, m_1; \vec{P}_{23}, \sqrt{s_{23}}, m_{J_{23}}, L_{23}, S_{23}, J_{23}\rangle$$

$$Y_{Lm_L}(\hat{k})\langle\frac{1}{2}J_{23}m_1' m_{J_{23}}'|Sm_S\rangle$$

$$\langle SLm_S m_L|Jm_J\rangle D^{\frac{1}{2}}_{m_1{}' m_1}(Pp_1) D^{J_{23}}_{m_{J_{23}}{}' m_{J_{23}}}(PP_{23}) \ . \qquad (12)$$

Vectors $|\vec{P}, m_J; \sqrt{s}, \sqrt{s_{23}}, L, S, J, (L, S, J)_2\rangle$ of the analogue options to free particles with $\vec{P}$ momentum and $\vec{J}$ spin. Particulary under pressure of $U(\Lambda)$ operator there behave as free particle vectors:

$$|\vec{P}, m_J; \sqrt{s}, \sqrt{s_{23}}, L, S, J, (L, S, J)_{23}\rangle =$$

$$= \sum_{m_J} |\Lambda\vec{P}, m_J'; \sqrt{s}, \sqrt{s_{23}}, L, S, J, (L, S, J)_{23}\rangle D^J_{m_J' m_J}(\Lambda P, P) \ . \quad (13)$$

Three interaction particles system wave-function may be presented as:

$$\langle\vec{P}, m_J; \sqrt{s}, \sqrt{s_{23}}, L, S, J, (L, S, J)_{23}|\vec{p}_c, \mu_c\rangle = N_c\, \delta(\vec{P} - \vec{p}_c)\delta_{m_J m_c}\varphi^J_\gamma(\sqrt{s}, \sqrt{s_{23}})\ , \tag{14}$$

where $\gamma \equiv \{L, S, (L, S, J)_{23}\}$, N_c - normalization factor.

Parametrization of current operator

Current operator of single particle:

$$\langle\vec{p}, M, j, m|J_\mu(0)|\vec{p}', M, j, m'\rangle = \sum_{m''}\langle m|D^j(pp')|m''\rangle$$

$$\langle m''|F_1 K'_\mu + F_2\Gamma_\mu(p') + F_3 R_\mu + F_4 K_\mu|m'\rangle \ , \qquad (15)$$

$$F_i = \sum_{n=0}^{2j} f_{in}(Q^2)(ip_\mu\Gamma^\mu(p'))^n \ ,$$

$$K'_\mu = (p+p')_\mu,\ K_\mu = (p-p')_\mu = Q_\mu,\ R_\mu = \epsilon_{\mu\nu\lambda\rho}p^\nu p'^\rho\Gamma^\rho(p') \ .$$

$$D^{\frac{1}{2}}(pp') = Cos\frac{\omega}{2} - 2i(\mathbf{nj})Sin\frac{\omega}{2}\ , \tag{16}$$

$$D^{1}(pp') = I - i(\mathbf{nj})Sin\omega + (\mathbf{nj})^2(Cos\omega - 1)\ , \tag{17}$$

$$\mathbf{n} = \frac{[\mathbf{pp}']}{|[\mathbf{pp}']|}\ ,\ \omega = 2arctn\frac{|[\mathbf{pp}']|}{(p_{01}+M_1)(p_{02}+M_2) - (\mathbf{pp}')}\ .$$

Conditions additional constraint by operator:

Self-conjugacy:

$$\Gamma_\mu(p') \to \Gamma_\mu(p') - \frac{K'_\mu}{K'^2}(p_\nu \Gamma^\nu(p'))\ ,\ n = 0\ ,$$

$$F_i A_\mu \to \frac{1}{2}\{F_i, A_\mu\}_+\ ,\ i = 2,3\ ,\ n \neq 0\ ,$$

$$F_i \to \mathbf{i} F_i\ ,\ i = 3,4\ .$$

Orthogonality:

$$\Gamma_\mu(p') \to \Gamma_\mu(p') - \left(\frac{K_\mu}{K^2} + \frac{K'_\mu}{K'^2}\right)(p_\nu \Gamma^\nu(p'))\ ,\ i = 2\ ,$$

Conditions additional constraint by operator:

Conservation of parity:
$F_i A_\mu$ contain $\Gamma_\mu(p')$ no more than 2j.

Conditions conservation:

$$j_\mu K^\mu = j_\mu Q^\mu = 0 \Rightarrow F_4 = 0\ .$$

$$\langle\vec{p},M,j,m|J_\mu(0)|\vec{p'},M,j,m'\rangle = \sum_{m''}\langle m|D^j(pp')|m''\rangle$$

$$\langle m''|F_1K'_\mu+\{F_2\Big(\Gamma_\mu(p')-\Big(\frac{K_\mu}{K^2}+\frac{K'_\mu}{K'^2}\Big)(p_\nu\Gamma^\nu(p'))\Big)\}_+ + \mathbf{i}\{F_3R_\mu\}_+|m'\rangle\ . \qquad (18)$$

In case $j=\frac{1}{2}$ we are have:

$$\langle\vec{p},M,\frac{1}{2},m|J_\mu(0)|\vec{p'},M,\frac{1}{2},m'\rangle = \sum_{m''}\langle m|D^{\frac{1}{2}}(pp')|m''\rangle$$

$$\langle m''|f_{10}(Q^2)K'_\mu + \mathbf{i}f_{30}(Q^2)R_\mu|m'\rangle\ . \qquad (19)$$

Connections with Sachs formfactors:

$$f_{10}(Q^2) = \frac{G_E(Q^2)}{\sqrt{1+\frac{Q^2}{4M^2}}}\ ,\ f_{30}(Q^2) = \frac{2G_M(Q^2)}{M^2\sqrt{1+\frac{Q^2}{4M^2}}}$$

Parametrization of electromagnetic current matrix element for free three-body system

Current operator of free three system:

$$J^0_\mu(0) = J^1_\mu(0) \otimes I^{23} \oplus J^2_\mu(0) \otimes I^{13} \oplus J^3_\mu(0) \otimes I^{12} \ .$$

$$\langle \vec{p}_a, m_a; \vec{P}_{bc}, \sqrt{s_{bc}}, m_{J_{bc}}, \gamma_{bc} | J^0_\mu(0) | \vec{p'}_a, m'_a; \vec{P'}_{bc}, \sqrt{s'_{bc}}, m'_{J_{bc}}, \gamma'_{bc} \rangle =$$

$$= \sum^{P(123)} \langle \vec{P}_{bc}, \sqrt{s_{bc}}, m_{J_{bc}}, \gamma_{bc} | \vec{P'}_{bc}, \sqrt{s'_{bc}}, m'_{J_{bc}}, \gamma'_{bc} \rangle \langle \vec{p}_a, m_a | J^a_\mu | \vec{p'}_a, m'_a \rangle \ , \tag{20}$$

where $\gamma_{bc} \equiv \{L_{bc}, S_{bc}, J_{bc}\}$.

$$\langle \vec{P}, m_J; \sqrt{s}, \sqrt{s_{23}}, \gamma | J^0_\mu(0) | \vec{P}', m'_J; \sqrt{s'}, \sqrt{s'_{23}}, \gamma \rangle =$$

$$= \sum_{m''_J} \langle m_J | D^{\frac{1}{2}}(PP') | m''_J \rangle \langle m''_J | F_E^{\gamma\gamma'} A^1_\mu + F_M^{\gamma\gamma'} A^3_\mu | m'_J \rangle \ , \qquad (21)$$

where

$$F_i^{\gamma\gamma'} \equiv F_i^{\gamma\gamma'}(s, s_{23}, Q^2, s', s'_{23}) \ ,$$

$$\gamma \equiv \{L, S, J, (L, S, J)_{23}\} \ ,$$

$$A^1_\mu = \frac{1}{Q^2}\Big((s - s' + Q^2)P_\mu + (s' - s + Q^2)P'_\mu\Big) \ ,$$

$$A^3_\mu = \frac{i}{\sqrt{s'}} R_\mu \ .$$

Free formfactors of three body systems

$$F_E^{00} = \sum^{P(123)} \frac{3Q^2(s+s'+Q^2)}{2\lambda^{\frac{3}{2}}(s,s',-Q^2)} \frac{k_{23}\delta(\sqrt{s_{23}}-\sqrt{s'_{23}})}{kk'} \times$$
$$\times \left[Af^a_{10}(Q^2)Cos\frac{\omega 1+\omega 2}{2} - \frac{M}{2}Bf^a_{30}(Q^2)Sin\frac{\omega 1+\omega 2}{2} \right],$$

$$F_E^{11} = \sum^{P(123)} \frac{Q^2(s+s'+Q^2)}{4\lambda^{\frac{3}{2}}(s,s',-Q^2)} \frac{k_{23}\delta(\sqrt{s_{23}}-\sqrt{s'_{23}})}{kk'} \times$$
$$\times \left[2Af^a_{10}(Q^2)\left\{ Cos\frac{\omega 1+\omega 2}{2} + 2Cos\frac{\omega 1+\omega 2+2\omega 3}{2} \right\} - \right.$$
$$\left. -\frac{M}{4}Bf^a_{30}(Q^2)\left\{ Sin\frac{\omega 1+\omega 2}{2} + 2Sin\frac{\omega 1+\omega 2+2\omega 3}{2} \right\} \right],$$

$$A = s + s' + Q^2 + 2(M^2 - s_{23})\ ,$$

$$B = \left(-M^2\lambda(s, s', -Q^2) + ss'Q^2 - s_{23}Q^4 - (s_{23} - M^2)Q^2(s + s') + \right.$$

$$\left. + Q^2(s_{23} - M^2)^2\right)^{\frac{1}{2}}\ .$$

$$F_M^{00} = \sum^{P(123)} \frac{Q^2(s+s'+Q^2)}{8\lambda^{\frac{3}{2}}(s,s',-Q^2)} \frac{k_{23}\delta(\sqrt{s_{23}}-\sqrt{s'_{23}})}{kk'} \times$$
$$\times\Big[-f_{10}^a(Q^2)K_1 Sin\frac{\omega_1+\omega_2}{2} + f_{30}^a(Q^2)(R_1 Cos\frac{\omega_1+\omega_2}{2} + R_2 Sin\frac{\omega_1+\omega_2}{2})$$

$$F_M^{11} = \sum^{P(123)} \frac{(4Cos\omega_3-1)Q^2(s+s'+Q^2)}{4\lambda^{\frac{3}{2}}(s,s',-Q^2)} \frac{k_{23}\delta(\sqrt{s_{23}}-\sqrt{s'_{23}})}{kk'} \times$$
$$\times\Big[\frac{2B_2+\lambda(s,s',-Q^2)(1-\frac{B_1}{s'\sqrt{2}})}{Q^2} f_{10}^a(Q^2) Cos\frac{\omega 1+\omega 2}{2} +$$
$$+\frac{MB_3(\lambda(s,s',-Q^2)-2s'Q^2)}{8\sqrt{s'}} f_{30}^a(Q^2) Sin\frac{\omega 1+\omega 2}{2}\Big],$$

$$B_1 = (\lambda(s, s', -Q^2) + \lambda(s_{23}, M^2, Q^2)+$$

$$+(Q^2 + M^2)(M^2 + 2s) + 2M^2(Q^2 - s'))^{\frac{1}{2}} \ ,$$

$$B_2 = \frac{s(s + s') + (s + Q^2)(Q^2 - 4s_23 + 6M^2) + s'(3Q^2 - s')}{4} -$$

$$-\frac{s'(2s_{23} + 4M^2 - Q^2)}{4} - \frac{(s + Q^2)^3 - 2(s_{23} - M^2)(s + Q^2)^2}{4s'}$$

$$B_3 = s'(s' - 2s_{23} + 4M^2) - (s + Q^2)^2 + 2(s + Q^2)(s_{23} - M^2) \ .$$

Impulse approximation

$$G_E(Q^2) = \sum_{\gamma\gamma'} \int d\sqrt{s_{23}}\, d\sqrt{s'_{23}}\, d\sqrt{s}\, d\sqrt{s'} \times$$

$$\times \psi^{\gamma}(\sqrt{s}\sqrt{s_{23}}) F_E^{\gamma\gamma'}(s, s_{23}, Q^2, s', s'_{23}) \psi^{\gamma'}(\sqrt{s'}\sqrt{s'_{23}})\ ,$$

$$G_M(Q^2) = \sum_{\gamma\gamma'} \int d\sqrt{s_{23}}\, d\sqrt{s'_{23}}\, d\sqrt{s}\, d\sqrt{s'} \times$$

$$\times \psi^{\gamma}(\sqrt{s}\sqrt{s_{23}}) F_M^{\gamma\gamma'}(s, s_{23}, Q^2, s', s'_{23}) \psi^{\gamma'}(\sqrt{s'}\sqrt{s'_{23}})\ .$$

Polarization behavior (in progress...)

Explain the polarization behavior is violating some discrete symmetry, it will lead to additional form factors in the parametrization.

Summary

1. In the framework of RQM the electromagnetic current matrix element for three-body system is constructed at conditions of conservation and Lorentz-covariance,
2. The electromagnetic form factors for free body system with $S = 1/2$ are obtained in relativistic impulse approximation.