Mixing of fermion fields of opposite parities and baryon resonances

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Plans

- ✓ Introduction
- ✓ Technics (off-shell projection operators)
- ✓ Mixing of fermion fields of opposite parities (OPF-mixing)
- ✓ Estimates for $\pi N \to \pi N$ and PWA results
- ✓ Identification of OPF-mixing in system of baryons $J^P = 3/2^{\pm}$
- ✓ Conclusions

Mixing of states (fields) is a well-known phenomenon existing in the systems of neutrinos, quarks and hadrons. In hadron systems the mixing effects are essential not only for K^0- and D^0-mesons but also for the broad overlapping resonances. As for theoretical description of mixing phenomena, a general tendency with time and development of experiment consists in transition from a simplified quantum-mechanical description to the quantum field theory methods.

Mixing of fermion fields has some specifics as compared with boson case. Firstly, there exists γ-matrix structure in a propagator. Secondly, fermion and antifermion have the opposite P-parity, so fermion propagator contains contributions of different parities. As a result, besides a standard mixing of fields with the same quantum numbers, for fermions there exists a mixing of fields with opposite parities (OPF-mixing) at loop level, even if the parity is conserved in Lagrangian (Kaloshin,Lomov (2006)).

We will say about non-standard effect of OPF-mixing and its manifestation in systems of baryon resonances.

Projection operators

We will use the off-shell projection operators $\Lambda^{\pm}$:

$$\Lambda^{\pm} = \frac{1}{2}\left(1 \pm \frac{\hat{p}}{W}\right),$$

where $W = \sqrt{p^2}$ is energy in the rest frame.
Main properties of projection operators are:

$$\Lambda^{\pm}\Lambda^{\pm} = \Lambda^{\pm}, \quad \Lambda^{\pm}\Lambda^{\mp} = 0, \quad \Lambda^{\pm}\gamma^5 = \gamma^5\Lambda^{\mp},$$

$$\Lambda^{+} + \Lambda^{-} = 1, \quad \Lambda^{+} - \Lambda^{-} = \frac{\hat{p}}{W}.$$

Dyson–Schwinger equation for dressed propagator $G(p)$:

$$G(p) = G_0 + G\Sigma G_0, \tag{1}$$

where G_0 is a bare propagator and Σ is a self-energy.
Let us expand all elements in eq. (1) in the basis of projection operators:

$$G = \sum_{M=1}^{2} \mathcal{P}_M G^M, \qquad \mathcal{P}_1 \equiv \Lambda^+, \quad \mathcal{P}_2 \equiv \Lambda^-. \tag{2}$$

Dyson–Schwinger equation is reduced to equations on scalar functions:

$$G^M = G_0^M + G^M \Sigma^M G_0^M, \qquad M = 1, 2, \tag{3}$$

or

$$\left(G^{-1}\right)^M = \left(G_0^{-1}\right)^M - \Sigma^M. \tag{4}$$

Dressing of fermion

So we have inverse dressed propagator:

$$G^{-1} = \mathcal{P}_1\ (W - m - \Sigma^1) + \mathcal{P}_1\ (-W - m - \Sigma^2). \tag{5}$$

Usual form of the self-energy is

$$\Sigma(p) = A(p^2) + \widehat{p}B(p^2), \tag{6}$$

and its decomposition in projection basis:

$$\Sigma^1 = A(W^2) + WB(W^2), \qquad \Sigma^2 = A(W^2) - WB(W^2). \tag{7}$$

Note the property of coefficients in the projection basis:

$$\Sigma^2(W) = \Sigma^1(-W).$$

So the dressed propagator:

$$G = \mathcal{P}_1\ \frac{1}{(W - m - \Sigma^1)} + \mathcal{P}_1\ \frac{1}{(-W - m - \Sigma^2)}. \tag{8}$$

Mixing of fermion field with the same quantun numbers

When we have two fermion fields Ψ_i , the including of interaction leads also to mixing of these fields. In this case the Dyson–Schwinger equation (1) acquire matrix indices:

$$G_{ij} = (G_0)_{ij} + G_{ik}\Sigma_{kl}(G_0)_{lj}, \quad i,j,k,l = 1,2. \tag{9}$$

Therefore we have the same equation, but the coefficients are matrices.

$$G(p) = G_0 + G\Sigma G_0, \tag{10}$$

The simplest variant is when the fermion fields Ψ_i have the same quantum numbers and the parity is conserved in the Lagrangian. Inverse propagator in this case:

$$G^{-1} = \mathcal{P}_1 S^1(W) + \mathcal{P}_2 S^2(W) =$$

$$= \mathcal{P}_1 \begin{pmatrix} W - m_1 - \Sigma^1_{11} & -\Sigma^1_{12} \\ -\Sigma^1_{21} & W - m_2 - \Sigma^1_{22} \end{pmatrix} + \mathcal{P}_2 S^1(-W). \tag{11}$$

Mixing of fermion field with the same quantun numbers

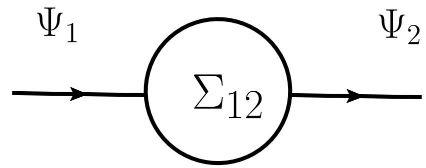
The matrix coefficients as before have the symmetry property $S^2(W) = S^1(-W)$. To obtain the matrix dressed propagator $G(p)$ one should reverse the matrix coefficients:

$$G(p) = \mathcal{P}_1(S^1(W))^{-1} + \mathcal{P}_2(S^2(W))^{-1} \tag{12}$$

We see that with use of projection basis the problem of fermion mixing is reduced to studying of the same mixing matrix as for bosons besides the obvious replacement $s - m^2 \to W - m$.

Opposite Parity Fiels (OPF) mixing

Look at the non-diagonal self-energy:



Let parity is conserved in Lagrangian.
Mixing of field with the same quantum numbers:

$$\begin{aligned}\Sigma_{12} &= A(p^2) + \hat{p}B(p^2) = \\ &= \Lambda^+ \ \left[A(W^2) + WB(W^2)\right] + \Lambda^- \ \left[A(W^2) - WB(W^2)\right]\end{aligned}$$

Mixing of field with opposite parities:

$$\begin{aligned}\Sigma_{12} &= \gamma^5 C(p^2) + \hat{p}\gamma^5 D(p^2) = \\ &= \Lambda^+\gamma^5 \ \left[C(W^2) + WD(W^2)\right] + \Lambda^-\gamma^5 \ \left[C(W^2) - WD(W^2)\right]\end{aligned}$$

Main statement: $\Sigma_{12} \neq 0$ for mixing of opposite parities fields.
Fermion specifics !

OPF-mixing

Let us consider the joint dressing of two fermion fields of opposite parities provided that the parity is conserved in a vertex. In this case the diagonal transition loops Σ_{ii} contain only I and $\hat{p}$ matrices, while the off-diagonal ones Σ_{12}, Σ_{21} must contain γ^5.
Projection basis should be supplemented by elements containing γ^5, it is convenient to choose the γ-matrix basis as:

$$\mathcal{P}_1 = \Lambda^+, \quad \mathcal{P}_2 = \Lambda^-, \quad \mathcal{P}_3 = \Lambda^+\gamma^5, \quad \mathcal{P}_4 = \Lambda^-\gamma^5. \tag{13}$$

In this case the γ-matrix decomposition has four terms:

$$S = \sum_{M=1}^{4} \mathcal{P}_M S^M, \tag{14}$$

where the coefficients S^M are matrices and have the obvious symmetry properties $S^2(W) = S^1(-W)$, $S^4(W) = S^3(-W)$.

Inverse propagator in this basis looks as:

$$S(p) = \mathcal{P}_1 \begin{pmatrix} W - m_1 - \Sigma^1_{11} & 0 \\ 0 & W - m_2 - \Sigma^1_{22} \end{pmatrix} + \\ + \mathcal{P}_2 \begin{pmatrix} -W - m_1 - \Sigma^2_{11} & 0 \\ 0 & -W - m_2 - \Sigma^2_{22} \end{pmatrix} + \\ + \mathcal{P}_3 \begin{pmatrix} 0 & -\Sigma^3_{12} \\ -\Sigma^3_{21} & 0 \end{pmatrix} + \mathcal{P}_4 \begin{pmatrix} 0 & -\Sigma^4_{12} \\ -\Sigma^4_{21} & 0 \end{pmatrix}, \tag{15}$$

where the indexes $i, j = 1, 2$ in the self-energy Σ^M_{ij} numerate dressing fermion fields and the indexes $M = 1, \ldots 4$ are refered to the γ-matrix decomposition (14).

Reversing of (15) gives the matrix dressed propagator (Kaloshin,Lomov Yad.Fiz.(2006)):

$$
\begin{aligned}
G =& \mathcal{P}_1 \begin{pmatrix} \dfrac{-W-m_2-\Sigma^2_{22}}{\Delta_1} & 0 \\ 0 & \dfrac{-W-m_1-\Sigma^2_{11}}{\Delta_2} \end{pmatrix} + \\
&+\mathcal{P}_2 \begin{pmatrix} \dfrac{W-m_2-\Sigma^1_{22}}{\Delta_2} & 0 \\ 0 & \dfrac{W-m_1-\Sigma^1_{11}}{\Delta_1} \end{pmatrix} + \\
&+\mathcal{P}_3 \begin{pmatrix} 0 & \dfrac{\Sigma^3_{12}}{\Delta_1} \\ \dfrac{\Sigma^3_{21}}{\Delta_2} & 0 \end{pmatrix} + \mathcal{P}_4 \begin{pmatrix} 0 & \dfrac{\Sigma^4_{12}}{\Delta_2} \\ \dfrac{\Sigma^4_{21}}{\Delta_1} & 0 \end{pmatrix}.
\end{aligned} \tag{16}
$$

$$\Delta_1 = \big(W - m_1 - \Sigma^1_{11}\big)\big(-W - m_2 - \Sigma^2_{22}\big) - \Sigma^3_{12}\Sigma^4_{21},$$

$$\Delta_2 = \big(-W - m_1 - \Sigma^2_{11}\big)\big(W - m_2 - \Sigma^1_{22}\big) - \Sigma^4_{12}\Sigma^3_{21} = \Delta_1\big(W \to -W\big).$$

Effective interaction Lagrangians have the form:

$$\mathscr{L}_{int} = i g_1 \bar{N}_1(x)\gamma^5 N(x)\phi(x) + \text{h.c.} \quad \text{for} \quad J^P(N_1) = 1/2^+$$

$$\mathscr{L}_{int} = g_2 \bar{N}_2(x) N(x)\phi(x) + \text{h.c.} \quad \text{for} \quad J^P(N_2) = 1/2^-.$$

In n-channel case, the scattering amplitude is a matrix of dimension n:

$$T = \bar{u}(p_2, s_2) R u(p_1, s_1), \tag{17}$$

where $\bar{u}(p_2, s_2)$, $u(p_1, s_1)$ are spinors, of final and initial nucleon, R is matrix of dimension n consisting of the propagator and coupling constants.
In the two-channel case (πN and ηN channel) matrix R is:

$$R = -\begin{pmatrix} i g_{1,\pi}\gamma^5 & g_{2,\pi} \\ i g_{1,\eta}\gamma^5 & g_{2,\pi} \end{pmatrix} G \begin{pmatrix} i g_{1,\pi}\gamma^5 & i g_{1,\eta}\gamma^5 \\ g_{2,\pi} & g_{2,\eta} \end{pmatrix}, \tag{18}$$

and generalization for n channels and m mixed states is obvious. Here G is dressed propagator (16), and we have introduced the short notations for coupling constants: $g_{1,\pi} \equiv g_{N_1 \pi N}$, $g_{2,\pi} \equiv g_{N_2 \pi N}$.

After some algebra the matrix R turns into into the standard form

$$R = \Lambda^{+} R_1 + \Lambda^{-} R_2, \tag{19}$$

where R_1 and R_2 are dimension 2 matrices. Note that the γ^5 matrix has been disappeared after multiplication in (18), since parity is not violated. After it we obtain from (17) the two-channel s- and p- partial waves.
s-waves amplitudes (produced resonances have $J^P = 1/2^-$):

$$\begin{aligned} f_{s,+}(\pi N \to \pi N) &= \frac{(E_1 + m_N)}{8\pi W \Delta_2}[g_{1,\pi}^2(W - m_2 - \Sigma_{22}^1) - \\ &- g_{2,\pi}^2(-W - m_1 - \Sigma_{11}^2) - i g_{1,\pi} g_{2,\pi}(\Sigma_{21}^3 + \Sigma_{12}^4)], \\ f_{s,+}(\pi N \to \eta N) &= \frac{\sqrt{(E_1 + m_N)(E_2 + m_N)}}{8\pi W \Delta_2}[g_{1,\pi} g_{1,\eta}(W - m_2 - \Sigma_{22}^1) - \\ &- g_{2,\pi} g_{2,\eta}(-W - m_1 - \Sigma_{11}^2) - i g_{2,\eta} g_{1,\pi} \Sigma_{21}^3 - i g_{1,\eta} g_{2,\pi} \Sigma_{12}^4], \end{aligned} \tag{20}$$

$$f_{s,+}(\eta N \to \eta N) = \frac{(E_2 + m_N)}{8\pi W \Delta_2}[g_{1,\eta}^2(W - m_2 - \Sigma_{22}^1) - \\ - g_{2,\eta}^2(-W - m_1 - \Sigma_{11}^2) - i g_{1,\eta} g_{2,\eta}(\Sigma_{21}^3 + \Sigma_{12}^4)],$$

$$\Delta_2 = \big(- W - m_1 - \Sigma_{11}^2 \big) \big(W - m_2 - \Sigma_{22}^1 \big) - \Sigma_{12}^4 \Sigma_{21}^3,$$

where E_1 and E_2 are nucleon energy in the c.m.s. for πN and ηN respectively.
For comparison, we write down the amplitude $\pi N \to \pi N$ in a tree approximation:

$$f_{s,+}^{\mathrm{tree}}(\pi N \to \pi N) = \frac{(E_1 + m_N)}{8\pi W} \left[\frac{g_{1,\pi}^2}{(-W - m_1)} - \frac{g_{2,\pi}^2}{(W - m_2)} \right]. \qquad (21)$$

Simultaneous calculation of p-wave amplitudes ($J^P = 1/2^+$) gives:

$$f_{p,-}(\pi N \to \pi N) = -\frac{(E_1 - m_N)}{8\pi W \Delta_1}[g^2_{1,\pi}(-W - m_2 - \Sigma^2_{22}) - $$
$$- g^2_{2,\pi}(W - m_1 - \Sigma^1_{11}) - ig_{1,\pi}g_{2,\pi}(\Sigma^4_{21} + \Sigma^3_{12})],$$
$$f_{p,-}(\pi N \to \eta N) = -\frac{\sqrt{(E_1 - m_N)(E_2 - m_N)}}{8\pi W \Delta_1}[g_{1,\pi}g_{1,\eta}(-W - m_2 - \Sigma^2_{22}) - $$
$$- g_{2,\pi}g_{2,\eta}(W - m_1 - \Sigma^1_{11}) - ig_{2,\eta}g_{1,\pi}\Sigma^4_{21} - ig_{1,\eta}g_{2,\pi}\Sigma^3_{12}],$$
$$f_{p,-}(\eta N \to \eta N) = -\frac{(E_2 - m_N)}{8\pi W \Delta_1}[g^2_{1,\eta}(-W - m_2 W - \Sigma^2_{22}) - $$
$$- g^2_{2,\eta}(W - m_1 - \Sigma^1_{11}) - ig_{1,\eta}g_{2,\eta}(\Sigma^4_{21} + \Sigma^3_{12})],$$
$$\Delta_1 = \big(W - m_1 - \Sigma^1_{11}\big)\big(-W + m_2 - \Sigma^2_{22}\big) - \Sigma^3_{12}\Sigma^4_{21}.$$

In tree approximation:

$$f^{\text{tree}}_{p,-}(\pi N \to \pi N) = \frac{(E_1 - m)}{8\pi W}\left[-\frac{g^2_{1,\pi}}{(W - m_1)} + \frac{g^2_{2,\pi}}{(-W - m_2)}\right]. \qquad (22)$$

Two remarks:

✓ One can check that the constructed partial amplitudes satisfy the multi-channel unitary condition:

$$Im\ f_{ij} = \sum_k \mid \mathbf{p}_k \mid \ f_{ik} \cdot f^*_{kj}, \tag{23}$$

where $\mathbf{p}_k$ is the c.m.s. spatial momentum of particles in k-th intermediate states.

✓ The obtained partial amplitudes satisfy the known MacDowell's symmetry (S.W.MacDowell. PR 116 (1959) 774), connecting different partial waves of πN scattering

$$f_{l,+}(-W) = -f_{l+1,-}(W). \tag{24}$$

In our approach it is a consequence of the symmetry properties of coefficients in the projection basis:

$$G^2(W) = G^1(-W), \quad G^4(W) = G^3(-W). \tag{25}$$

The self-energy (before renormalization) is expressed through the components of the standard loop functions $\Sigma_\pi(W)$ and $\Sigma_\eta(W)$:

$$\begin{aligned}
\Sigma^1_{11} &= -g^2_{1,\pi}\Sigma^2_\pi - g^2_{1,\eta}\Sigma^2_\eta,\\
\Sigma^2_{11} &= -g^2_{1,\pi}\Sigma^1_\pi - g^2_{1,\eta}\Sigma^1_\eta,\\
\Sigma^1_{22} &= g^2_{2,\pi}\Sigma^1_\pi + g^2_{2,\eta}\Sigma^1_\eta,\\
\Sigma^2_{22} &= g^2_{2,\pi}\Sigma^2_\eta + g^2_{2,\pi}\Sigma^2_\eta,\\
\Sigma^3_{12} &= ig_{1,\pi}g_{2,\pi}\Sigma^2_\pi + ig_{1,\eta}g_{2,\eta}\Sigma^2_\eta,\\
\Sigma^4_{12} &= ig_{1,\pi}g_{2,\pi}\Sigma^1_\pi + ig_{1,\eta}g_{2,\eta}\Sigma^1_\eta,\\
\Sigma^3_{21} &= \Sigma^4_{12},\\
\Sigma^4_{21} &= \Sigma^3_{12},
\end{aligned} \tag{26}$$

where function $\Sigma_\pi(p)$ corresponding πN intermediate state has the form:

$$\Sigma_\pi(p) = \frac{i}{(2\pi)^4}\int \frac{d^4k}{(\widehat{p}-\widehat{k}-m_N)(k^2-m_\pi^2)} = \Lambda^+\Sigma^1_\pi(W) + \Lambda^-\Sigma^2_\pi(W).$$

Self-energy

It is convenient to calculate first A and B and then pass to the projections $\Sigma^{1,2}$. So, we calculate discontinuities using Landau–Cutkosky rule:

$$\Delta A(p^2) = -\frac{i m_N |\mathbf{p}_\pi|}{4\pi W}, \quad \Delta B(p^2) = -\frac{i|\mathbf{p}_\pi|(p^2 + m_N^2 - m_\pi^2)}{8\pi p^2 W},$$

then restore functions $A(p^2)$ and $B(p^2)$ through dispersion relation, and finally calculate $\Sigma^{1,2}$:

$$\Sigma^1 = A(W^2) + WB(W^2), \quad \Sigma^2 = A(W^2) - WB(W^2).$$

Let us write down expression for imaginary parts $\Sigma^{1,2}$:

$$Im\ \Sigma_\pi^1 = -\frac{|\mathbf{p}_\pi|(E_1 + m_N)}{8\pi W}, \quad Im\ \Sigma_\pi^2 = \frac{|\mathbf{p}_\pi|(E_1 - m_N)}{8\pi W}, \qquad (27)$$

where $\mathbf{p}_\pi$ is momentum of pion in the c.m.s.

Recall that decomposition coefficients in the projection basis are related with each other by the substitution $W \to -W$. So to renormalize the self-energy, it is sufficient to define an exact form of $\Sigma^1(W)$ and $\Sigma^3(W)$, then the components $\Sigma^2(W)$, $\Sigma^4(W)$ are fixed by symmetry. We will use the on-mass-shell subtraction method of renormalization of resonance contribution (K.I.Aoki et al. Prog.Theor.Phys.Suppl. **73** (1982) 1; A.Denner. Fort.Phys. **41** (1993) 307).

According to this recipe, subtraction conditions for the self-energy included in the s-wave amplitudes have the form:

$$\begin{aligned} &Re\ \Sigma^1_{22}(W) && \text{has zero of second order at } W = m_2, \\ &Re\ \Sigma^2_{11}(W) && \text{has zero of second order at } W = -m_1, \\ &Im\ \Sigma^3_{21} && \text{has zeros at } W = -m_1 \text{ and } W = m_2. \end{aligned} \tag{28}$$

After it all is defined in partial waves besides parameters (masses and coupling constants)

Estimates of effects

Let us use our amplitudes (20),(22) to calculate πN partial s- and p-waves, where baryons $J^P = 1/2^\pm$ can be produced. We are interested here only in estimates of the observed effects, so we restrict ourselves by the single-channel approach and fix the parameters (masses and coupling constants) from rough correspondence to parameters of the observed baryon resonances $I = 1/2$

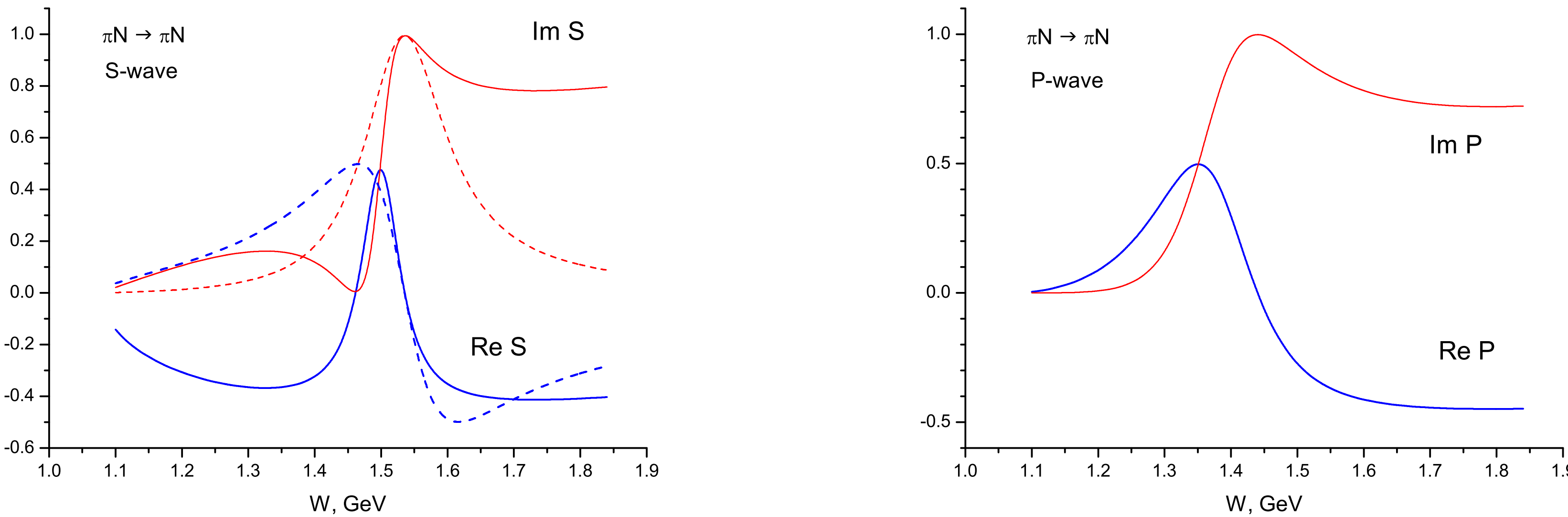
$$P_{11}(1440),\ \ J^P = 1/2^+ : \quad M_1 = 1.440 \text{ GeV},\ \ \Gamma_1 = 300 \text{ MeV} \Rightarrow g_{1,\pi} = 13.0 \text{ GeV}$$
$$S_{11}(1535),\ \ J^P = 1/2^- : \quad M_2 = 1.535 \text{ GeV},\ \ \Gamma_2 = 150 \text{ MeV} \Rightarrow g_{2,\pi} = 1.77 \text{ GeV}. \tag{29}$$

For estimates we used the relations of the widths and coupling constants in the absence of mixing.

$$\begin{aligned} \Gamma(N_1(1/2^+) \to \pi N) &= \frac{g_{1,\pi}^2}{4\pi} \cdot \frac{|\mathbf{p}_\pi|(E_1 - m_N)}{M}, \\ \Gamma(N_2(1/2^-) \to \pi N) &= \frac{g_{2,\pi}^2}{4\pi} \cdot \frac{|\mathbf{p}_\pi|(E_1 + m_N)}{M}. \end{aligned} \tag{30}$$

Estimates of effects

The results of calculations of πN s- and p-wave partial wave.



Solid lines — real and imaginary parts of our partial amplitude (20), (22) *in the single-channel approach with the parameters* (29). *Dashed lines — our amplitudes with neglecting the mixing effect:* $\Sigma_{12} = \Sigma_{21} = 0$. *For p-wave the solid and dashed lines coincide with each other. All variants of amplitudes satisfy the single-channel unitary condition* $Im\ f = |f|^2$.

Two features are seen:

- ✓ Considered OPF-mixing is more essential for lowest l wave
- ✓ For s-wave we observe the (unitary) interference of resonance and negative background

Both features are seen from tree amplitudes

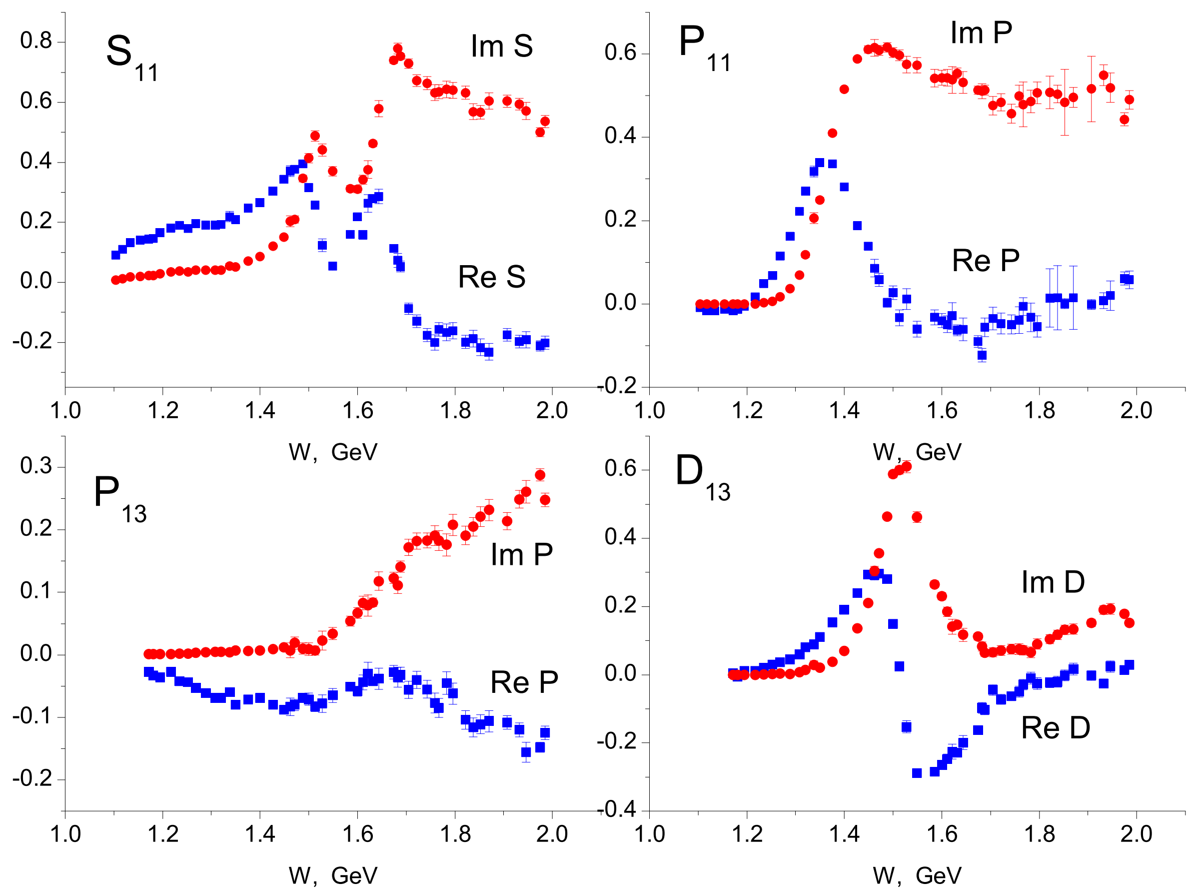
$$f^{\text{tree}}_{s,+}(\pi N \to \pi N) = \frac{(E_1 + m_N)}{8\pi W}\left[\frac{g^2_{1,\pi}}{(-W - m_1)} - \frac{g^2_{2,\pi}}{(W - m_2)}\right] \qquad (31)$$

$$f^{\text{tree}}_{p,-}(\pi N \to \pi N) = \frac{(E_1 - m)}{8\pi W}\left[-\frac{g^2_{1,\pi}}{(W - m_1)} + \frac{g^2_{2,\pi}}{(-W - m_2)}\right] \qquad (32)$$

Since we have normalized the coupling constants on the resonance width, inequality between the coupling constants ($|g_{1,\pi}| \gg |g_{2,\pi}|$) is a consequence of the inequality between the s- and p-wave phase volumes.

Partial wave analysis (PWA) of $\pi N \to \pi N$ with $I = 1/2$

R.A.Arndt et al. PR C74 (2006) 045205; (gwdac.phys.gwu.edu) (current solution)



The pair of partial waves P_{13}, D_{13} looks as a most suitable place for identification of the discussed OPF-mixing effect.

OPF-mixing for baryons $3/2^{\pm}$

The same effect of OPF-mixing arises for vector-spinor fields Ψ^{μ}, which describe the spin-3/2 particles.
Here we present only the results of calculations: the hadron partial amplitudes in two-channel (πN, ηN) approach.

p-wave amplitudes ($J^P = 3/2^+$) have the form:

$$\begin{aligned}
f_{p,+}(\pi N \to \pi N) &= |\mathbf{p}_\pi|^2 \frac{(E_1+m)}{24\pi W \Delta_2}[g_{1,\pi}^2(W - m_2 - \Sigma_{22}^1) - \\
&\quad - g_{2,\pi}^2(-W - m_1 - \Sigma_{11}^2) + i g_{1,\pi} g_{2,\pi}(\Sigma_{21}^3 + \Sigma_{12}^4)], \\
f_{p,+}(\pi N \to \eta N) &= |\mathbf{p}_\pi||\mathbf{p}_\eta| \frac{\sqrt{(E_1+m)(E_2+m)}}{24\pi W \Delta_2}[g_{1,\pi} g_{1,\eta}(W - m_2 - \Sigma_{22}^1) - \\
&\quad - g_{2,\pi} g_{2,\eta}(-W - m_1 - \Sigma_{11}^2) + i g_{1,\pi} g_{2,\eta} \Sigma_{12}^4 + i g_{2,\pi} g_{1,\eta} \Sigma_{21}^3], \\
f_{p,+}(\eta N \to \eta N) &= |\mathbf{p}_\eta|^2 \frac{(E_2+m)}{24\pi W \Delta_2}[g_{1,\eta}^2(W - m_2 - \Sigma_{22}^1) - \\
&\quad - g_{2,\eta}^2(-W - m_1 - \Sigma_{11}^2) + i g_{1,\eta} g_{2,\eta}(\Sigma_{21}^3 + \Sigma_{12}^4)], \\
\Delta_2 &= (-W - m_1 - \Sigma_{11}^2)(W - m_2 - \Sigma_{22}^1) - \Sigma_{12}^4 \Sigma_{21}^3.
\end{aligned} \tag{33}$$

d-wave amplitudes ($J^P = 3/2^-$):

$$f_{d,-}(\pi N \to \pi N) = |\mathbf{p}_\pi|^2 \frac{(E_1 - m)}{24\pi W \Delta_1}[-g^2_{1,\pi}(-W - m_2 - \Sigma^2_{22}) + \\ + g^2_{2,\pi}(W - m_1 - \Sigma^1_{11}) - i g_{1,\pi} g_{2,\pi}(\Sigma^4_{21} + \Sigma^3_{12})],$$

$$f_{d,-}(\pi N \to \eta N) = |\mathbf{p}_\pi||\mathbf{p}_\eta| \frac{\sqrt{(E_1 - m)(E_2 - m)}}{24\pi W \Delta_1}[-g_{1,\pi} g_{1,\eta}(-W - m_2 - \Sigma^2_{22}) + \\ + g_{2,\pi} g_{2,\eta}(W - m_1 - \Sigma^1_{11}) - i g_{1,\pi} g_{2,\eta} \Sigma^3_{12} - i g_{2,\pi} g_{1,\eta} \Sigma^4_{21}],$$

$$f_{d,-}(\eta N \to \eta N) = |\mathbf{p}_\eta|^2 \frac{(E_2 - m)}{24\pi W \Delta_1}[-g^2_{1,\eta}(-W - m_2 - \Sigma^2_{22}) + \\ + g^2_{2,\eta}(W - m_1 - \Sigma^1_{11}) - i g_{1,\eta} g_{2,\eta}(\Sigma^4_{21} + \Sigma^3_{12})],$$

$$\Delta_1 = (W - m_1 - \Sigma^1_{11})(-W - m_2 - \Sigma^2_{22}) - \Sigma^3_{12}\Sigma^4_{21}.$$

where E_1 and E_2 are nucleon energies for πN and ηN states respectively. The obtained p- and d- partial amplitudes satisfy the two-channel unitary condition.

OPF-mixing for baryons $3/2^{\pm}$

Besides, we should take into account the W-dependent form-factor in a vertex (the so called centrifugal barrier factor). We choose it in the simplest pole form:

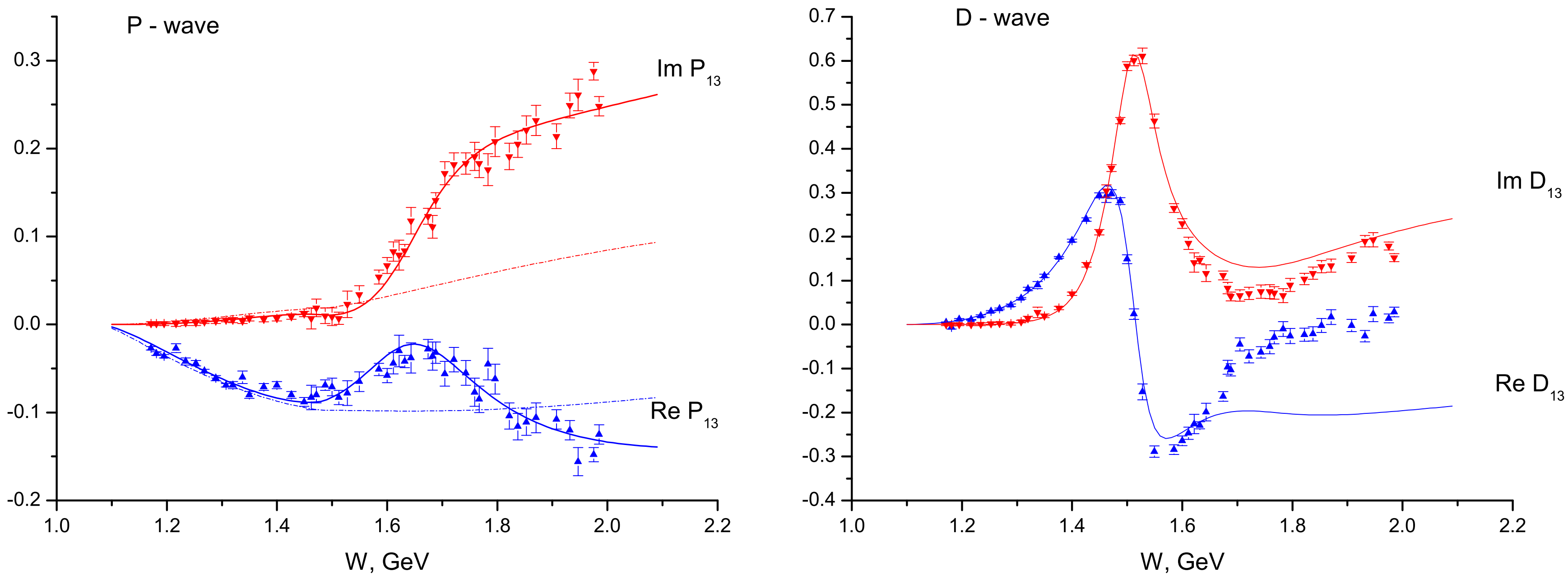
$$g \to g \cdot F(W) = g \cdot \frac{M^2 + a}{W^2 + a}, \quad M = 1.5 \quad \text{GeV}. \tag{34}$$

But in fact in considered region of energy $W < 2$ GeV the two-channel approximation is not valid because there exist at least five open channels. In this situation we will follow the way suggested in
M.Batinic et al. PR C51 (1995) 2310,
S.Ceci at al. PR D77 (2008) 116007:
we will restrict ourselves by the three-channel approach (πN, ηN and σN). As for third channel ($\sigma N = \pi\pi N$), it will be some "effective" channel and its threshold will be a free parameter in a fit.

Three-channel amplitudes may be obtained from by the same manner. We use the same procedure of renormalization as for spin $1/2$.

Fit of PWA results $\pi N \to \pi N$

The result of simultaneous fitting of P_{13} and D_{13} waves:



The points are the result of the partial wave analysis **R.A.Arndt et al. PR C74 (2006) 045205** *for real and imaginary parts of amplitude, the solid curves are the result of fitting by our amplitudes with account of OPF-mixing, dotted line is the background contribution at zeroth coupling constants of the resonance* $3/2^+$*.*

Fit of PWA results $\pi N \rightarrow \pi N$

When fitting we restrict ourselves by the region $W < 2$ GeV for P_{13} partial wave and $W < 1.6$ GeV for D_{13}.

The best result of **simultaneous** fitting of P_{13} and D_{13} is:

$$\chi^2/DOF = 338/136 = 2.48. \tag{35}$$

The fitting parameters are the masses and coupling constants of two resonances, the mass of "effective particle" σ and the cutoff parameter a in the vertex form-factor.

Conclusions

We have analyzed the non-standard mixing effect, when two fermion fields of opposite parities are mixed at loop level. For fermions it is possibly, even if the parity is conserved in a vertex.

- ✓ As a result we have a pair of πN partial waves (same J but different parities) with strongly correlated parameters. Namely, the resonance in one partial wave is connected with background contribution in another wave and vice versa.
- ✓ We used the obtained amplitudes for the simultaneous description of two πN partial waves P_{13} and D_{13}. The discussed effect reproduces all the observed features of these partial waves and leads to the good quality description of PWA results. So, we see manifestation of this effect in πN partial waves with $J^P = 3/2^{\pm}$ and $I = 1/2$.
- ✓ We suppose that the most interesting application of this effect is related with the problem of Roper resonance $N(1440)$, $1/2^+$.

Multiplicative properties of elements of basis

	$\mathcal{P}_1$	$\mathcal{P}_2$	$\mathcal{P}_3$	$\mathcal{P}_4$
$\mathcal{P}_1$	$\mathcal{P}_1$	0	$\mathcal{P}_3$	0
$\mathcal{P}_2$	0	$\mathcal{P}_2$	0	$\mathcal{P}_4$
$\mathcal{P}_3$	0	$\mathcal{P}_3$	0	$\mathcal{P}_1$
$\mathcal{P}_4$	$\mathcal{P}_4$	0	$\mathcal{P}_2$	0