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Violation of unitarity of the tree approximation to the Glauber AA-scattering amplitude.

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XX international Baldin seminar on high energy physics problems.

Dubna. 7 october 2010.

Contents:

The Glauber formula for elastic nucleus-nucleus scattering;

- Approximations for the nucleus-nucleus elastic scattering amplitude in the limit of heavy nuclei;
- The integral representation for the tree S-matrix and violation of unitarity;
- The comparison with the previous results;
- S-matrix in the saddle-point approximation and its unitarity.

The Glauber formula for the elastic scattering:

$$S(b) = \left\langle \prod_{i=1}^{A} \prod_{k=1}^{B} s(x_i - x'_k - b) \right\rangle_{A,B}$$

$$s(x) = 1 + ia(x)$$

$|S(b)| < 1$ - The condition of unitarity

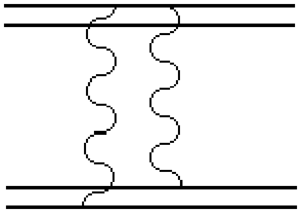
$a(x)$ – the nucleon-nucleon elastic scattering amplitude.

Diagram expansion for AB-amplitude:

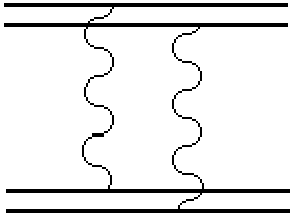
$$iA(b) = \left\langle \prod_{i=1}^{A} \prod_{k=1}^{B} \left(1 + ia(x_i - x'_k - b)\right) - 1 \right\rangle_{A,B}$$

$$\langle F \rangle_{A,B} = \int \prod_{i=1}^{A} d^2 x_i T_A(x_i) \int \prod_{k=1}^{B} d^2 x'_k \, T_B(x'_k) F\left(x_i, x'_k\right)$$

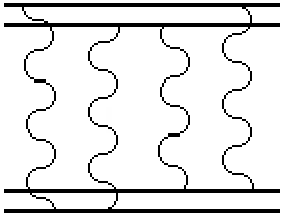
Diagrams for Glauber AB-amplitude:

a)  =

$$\int d^2x_1 d^2x'_1 d^2x'_2\, T_A(x_1) T_B(x'_1) T_B(x'_2) \times$$
$$\times\, ia\left(x_1 - x'_1 - b\right) ia\left(x_1 - x'_2 - b\right)$$

b)  =

$$\left(\int d^2x_1 d^2x'_1\, T_A(x_1) T_B(x'_1)\, ia\left(x_1 - x'_1 - b\right)\right)^2$$

c)  =

$$\int d^2x_1 d^2x_2 d^2x'_1 d^2x'_2\, T_A(x_1) T_A(x_2) T_B(x'_1) T_B(x'_2) \times$$
$$\times\, ia\left(x_1 - x'_1 - b\right) ia\left(x_1 - x'_2 - b\right) ia\left(x_2 - x'_1 - b\right) \times$$
$$\times\, ia\left(x_2 - x'_2 - b\right)$$

Optical approximation:

- Optical approximation is determined by the sum of diagrams with all the connected parts to be the simplest (only single collisions, like (b));
- It is obviously unitary in the limit of heavy nuclei:

$$\mathrm{A, B} >> 1$$

$$S(b) = e^{F(b)}$$

$$F(b) = (ia)AB\int d^2x T_A(x) T_B(b-x)$$

$$a = \int d^2b\, a(b) \qquad a = \frac{i}{2}\sigma_{NN}$$

The tree approximation in the limit A,B>>1

1. The result by Pak, Tarasov, Uzhinsky and Tseren [1]:

$$S(b) = e^{F(b)}, \quad F(b) = \frac{i}{a} \int d^2x \, f(\gamma_A, \gamma_B)$$

$$\gamma_A(x) = -iaAT_A(x), \quad \gamma_B(x) = -iaBT_B(b-x)$$

$$u = \gamma_A e^{-v}$$

$$v = \gamma_B e^{-u}$$

$$f(\gamma_A, \gamma_B) = \sum_{l=1}^{k} (-1)^{l+1} (u_l + v_l + u_l v_l) - \gamma_A - \gamma_B$$

$$k = 1 \; or \; k = 3. \quad u_1 > u_2 > u_3, \; v_1 < v_2 < v_3$$

[1] A.S. Pak, A. V. Tarasov, V. V. Uzhinsky, Ch. Tseren, Yad. Fiz. 30, 102 (1979).

The tree approximation in the limit A,B>>1

2. The formula by Boreskov and Kaidalov [2]:

$$S(b)=\sqrt{1-n_A n_B\left(\sigma_{NN}\right)^2}\, e^{-\frac{1}{2}G(b)n_A n_B \sigma_{NN}}$$

with the nucleon distribution:

$$n_A=\frac{A}{\pi R_A^2}\Theta\left(R_A-|x|\right)$$

The result is just the optical approximation.

[2]. K.G. Boreskov and A. B. Kaidalov. Yad. Fiz. 48, 575 (1988)

Integral representation for the tree Glauber amplitude [3].

$$S(b)=\frac{A!\,B!}{4\pi^2 i^{A+B+2}}\oint\frac{d\tau\, d\tau'}{\tau^{A+1}\tau'^{B+1}}e^{i(\tau+\tau')+i\int d^2x\, d^2x'\,\delta^{(2)}(b-x-x')\cdot W}$$

$$W=\frac{1}{a}uv+\tau T_A(x)\left(e^{-u}-1\right)+\tau' T(x')\left(e^{-v}-1\right)$$

$$u=a\tau' T_B(x')e^{-v},\qquad v=a\tau T_A(x)e^{-u}.$$

The model of cylindrical nuclei:

$$T_A(x)=\frac{1}{\pi R_A^2}\Theta\left(R_A-|x|\right),\qquad R_A=A^{\frac{1}{3}}R_0$$

[3]. M. A. Braun, Yad. Fiz. 45, 1625 (1987).

We study the special case of two identical nuclei: A=B

$$S(b)=\frac{(A!)^2}{4\pi^2 i^{2A-2}}\kappa^{2A}\oint\frac{du\,dv}{(uv)^{A+1}}(1-uv)e^{-A(u+v)}\cdot$$

$$\cdot\, e^{\frac{i}{\kappa}ve^{u}(1-\lambda)}e^{\frac{i}{\kappa}ue^{v}(1-\lambda)}e^{i\frac{\lambda}{\kappa}(uv+u+v)};$$

$$\kappa=\frac{a}{\pi R^2},\qquad \lambda=\frac{G(b)}{\pi R^2},\qquad \gamma=-iA\kappa.$$

An exact expression for S-matrix for b=0:

$$S(0)=(A!)^2(1-\gamma)^{2A}\cdot\left\{\sum_{n=0}^{A}\frac{1}{n![(A-n)!]^2}\left(\frac{\gamma}{(1-\gamma)^2}\right)^n-\right.$$

$$\left.-\frac{\gamma^2}{A^2(1-\gamma)^2}\sum_{n=0}^{A-1}\frac{1}{n![(A-n-1)!]^2}\left(\frac{\gamma}{(1-\gamma)^2}\right)^n\right\}$$

Violation of unitarity.

S(0) contains terms $(1-\gamma)^k = (1+iA\kappa)^k, \quad \gamma > 0$

$$A\kappa = \frac{aA}{\pi R_A^2} \sim A^{\frac{1}{3}}, \qquad (1-\gamma)^k \sim A^{\frac{k}{3}}, 0 \le k \le 2A$$

For A>>1 |S(0)|>>1, unitarity is violated

The region of unitarity: $0 < \gamma < 1.42$

$$a = \frac{i}{2}\sigma_{NN},$$

$$\frac{A^{\frac{1}{3}}\sigma_{NN}}{2\pi R_0^2} < 1{,}42$$ - is not satisfied for large A

For nucleon-nucleus scattering $\frac{\sigma_{NN}}{2\pi R_0^2 A^{2/3}} < 1$

Estimations of A for tree approximation to be non-unitary.

$$A \geq \left(\frac{2\pi R_0^2 \gamma_{\max}}{\sigma_{NN}} \right)^3; \quad \gamma_{\max} = 1{,}42$$

For the estimations total pp cross section was used.

If $\sigma_{pp} \approx 40 mbn$ (energies about $\sqrt{s} \approx 3 \div 30\, Gev$) the boundary value of A for tree S-matrix to be unitary is $A = 53$. So, for heavy nuclei, the tree approximation can't describe even the region of small energies without getting in contradiction with causality principle.

For LHC energies $\sigma_{pp} \approx 90 \div 100\, mbn$ and $A = 3$.

Asymptotic in the limit A,B>>1

Stationary points for b=0: $u = v = \gamma, \quad u = v = -1$

$$S(0) = S^{(1)}(0) + S^{(2)}(0)$$

$\gamma < 1$

$$S^{(1)}(0) = \sqrt{1-\gamma^2}\, e^{-A\gamma}$$

$$S^{(2)}(0) = \pm i \frac{\gamma^2}{A(\gamma+1)^{5/2}(1-\gamma)^{1/2}} e^{A\left(2\ln\gamma - \frac{1}{\gamma}\right)}$$

$\gamma > 1$

$$S^{(1)}(0) = -\frac{\gamma^2}{A(\gamma+1)^{5/2}(\gamma-1)^{1/2}} e^{A\left(2\ln\gamma - \frac{1}{\gamma}\right)}$$

$$S^{(2)}(0) = \pm i \sqrt{\gamma^2 - 1}\, e^{-A\gamma}$$

When $\gamma > 1{,}42, \quad 2\ln\gamma - \frac{1}{\gamma} > 0$ and S-matrix is non-unitary

The previous unexact results and our integral representation for the S-matrix.

$$S(b) = \frac{(A!)^2}{4\pi i^{2A+2}} \oint \frac{d\tau d\tau'}{(\tau\tau')^{A+1}} e^{i(1-\lambda)(\tau+\tau')} e^{\frac{i\lambda}{\kappa}(u+v+uv)}$$

$$u = \kappa\tau' e^{-v} \qquad v = \kappa\tau e^{-u}$$

If we **consider** only $e^{i(\tau+\tau')-A\ln(\tau\tau')}$ to change rapidly with A, we get:

$i\tau = i\tau' = A$ - the one stationary point.

$$S(b) = e^{\frac{i\lambda}{\kappa}(uv+u+v)-2A\lambda}$$

$$u = -iA\kappa e^{-v}, \quad v = -iA\kappa e^{-u}$$

- The formula of Pak et al for our simplified model

More accurate calculation of the asymptotic:

$$i\tau = At, \qquad i\tau' = At',$$

$$S(b) = \left(\frac{A!}{A^A}\right)^2 \frac{1}{4\pi^2} \int \frac{dt\,dt'}{tt'} e^{AP(t,t')}$$

$$P(t,t') = -\ln(tt') + t + t' + \frac{\lambda}{\gamma}(uv + u + v - \gamma t - \gamma t')$$

$$u = \gamma t' e^{-v}, \qquad v = \gamma t e^{-u}.$$

$u = v = \gamma$ - the one stationary point for $\lambda = 1.$

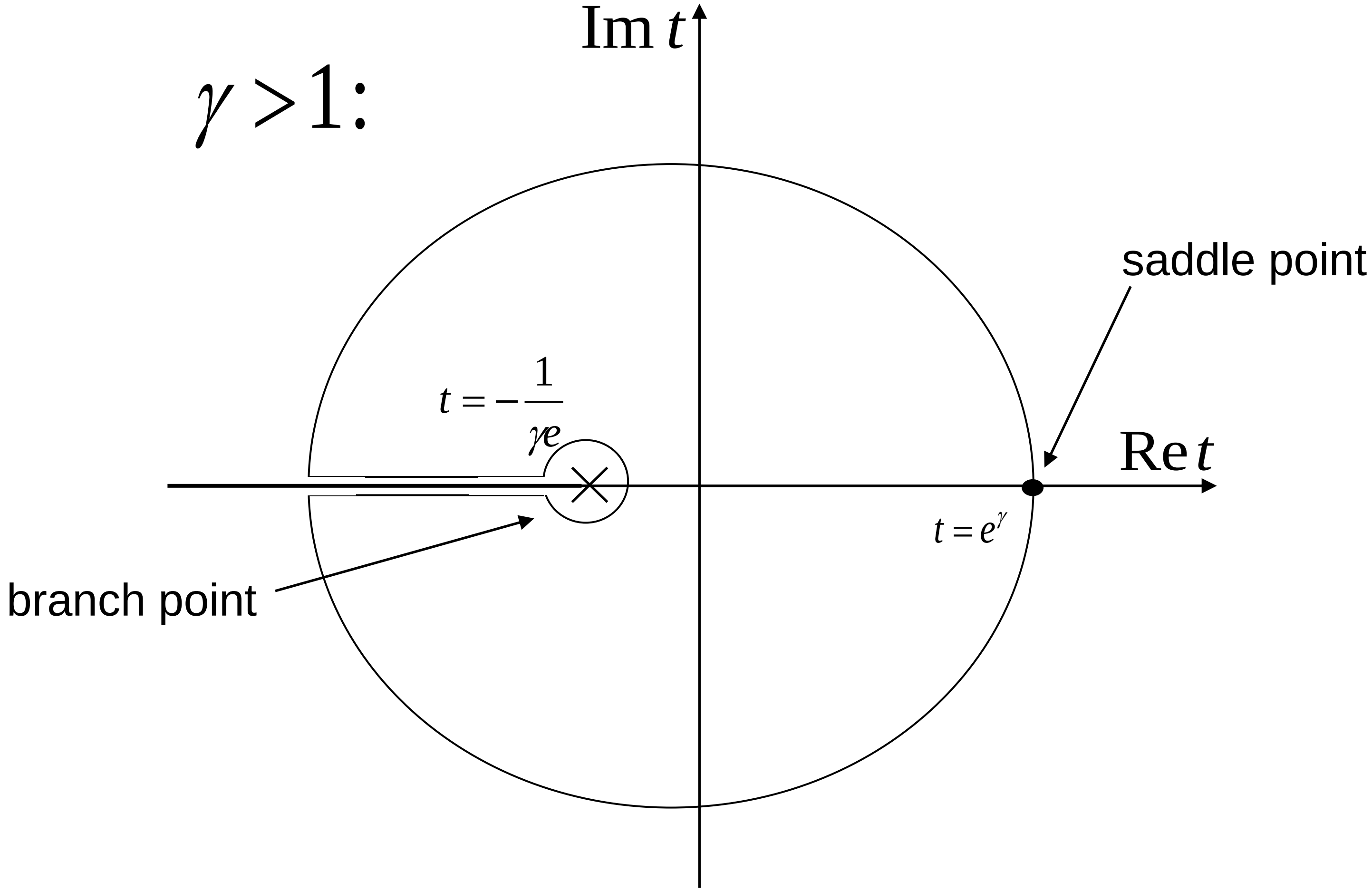
$S(0) = \sqrt{1-\gamma^2}\, e^{-A\gamma}$ - The result of Boreskov and Kaidalov.

$u = v = -1 \quad \rightarrow$ Singularities on the t-t'-plane, which destroy unitarity.

Integration contour for the asymptotic:

Central collisions: $\lambda = 0$

$\gamma > 1$:



Saddle-point approximation to the Glauber S-matrix.

An alternative representation via functional integral:

$$S_{AB}(b) = \int D\phi D\phi^{+} e^{R(b)}$$

A formula for the eikonal:

$$R(b) = \frac{1}{ia}\int d^2x\, \phi^{+}(x)\phi(x) +$$
$$+ A \cdot \ln \int d^2x\, T_A(x) e^{\phi(x)} + B \cdot \ln \int d^2y\, T_B(b-y) e^{-\phi^{+}(y)}$$

Saddle-point approximation => fields are determined by the saddle-point equations:

$$\frac{\delta R}{\delta\phi(x)} = 0; \qquad \frac{\delta R}{\delta\phi^{+}(x)} = 0;$$

The results for the model of cilindrical nuclei and A=B:

$$T_A(x) = \frac{1}{\pi R_A^2}\Theta\left(R_A - |x|\right), \quad R_A = A^{\frac{1}{3}} R_0$$

1. For arbitrary b:

$$R(b) = -\frac{G(b)}{ia}\phi_3^2 + 2A \cdot \ln\left(1 - \lambda + \lambda e^{\phi_3}\right)$$

$$\lambda = \frac{G(b)}{\pi R_A^2};$$

ϕ_3 - the root of $\phi_3 = \dfrac{Aia\, e^{\phi_3}}{\pi R_A^2\left(1 - \lambda + \lambda e^{\phi_3}\right)}$

This expression turns out to be unitary for all b.

2. For b=0:

$R = \dfrac{A^2 ia}{\pi R_A^2}$ - optical approximation.

Summary:

- The tree approximation for Glauber AA-amplitude is non-unitary for large atomic numbers.
- Unitarity is fulfilled only for not very large A and for the rather low energies, when the NN-cross section σ has its minimal value.
- It is necessary to take into consideration Glauber loop diagrams for the correct description of scattering process.
- The saddle-point approximation gives an unitary S-matrix for any values of atomic number.

Thank you.