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TWO-PHOTON EXCHANGE IN ELECTRON-DEUTERON SCATTERING

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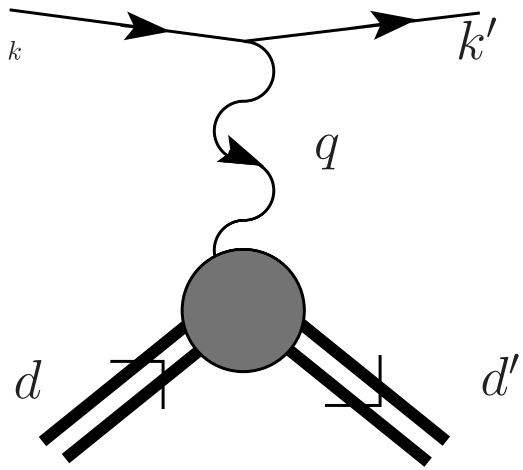
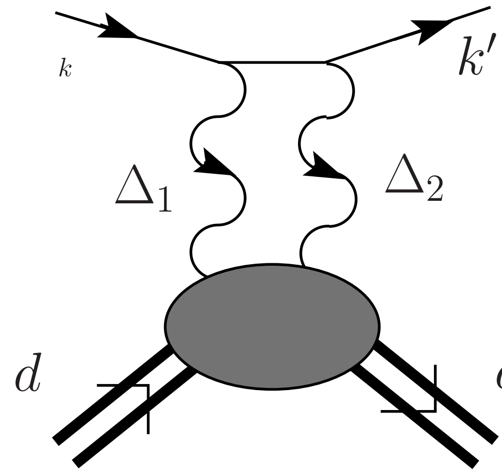
The study of electron scattering on the nucleon and the light nuclei provides a convenient tool to study the structure of strongly interacting systems. Due to the smallness of the fine structure constant $\alpha \approx \frac{1}{137}$, one may expect that the Born approximation (one-photon exchange, OPE) should describe such processes with an accuracy of a few percent.



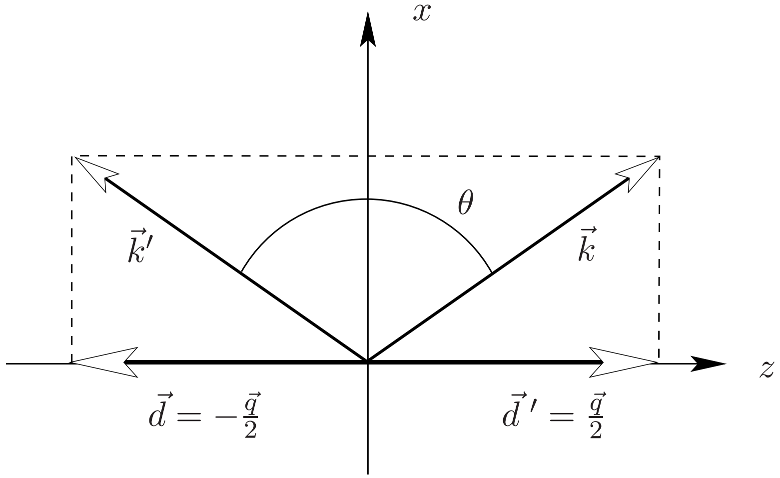


Nevertheless, for precise studies of these nuclei a quantitative theoretical investigation of the second order perturbative effects should be taken into account.

The aim of this study is to estimate the Two-Photon Exchange (TPE) amplitude for *ed*-scattering in the framework of semi-relativistic calculations, with deuteron wave functions from "realistic" NN potentials, Paris, CD-Bonn, etc.

A.P.K., Ya.D.Krivenko-Emetov and S.Dubnička, Phys. Rev., C81, 054001 (2010)

KINEMATICS AND DEFINITIONS



Breit frame

$$d_0 = d'_0 = E_d = \sqrt{M^2 + Q^2/4}$$

$$\vec{d}_\perp = \vec{d}\,'_\perp = 0, \qquad d_3 = -d'_3 = -Q/2$$

$$q_0 = q_1 = q_2 = 0, \qquad q_3 = Q$$

$$k_0 = k'_0 \equiv E_e, \qquad \vec{k}_\perp = \vec{k}\,'_\perp, \qquad k_3 = -k'_3 = Q/2$$

$$\eta = \frac{Q^2}{4M_d^2}$$

Polarization parameter $\epsilon = \frac{\cos^2\frac{\theta}{2}}{1+\sin^2\frac{\theta}{2}}$

Instead of the usual reaction amplitude $\mathcal{M}$ it is useful to introduce the reduced amplitude $T_{\lambda'\lambda,h}$ by

$$\mathcal{M} = \frac{16\pi\alpha}{Q^2} E_e E_d T_{\lambda'\lambda,h}.$$

It follows from P and T invariance that, in the Breit frame, this amplitude must have the following properties

$$T_{\lambda'\lambda;h} = (-1)^{\lambda-\lambda'} T_{-\lambda'-\lambda;-h} \qquad \textbf{and} \qquad T_{\lambda'\lambda;h} = T_{-\lambda-\lambda';h}$$

The reaction amplitude is determined by six independent invariant amplitudes (form factors), which are specified by the following parametrization:

$$T_{\lambda'\lambda;h} = \begin{pmatrix} \mathcal{G}_{11}\cos\frac{\theta}{2} & -\sqrt{\frac{\eta}{2}}\mathcal{G}^{h}_{10} & \mathcal{G}^{h}_{1,-1} \\ \sqrt{\frac{\eta}{2}}\mathcal{G}^{-h}_{10} & \mathcal{G}_{00}\cos\frac{\theta}{2} & -\sqrt{\frac{\eta}{2}}\mathcal{G}^{h}_{10} \\ \mathcal{G}^{-h}_{1,-1} & \sqrt{\frac{\eta}{2}}\mathcal{G}^{-h}_{10} & \mathcal{G}_{11}\cos\frac{\theta}{2} \end{pmatrix},$$

$$\mathcal{G}^{h}_{10} = f_1 + h\sin\frac{\theta}{2} f_2, \qquad \mathcal{G}^{h}_{1,-1} = f_3 + h\sin\frac{\theta}{2} f_4.$$

Some authors (G.Gakh and E.Tomasi-Gustaffson) use another parametrization of the amplitude.

The form factors $\mathcal{G}_{11}$, $\mathcal{G}_{00}$, f_1, ..., f_4 are complex functions of the two independent kinematical variables, for example Q^2 and θ.

This amplitudes can be related with the generalized electric, quadrupole and magnetic from factors

$$\mathcal{G}_C(Q^2,\theta), \qquad \mathcal{G}_Q(Q^2,\theta), \qquad \mathcal{G}_M(Q^2,\theta)$$

and the additional form factors

$$g_1(Q^2,\theta), \qquad g_2(Q^2,\theta), \qquad g_3(Q^2,\theta)$$

as follows:

$$\mathcal{G}_{11} = \mathcal{G}_C - \tfrac{2}{3}\eta\mathcal{G}_Q, \qquad \mathcal{G}_{00} = \mathcal{G}_C + \tfrac{4}{3}\eta\mathcal{G}_Q,$$

$$f_1 = \mathcal{G}_M + g_1 \sin^2\tfrac{\theta}{2}, \qquad f_2 = \mathcal{G}_M - g_1,$$

$$f_3 = g_2, \qquad f_4 = g_3.$$

In OPE+TPE approximation the form factors can be written as:

$$\mathcal{G}_C = G_C + \delta\mathcal{G}_C, \qquad \mathcal{G}_Q = G_Q + \delta\mathcal{G}_Q,$$

$$\mathcal{G}_M = G_M + \delta\mathcal{G}_M,$$

where δ stands for the terms of order α;

likewise, the form factors $g_{1,2,3}$ are also proportional to α.

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_{\rm M}}{\cos^2\frac{\theta}{2}}\overline{|T|^2},$$

$\sigma_{\rm M}$ **is the Mott cross section**

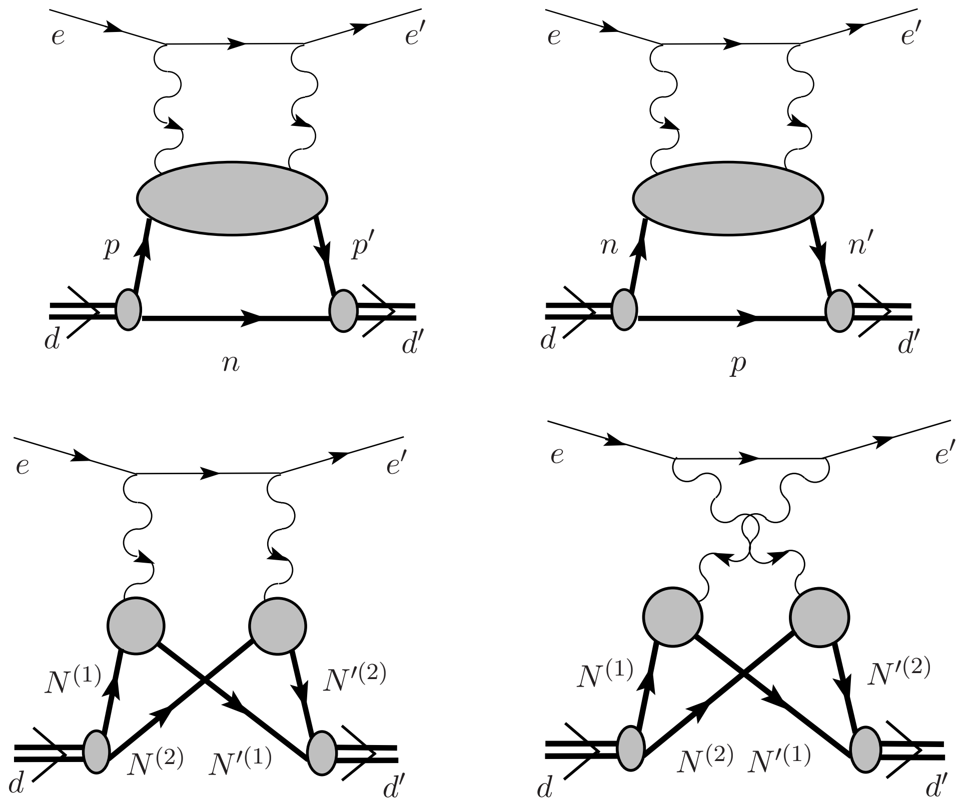
$$\overline{|T|^2} = \frac{1}{6}\sum_{\lambda,\lambda',h} |T_{\lambda'\lambda;h}|^2 = \cos^2\frac{\theta}{2}\left[\mathcal{A}(Q^2,\theta) + {\rm tg}^2\frac{\theta_{\rm lab}}{2}\mathcal{B}(Q^2,\theta)\right] + \mathcal{O}(\alpha^2) =$$

$$= \left(1+\sin^2\frac{\theta}{2}\right)\left[\epsilon|\mathcal{G}_E(Q^2,\theta)|^2 + \frac{2}{3}\eta|\mathcal{G}_M(Q^2,\theta)|^2\right] + \mathcal{O}(\alpha^2),$$

$$\mathcal{A}(Q^2,\theta) = |\mathcal{G}_C(Q^2,\theta)|^2 + \frac{8}{9}\eta^2|\mathcal{G}_Q(Q^2,\theta)|^2 + \frac{2}{3}\eta|\mathcal{G}_M(Q^2,\theta)|^2,$$
$$\mathcal{B}(Q^2,\theta) = \frac{4}{3}(1+\eta)\eta|\mathcal{G}_M(Q^2,\theta)|^2,$$
$$\mathcal{G}_E^2 = |\mathcal{G}_C(Q^2,\theta)|^2 + \frac{8}{9}\eta^2|\mathcal{G}_Q(Q^2,\theta)|^2.$$

The cross section has the same form as the Rosenbluth formula; nevertheless the Rosenbluth separation of the structure functions $\mathcal{A}(Q^2,\theta)$ and $\mathcal{B}(Q^2,\theta)$ can no longer be done because they depend on two variables.

CALCULATION OF TPE



Two-photon exchange diagrams. The top diagrams correspond to the amplitudes $\mathcal{M}_p^{\mathrm{I}}$ and $\mathcal{M}_n^{\mathrm{I}}$, the bottom diagrams to the amplitudes $\mathcal{M}_{\mathrm{P}}^{\mathrm{II}}$ and $\mathcal{M}_{\mathrm{X}}^{\mathrm{II}}$.

$$\mathcal{M}_2 = \mathcal{M}^{\mathrm{I}} + \mathcal{M}^{\mathrm{II}}$$

$\mathcal{M}^{\mathrm{I}}$ diagram

The TPE amplitude for a nucleon N has the following structure

$$\mathcal{M}_{2\gamma N} = \frac{4\pi\alpha}{Q^2}\bar{u}'_h\gamma_\mu u_h \left\langle \vec{p}\,'_N\sigma' \left| \widehat{H}^\mu_N \right| \vec{p}_N\,\sigma \right\rangle ,$$

where $\widehat{H}^\mu_N$ is the "effective hadron current"

$$\widehat{H}^\mu_N = \Delta\widetilde{F}^N_1\gamma^\mu - \Delta\widetilde{F}^N_2[\gamma^\mu,\gamma^\nu]\frac{q_\nu}{4m} + \widetilde{F}^N_3 K_\nu\gamma^\nu\frac{P^\mu}{m^2}.$$

$$K = (k+k')/2,\ P = (p_N + p'_N)/2$$

$\Delta\widetilde{F}^N_1$ and $\Delta\widetilde{F}^N_2$ may be called corrections to the Dirac and Pauli form factors and $\widetilde{F}^N_3$ is a new form factor.

$$\delta\mathcal{G}_C^{\mathrm{I}} = 2\delta\mathcal{G}_E^S \left[I_{00}^0(Q) + I_{22}^0(Q)\right]$$

$$\delta\mathcal{G}_Q^{\mathrm{I}} = \frac{3\sqrt{2}}{\eta}\delta\mathcal{G}_E^S \left[I_{20}^2(Q) - \frac{1}{2\sqrt{2}}I_{22}^2(Q)\right]$$

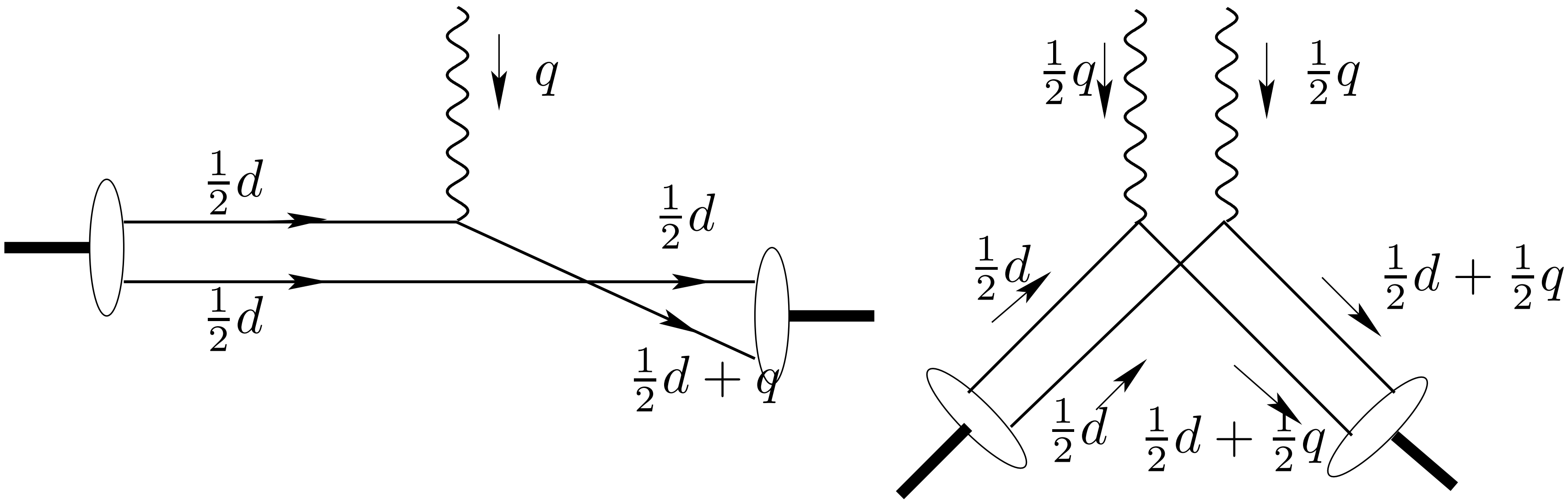
$$\delta\mathcal{G}_M^{\mathrm{I}} = \frac{M}{m}\left\{\frac{3}{2}\delta\mathcal{G}_E^S \left[I_{22}^0(Q) + I_{22}^2(Q)\right] + 2\delta\mathcal{G}_M^S \left[I_{00}^0(Q) - \tfrac{1}{2}I_{22}^0(Q) + \sqrt{\tfrac{1}{2}}I_{20}^2(Q) + \tfrac{1}{2}I_{22}^2(Q)\right]\right\}$$

$$g_1^{\mathrm{I}} = -\epsilon\tfrac{E_e}{m}\mathcal{F}_3, \qquad g_2^{\mathrm{I}} = g_3^{\mathrm{I}} = 0$$

$$\mathcal{F}_3 = 2\frac{M}{m}\widetilde{F}_3^S \left[I_{00}^0(Q) - \tfrac{1}{2}I_{22}^0(Q) + \sqrt{\tfrac{1}{2}}I_{20}^2(Q) + \tfrac{1}{2}I_{22}^2(Q)\right].$$

$$I_{\ell'\ell}^L(Q) = \int_0^\infty dr\, j_L\left(\tfrac{1}{2}Qr\right) u_{\ell'}(r)u_\ell(r)$$

$\mathcal{M}^{\text{II}}$ diagram



The configuration where each intermediate photon carries about half of the transferred momentum (hard-photon approximation) becomes mostly important at high Q^2

$$\Delta_1 \sim \Delta_2 \sim \frac{q}{2} \ .$$

$$\delta\mathcal{G}_C^{\rm II} = \kappa\left(G_{EE} - \tfrac{1}{3}\eta G_{MM}\right), \qquad \delta\mathcal{G}_Q^{\rm II} = -\tfrac{\kappa}{2}G_{MM}, \qquad \delta\mathcal{G}_M^{\rm II} = \frac{2\kappa G_{EM}}{1+\sin^2\frac{\theta}{2}}$$

$$g_1^{\rm II} = \frac{\kappa G_{EM}\cos^2\frac{\theta}{2}}{1+\sin^2\frac{\theta}{2}}, \qquad g_2^{\rm II} = g_3^{\rm II} = \kappa\eta\cos\tfrac{\theta}{2}G_{MM}$$

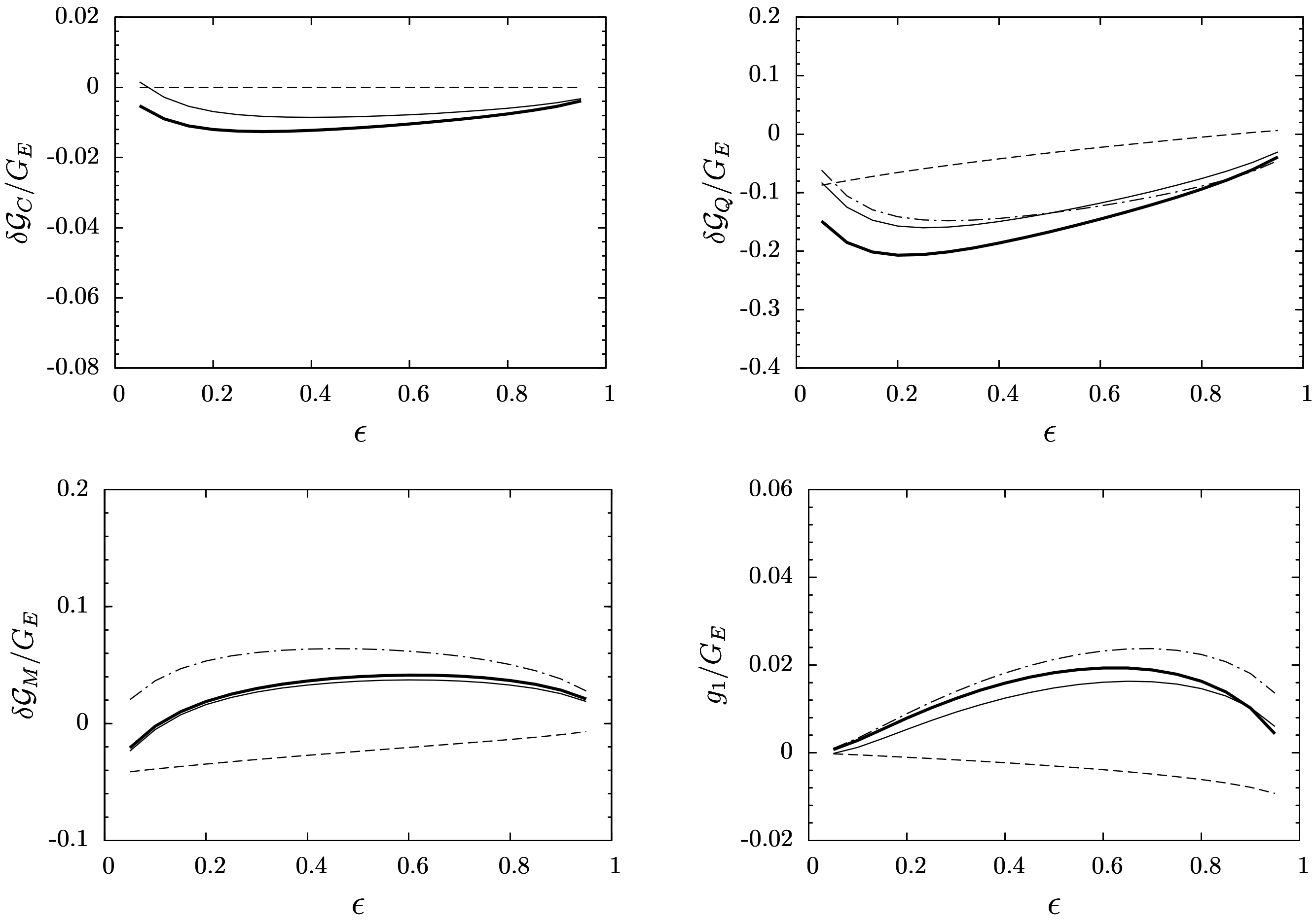
$$G_{EE} = G_E^p(\tfrac{1}{4}Q^2)G_E^n(\tfrac{1}{4}Q^2), \qquad G_{MM} = G_M^p(\tfrac{1}{4}Q^2)G_M^n(\tfrac{1}{4}Q^2)$$

$$G_{EM} = \tfrac{1}{2}\left[G_E^p(\tfrac{1}{4}Q^2)G_M^n(\tfrac{1}{4}Q^2) + G_M^p(\tfrac{1}{4}Q^2)G_E^n(\tfrac{1}{4}Q^2)\right]$$

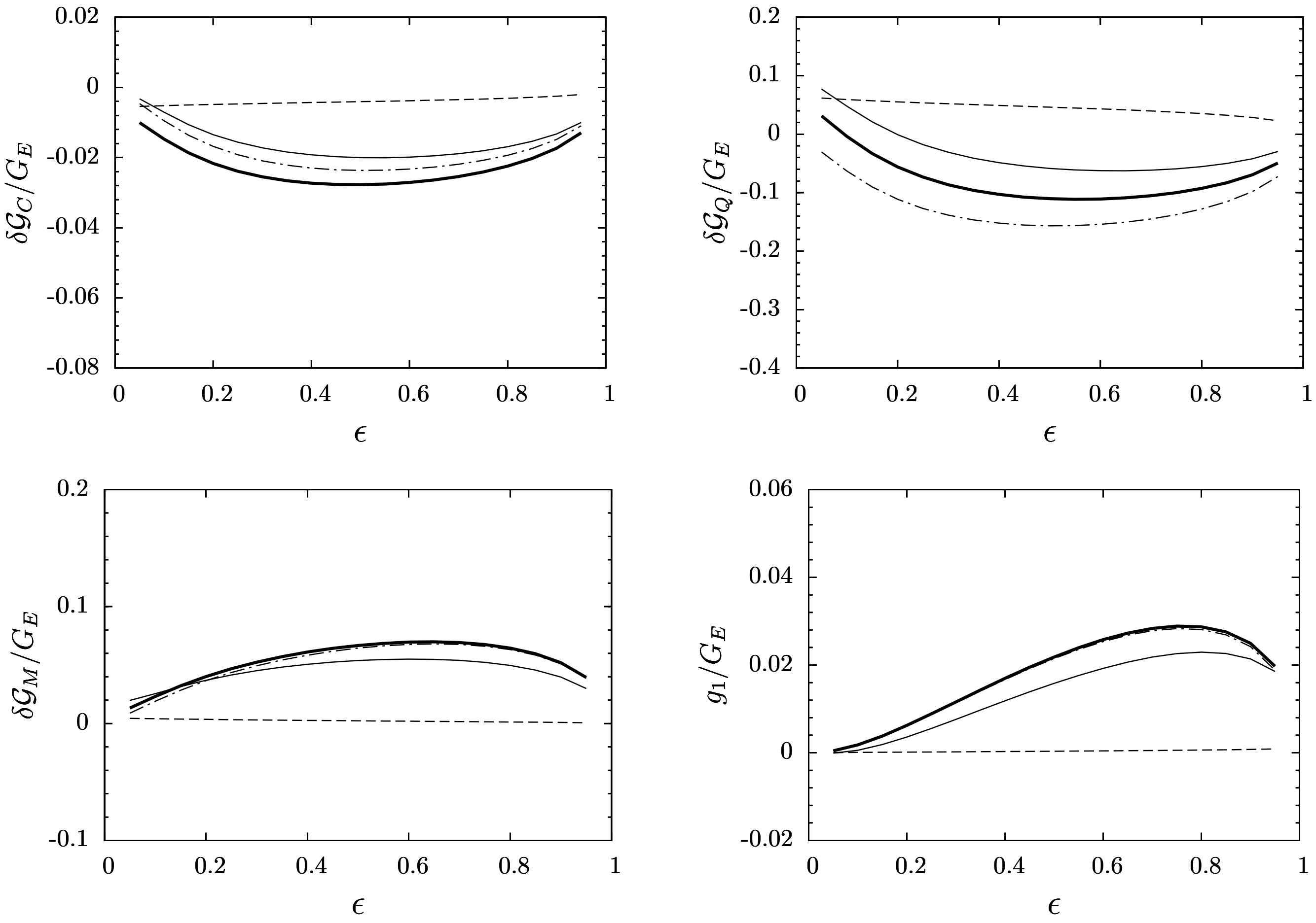
$$\kappa = -\frac{128\alpha\mathcal{S}E_e}{Q^4}$$

$$\mathcal{S} = -if\int_0^\infty \frac{dy}{y^2}e^{ify}u_0^2(y) \qquad\qquad f = \frac{Q^2}{4E_e\sqrt{4\cos^2\frac{\theta}{2} + \frac{Q^2}{M^2}}}$$

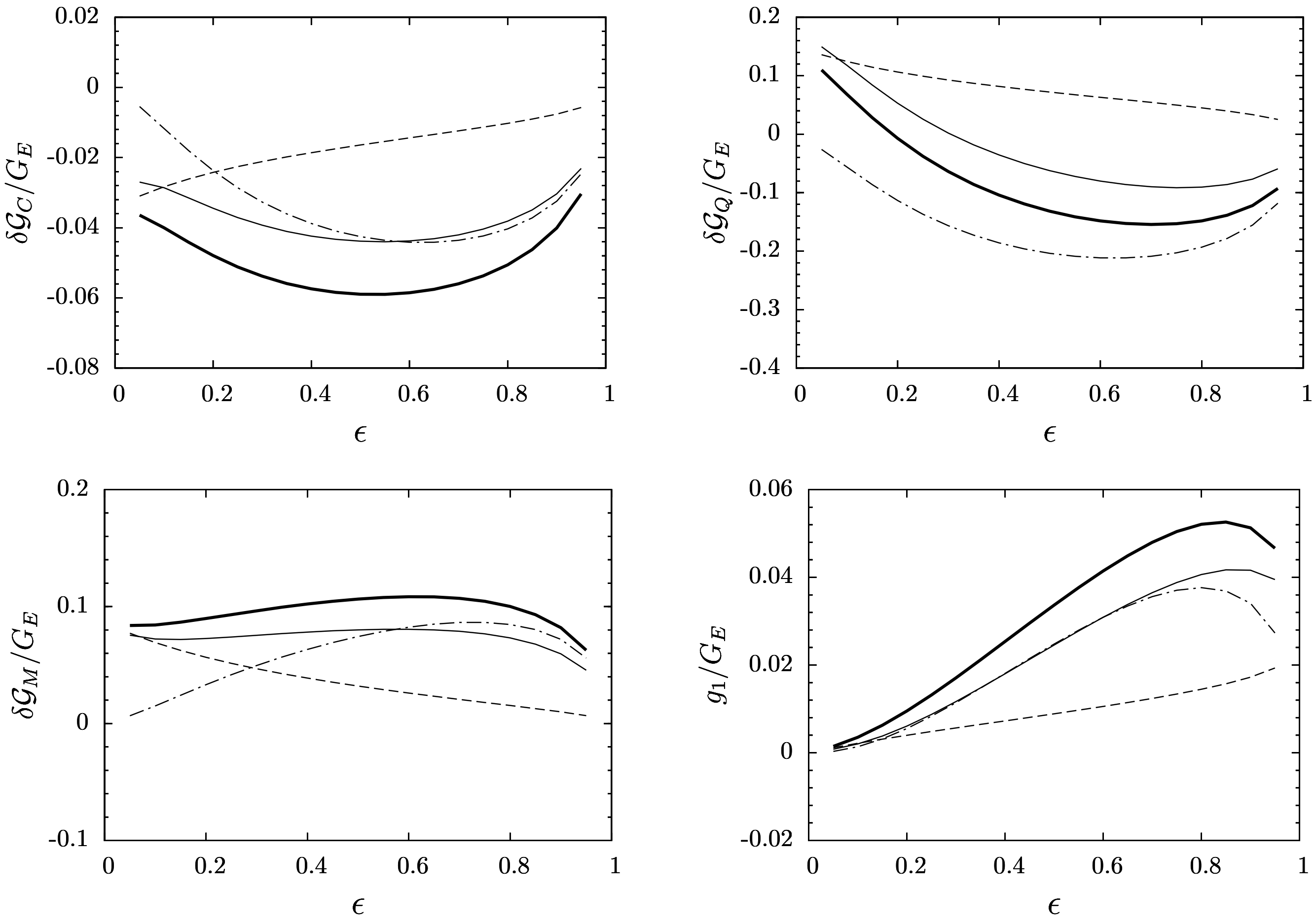
NUMERICAL RESULTS



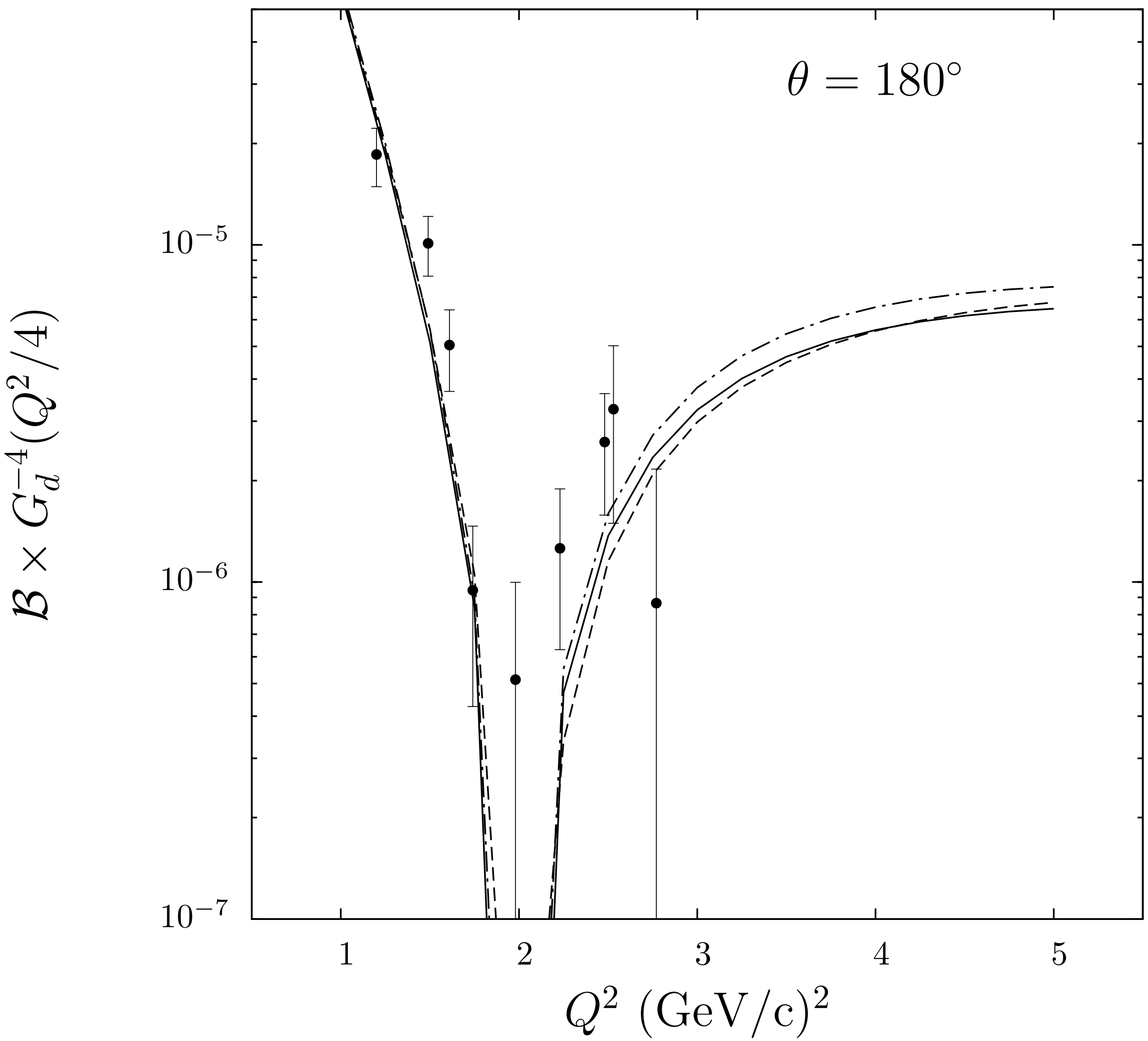
Two-photon exchange corrections $\delta\mathcal{G}_C/G_E$, $\delta\mathcal{G}_Q/G_E$, $\delta\mathcal{G}_M/G_E$ and g_1/G_E at Q^2 =1 GeV^2. Dashed, dot-dashed and solid (bold) curves are for $\mathcal{M}^{\rm I}$, $\mathcal{M}^{\rm II}$ and $\mathcal{M}^{\rm I}+\mathcal{M}^{\rm II}$, respectively, calculated with the CD-Bonn potential. These solid (thin) curves depict $\mathcal{M}^{\rm I}+\mathcal{M}^{\rm II}$ calculated with the Paris potential.



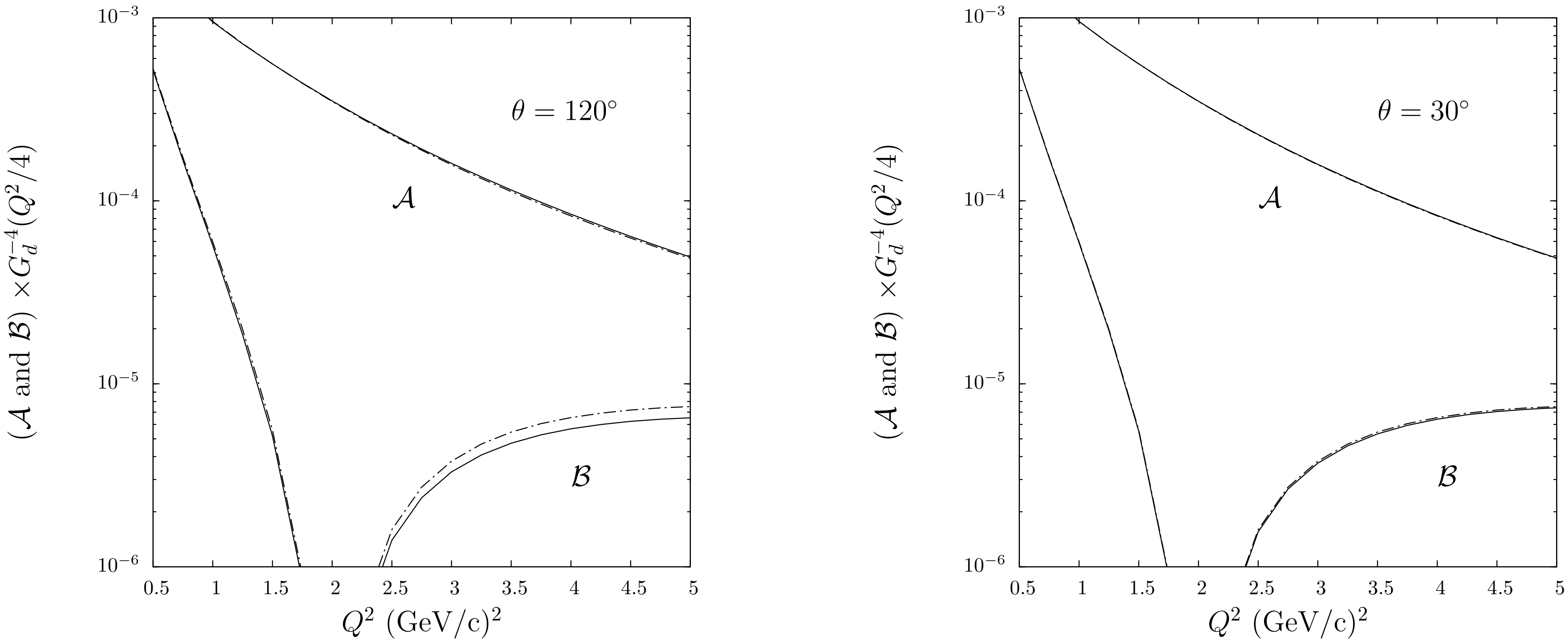
Same for Q^2 =2 GeV^2.



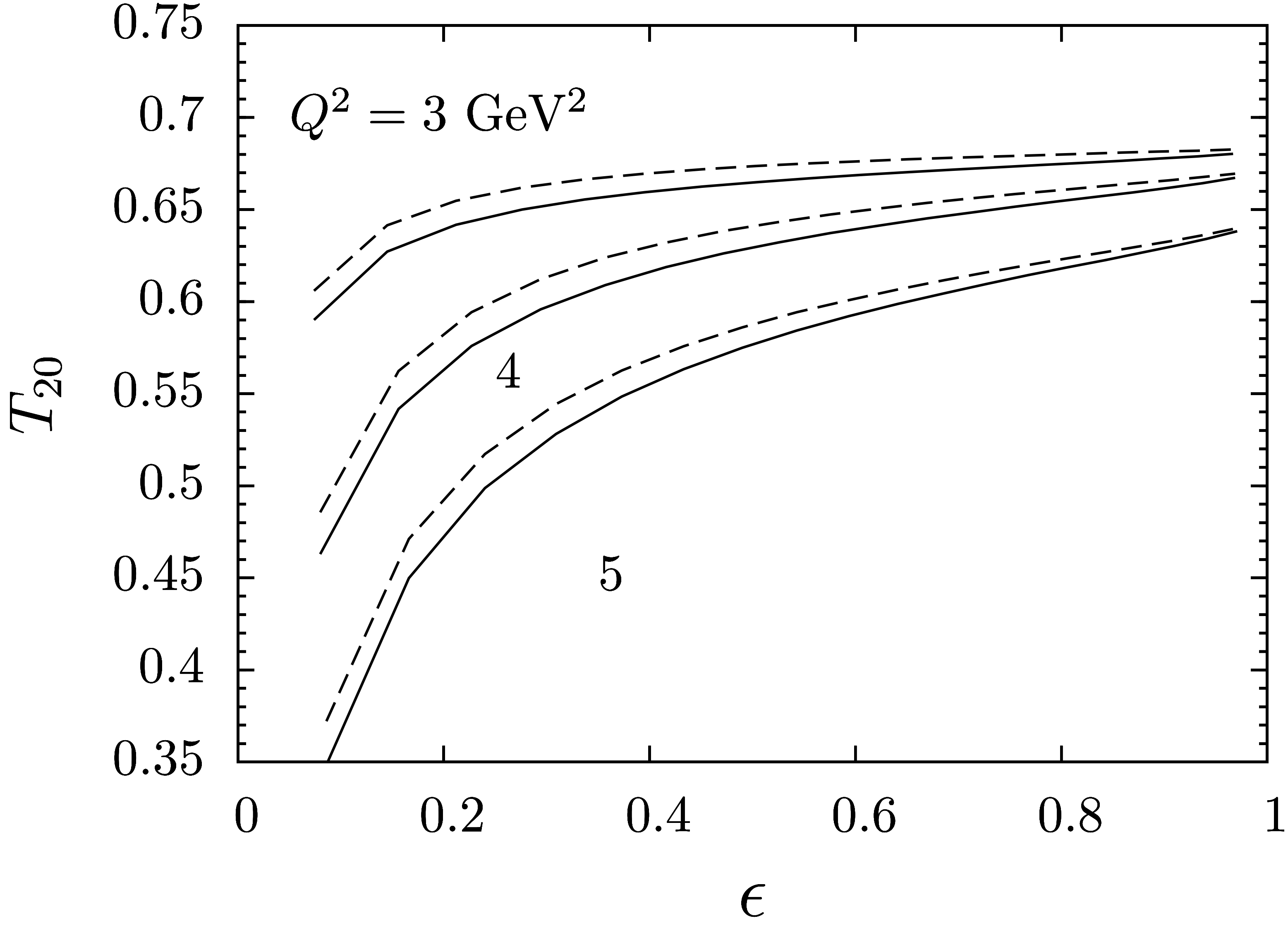
Same for Q^2 =3 GeV^2.



$\theta = 180°$
10^{-5}
10^{-6}
10^{-7}
1
2
3
4
5
$\mathcal{B} \times G_d^{-4}(Q^2/4)$
Q^2 (GeV/c)^2



10^{-3}
10^{-4}
10^{-5}
10^{-6}
$(\mathcal{A}$ and $\mathcal{B})\ \times G_d^{-4}(Q^2/4)$
$\theta = 120^\circ$
$\mathcal{A}$
$\mathcal{B}$
0.5 1 1.5 2 2.5 3 3.5 4 4.5 5
Q^2 (GeV/c)^2
10^{-3}
10^{-4}
10^{-5}
10^{-6}
$(\mathcal{A}$ and $\mathcal{B})\ \times G_d^{-4}(Q^2/4)$
$\theta = 30^\circ$
$\mathcal{A}$
$\mathcal{B}$
0.5 1 1.5 2 2.5 3 3.5 4 4.5 5
Q^2 (GeV/c)^2



0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
T_{20}
$Q^2 = 3$ GeV^2
4
5
0
0.2
0.4
0.6
0.8
1
ϵ

CONCLUSIONS

- **We estimated TPE amplitude in elastic ed scattering. There are six independent form factors which determine this amplitude, but only three of them contribute to the cross section in second order perturbation theory.**
- **There are two types of TPE diagrams ($\mathcal{M}^{\mathrm{I}}$ and $\mathcal{M}^{\mathrm{II}}$). For the first type two intermediate photons interact with the same nucleon. For the second type the intermediate photons interact with different nucleons.**
- **Role of TPE effects increased with increasing of Q^2 mainly due to diagrams of $\mathcal{M}^{\mathrm{II}}$ diagrams. But at intermediate $Q^2 \sim$1 GeV^2 both diagram must be taken into account.**