UNIVERSAL CONSTITUENTS FOR DESCRIPTION OF A STRUCTURES BOTH THE LEPTONS AND HADRONS

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MAIN RESULTS OF LEPTON SECTOR ANALYSIS

- **I. All leptons possess structural individual characteristics. They are related with properties of lepton relativistic wave equations.**
- **II. Simplest constituents of these structures are four connected components of homogeneous Lorentz group. Four groups, which realized these connected components was obtained in explicit form.**

These groups are:

1. **group $\mathrm{d}_\gamma \rightarrow$ proper orthochronous representation;**
2. **group $\mathrm{f}_\gamma \rightarrow$ improper orthochronous, P-conjugate representation;**
3. **group $\mathrm{b}_\gamma \rightarrow$ proper antichronous, T-cojugate representation;**
4. **group $\mathrm{c}_\gamma \rightarrow$ improper antichronous, (PT)-cojugate representation.**

- **III. So-called intrinsic symmetries of the leptons are reflection or conversion space-time symmetries taking into account (P)-,(T)-, (PT)-inversion and symmetries of relativistic quantum mechanics.**
- **IV.All distinctions between leptons and possibilities of their specification by means of quantum numbers are determined by individual structure of every lepton equation.**

Necessary and sufficient conditions for obtaining of lepton equations.

- **1. The equations must be invariant and covariant under homogeneous Lorentz transformations taken into account all four connected components.**
- **2. The equations must be formulated on the base of irreducible representations of the groups determining every lepton equation.**
- **3. Conservation of four-vector of probability current must be fulfilled and fourth component of the current must be positively defined.**
- **4. The lepton spin is supposed equal to 1/2.**
- **5. Every lepton equation must be reduced to Klein-Gordon equation.**

We see here different kinds of symmetries: with respect to the homogeneous Lorentz group, relativistic wave equations, quantum mechanics.

Structures of the stable lepton groups.

- **The Dirac equation** — $D_\gamma(II)$: $\boxed{\mathbf{d}_\gamma,\ \mathbf{b}_\gamma,\ \mathbf{f}_\gamma,}$
  structural invariant $In[D_\gamma(II)] = -1$.
- **The equation for a doublet of massive neutrinos** — $D_\gamma(I)$: $\boxed{\mathbf{d}_\gamma,\ \mathbf{c}_\gamma,\ \mathbf{f}_\gamma}$,
  structural invariant $In[D_\gamma(I)] = 1$.
- **The equation for a quartet of massless neutrinos** —
  $D_\gamma(III)$: $\boxed{\mathbf{d}_\gamma,\ \mathbf{b}_\gamma,\ \mathbf{c}_\gamma,\ \mathbf{f}_\gamma}$,
  structural invariant $In[D_\gamma(III)] = 0$.
- **The equation for a massless T-singlet** — $D_\gamma(IV)$: $\boxed{\mathbf{b}_\gamma}$,
  structural invariant $In[D_\gamma(IV)] = -1$.
- **The equation for a massless (PT)-singlet** — $D_\gamma(V)$: $\boxed{\mathbf{c}_\gamma}$,
  structural invariant $In[D_\gamma(V)] = 1$.

Every group related with corresponding equation has

nonrecurrent composition.

Structures of the unstable lepton groups.

Group Δ_1 has the following defining relations:

$$\Gamma_\mu\Gamma_\nu + \Gamma_\nu\Gamma_\mu = 2\delta_{\mu\nu}, \quad (\mu, \nu = 1, 2, 3, 4, 5) \tag{1}$$

As a result we obtain the following composition:

$$\boxed{\Delta_1\{D_\gamma(II), \quad D_\gamma(III), \quad D_\gamma(IV)\}} \qquad In[\Delta_1] = -1.$$

Group Δ_3 has the following defining relations:

$$\begin{array}{ll} \Gamma_s\Gamma_t + \Gamma_t\Gamma_s = 2\delta_{st}, & (s, t = 1, 2, 3, 4), \\ \Gamma_s\Gamma_5 + \Gamma_5\Gamma_s = 0, & (s = 1, 2, 3, 4), \\ \Gamma_5^2 = -I. & \end{array}$$

It follows from here:

$$\boxed{\Delta_3\{D_\gamma(II), \quad D_\gamma(I), \quad D_\gamma(III)\}} \qquad In[\Delta_3] = 0.$$

Structures of the unstable lepton groups.

Group Δ_2 has the following defining relations:

$$\begin{array}{ll} \Gamma_s\Gamma_t+\Gamma_t\Gamma_s=2\delta_{st}, & (s,t=1,2,3), \\ \Gamma_s\Gamma_4+\Gamma_4\Gamma_s=0, & (s=1,2,3), \\ \Gamma_4^2=-I. & \\ \Gamma_u\Gamma_5+\Gamma_5\Gamma_u=0, & (u=1,2,3,4), \\ \Gamma_5^2=-I. & \end{array}$$

We obtain in this case:

$$\boxed{\Delta_2\{D_\gamma(I), \quad D_\gamma(III), \quad D_\gamma(V)\}} \qquad In[\Delta_2]=1.$$

All three groups have its own structures.

We see that four conjugate components of Lorentz group allowed to describe different leptons due to complication of structural constituents.

Significance and importance of structural notions increased after successes of quark model.

Shortly after $\Longrightarrow$ $\boxed{\text{no-go theorems}}$

The most strong statement is known as O'Raifeartaigh theorem (1965).

XXIII International colloquium on group theoretical methods in physics
(Dubna, Russia, 2000)

Here Professor **Laughlin O'Raifeartaigh** was awarded by Wigner medal

$\boxed{\textbf{It means that problem is important and unresolved}}$

A.Pais (1966): Are there any alternatives left to the internal symmetry $\otimes$ Poincare group picture in face of the no-go theorems?

$\boxed{\textbf{We suggest alternative based on structural approach}}$

The whole lepton sector was left out structural possibilities of quark model and unitary scheme.

Reasons:

- **Lepton sector can not be described beyond the framework of consistent relativism, while quark model does not satisfy this requirement.**
- **At the same time it is not excluded possibility, that quarks are <u>more complex and extensive formations</u> then constituents for leptons.**

EXPERIENCE OF THE WORK WITH THE LEPTON SECTOR CREATES PREREQUISITES FOR DETAILED ELABORATION AND GENERALIZATION MOST ESSENTIAL NOTIONS BOTH THE HADRONS AND THEIR CONSTITUENTS

SPIN

It is kinematic manifestation of relativity having two sides. First one is related with the form of wave equation and second side is related with observable value of spin momentum. Observed spin manifestation is determined by concrete condition of interactions. Any spin states can be obtained with the help of half spin states, that is $\mathbf{d}_\gamma$, $\mathbf{b}_\gamma$, $\mathbf{c}_\gamma$, $\mathbf{f}_\gamma$.

QUANTUM NUMBERS

G.Weyl: Quantum numbers are indexes characterizing group representations.

One can see on example of unstable leptons how complication of structure leads to additional properties i.e. to additional quantum numbers.

ANTIMATTER

Exhaustive analysis of the lepton equation groups shows, that necessary condition for description particle and antiparticle in the frame of the same equation is a presence of T-conjugate representations in the group of this equation. Conjugate components allowed to generalize this notion on any structural combination of the components including any components of hadron. All conjugate components of Lorentz group are connected between themselves by means of P-, T- and (PT)-inversions.

$$\begin{gathered}
\langle T\rangle d_\gamma = b_\gamma, \quad \langle P\rangle d_\gamma = f_\gamma, \quad \langle PT\rangle d_\gamma = c_\gamma, \\
\langle T^{-1}\rangle b_\gamma = d_\gamma, \quad \langle P\rangle b_\gamma = c_\gamma, \quad \langle T^{-1}P\rangle b_\gamma = f_\gamma, \\
\langle T^{-1}\rangle c_\gamma = f_\gamma, \quad \langle P^{-1}\rangle c_\gamma = b_\gamma, \quad \langle T^{-1}P^{-1}\rangle c_\gamma = d_\gamma, \\
\langle T\rangle f_\gamma = c_\gamma, \quad \langle P^{-1}\rangle f_\gamma = d_\gamma, \quad \langle P^{-1}T\rangle f_\gamma = b_\gamma.
\end{gathered}$$

Here:

$\langle T\rangle$ means transition $b_k \to b'_k = ib_k \quad (k = 1, 2, 3)$,
$\langle P\rangle$ means transition $a_2 \to a'_2 = ia_2$.

Moreover decisive and essential factor is agreement between internal structure and adequate form of four-vector current for real observed particle.

Because of called reasons conjugate components of Lorentz group possess larger possibilities for more detailed descriptions leptons as well as hadrons and their substructures.

Summary

- We obtained all possible kinds of lepton equations in the frame work of few fixed suppositions.
- It appears a possibility for primary structural classification of leptons: singlets, doublets, quartets. This lepton separation is connected with relations particle-antiparticle. It provides leptons with their own structures and hence necessary quantum numbers.
- There is no principled obstacles for extension of the method on the hadron sector by means of composition complications on the base of connected components. It will be further more precise definition and clarity of hadron structure.

Bridging structures of the leptons and hadrons on a common structural base is bridging electroweak and strong interactions.

APPENDICES

A new (for physical applications) and effective tool for analysis and constructing lepton equations was found, i.e. numerical characteristic of irreducible matrix group.

Theorem. If $D = \{\gamma_1, ..., \gamma_\rho\}$ is an irreducible matrix group, then

$$In[D] = \frac{1}{\rho}\sum_{i=1}^{\rho}\chi(\gamma_i^2) = \left\{\begin{array}{c} 1 \\ -1 \\ 0 \end{array}\right. \qquad (2)$$

Here ρ - is order of the group, $\chi(\gamma_i^2)$ - is a trace of i-matrix squared. $In[D]$ — will be called **structural invariant** of D-group.

Lie algebra of d_γ-group (proper representation) is:

$$
\begin{array}{lll}
[a_1, a_2] = 2a_3, & [a_2, a_3] = 2a_1, & [a_3, a_1] = 2a_2, \\
[b_1, b_2] = -2a_3, & [b_2, b_3] = -2a_1, & [b_3, b_1] = -2a_2, \\
[a_1, b_1] = 0, & [a_2, b_2] = 0, & [a_3, b_3] = 0, \\
[a_1, b_2] = 2b_3, & [a_1, b_3] = -2b_2, & \\
[a_2, b_3] = 2b_1, & [a_2, b_1] = -2b_3, & \\
[a_3, b_1] = 2b_2, & [a_3, b_2] = -2b_1. &
\end{array}
$$

Lie algebra of f_γ-group (P-conjugate representation) is:

$$
\begin{array}{lll}
[a_1, a_2'] = 2a_3', & [a_2', a_3'] = -2a_1, & [a_3', a_1] = 2a_2', \\
[b_1', b_2'] = -2a_3', & [b_2', b_3'] = 2a_1, & [b_3', b_1'] = -2a_2', \\
[a_1, b_1'] = 0, & [a_2', b_2'] = 0, & [a_3', b_3'] = 0, \\
[a_1, b_2'] = 2b_3', & [a_1, b_3'] = -2b_2', & \\
[a_2', b_3'] = -2b_1', & [a_2', b_1'] = -2b_3', & \\
[a_3', b_1'] = 2b_2', & [a_3', b_2'] = 2b_1'. &
\end{array}
$$

Lie algebra of b_γ-group (T-conjugate representation) is:

$$\begin{array}{lll}
[a_1, a_2] = 2a_3, & [a_2, a_3] = 2a_1, & [a_3, a_1] = 2a_2, \\
[b'_1, b'_2] = 2a_3, & [b'_2, b'_3] = 2a_1, & [b'_3, b'_1] = 2a_2, \\
[a_1, b'_1] = 0, & [a_2, b'_2] = 0, & [a_3, b'_3] = 0, \\
[a_1, b'_2] = 2b'_3 & [a_1, b'_3] = -2b'_2, & \\
[a_2, b'_3] = 2b'_1, & [a_2, b'_1] = -2b'_3, & \\
[a_3, b'_1] = 2b'_2, & [a_3, b'_2] = -2b'_1, &
\end{array}$$

Lie algebra of b_γ-group ((PT)-conjugate representation) is:

$$
\begin{array}{lll}
[a_1, a'_2] = 2a'_3, & [a'_2, a'_3] = -2a_1, & [a'_3, a_1] = 2a'_2, \\
[b^*_1, b^*_2] = 2a'_3, & [b^*_2, b^*_3] = -2a_1, & [b^*_3, b^*_1] = 2a'_2, \\
[a_1, b^*_1] = 0, & [a'_2, b^*_2] = 0, & [a'_3, b^*_3] = 0, \\
[a_1, b^*_2] = 2b^*_3 & [a_1, b^*_3] = -2b^*_2, & \\
[a'_2, b^*_3] = -2b^*_1, & [a'_2, b^*_1] = -2b^*_3, & \\
[a'_3, b^*_1] = 2b^*_2, & [a'_3, b^*_2] = 2b^*_1. &
\end{array}
$$

Here (PT)=(P)(T)=(T)(P) means sequential action (P)- and (T)-conjugation.

The defining relations for the groups of stable leptons

Dirac

$$\begin{aligned} &D_\gamma(II) : d_\gamma, b_\gamma, f_\gamma. \\ &\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\delta_{\mu\nu}, \\ &\mu, \nu = 1, 2, 3, 4. \end{aligned} \tag{3}$$

Majorana

$$\begin{aligned} &D_\gamma(I) : d_\gamma, c_\gamma, f_\gamma. \\ &\gamma_s\gamma_t + \gamma_t\gamma_s = 2\delta_{st}, \\ &\gamma_4\gamma_s + \gamma_s\gamma_4 = 0 \\ &\gamma_4^2 = -1, s, t = 1, 2, 3 \end{aligned} \tag{4}$$

Pauli

$$\begin{aligned} &D_\gamma(III) : d_\gamma, b_\gamma, c_\gamma, f_\gamma. \\ &\gamma_s\gamma_t + \gamma_t\gamma_s = 2\delta_{st}, \\ &\gamma_4\gamma_s - \gamma_s\gamma_4 = 0 \\ &\gamma_4^2 = 1, s, t = 1, 2, 3 \end{aligned} \tag{5}$$

The defining relations for the groups of stable leptons

T-singlet

$$\begin{array}{l} D_\gamma(IV) : b_\gamma. \\ \gamma_s\gamma_t + \gamma_t\gamma_s = -2\delta_{st}, s, t = 1, 2, 3 \\ \gamma_4\gamma_s - \gamma_s\gamma_4 = 0, s = 1, 2, 3, \gamma_4^2 = 1 \end{array} \tag{6}$$

P-singlet

$$\begin{array}{l} D_\gamma(V) : c_\gamma. \\ \gamma_s\gamma_t + \gamma_t\gamma_s = 0, s \neq t, s, t = 1, 2, 3 \\ \gamma_1^2 = \gamma_2^2 = 1, \gamma_3^2 = -1, \\ \gamma_4\gamma_s - \gamma_s\gamma_4 = 0, \gamma_4^2 = 1, s = 1, 2, 3. \end{array} \tag{7}$$