A.V.Kotikov, JINR, Dubna

(in collab. with A.Yu. Illarionov, Trento Uni., Italy)

XX International Baldin Seminar "Relativistic Nuclear Physics & Quantum Chromodynamics"

4 - 9 October 2010, Dubna

Small x behavior of the structure functions F_2 and F_2^c

OUTLINE

1. Introduction

2. Results

3. Conclusions and Prospects

1. Introduction

A. The knowledge of parton densities (the quark one $f_q(x,Q^2)$ and the gluon one $f_g(x,Q^2)$) is very important for many processes.

B. The deep-inelastis scattering (DIS) process is the basic one to extract the parton densities (PD), because the DIS structure functions (SF) $F_k(x,Q^2)$ $(k=2,3,L)$ relates with the parton densties

$$F_k(x,Q^2) \;=\; \sum_{i=q,g} C_{k,i}(x) \otimes f_i(x,Q^2), \qquad (1)$$

where the simbol $\otimes$ marks the Mellin convolution

$$f_1(x) \otimes f_2(x) \;\equiv\; \int_x^1 \frac{dy}{y} f_1(x/y) f_2(y) \qquad (2)$$

The best measured SF $F_2(x,Q^2)$ and $F_3(x,Q^2)$ relate directly with the quarks density at the leading order (LO)

$$F_{2,3}(x,Q^2) \;=\; \sum_q e_q^2 f_q(x,Q^2) + O(\alpha_s) \qquad (3)$$

C. Charm part of F_2: F_2^c. There are new (preliminary) experimental (H1+ZEUS) data (K.Lipka, 2009), (A. M. Cooper-Sarkar, 2010)

Theoretical approaches for F_2^c:

1. Photon-gluon fuzion (will be used here):

charm is pure perturbative [it is good above the charm threshold]

good agreement with HERA data

$$F_2^c(x,Q^2) = \alpha_s(Q^2) B_{2,g}^{(0)}(x,m_c^2) \otimes f_g(x,Q^2) + O(\alpha_s^2) \tag{4}$$

2. Charm is nonperturbative one (as light quarks) [it is good for $Q^2 \to \infty$]

$$F_2^c(x,Q^2) = e_c^2 f_c(x,Q^2) + O(\alpha_s) \tag{5}$$

3. There are intermediate schemes (F. I. Olness and W. K. Tung,, 1988), (M. A. G. Aivazis et al., 1994)

Here I will present simple formulae to find F_2 and F_2^c using approximated formulas for Mellin convolution at low x values.

So, if $f_1(x) = B_k(x, Q^2)$ is perturbatively calculated Wilson kernel and $f_2(x)$ is the some parton density with its property: $f_2(x) = x f_i(x, Q^2) \sim x^{-\delta}$ at $x \rightarrow 0$, then

$$f_1(x) \otimes f_2(x) \approx M_k(1+\delta, Q^2)\, f_2(x) \tag{6}$$

where $M_k(1+\delta, Q^2)$ is the analytical continuation to non-integer arguments of the Mellin moment $M_k(n, Q^2)$ of the Wilson kernel $B_k(x, Q^2)$:

$$M_k(n, Q^2) = \int_0^1 x^{n-2} B_k(x, Q^2) \tag{7}$$

So, we have (for F_2, F_3 and F_L, for example)

$$F_k(x, Q^2) \approx \sum_{l=q,g} M_{k,l}(1+\delta, Q^2)\, x f_l(x, Q^2). \tag{8}$$

where hereafter $k = 2, L$.

The situation is same also for heavy-quark parts of F_2, but in the case $M_2(n, Q^2) \to M_2(n, Q^2, m_i^2)$ ($i = c, b$).

2. Method

The method leads to the possibility to replace the Mellin convolution of two functions by a simple products at small x.

A. Firstly I consider only the case of regular behavior of kernel moments at $n \to 1$.

Let us to consider the set of PD with have the different forms:

- Regge-like form $f_R(x) = x^{-\delta}\tilde{f}(x)$,
- Logarithmic-like form $f_L(x) = x^{-\delta} ln(1/x)\tilde{f}(x)$,
- Bessel-like form $f_I(x) = x^{-\delta} I_k(2\sqrt{\hat{d} ln(1/x)})\tilde{f}(x)$,

where $\tilde{f}(x)$ and its derivative $\tilde{f}'(x) \equiv d\tilde{f}(x)/dx$ are smooth at $x = 0$ and both are equal to zero at $x = 1$:

$$\tilde{f}(1) \; = \; \tilde{f}'(1) \; = \; 0$$

1. Consider the basic integral with the integer $m > 1$:

$$J_{\delta,i}(m,x) = x^m \otimes f_i(x) \equiv \int_x^1 \frac{\mathrm{d}y}{y}\, y^m\, f_i(\frac{z}{y})\ , \qquad i = R, L, I$$

a) *Regge-like case.* Expanding $\tilde{f}(x)$ near $\tilde{f}(0)$, we have

$$\begin{aligned} J_{\delta,R}(m,x) &= x^{-\delta} \int_x^1 \mathrm{d}y\, y^{m+\delta-1}[\tilde{f}(0) + \frac{x}{y}\, \tilde{f}^{(1)}(0) + \ldots \\ &\quad + \frac{1}{k!}\left(\frac{x}{y}\right)^k \tilde{f}^{(k)}(0) + \ldots] \\ &= x^{-\delta}\left[\frac{1}{m+\delta}\, \tilde{f}(0) + O(x)\right] \\ &- x^m[\frac{1}{m+\delta}\, \tilde{f}(0) + \frac{1}{m+\delta-1}\, \tilde{f}^{(1)}(0) + \ldots \\ &\quad + \frac{1}{k!}\, \frac{1}{m+\delta-k}\, \tilde{f}^{(k)}(0) + \ldots] \end{aligned}$$

The second term in the r.h.s. can be summed:

$$J_{\delta,R}(m,x) = x^{-\delta}\left[\frac{1}{m+\delta}\ \tilde{f}(0) + O(x)\right] + \ x^{m}\ \frac{\Gamma(-(m+\delta))\Gamma(1+\nu)}{\Gamma(1+\nu-m-\delta)}\ \tilde{f}(0)$$

Because now our interest is limited by the nonsingular case ($n \geq 1$), we can neglect here the second term and obtain:

$$J_{\delta,R}(m,x) = x^{-\delta}\frac{1}{m+\delta}\ \tilde{f}(x) + O(x^{1-\delta})$$

B. Now I consider the case of singular behavior of kernel moments at $n \to 1$.

1. Really it is nedded to study only the above basic integral $J_{\delta,i}(m,x)$ considering the case $m \to 0$:

$$J_{\delta,R}(m \to 0, x) = \frac{1}{\tilde{\delta}_R} x^{-\delta} \tilde{f}(0) + O(x^{1-\delta}),$$

where

$$\frac{1}{\tilde{\delta}_R} = \frac{1}{\delta}[1 - x^{\delta} \, \frac{\Gamma(1-\delta)\Gamma(1+\nu)}{\Gamma(1+\nu-\delta)}],$$

i.e.

$$\frac{1}{\tilde{\delta}_R} = \frac{1}{\delta} \quad \text{if} \quad x^{\delta} << 1$$

and

$$\frac{1}{\tilde{\delta}_R} = \ln\frac{1}{x} - [\Psi(1+\nu) - \Psi(1)] \quad \text{if} \quad \delta = 0$$

Analogously, at $\delta \longrightarrow 0$

$$\frac{1}{\tilde{\delta}_L} = \frac{1}{2}\ln\frac{1}{x} + O(1/ln(1/x)),$$

$$\frac{1}{\tilde{\delta}_I} = \sqrt{\frac{\hat{d}}{ln(1/x)}}\,\frac{I_{k+1}(2\sqrt{\hat{d}ln(1/x)})}{I_k(2\sqrt{\hat{d}ln(1/x)})}$$

For arbitrary PD $f^{(l)}(x,Q^2) = x^{-\delta}\tilde{f}^{(l)}(x,Q^2)$:

$$\frac{1}{\delta_l} = \frac{1}{\tilde{f}^{(l)}(x,Q^2)}\int_x^1 \frac{dy}{y}\tilde{f}^{(l)}(y,Q^2), \tag{9}$$

2. In the general case, if $M(n)$ contains the singularity at $n \longrightarrow 1$, then $(i = R, L, I)$

$$I_{\delta,i}(n,x) = \tilde{M}_{1+\delta,i}\; f_L(x) + ...,$$

where $\tilde{M}_{1+\delta,i} = M_{1+\delta}$ with $1/\delta \longrightarrow 1/\tilde{\delta}_i$

3. Double-logarithmic approach

1 Leading order without quarks

First of all, we consider the LO approximation without quarks as a pedagogical example of the more cumbersome calculations below. This case is at the same time very simple and very closed to the real situation, because gluons give the basic contribution at small x.

At the momentum space, the solution of the DGLAP equation in this case has the form

$$M_g(n,Q^2) \;=\; M_g(n,Q_0^2)e^{-d_{gg}(n)s},$$

where $M_g(n,Q^2)$ are the moments of the gluon distribution,

$$s = ln\left(\frac{\alpha(Q_0^2)}{\alpha(Q^2)}\right) \quad \text{and} \quad d_{gg} = \frac{\gamma_{gg}^{(0)}(n)}{2\beta_0}$$

The terms $\gamma_{gg}^{(0)}(n)$ and β_0 are respectively the LO coefficients of the gluon-gluon AD and the QCD β-function. Through this work we use the short notation $\alpha(Q^2) = \alpha_s(Q^2)/(4\pi)$.

At LO, s can be written in terms of the QCD scale Λ as:

$$s_{LO} = ln\left(\frac{ln(Q^2/\Lambda_{LO}^2)}{ln(Q_0^2/\Lambda_{LO}^2)}\right)$$

For any perturbatively calculable variable $K(n)$, it is very convenient to separate the singular part when $n \to 1$ (denoted by "$\widehat{K}$") and the regular part (marked as "$\overline{K}$"). Then, the above equation can be represented by the form

$$M_g(n, Q^2) \; = \; M_g(n, Q_0^2) e^{-\hat{d}_{gg} s_{LO}/(n-1)} e^{-\overline{d}_{gg}(n) s_{LO}},$$

with $\hat{\gamma}_{gg} = -8C_A$ and $C_A = N$ for $SU(N)$ group.

Finally, if one takes the flat boundary conditions

$$x f_a(x, Q_0^2) \; = \; A_a,$$

the coefficient $M_a(n, Q_0^2$ becomes

$$M_a(n, Q_0^2) \; = \; \frac{A_a}{n-1} \tag{10}$$

As a first step, we consider the classical double-logarithmic case which corresponds to the acse $\overline{d}_{gg}(n) = 0$.

Then, expanding the second exponential in the above equation

$$M_g^{cdl}(n, Q^2) \;=\; A_g \sum_{k=0}^{\infty} \frac{1}{k!} \frac{\left(-\hat{d}_{gg} s_{LO}\right)^k}{(n-1)^{k+1}}$$

and using the Mellin transformation for $(ln(1/x))^k$:

$$\int_0^1 dx x^{n-2} (ln(1/x))^k \;=\; \frac{k!}{(n-1)^{k+1}}$$

we immediately obtain the well known double-logarithmic behavior

$$f_g^{cdl}(z, Q^2) \;=\; A_g \sum_{k=0}^{\infty} \frac{1}{(k!)^2} \left(-\hat{d}_{gg} s_{LO}\right)^k (ln(1/x))^k \;=\; A_g I_0(\sigma_{LO}),$$

where $I_0(\sigma_{LO})$ is the modified Bessel function with argument $\sigma_{LO} = 2\sqrt{\hat{d}_{gg} s_{LO} ln(x)}$.

1.2 The more general case

For a regular kernel $\tilde{K}(x)$, having Mellin transform

$$K(n) \; = \; \int_0^1 dx x^{n-2} \tilde{K}(x)$$

and the PD $f_a(x)$ in the form $I_\nu(\sqrt{\hat{d} ln(1/x)})$ we have the following equation

$$\tilde{K}(x) \otimes f_a(x) \; = \; K(1) f_a(x) + O(\sqrt{\frac{\hat{d}}{ln(1/x)}})$$

So, one can find the general solution for the LO gluon density without the influence of quarks

$$f_g(z,Q^2) \;=\; A_g I_0(\sigma_{LO}) e^{-\overline{d}_{gg}(1)s_{LO}} \;+\; O(\rho_{LO}),$$

where

$$\rho_{LO} = \sqrt{\frac{\hat{d}_{gg}s_{LO}}{ln(z)}} = \frac{\sigma_{LO}}{2ln(1/z)}\,, \qquad \overline{\gamma}^{(0)}_{gg}(1) = 22 + \frac{4}{3}f$$

and

$$\overline{d}_{gg}(1) = 1 + \frac{4f}{3\beta_0}$$

with f as the number of active quarks.

At the momentum space, the solution of the DGLAP equation at LO has the form (*after diagonalization*)

$$M_a(n,Q^2) = M_a^{+}(n,Q^2) + M_a^{-}(n,Q^2) \text{ and}$$
$$M_a^{\pm}(n,Q^2) = M_a^{\pm}(n,Q_0^2)e^{-d_{\pm}(n)s} = M_a^{\pm}e^{-\hat{d}_{\pm}s/(n-1)}e^{-\overline{d}_{\pm}(n)s},$$

where

$$\begin{aligned}
M_a^{\pm}(n,Q^2) &= \varepsilon_{ab}^{\pm}(n)M_b(n,Q^2), \qquad d_{ab} = \frac{\gamma_{ab}^{(0)}(n)}{2\beta_0},\\
d_{\pm}(n) &= \frac{1}{2}[(d_{gg}(n)+d_{qq}(n))\\
&\pm (d_{gg}(n)-d_{qq}(n))\sqrt{1+\frac{4d_{qg}(n)d_{gq}(n)}{(d_{gg}(n)-d_{qq}(n))^2}}]\\
\varepsilon_{qq}^{\pm}(n) &= \varepsilon_{gg}^{\mp}(n) = \frac{1}{2}(1+\frac{d_{qq}(n)-d_{gg}(n)}{d_{\pm}(n)-d_{\mp}(n)}),
\end{aligned}$$

$$\varepsilon_{ab}^{\pm}(n) = \frac{d_{ab}(n)}{d_{\pm}(n) - d_{\mp}(n)}(a \neq b)$$

As the singular (when $n \to 1$) part of the $+$ component of the anomalous dimension is $\hat{d}_+ = \hat{d}_{gg} = -4C_A/\beta_0$ while the $-$ component does not exist ($\hat{d}_- = 0$), we consider below both cases separately.

2.1 The "+" component

The analysis of the "+" component is practically identical to the case studied before. The only difference lies in the appearance of new terms $\varepsilon^{+}_{ab}(n)$. If they are expanded in the vicinity of $n = 1$ in the form $\varepsilon^{+}_{ab}(n) = \overline{\varepsilon}^{+}_{ab} + (n-1)\tilde{\varepsilon}^{+}_{ab}$, then for the terms $\overline{\varepsilon}^{+}_{ab}$ multiplying $M_b(n, Q^2)$, we have the same results as in previous section:

$$\overline{\varepsilon}^{+}_{ab} M_b(n, Q^2) \stackrel{\mathcal{M}^{-1}}{\longrightarrow} \overline{\varepsilon}^{+}_{ab} A_b I_0(\sigma_{LO}) e^{-\overline{d}_{+}(1) s_{LO}} \; + \; O(\rho_{LO}),$$

where the symbol $\stackrel{\mathcal{M}^{-1}}{\longrightarrow}$ denotes the inverse Mellin transformation. The values of σ and ρ coincide with those defined in the previous section because $\hat{d}_{+} = \hat{d}_{gg}$.

The terms $\tilde{\varepsilon}^+_{ab}$ that come with the additional factor $(n-1)$ in front, lead to the following results

$$(n-1)\tilde{\varepsilon}^+_{ab}\frac{A_b}{(n-1)}e^{-\hat{d}_+ s_{LO}/(n-1)} = \tilde{\varepsilon}^+_{ab}A_b \sum_{k=0}^{\infty}\frac{1}{k!}\frac{\left(-\hat{d}_+ s_{LO}\right)^k}{(n-1)^k}$$

$$\xrightarrow{\mathcal{M}^{-1}} \tilde{\varepsilon}^+_{ab}A_b \sum_{k=0}^{\infty}\frac{1}{k!}\frac{1}{(k-1)!}\left(-\hat{d}_+ s_{LO}\right)^k (ln(1/z))^{k-1}$$

$$= \tilde{\varepsilon}^+_{ab}A_b\rho_{LO}I_1(\sigma_{LO}),$$

i.e. the additional factor $(n-1)$ in momentum space leads to replacing the Bessel function $I_0(\sigma_{LO})$ by $\rho_{LO}I_1(\sigma_{LO})$ in x-space.

Thus, we obtain that the term $\varepsilon^+_{ab}(n)M_b(n,Q^2)$ leads to the following contribution in x space:

$$(\overline{\varepsilon}^+_{ab}I_0(\sigma_{LO}) + \tilde{\varepsilon}^+_{ab}\rho_{LO}I_1(\sigma_{LO}))A_b e^{-\overline{d}_+(1)s_{LO}} + O(\rho_{LO})$$

Because the Bessel function $I_\nu(\sigma)$ has the ν-independent asymptotic behavior $e^\sigma/\sqrt{\sigma}$ at $\sigma \to \infty$ (i.e. $x \to 0$), the second term is $O(\rho)$ and must be kept only when $\overline{\varepsilon}^+_{ab} = 0$. This is the case for the quark distribution at the LO approximation.

Using the concrete AD values, one has

$$f_g^+(x,Q^2) \;=\; (A_g + \frac{4}{9}A_q)I_0(\sigma_{LO})e^{-\overline{d}_+(1)s_{LO}} \;+\; O(\rho_{LO}) \quad \text{and}$$
$$f_q^+(x,Q^2) \;=\; \frac{f}{9}(A_g + \frac{4}{9}A_q)\rho_{LO}I_1(\sigma_{LO})e^{-\overline{d}_+(1)s_{LO}} \;+\; O(\rho_{LO})$$

where $\overline{d}_+(1) = 1 + 20f/(27\beta_0)$.

In this case the anomalous dimension is regular and one has

$$\varepsilon^{-}_{ab}(n) A_b e^{-d_-(n)s} \xrightarrow{\mathcal{M}^{-1}} \bar{\varepsilon}^{-}_{ab}(1) A_b e^{-d_-(1)s_{LO}} + O(x)$$

Using the concrete AD values, we have

$$f_g^-(x, Q^2) = -\frac{4}{9} A_q e^{-d_-(1)s_{LO}} + O(x) \text{ and}$$
$$f_q^-(x, Q^2) = A_q e^{-d_-(1)s_{LO}} + O(x),$$

where $d_-(1) = 16f/(27\beta_0)$.

Finally we present the full small x asymptotic results for PD and F_2 structure function at LO of perturbation theory:

$$f_a(x,Q^2) = f_a^+(x,Q^2) + f_a^-(x,Q^2) \quad \text{and}$$
$$F_2(x,Q^2) = e \cdot f_q(z,Q^2)$$

where f_q^+,f_g^+, f_q^- and f_g^- were already given before and $e = \Sigma_1^f e_i^2/f$ is the average charge square of the f active quarks.

Let us now describe the main conclusions

- The "$+$" and "$-$" components are presented explicitly separated. The "$-$" component $\sim Const$ is negligible at small x (and large Q^2) in comparison with $\rho_{LO} I_1(\sigma_{LO})$ and the LO quark distribution is "driven" by the gluons: $f_q^+(z,Q^2) \approx (f/9)\rho f_g^+(z,Q^2)$. However, at intermediate Q^2, the "$-$" component is essential Thus, in order to give the more general result valid for a wide Q^2 range, we consider PD as the combinations of the "$+$" and "$-$" components, where every component evolves *independently*.

- The separation of the singular and regular parts of the AD performed above leads to the possibility of avoiding complicated methods for evaluating the inverse Mellin convolution or special analyses of DGLAP equations.
  In our case, we use the exact solution to get the moments of the PD. The simple form of the singular part of this exact solution is easily transformed to the x-space. The non-singular part is added by the method of replacing Mellin convolution by usual product. In this case the non-singular part in the x-space is equal to the corresponding contribution for the first moment $n = 1$.

So, we resume the steps we have followed to reach the small x approximate solution of DGLAP shown above:

- Use the n-space exact solution.
- Expand the perturbatively calculated parts (AD and coefficient functions) in the vicinity of the point $n = 1$.
- The singular part with the form

$$A_a(n-1)^k e^{-\hat{d}s_{LO}/(n-1)}$$

leads to Bessel functions in the x-space in the form

$$A_a\left(\frac{\hat{d}s_{LO}}{lnx}\right)^{(k+1)/2} I_{k+1}(2\sqrt{\hat{d}s_{LO}lnx})$$

- The regular part $B(n)\exp\left(-\overline{d}(n)s_{LO}\right)$ leads to the additional coefficient

$$B(1)exp(-\overline{d}(1)s_{LO}) + O(\sqrt{\hat{d}s_{LO}/lnx})$$

behind of the Bessel function in the x-space. Because the accuracy is $O(\sqrt{\hat{d}s_{LO}/lnx})$, it is necessary to use only the first nonzero term, i.e. all terms $(n-1)^k$ in front of $\exp\left(-\hat{d}/(n-1)\right)$, with the exception of one with the smaller k value, can be neglected.

- If the singular part at $n \to 1$ is absent, i.e. $\hat{d} = 0$, the result in the x-space is determined by $B(1)exp(-\overline{d}(1)s_{LO})$ with accuracy $O(x)$.

3. Fits of HERA data

At low x, the structure function $F_2(x,Q^2)$ is related to parton densities as (A.V.K. and G.Parente, 1998)

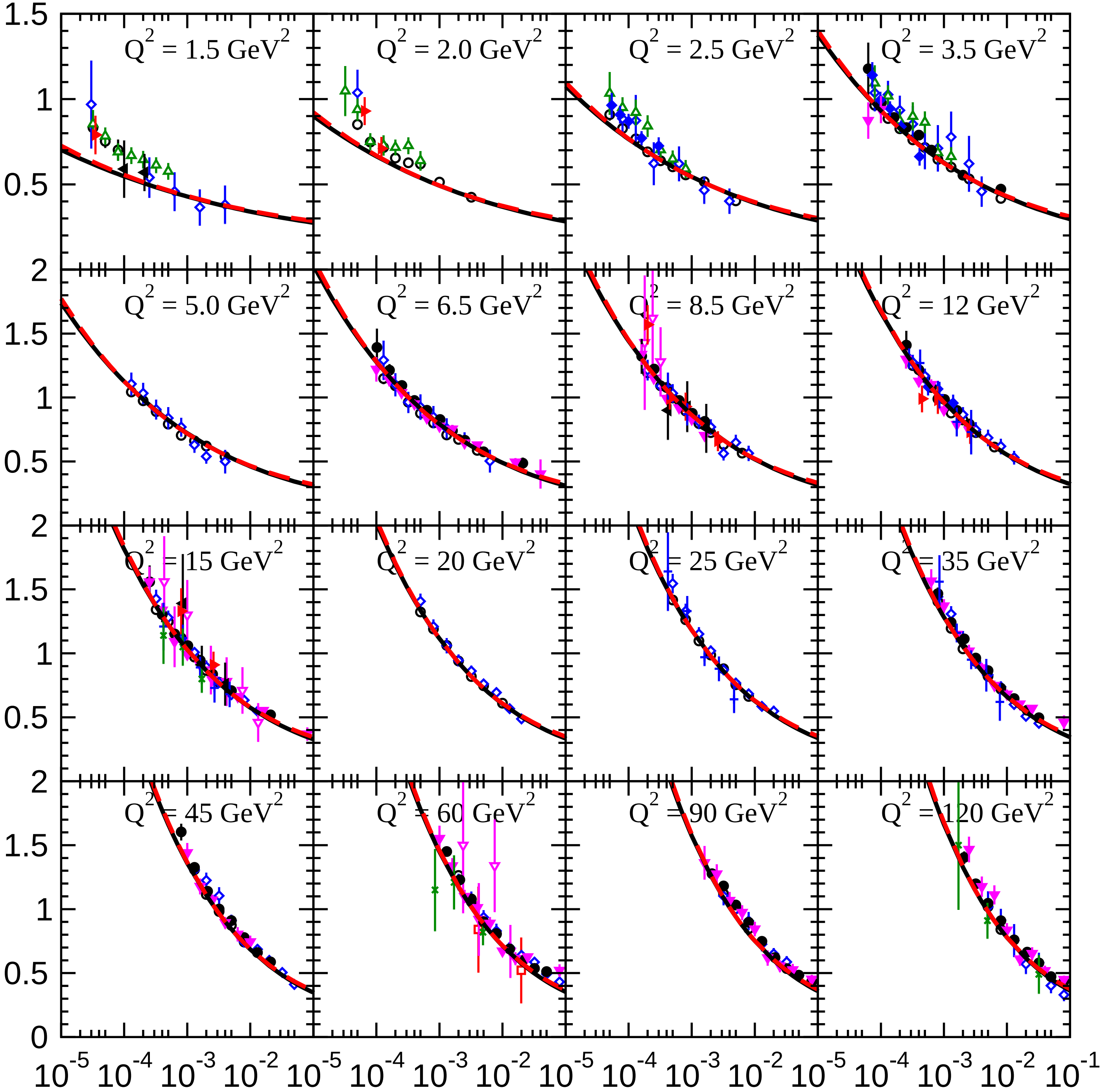
at LO

$$F_2(x,Q^2) = \frac{5}{18} f_q(x,Q^2)$$

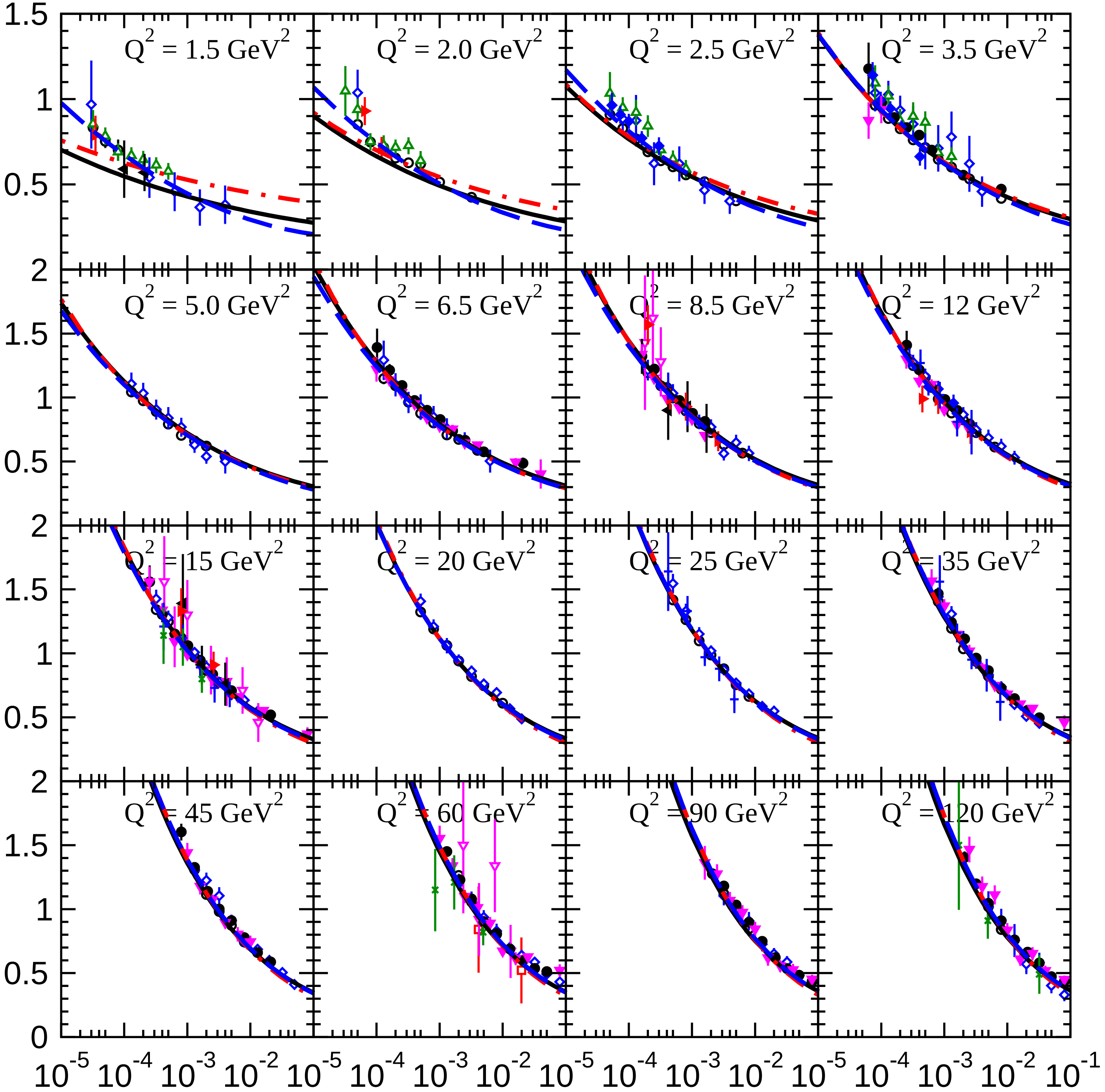
at NLO

$$F_2(x,Q^2) = \frac{5}{18}\left[f_q(x,Q^2) + \frac{2f}{3} a_s(Q^2) f_g(x,Q^2)\right].$$

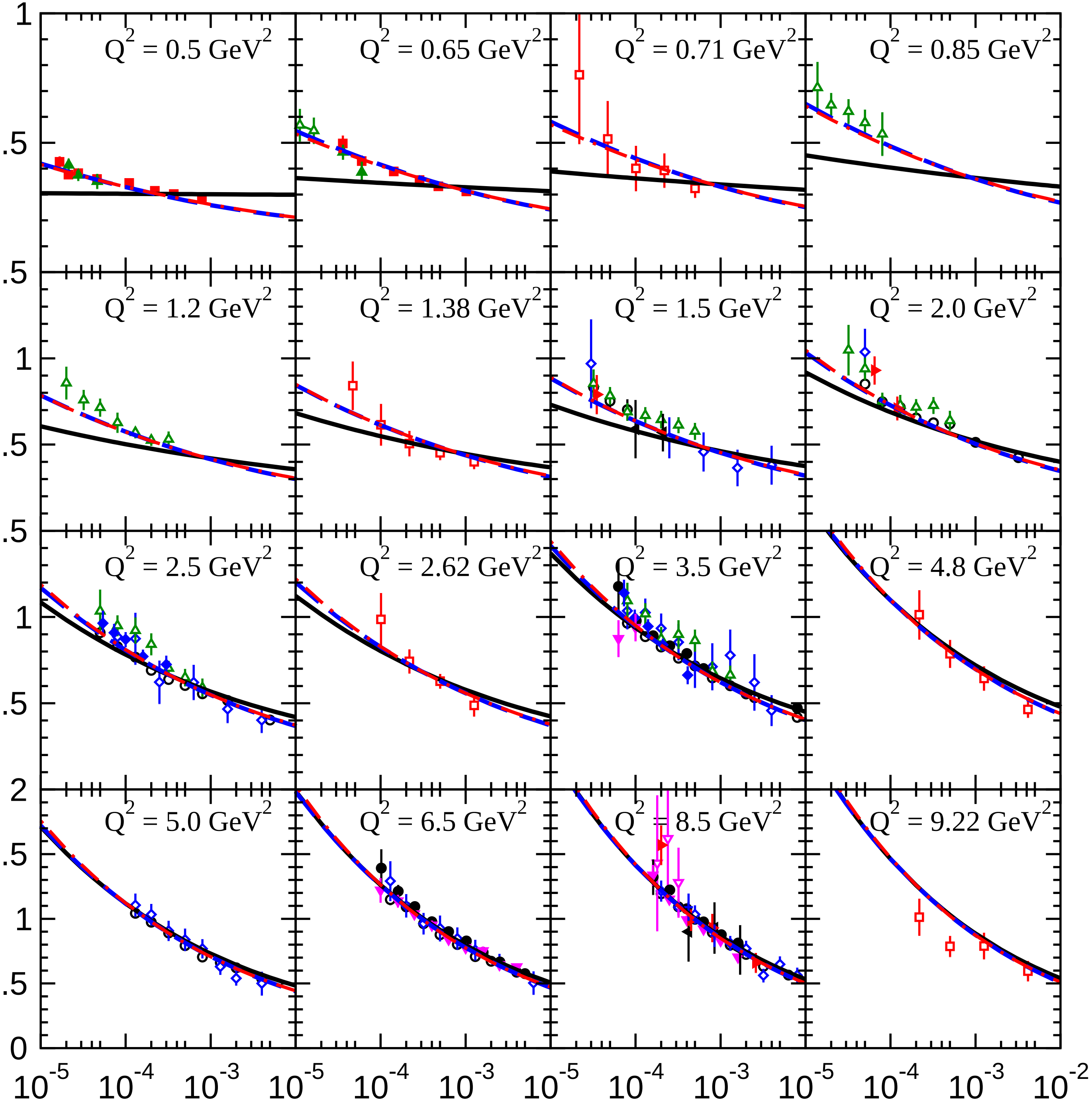
Fits of HERA experimental data of the structure function $F_2(x,Q^2)$ (A.Yu.Illarionov, A.V.K. and G.Parente, 2004): for $f = 4$, NLO twist-two fit of H1 data for F_2 with Q^2 cut: $Q^2 > 1.5$ GeV^2 produces $Q_0^2 = 0.523$ GeV^2, $A_g = 0.060$ and $A_q = 0.844$.



Q^2 = 1.5 GeV^2
Q^2 = 2.0 GeV^2
Q^2 = 2.5 GeV^2
Q^2 = 3.5 GeV^2
Q^2 = 5.0 GeV^2
Q^2 = 6.5 GeV^2
Q^2 = 8.5 GeV^2
Q^2 = 12 GeV^2
Q^2 = 15 GeV^2
Q^2 = 20 GeV^2
Q^2 = 25 GeV^2
Q^2 = 35 GeV^2
Q^2 = 45 GeV^2
Q^2 = 60 GeV^2
Q^2 = 90 GeV^2
Q^2 = 120 GeV^2
0
0.5
1
1.5
2
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}



Q^2 = 1.5 GeV^2
Q^2 = 2.0 GeV^2
Q^2 = 2.5 GeV^2
Q^2 = 3.5 GeV^2
Q^2 = 5.0 GeV^2
Q^2 = 6.5 GeV^2
Q^2 = 8.5 GeV^2
Q^2 = 12 GeV^2
Q^2 = 15 GeV^2
Q^2 = 20 GeV^2
Q^2 = 25 GeV^2
Q^2 = 35 GeV^2
Q^2 = 45 GeV^2
Q^2 = 60 GeV^2
Q^2 = 90 GeV^2
Q^2 = 120 GeV^2
1.5
1
0.5
2
0
10^-5 10^-4 10^-3 10^-2 10^-1



Q^2 = 0.5 GeV^2
Q^2 = 0.65 GeV^2
Q^2 = 0.71 GeV^2
Q^2 = 0.85 GeV^2
Q^2 = 1.2 GeV^2
Q^2 = 1.38 GeV^2
Q^2 = 1.5 GeV^2
Q^2 = 2.0 GeV^2
Q^2 = 2.5 GeV^2
Q^2 = 2.62 GeV^2
Q^2 = 3.5 GeV^2
Q^2 = 4.8 GeV^2
Q^2 = 5.0 GeV^2
Q^2 = 6.5 GeV^2
Q^2 = 8.5 GeV^2
Q^2 = 9.22 GeV^2
1
0.5
1.5
1
0.5
1.5
1
0.5
2
1.5
1
0.5
0
10^{-5}
10^{-4}
10^{-3}
10^{-2}

3. Heavy quarks

Recently, the H1 and ZEUS Collaborations at HERA presented new data on F_2^c and F_2^b (H1+ZEUS) data (K.Lipka, 2009), (A. M. Cooper-Sarkar, 2010).

At small x values, of order 10^{-4}, F_2^c was found to be around 25% of F_2, which is considerably larger than what was observed by the European Muon Collaboration (EMC) at CERN at larger x values, where it was only around 1% of F_2. Extensive theoretical analyses in recent years have generally served to establish that the F_2^c data can be described through the perturbative generation of charm within QCD.

Here I demonstrate the compact low-x approximation formulae for the SF F_2^c

$$F_2^c(x, Q^2) \approx M_{2,g}^c(1+\delta, Q^2, m_c^2)\, x f_g(x, Q^2). \tag{11}$$

Through NLO, $M_{2,g}(1, Q^2)$ exhibits the structure

$$\begin{aligned} M_{2,g}(1, Q^2) &= e_i^2 \alpha(\mu)\{M_{2,g}^{(0)}(1, a_c) + \alpha(\mu)[M_{2,g}^{(1)}(1, a_c). \\ &\quad + M_{2,g}^{(2)}(1, a_i) \ln(\mu^2/m_i^2)]\} + \mathcal{O}(\alpha^3), \end{aligned} \tag{12}$$

where e_i is the fractional electric charge of heavy quark i and $\alpha(\mu) = \alpha_s(\mu)/(4\pi)$ is the couplant.

The LO coefficient functions of PGF can be obtained from the QED case by adjusting coupling constants and colour factors, and they read

$$\begin{aligned} C_{2,g}^{(0)}(x,a) = & -2x\{[1-4x(2-a)(1-x)]\beta - [1-2x(1-2a) \\ & + 2x^2(1-6a-4a^2)]L(\beta)\}, \end{aligned}$$

where

$$a = \frac{m^2}{Q^2}, \qquad \beta = \sqrt{1-\frac{4ax}{1-x}}, \qquad L(\beta) = \ln\frac{1+\beta}{1-\beta}.$$

Using the auxiliary formulas

$$\int_0^b dx\, x^r \beta = \begin{cases} 1 - 2aJ(a), & \text{if } r = 0 \\ \frac{b}{2}[1 - 2a - 4a(1+3a)J(a)], & \text{if } r = 1 \\ \frac{b^2}{3}[(1+3a)(1+10a) - 6a(1+6a+10a^2)J(a)], & \text{if } r = 2 \end{cases},$$

$$\int_0^b dx\, x^r L(\beta) = \begin{cases} J(a), & \text{if } r = 0 \\ -\frac{b}{2}[1 - (1+2a)J(a)], & \text{if } r = 1 \\ -\frac{b^2}{3}[3(1+2a) - 2(1+4a+6a^2)J(a)], & \text{if } r = 2 \end{cases},$$

where

$$b = \frac{1}{1+4a}, \qquad J(a) = -\sqrt{b}\ln t, \qquad t = \frac{1-\sqrt{b}}{1+\sqrt{b}}, \tag{13}$$

we perform the Mellin transformation to find

$$M_{2,g}^{(0)}(1,a) = \frac{2}{3}[1 + 2(1-a)J(a)]. \tag{14}$$

The NLO coefficient functions of PGF are rather lengthy and not published in print; they are only available as computer codes. The high-energy asymptorics (Catani et al., 1992):

$$C_{2,g}^{(j)}(x,a) = \beta R_{2,g}^{(j)}(1,a), \tag{15}$$

with

$$\begin{aligned} R_{2,g}^{(1)}(1,a) &= \frac{8}{9} C_A [5 + (13 - 10a) J(a) + 6(1-a) I(a)], \\ R_{2,g}^{(2)}(1,a) &= -4 C_A M_{2,g}^{(0)}(1,a), \quad C_A = N, \end{aligned}$$

where $J(a)$ is defined by Eq. (13), and

$$I(a) = -\sqrt{b}\left[\zeta(2) + \frac{1}{2}\ln^2 t - \ln(ab)\ln t + 2\,\mathrm{Li}_2(-t)\right],$$

Here $\mathrm{Li}_2(x) = -\int_0^1 (dy/y)\ln(1-xy)$ is the dilogarithmic function.

As already mentioned before, the Mellin transforms of $C_{k,g}^{(j)}(x, a)$ exhibit singularities in the limit $\delta_l \to 0$ The terms involving $1/\delta_l$ depend on the exact form of the subasymptotic low-x behaviour encoded in $\tilde{f}_g^l(x, Q^2)$, as

$$\frac{1}{\delta_l} = \frac{1}{\tilde{f}_g^l(x, Q^2)} \int_{\hat{x}}^1 \frac{dy}{y} \tilde{f}_g^l(y, Q^2), \tag{16}$$

where $\hat{x} = x/b$.

The $+$ and $-$ components of the gluon PDF:

$$f_g^+(x, Q^2) \propto I_0(\sigma), \qquad f_g^-(x, Q^2) \propto \text{const.}, \tag{17}$$

where I_n denote the modified Bessel functions. Here and in the following, we employ the variables

$$\sigma = \sqrt{\frac{48s}{\beta_0} \ln \frac{1}{x}}, \qquad \rho = \frac{\sigma}{2 \ln(1/x)}, \tag{18}$$

where β_0 is the first coefficient of the QCD beta function and $s = \ln[\alpha(Q_0)/\alpha(Q)]$, with Q_0 being the initial scale of the DGLAP evolution. We thus have

$$\frac{1}{\delta_+} = \frac{1}{\hat{\rho}} \frac{I_1(\hat{\sigma})}{I_0(\hat{\sigma})}, \qquad \frac{1}{\delta_-} = \ln \frac{1}{\hat{x}}, \tag{19}$$

where $\hat{\sigma}$ and $\hat{\rho}$ are σ and ρ evaluated at $x = \hat{x}$, respectively.

Because the ratio $f_g^-(x,Q^2)/f_g^+(x,Q^2)$ is rather small at the Q^2 values considered,

$$F_2^c(x,Q^2) \approx \tilde{M}_{2,g}(1,Q^2)xf_g(x,Q^2), \tag{20}$$

where $\tilde{M}_{2,g}(1,Q^2)$ is obtained from $M_{2,g}(n,Q^2)$ by taking the limit $n \to 1$ and replacing $1/(n-1) \to 1/\delta_+$.

Using the identity

$$\frac{1}{\tilde{f}_g^+(x,Q^2)} \int_{\hat{x}}^1 \frac{dy}{y} \tilde{f}_g^+(y,Q^2)\beta(\hat{x}/y) = \frac{1}{\delta_+} - \ln(ab) - bJ(a), \tag{21}$$

we find the Mellin transform of Eq. (15) to be

$$\tilde{M}_{2,g}^{(j)}(1,a) = \left[\frac{1}{\delta_+} - \ln(ab) - bJ(a)\right] R_{k,g}^{(j)}(1,a) \quad (j=1,2).$$

The rise of the NLO terms as $x \to 0$ is in agreement with earlier investigations.

5 Results

We choose $m_c = 1.25$ GeV in agreement with Particle Data Group.

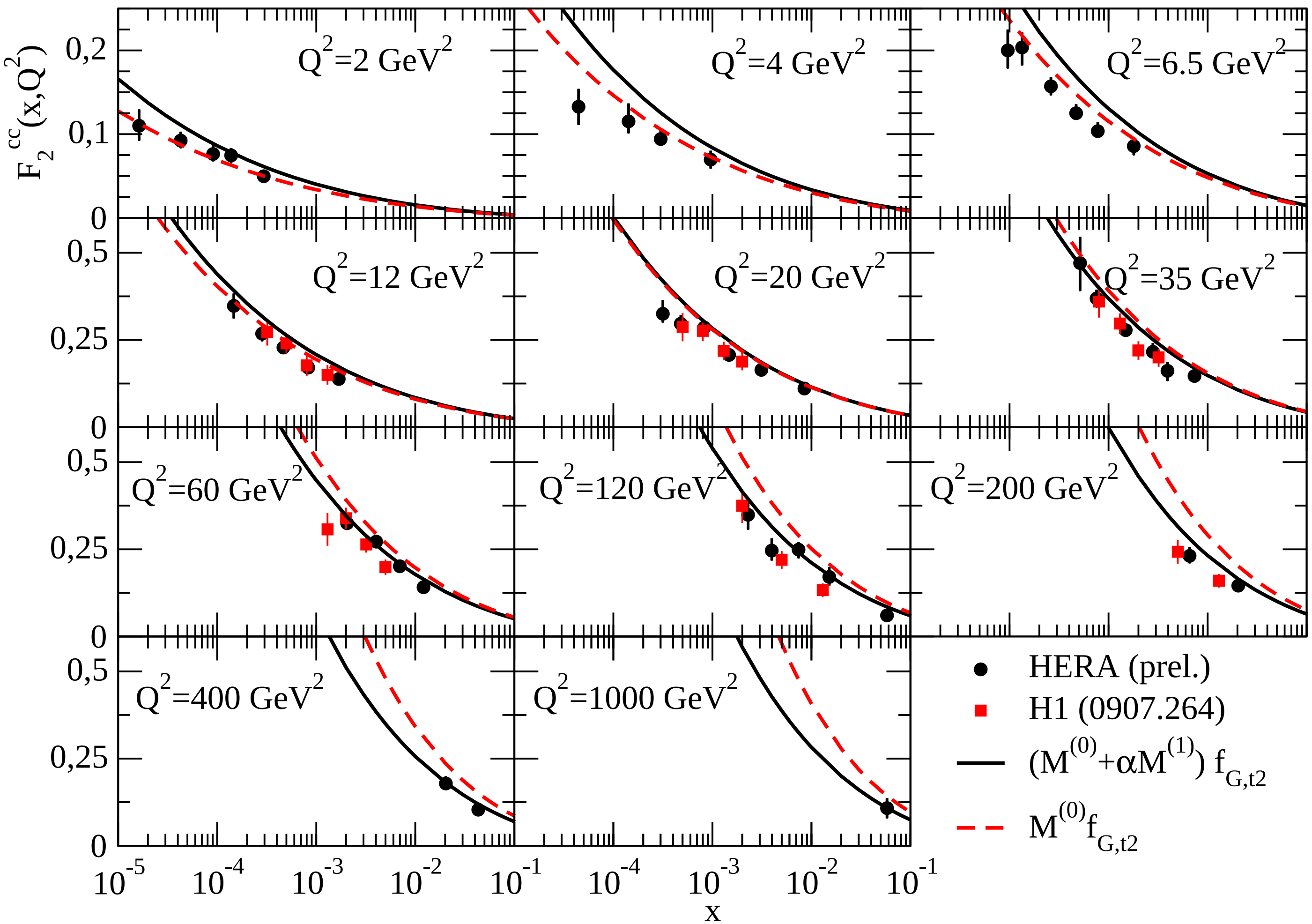
We put $\mu^2 = Q^2 + 4m_c^2$, which is the standart scale in heavy quark production.

The PDF parameters μ_0^2, A_q and A_g have been fixed in the fits of F_2 experimental data. Their values depend on conditions chosen in the fits: the order of perturbation theory and the number f of active quarks.

Below b-quark threshold, the scheme with $f = 4$ has been used in the fits of F_2 data. Note, that the F_2 structure function contains F_2^c as a part. In the fits, the NLO gluon density and the LO and NLO quark ones contribute to F_2^c, as the part of to F_2. Then, now in PGF scattering the LO coefficient function corresponds in $m \to 0$ limit to the standart NLO Wilson coefficient (together with the product of the LO anomalous dimension γ_{qg} and $\ln(m_c^2/Q^2)$. It is a general situation, i.e. the coefficient funstion of PGF scattering at some order of perturbation theory corresponds to the standart DIS Wilson coefficient with the one step higher order. The reason is following: the standart DIS analysis starts with handbag diagram of photon-quark scattering and photon-gluon interaction begins at one-loop level.

Thus, in our F_2^c analysis in the LO approximation of PGF process we should take $xf_a(x,Q^2)$ extracted from fits of F_2 data at $f=4$ and NLO approximation. In practice, we apply our $f=4$ NLO twist-two fit of H1 data for F_2 with Q^2 cut: $Q^2 > 1.5$ GeV^2, which produces $Q_0^2 = 0.523$ GeV^2, $A_g = 0.060$ and $A_q = 0.844$.

Correspondingly, the NLO approximation of PGF process needs the gluon density exracted from fits of F_2 data at NNLO approximation, which is not yet known in generalized double-asymptotic scalling regime. As we see, however, from the modern global fits (M. Dittmar et al., 2005), "Working Group I: Parton distributions: Summary report for the HERA LHC" , the difference between NLO and NNLO gluon densities is not so large. So, we can apply the NLO form of $xf_a(x,Q^2)$ for our NLO PGF analysis, too.



$F_2^{cc}(x,Q^2)$
Q^2=2 GeV^2
Q^2=4 GeV^2
Q^2=6.5 GeV^2
Q^2=12 GeV^2
Q^2=20 GeV^2
Q^2=35 GeV^2
Q^2=60 GeV^2
Q^2=120 GeV^2
Q^2=200 GeV^2
Q^2=400 GeV^2
Q^2=1000 GeV^2
0
0,1
0,2
0,25
0,5
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
x
HERA (prel.)
H1 (0907.264)
$(M^{(0)}+\alpha M^{(1)})\, f_{G,t2}$
$M^{(0)} f_{G,t2}$

Conclusion

- I have demonstrated

  the method to replace the Mellin convolution by usual products at low x;

  the structure function $F_2(x, Q^2)$ and parton density in the doble-logarithmic approximation;

- Low x double-logarithmic asymptotics of $F_2(x, Q^2)$ and $F_2^c(x, Q^2)$ are in good agreement with data from HERA.

Next steps:

- To study $F_2^b(x, Q^2)$ structure function (it is in progress)