

# Soft Pomeron structure and Chromomagnetic quark-gluon interaction

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# Quark-gluon interaction induced by instantons

## Instanton model for QCD vacuum

Instantons are strong fluctuations of vacuum gluon fields (review was done in the talk by Prof. Musakhanov)

$$A_{\mu}^a = \frac{2}{g_s} \eta_{a\mu\nu} \frac{(x - x_0)_{\nu}}{(x - x_0)^2 + \rho^2}$$

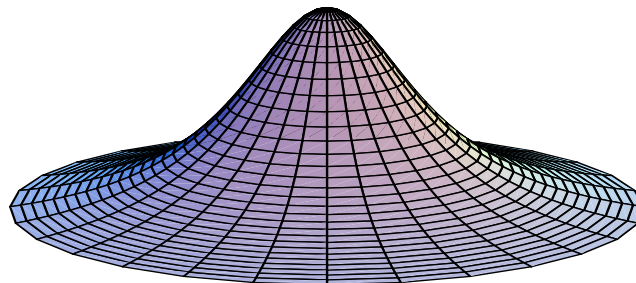
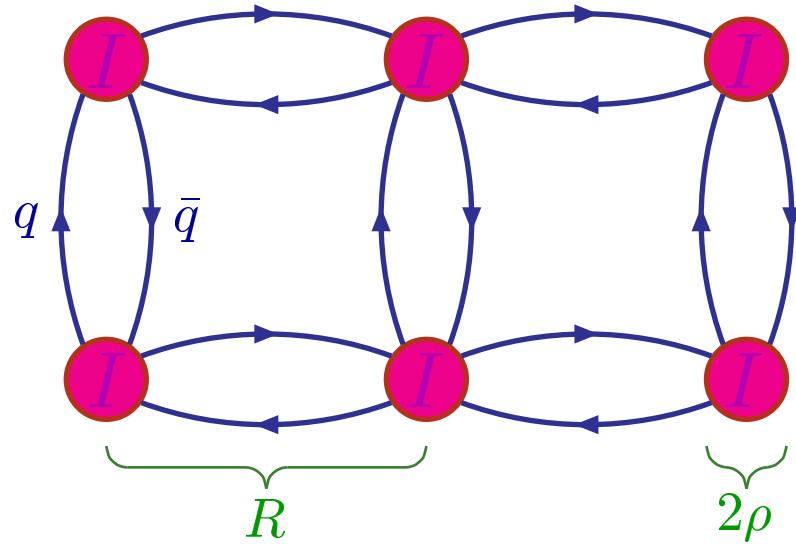


Figure 1: Instanton solution in QCD.

Instanton liquid model for QCD vacuum [Shuryak, Diakonov, Petrov ...]



One can introduce the “density” of instantons

$$n_I(\rho) \sim \exp\left(-\frac{2\pi}{\alpha_s(\rho)}\right)$$

$R$  - distance between instantons  
 $\rho \approx 0.3$  fm - instanton size

If  $R \gg \rho \rightarrow$  vacuum is a “gas” of instantons

If  $R \approx 3\rho \rightarrow$  vacuum is an instanton “liquid”

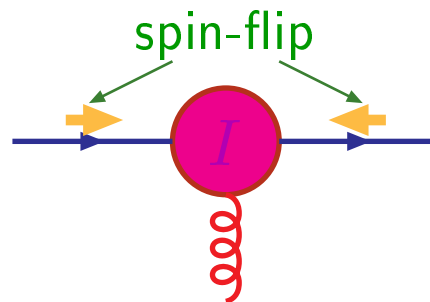
- In the general case, the interaction vertex of massive quark with gluon can be written in the following form:

$$V_\mu(k_1^2, k_2^2, q^2)t^a = -g_s t^a \left[ \gamma_\mu F_1(k_1^2, k_2^2, q^2) + \frac{\sigma_{\mu\nu} q_\nu}{2M_q} F_2(k_1^2, k_2^2, q^2) \right],$$

- Anomalous quark chromomagnetic moment (AQCM):

$$\mu_a = F_2(0, 0, 0).$$

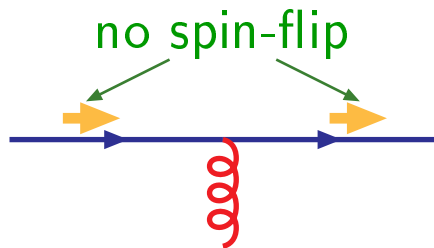
AQCM within instanton model for QCD vacuum [N.K. (1996)]



$$\Delta\mathcal{L} = -i\mu_a \frac{g_s}{4m_q^*} \bar{q} \sigma_{\mu\nu} t^a q G_{\mu\nu}^a$$

$$\mu_a \approx -0.4 \text{ [N.K. 1996]}$$

$$\mu_a \approx -1.6 \text{ [Diakonov 2002]}$$



pQCD quark-gluon vertex

$$\Delta\mathcal{L} = g_s \bar{q} \gamma_\mu t^a q A_\mu^a$$

Effective instanton induced quark-gluon vertex in local limit (small instanton size)  $\rho \rightarrow 0$

$$V_\beta^a = - \int dU d\rho \left\{ \frac{2\pi^2 \rho^2 n_I(\rho)}{M_q} \bar{q}_R U \left[ I + \frac{i\bar{\eta}_{b\mu\nu} \tau^b \sigma_{\mu\nu}}{4} \right] U^\dagger q_L \text{Tr}[\lambda^a \hat{A}_\beta] \right\},$$

where

$$\hat{A}_\beta = -iq_\alpha \bar{\eta}_{c\alpha\beta} \frac{2\pi^2 \rho^2}{g_s} U \tau^c U^\dagger$$

is amputated instanton field and  $U$  is present the orientation of the instanton in  $SU(3)_c$  color space.

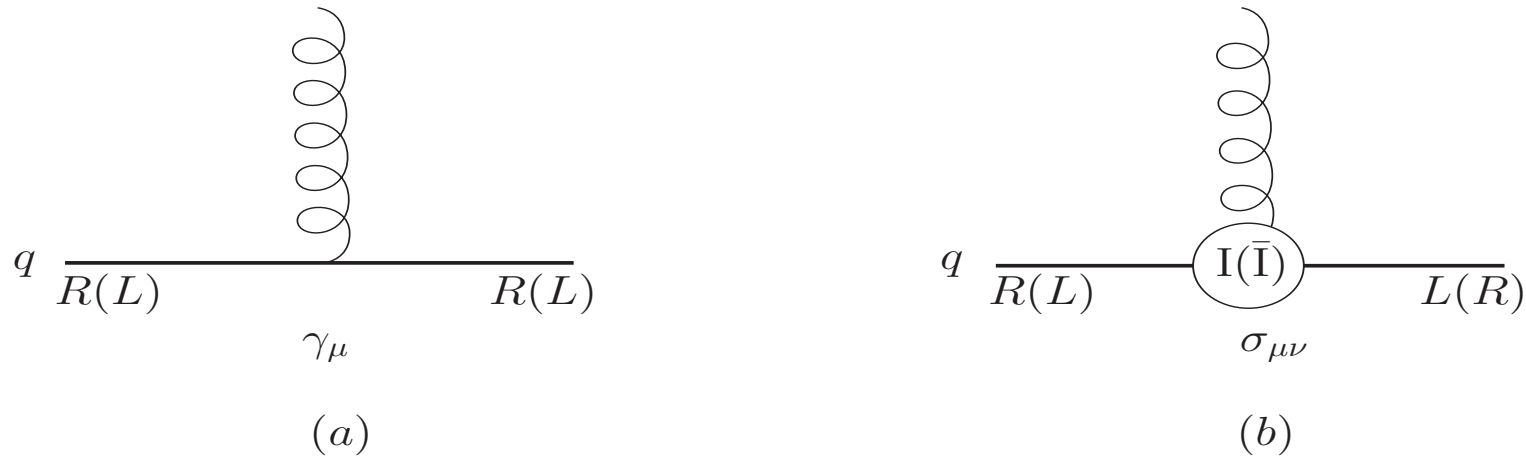


Figure 2: The quark-gluon coupling: a) perturbative and b) nonperturbative. Symbols  $R$  and  $L$  denote quark chirality and symbol  $I(\bar{I})$  denotes instanton (antiinstanton).

The shape of form factor  $F_2(k_1^2, k_2^2, q^2)$  is fixed:

$$F_2(k_1^2, k_2^2, q^2) = \mu_a \Phi_q(|k_1| \rho/2) \Phi_q(|k_2| \rho/2) F_g(|q| \rho) ,$$

where

$$\Phi_q(z) = -z \frac{d}{dz} (I_0(z)K_0(z) - I_1(z)K_1(z)),$$

$$F_g(z) = \frac{4}{z^2} - 2K_2(z)$$

are the Fourier-transformed quark zero-mode and instanton fields, respectively, and  $I_\nu(z)$ ,  $K_\nu(z)$ , are the modified Bessel functions and  $\rho$  is the instanton size.

The value of AQCM is determined by the effective density of the instantons  $n(\rho)$  in nonperturbative QCD vacuum (N.K. (1996))

$$\mu_a = -\pi^3 \int \frac{d\rho n(\rho) \rho^4}{\alpha_s(\rho)}.$$

Within Shuryak's instanton liquid model

$$n(\rho) = n_c \delta(\rho - \rho_c),$$



leads to AQCM which is proportional to the packing fraction of instantons  $f = \pi^2 n_c \rho_c^4$  in vacuum

$$\mu_a = -\frac{\pi f}{\alpha_s(\rho_c)}.$$

By using the following relation between parameters of the instanton model

$$f = \frac{3}{4}(M_q \rho_c)^2,$$

we obtain

$$\mu_a = -\frac{3\pi(M_q \rho_c)^2}{4\alpha_s(\rho_c)}.$$

The dimensionless parameter  $\delta = (M_q \rho_c)^2$  is one of the main parameters of the instanton model. It is proportional to the packing fraction of instantons in QCD vacuum  $\delta \propto f \ll 1$ , and is rather small. For a fixed value of average instanton size  $\rho_c^{-1} = 0.6$  GeV it changes from  $\delta^{MF} = 0.08$  for  $M_q = 170$  MeV in the mean field approximation to  $\delta^{DP} = 0.33$  for  $M_q = 345$  MeV within Diakonov-Petrov model. For the strong coupling constant at the scale of instanton average size

$$\alpha_s(\rho_c) \approx 0.5,$$

we obtain the following values for AQCM:

$$\mu_a^{MF} \approx -0.4, \quad \mu_a^{DP} \approx -1.6$$

in the mean field approximation and in the DP approach, respectively.

Formula for AQCM can be rewritten in the following form:

$$\mu_a = -\frac{3}{8}S_0\delta,$$

where  $S_0 = 2\pi/\alpha_s(\rho)$  is the Euclidean instanton action. The typical value of this action is large

$$S_0 \approx 10 \div 15$$

and leads to the compensation of the  $\delta$  smallness effect on AQCM.

Recent calculation within Dyson-Schwinger approach (Craig Roberts et al. (2010)) gives for AQCM  $\mu_a \approx -0.5$ .

# Fine pomeron structure

- Pomeron is effective colorless exchange between hadrons (quarks) which gives a dominated contribution to the high energy cross sections:

$$\sigma \approx s^{\alpha(0)-1}$$

- Two types of pomerons: "soft" (Landshoff and Nachtmann (low virtuality of quark and gluons and intercept  $\alpha_{soft}(0) \approx 1.08$  ) and "hard" Balitsky ,Kuraev, Lipatov and Fadin (BFKL) pomeron ( large virtualities and intercept  $\alpha_{hard}(0) \approx 1.4$  ).

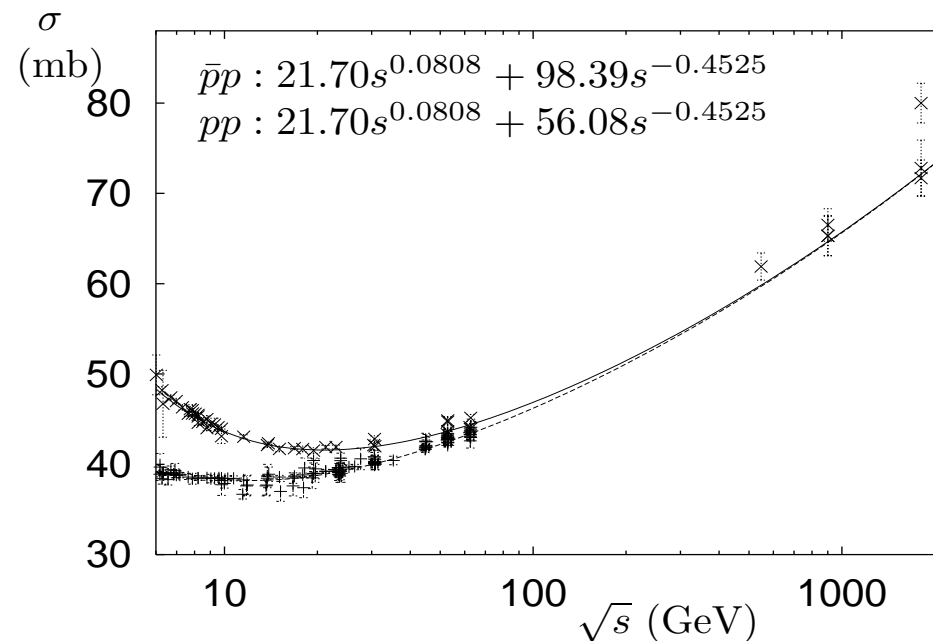


Figure 3: Soft Pomeron in  $pp$  and  $\bar{p}p$  total cross sections.

Form factor in the first term in general quark-gluon vertex might be chosen in the form

$$F_1(k_1^2, k_2^2, q^2) = \Theta(|k_1^2| - \mu^2) \Theta(|k_2^2| - \mu^2) \Theta(|q^2| - \mu^2),$$

where  $\mu$  is the factorization scale between perturbative and nonperturbative regimes. In our estimation below we will use  $\mu \approx 1/\rho_c \approx 0.6$  GeV.

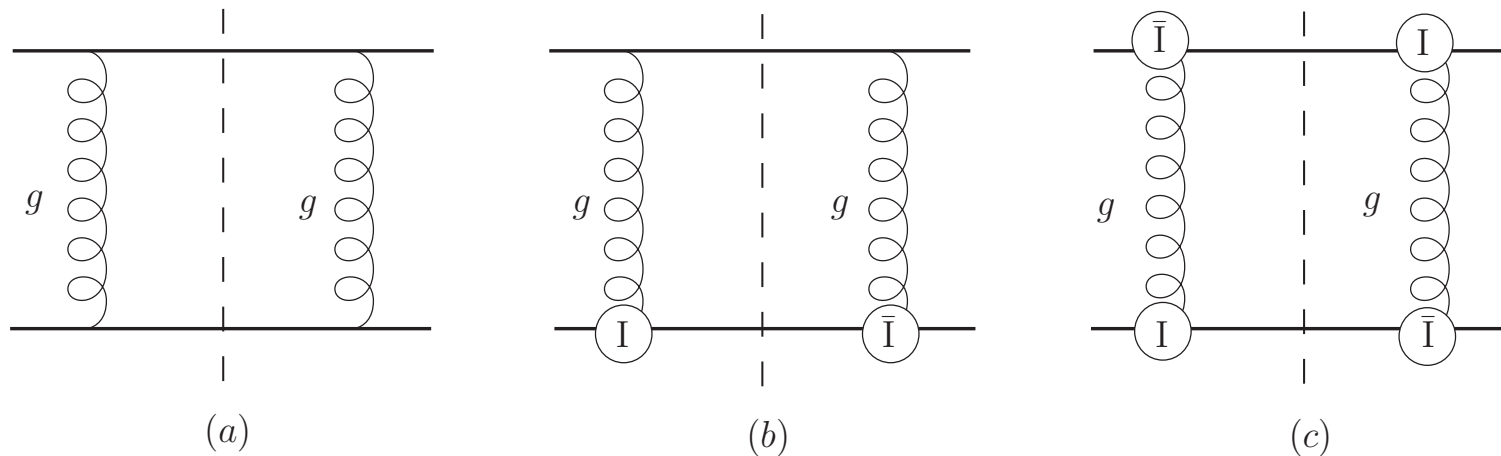


Figure 4: The fine pomeron structure in the model with perturbative and nonperturbative interactions: a) perturbative contribution, b) interference perturbative and nonperturbative vertices, c) nonperturbative contribution. The symbol  $I$  ( $\bar{I}$ ) denotes instanton (antiinstanton).

The total contribution to quark-quark cross section for the quarks with small virtualities is

$$\sigma^{total} = \sigma^{pert} + \sigma^{mix} + \sigma^{nonpert}, \quad (1)$$

where

$$\sigma^i = \int_{q_{min}^2}^{\infty} \frac{d\sigma^i(t)}{dt} dq^2, \quad (2)$$

$$\begin{aligned} \frac{d\sigma(t)^{pert}}{dt} &= \frac{8\pi\alpha_s^2(q^2)}{9q^4} \\ \frac{d\sigma(t)^{mix}}{dt} &= \frac{\alpha_s(q^2)\pi^2 |\mu_a| \rho_c^2 F_g^2(|q|\rho_c)}{3q^2} \\ \frac{d\sigma(t)^{nonpert}}{dt} &= \frac{\pi^3 \mu_a^2 \rho_c^4 F_g^4(|q|\rho_c)}{32}, \end{aligned} \quad (3)$$

where  $q^2 = -t$  and  $q_{min}^2 \approx 1/\rho_c^2$  for perturbative and mixed contributions and  $q_{min}^2 = 0$  for pure nonperturbative contribution.

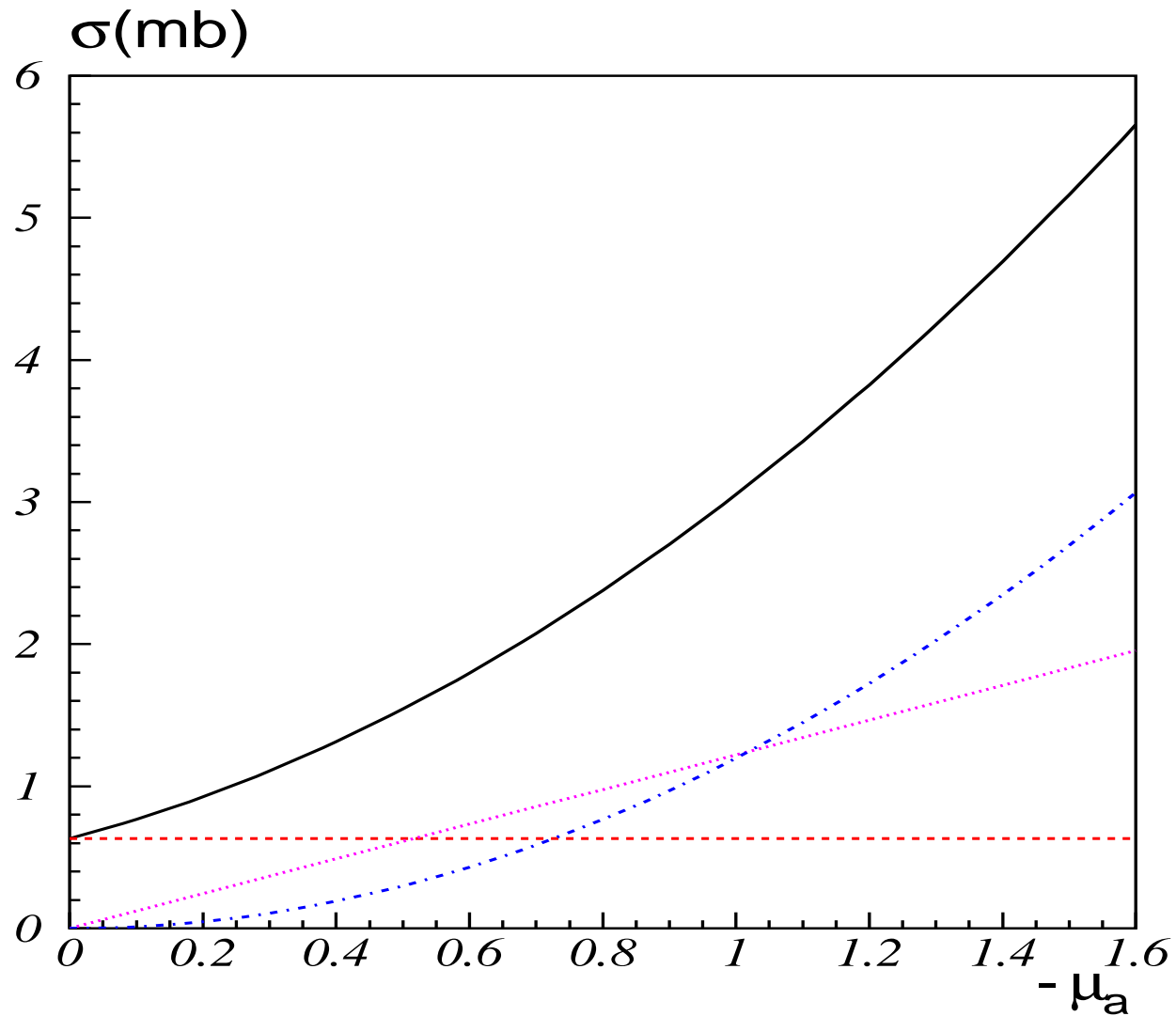


Figure 5: The contribution to the total quark-quark cross section as a function of AQCM: perturbative (dashed line), mixed (dotted line), nonperturbative (dashed-dotted line) and their sum (solid line).

For the strong coupling constant, the following parametrization was:

$$\alpha_s(q^2) = \frac{4\pi}{9 \ln((q^2 + m_g^2)/\Lambda_{QCD}^2)},$$

where  $\Lambda_{QCD} = 280$  MeV and the value  $m_g = 0.88$  GeV was fixed from the requirement  $S_0 = 2\pi/\alpha_s(q^2 = 1/\rho_c^2) \approx 12$  as in Shuryak-Diakonov-Petrov instanton liquid model. This form for  $\alpha_s(q^2)$  describes the frozen coupling constant in the infrared region,  $\alpha_s(q^2) \rightarrow constant$  as  $q^2 \rightarrow 0$ .

From hadron spectroscopy (Faustov, Galkin, Ebert) obtained  $\mu_a \approx -1$ . In our calculation it corresponds to the value of dynamical quark mass  $M_q = 280$  MeV. For such  $\mu_a$  we have got  $\sigma_{qq}^{tot} = 3.05$  mb.

That number is not far away from  $\sigma_{qq}^{exp} = 4$  mb ( inelastic  $\sigma_{pp(\bar{p})}^{exp} \approx 36$  mb in the energy range where  $pp(\bar{p})$  cross sections are approximately constant ).

## **Gluon distribution in nucleon induced by anomalous chromomagnetic interaction**

The anomalous quark-gluon interaction structure should also manifest itself in the structure of gluon distribution in nucleon. One of the ways to

show it is in the use of a DGLAP-like approach with the modified quark splitting function  $\mathcal{P}_{Gq}$  according with the general quark-gluon vertex.

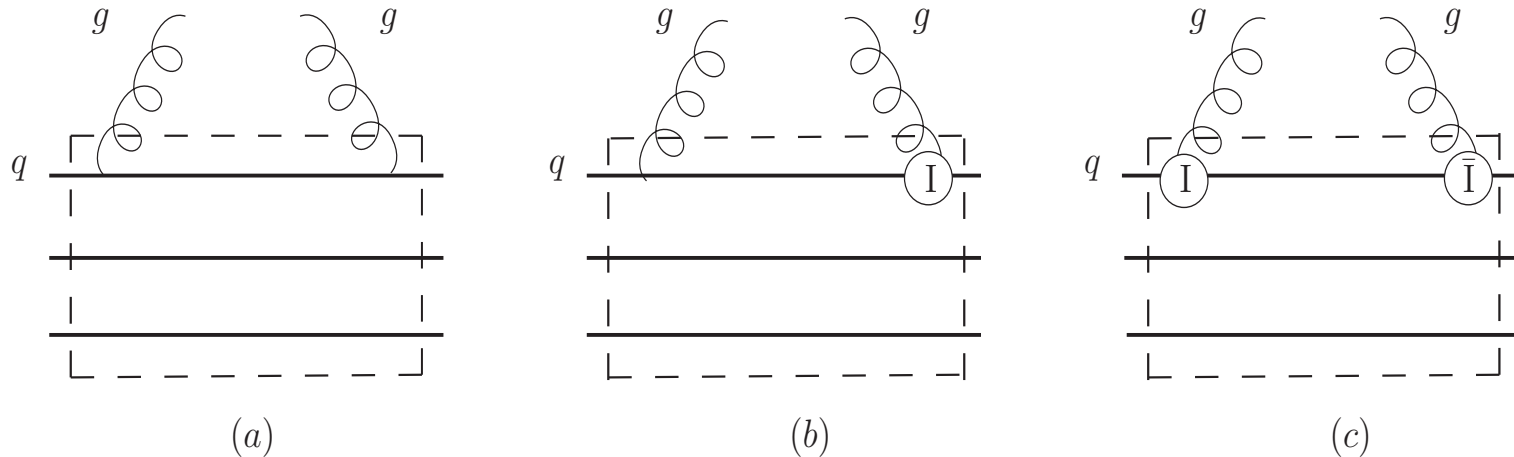


Figure 6: The diagrams contributing to nucleon gluon distribution: a) "hard"-perturbative, b) "hard-soft" interference perturbative and nonperturbative exchanges, c) "soft"-nonperturbative part.

To calculate unintegrated distribution, we use the convolution model formula

$$f(x, k_{\perp}^2) = N_q k_{\perp}^2 \int_x^1 \frac{dy}{y} \mathcal{P}_{Gq}(x/y, k_{\perp}^2) q_V(y),$$

where  $N_q = 3$  is the number of valence quarks in nucleon,  $q_V$  is the valence quark distribution function in nucleon,  $\mathcal{P}_{Gq}$  is the quark splitting



function as defined by Altarelli and Parisi. The splitting function is given by the formula

$$\mathcal{P}_{Gq}(z, k_{\perp}^2) = \frac{C_F z(1-z)k_{\perp}^2}{8\pi^2(k_{\perp}^2 + M_q^2 z^2)^2} \sum_{\lambda} \text{Tr}[(\hat{k}_C + M_q)U_{\mu}(t)(\hat{k}_A + M_q) \\ \times \bar{U}_{\rho}(t)]\epsilon_{\mu}(\lambda)\epsilon_{\rho}^*(\lambda),$$

where  $U_{\mu}(t) = V_{\mu}(0, 0, t)$ ,  $k_A$  ( $k_C$ ) is momentum of initial (final) quark,  $t = q^2 = (k_A - k_C)^2$ ,  $\bar{U} = \gamma_0 U^{\dagger} \gamma_0$  and  $\lambda$  is gluon helicity. In the infinite momentum frame

$$k_A = (P, P + \frac{M_q^2}{2P}, \vec{0}_{\perp})$$

$$k_C = ((1-z)P + \frac{k_{\perp}^2 + M_q^2}{2(1-z)P}, (1-z)P, -\vec{k}_{\perp})$$

$$q = (zP - \frac{k_{\perp}^2 + M_q^2 z}{2(1-z)P}, zP, \vec{k}_{\perp}),$$

the result for splitting function is

$$\begin{aligned}
 \mathcal{P}_{Gq}(z, k_{\perp}^2) &= \frac{C_F k_{\perp}^2}{2\pi z (k_{\perp}^2 + M_q^2 z^2)^2} \\
 &\times [(\sqrt{\alpha_s(|t|)} \Theta(|t| - \mu^2) + \sqrt{\alpha_s(1/\rho_c^2)} \mu_a F_g(|t|))^2 z^2 \\
 &+ 2((1-z)\alpha_s(|t|) \Theta(|t| - \mu^2) + \frac{\alpha_s(1/\rho_c^2) \mu_a^2 k_{\perp}^2}{4M_q^2} F_g^2(|t|))],
 \end{aligned}$$

where  $|t| = (k_{\perp}^2 + M_q^2 z^2)/(1-z)$  is the gluon virtuality. The integrated distribution is given by

$$g(x, Q^2) = \int_0^{Q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} f(x, k_{\perp}^2),$$

For estimation we use a simple form for valence quark distribution

$$q_V(x) = 1.09 \frac{(1-x)^3}{\sqrt{x}}$$

with the normalization  $\int_0^1 q_V(x) dx = 1$ .

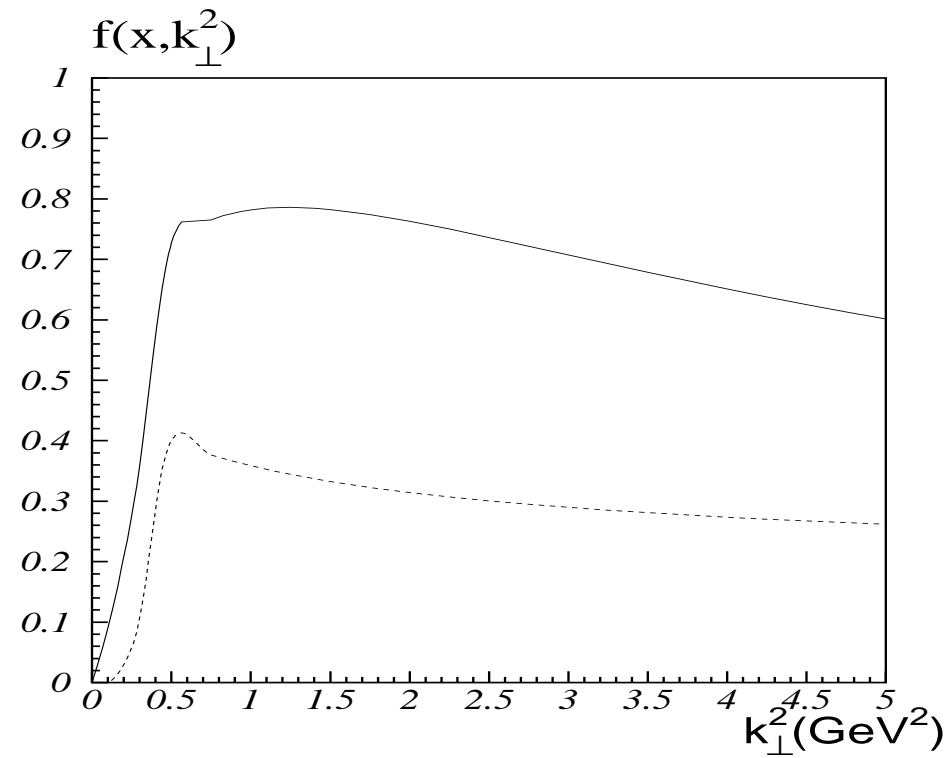


Figure 7: The unintegrated gluon distribution at  $x = 10^{-2}$ : solid (dashed) line is total (perturbative) contribution.

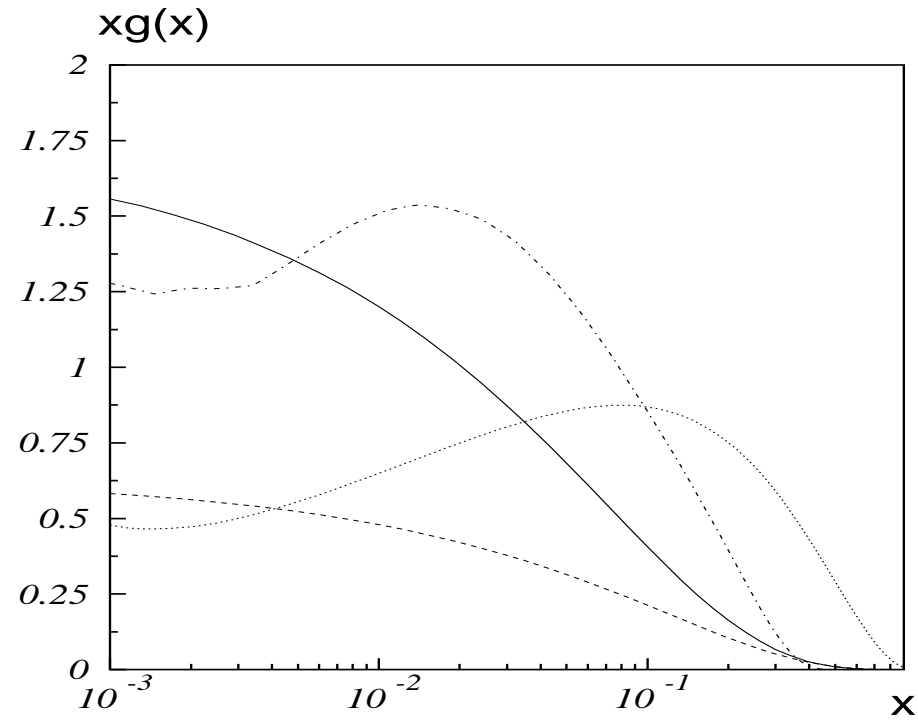


Figure 8: Perturbative (dashed line) and total (solid line) contributions to gluon distribution at  $Q^2 = 1\text{GeV}^2$  in comparison with some of the phenomenological fits: dotted line is ALEKHIN02LO set and dashed-dotted line is MSTW2008LO fit.

# CONCLUSION

- Instantons induce large anomalous quark-gluon chromomagnetic spin-flip interaction
- This interaction gives the dominating contribution to the soft pomeron exchange and to gluon distribution in nucleon for the small virtuality of quark and gluons
- Internal spin structure of soft and hard Pomerons is different
- That feature should give a strong influence to the spin effects in the diffraction ( N.K., N.Nikolaev, N.Korchagin work in progress)

**Thank you for your attention!**