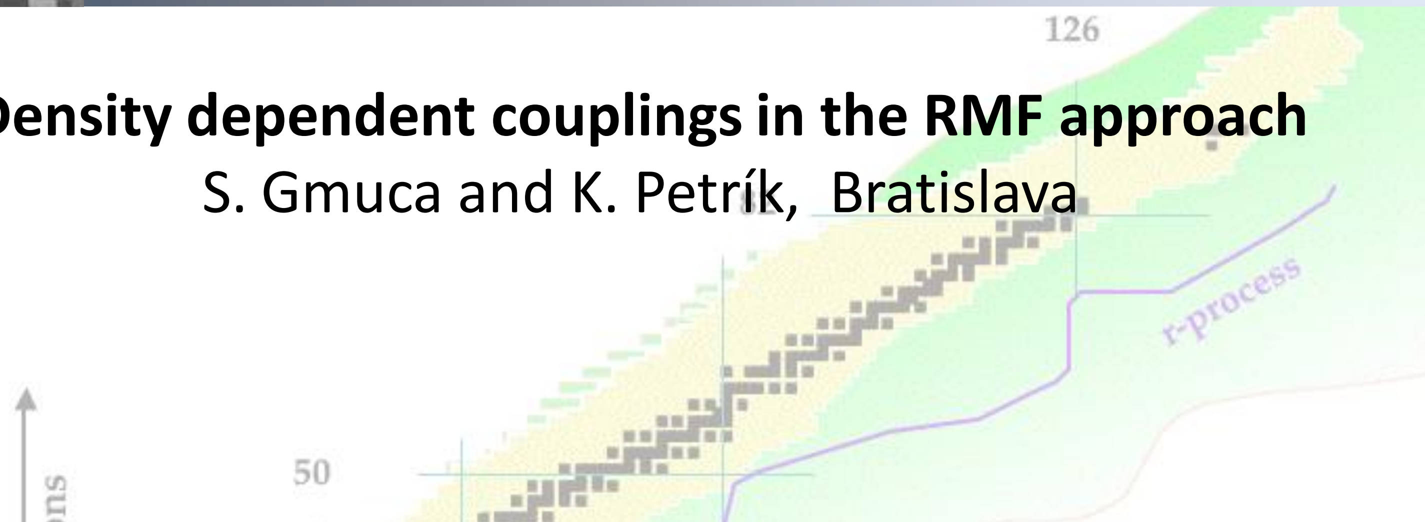
ISHEPP XX

International Baldin Seminar on High Energy Physics Problems

"Relativistic Nuclear Physics and Quantum Chromodynamics"

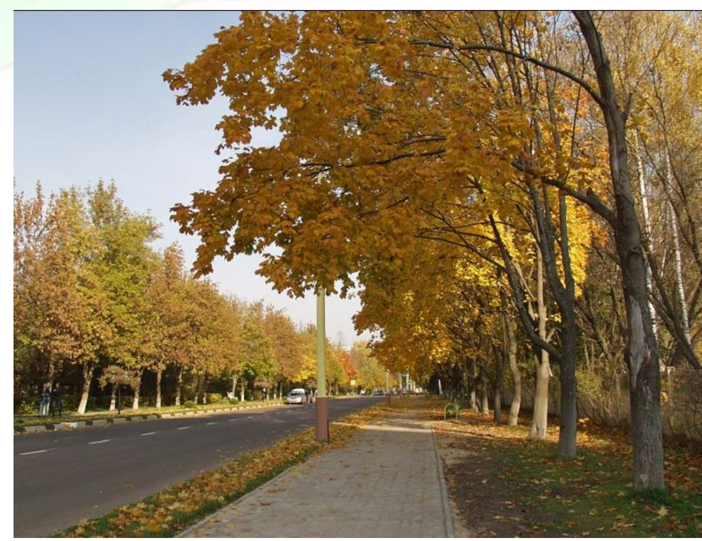
October 4-9, 2010, Dubna, Russia



Density dependent couplings in the RMF approach

S. Gmuca and K. Petrík, Bratislava

- Introduction
- Current approaches
- Dirac Hartree-Fock
- Effective RMF couplings
- Summary and outlook



Many-Body Approaches to Nuclear Matter EOS

- **Microscopic Many-Body Approaches**

  Non-relativistic Brueckner-Bethe-Goldstone (BBG) Theory
  Relativistic Dirac-Brueckner-Hartree-Fock (DBHF) approach
  Self-consistent Green's Function (SCGF) Theory
  Variational Many-Body (VMB) approach
  ……
- **Effective Field Theory**

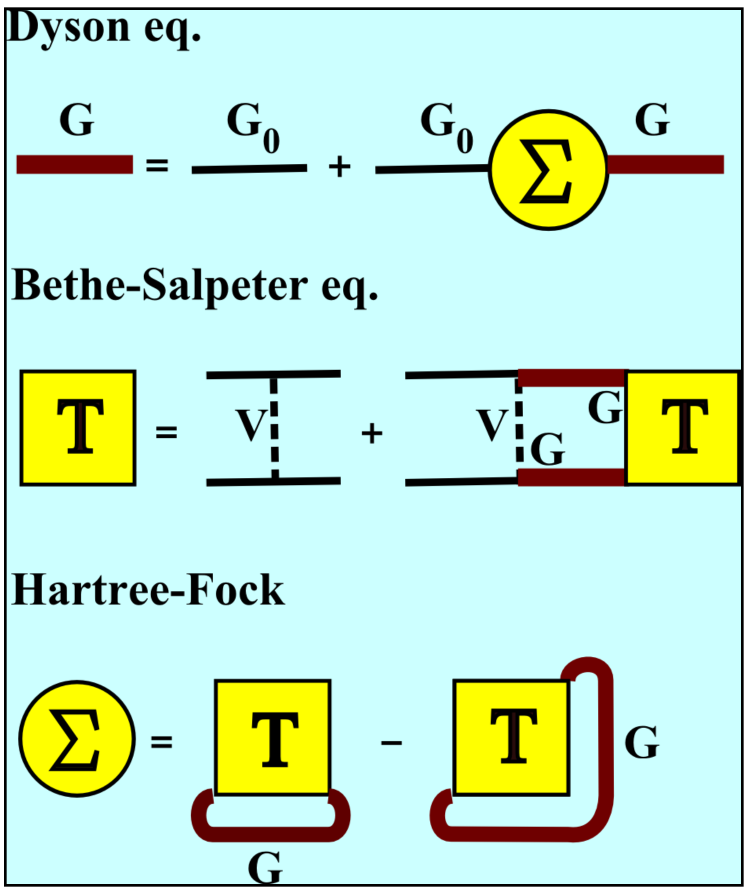
  Density Functional Theory (DFT)
  Chiral Perturbation Theory (ChPT)
  ……
- **Phenomenological Approaches**

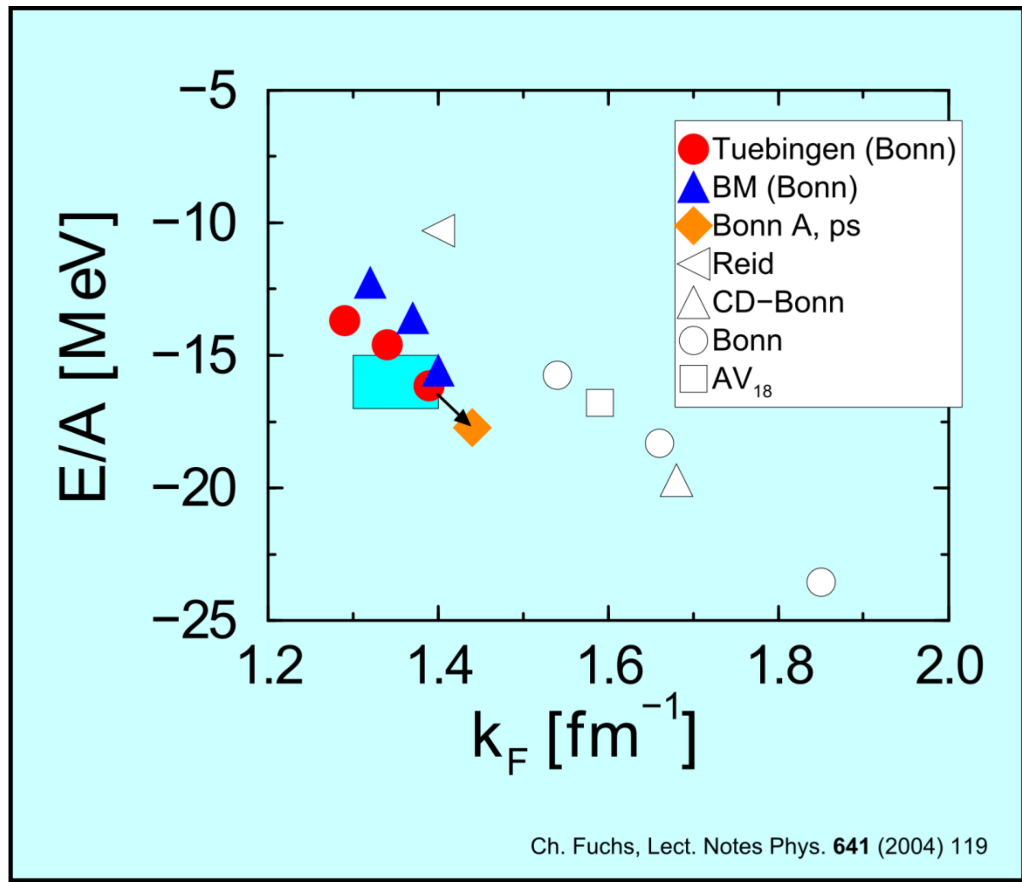
  Relativistic mean-field (RMF) theory
  Density dependent relativistic hadron field theory (DDRH)
  Relativistic Hartree-Fock (RHF)
  Non-relativistic Hartree-Fock (Skyrme-Hartree-Fock)
  Thomas-Fermi (TF) approximations
  Phenomenological potential models
  ……

Dirac-Brueckner-Hartree-Fock (DBHF)

- 2-N correlations in hole-line expansion;
- self-consistent sum of ladder diagrams

⇨ *realistic **in-medium** interaction by the use of realistic free (bare) interaction*





Dirac-Brueckner-Hartree-Fock (DBHF)
- 2-N correlations in hole-line expansion;
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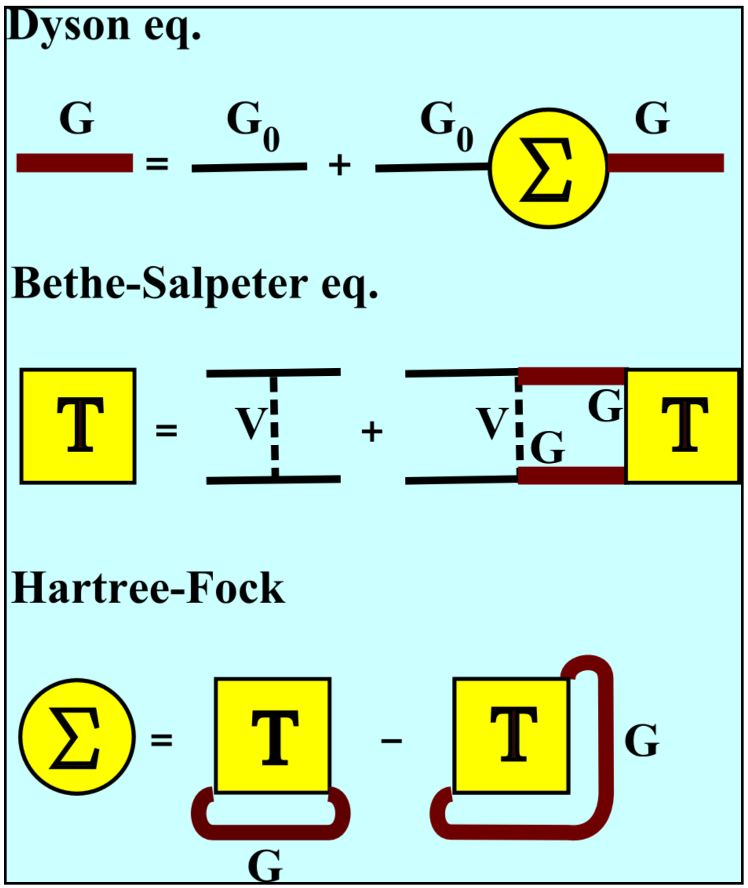
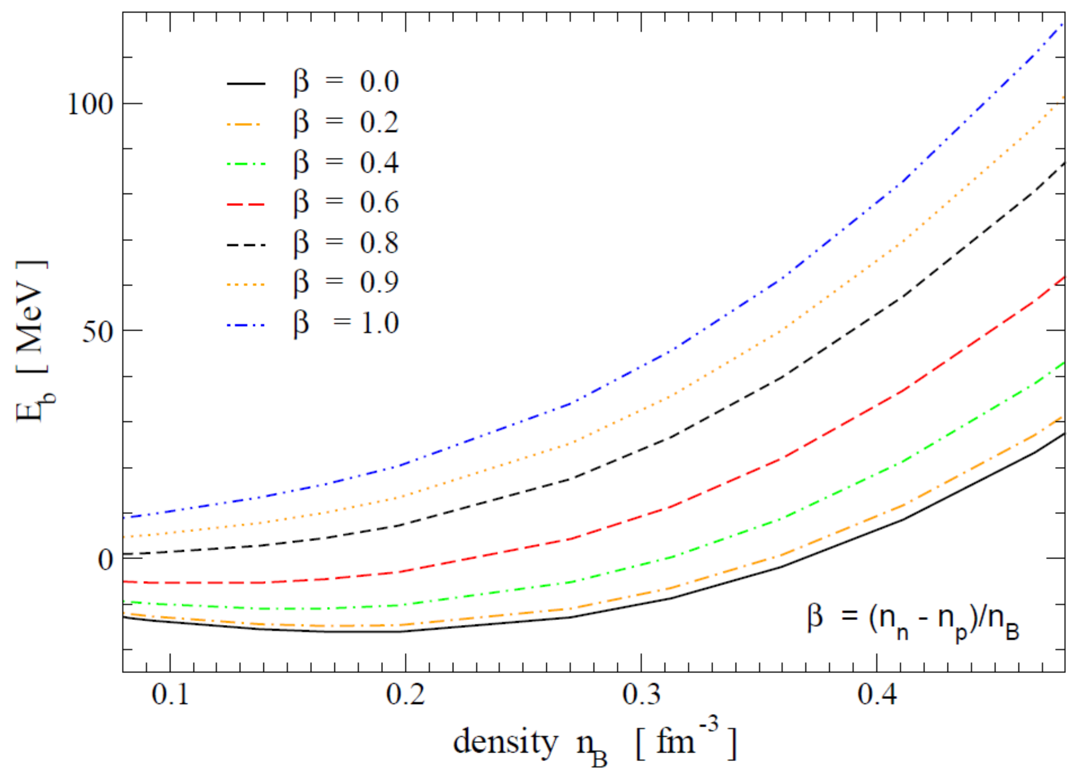


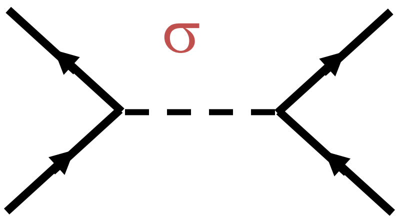
subtracted T matrix representation



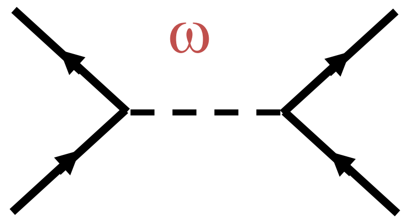
E.N.E. van Dalen, H. Muther. PRC **80**, 037303 (2009)

Nonlinear relativistic mean field model (RMF)

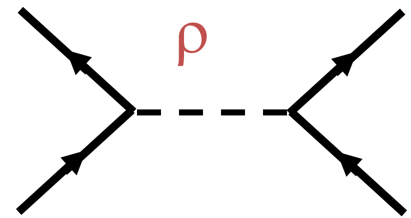
Nucleons are coupled by exchange of mesons via an effective Lagrangian



$(J^{\pi}\, T) = (0^{+}0)$



$(J^{\pi}\, T) = (1^{-}0)$



$(J^{\pi}\, T) = (1^{-}1)$

Sigma-meson:
attractive scalar field

Omega-meson:
Short-range repulsive

Rho-meson:
Isovector field

$$S(\mathbf{r}) = g_{\sigma}\sigma(\mathbf{r})$$

$$V(\mathbf{r}) = g_{\omega}\omega^{0}(\mathbf{r}) + g_{\rho}\tau_{3}\rho^{0}(\mathbf{r}) + e\frac{1-\tau_{3}}{2}A^{0}(\mathbf{r})$$

Lagrangian density of RMF theory

$$L=\bar{\psi}_i\left(i\gamma_\mu\partial^\mu-M-g_\sigma\sigma-g_\omega\gamma_\mu\omega^\mu-g_\rho\gamma_\mu\vec{\rho}^\mu\vec{\tau}-e\frac{1-\tau_3}{2}\gamma_\mu A^\mu\right)\psi_i$$

$$+\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma-U(\sigma)-\frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu}+U(\omega)$$

$$-\frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu}+U(\vec{\rho})-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Masses: $M, m_\sigma, m_\omega, m_\rho$

Nucleon-meson couplings: $g_\sigma, g_\omega, g_\rho$

Self-coupling constants: g_2, g_3, c_3, d_3

Nonlinear meson self-couplings

$$U(\sigma)=\frac{1}{2}m_\sigma^2\sigma^2+\frac{1}{3}g_2\sigma^3+\frac{1}{4}g_3\sigma^4$$

$$U(\omega)=\frac{1}{2}m_\omega^2\omega^\mu\omega_\mu+\frac{1}{4}c_3\left(\omega^\mu\omega_\mu\right)^2$$

$$U(\rho)=\frac{1}{2}m_\rho^2\vec{\rho}^\mu\cdot\vec{\rho}_\mu+\frac{1}{4}d_3\left(\vec{\rho}^\mu\cdot\vec{\rho}_\mu\right)^2$$

Many typical nonlinear parameterizations:

NL1, NL2, NL3, NLSH, TM1, TM2, PK1, PK2, …….

DDRH - Density Dependent Relativistic Hadron field theory

[Fuchs, Lenske, Wolter, PRC **52** (1995),
Keil, Hofmann, Lenske, PRC **61** (2000) 064309]

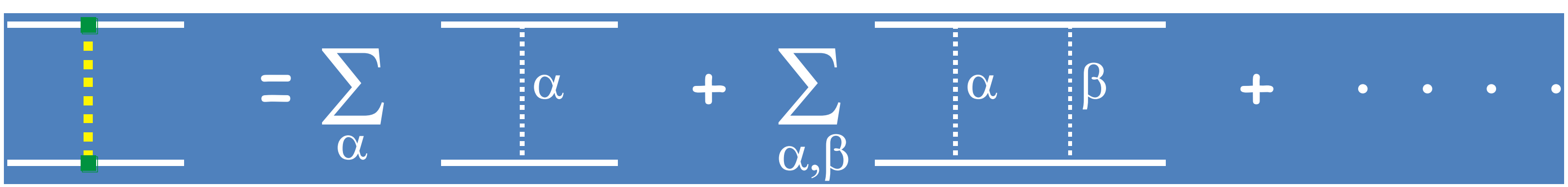
$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\text{int}} \ ,$$

$$L_0 = \bar{\psi}(i\gamma_\mu \partial^\mu - M)\psi + \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2 \omega_\mu \omega^\mu$$
$$+ \frac{1}{2}(\partial_\mu \boldsymbol{\delta} \partial^\mu \boldsymbol{\delta} - m_\delta^2 \boldsymbol{\delta}^2) - \frac{1}{4}\boldsymbol{\rho}_{\mu\nu}\boldsymbol{\rho}^{\mu\nu} + \frac{1}{2}m_\rho^2 \boldsymbol{\rho}_\mu \boldsymbol{\rho}^\mu$$

$$L_{\text{int}} = g^*_\sigma(\hat{\rho})\bar{\psi}\psi\sigma + g^*_\omega(\hat{\rho})\bar{\psi}\gamma_\mu\psi\omega^\mu$$
$$+ g^*_\delta(\hat{\rho})\bar{\psi}\boldsymbol{\tau}\psi\boldsymbol{\delta} + g^*_\rho(\hat{\rho})\bar{\psi}\boldsymbol{\tau}\gamma_\mu\psi\boldsymbol{\rho}^\mu$$

$$\hat{\rho} = f(\bar{\psi}, \psi), \ \text{Lorentz scalar}$$

- Walecka-type mean-field model
- ψ dependent functionals instead of constant couplings
- no meson self-interactions
- couplings give parameterization of G-matrix

G


Bonn A

DDRH - density dependent effective couplings

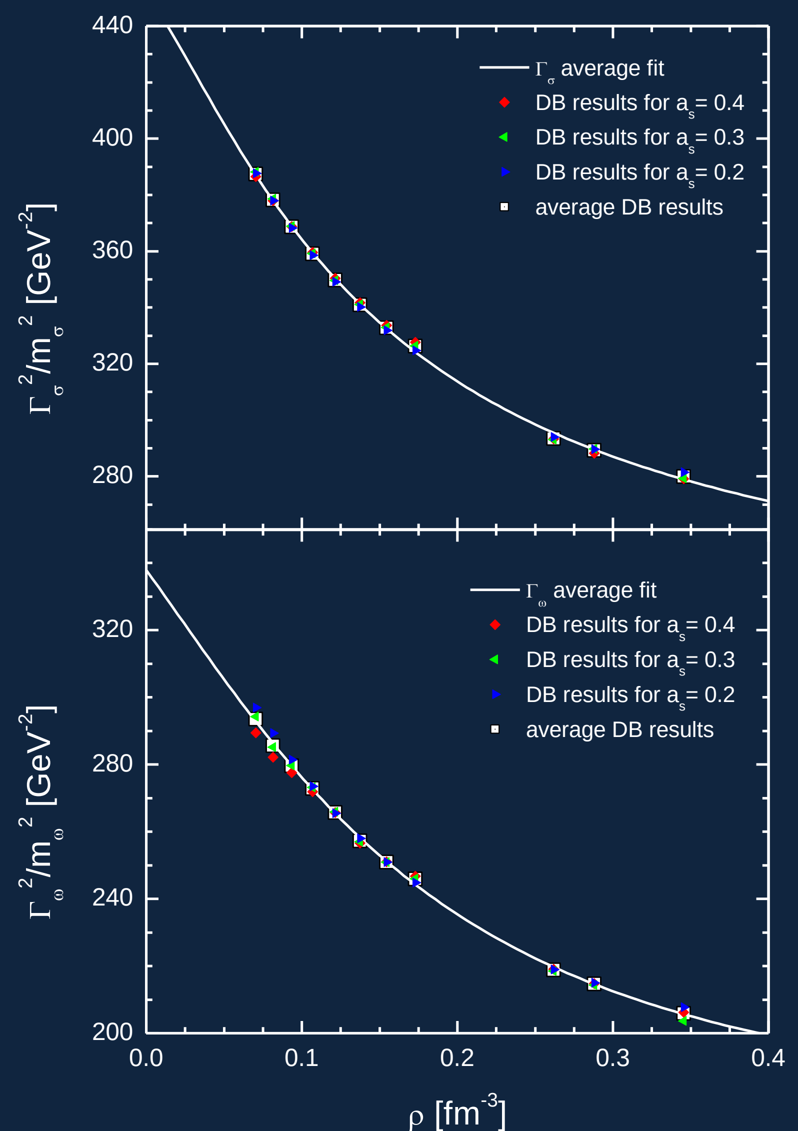
Decomposition of DBHF self-energy

$$\Sigma(p) = \Sigma_s(p) - \gamma^0 \Sigma_0(p) + \boldsymbol{\gamma} \cdot \hat{\boldsymbol{p}} \Sigma_v(p)$$

Density and momentum dependent couplings

$$\Gamma_i^2(p,\rho) = \left(\frac{g_i}{m_i}\right)^2 = -\frac{1}{2}\frac{\Sigma_i(p,\rho)}{\rho_i}$$

$$i = \sigma, \omega, \rho, \delta$$

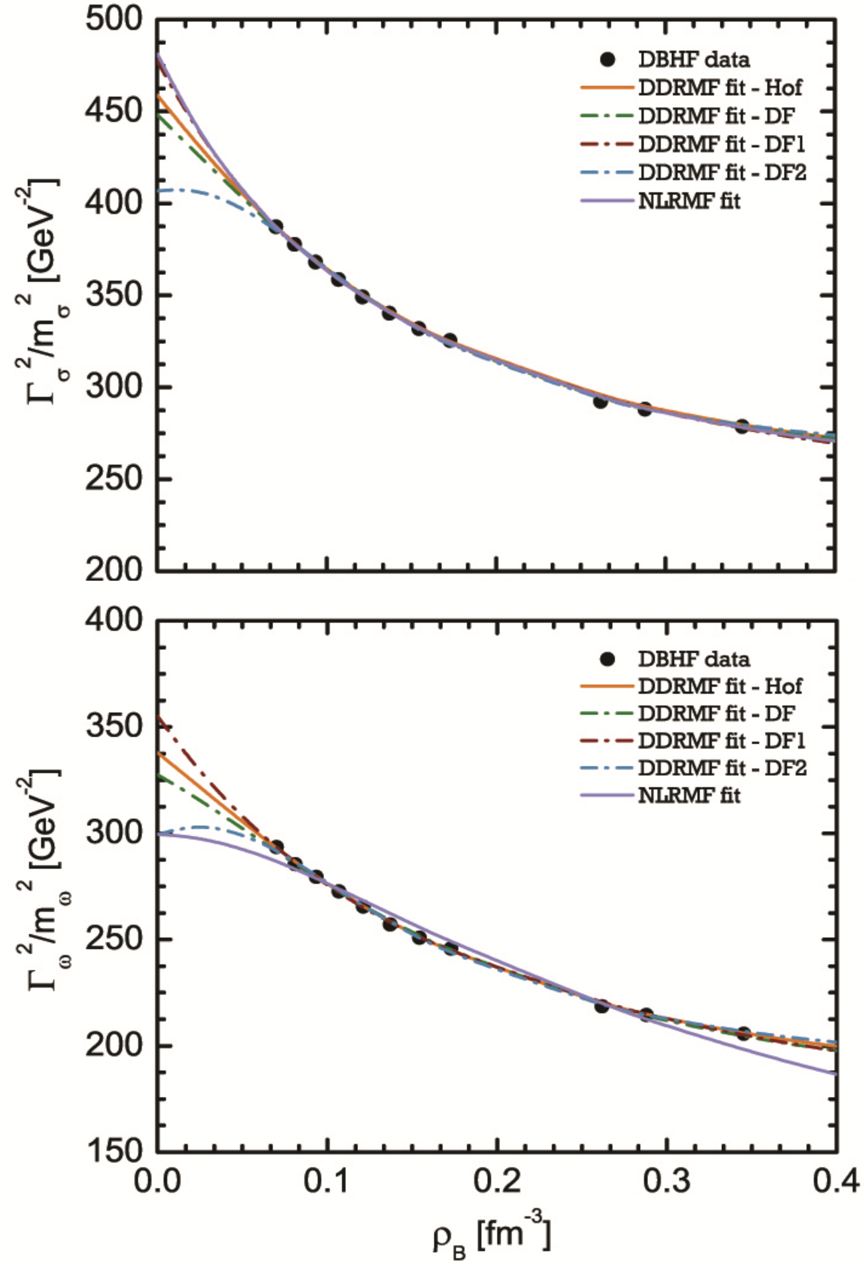


Low density behaviour of effective couplings

$$DF = a_i \frac{1+\left(\frac{\rho_B}{\rho_{sat}}+d_i\right)^2}{c_i+\left(\frac{\rho_B}{\rho_{sat}}+c_i\right)^2}$$

$$DF1 = a_i \frac{1+\left(\frac{\rho_B}{\rho_{sat}}+d_i\right)^2}{\left(\frac{\rho_B}{\rho_{sat}}+c_i\right)^2}$$

$$DF2 = a_i \frac{1+\left(\frac{\rho_B}{\rho_{sat}}+d_i\right)^2}{\left(\frac{\rho_B}{\rho_{sat}}\right)^2+c_i}$$

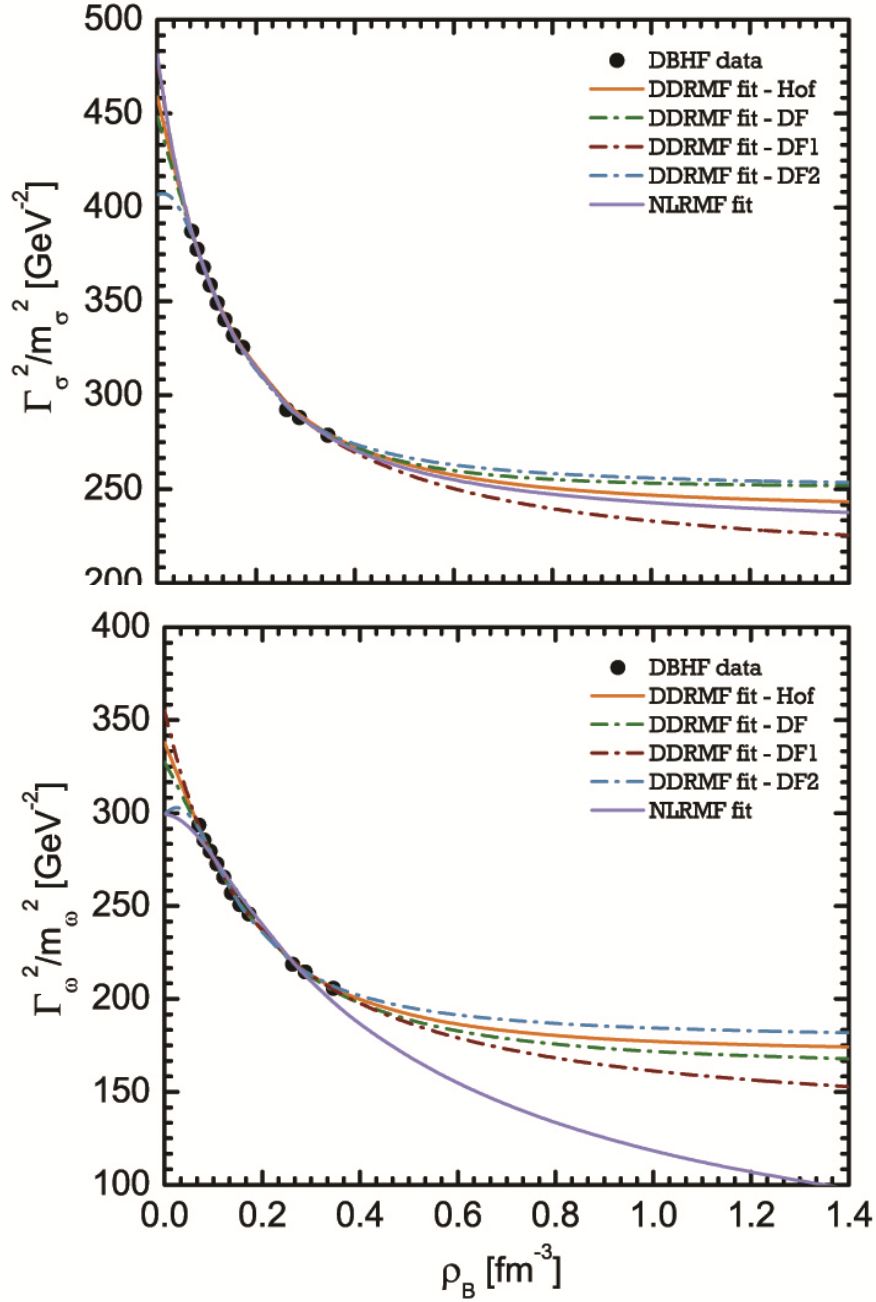


High density behaviour of effective couplings

$$DF = a_i \frac{1+\left(\frac{\rho_B}{\rho_{sat}}+d_i\right)^2}{c_i+\left(\frac{\rho_B}{\rho_{sat}}+c_i\right)^2}$$

$$DF1 = a_i \frac{1+\left(\frac{\rho_B}{\rho_{sat}}+d_i\right)^2}{\left(\frac{\rho_B}{\rho_{sat}}+c_i\right)^2}$$

$$DF2 = a_i \frac{1+\left(\frac{\rho_B}{\rho_{sat}}+d_i\right)^2}{\left(\frac{\rho_B}{\rho_{sat}}\right)^2+c_i}$$

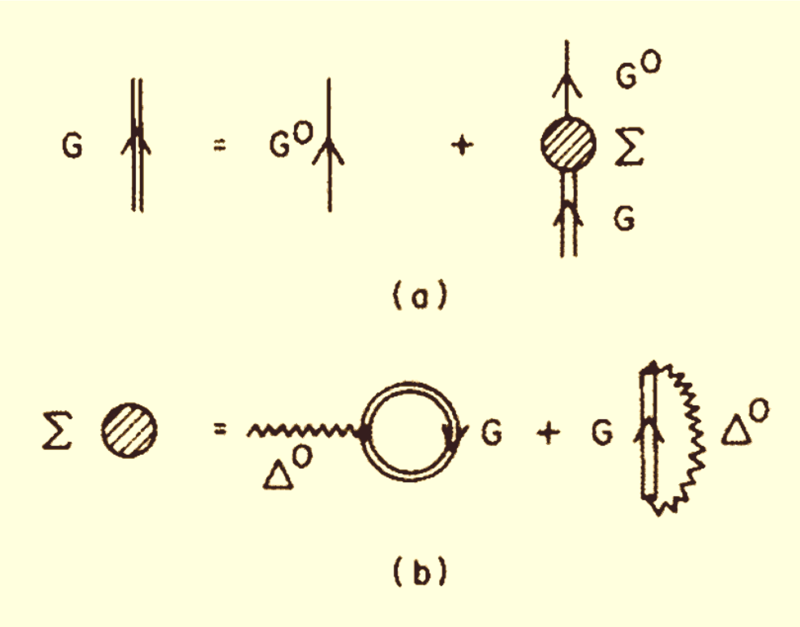


CJ Horowitz, BD Serot, NPA **399**, 529; PLB **108**, 377
A Bouyssy *et al.*, PRC **36**, 380

DHF for Nuclear Matter

$$L_0 = \bar{\psi}(i\gamma_\mu \partial^\mu - M)\psi + \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2)$$
$$-\frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2 \omega_\mu \omega^\mu + \frac{1}{2}(\partial_\mu \pi \partial^\mu \pi - m_\pi^2 \pi^2)$$

$$L_I = -g_\sigma \bar{\psi}\sigma\psi - g_\omega \bar{\psi}\gamma_\mu \omega^\mu \psi - \frac{f_\pi}{m_\pi}\bar{\psi}\gamma_5 \gamma_\mu (\partial^\mu \pi)\psi$$



Decomposition of self-energy

$$\Sigma(k) = \Sigma^s(k) - \gamma^0 \Sigma^0(k) + \boldsymbol{\gamma} \bullet \hat{\boldsymbol{k}} \Sigma^v(k)$$

DHF components of Σ in nuclear matter at Fermi momentum k_F

$$\Sigma^s(k_F,k) = \frac{-\gamma}{(2\pi)^3}\frac{g_\sigma^2}{m_\sigma^2}\int_0^{k_F} d^3q\,\frac{M^\star(q)}{E^\star(q)} + \frac{1}{(4\pi)^2 k}\int_0^{k_F} qdq\,\frac{M^\star(q)}{E^\star(q)}\left[g_\sigma^2\Theta_\sigma(k,q) - 4g_\omega^2\Theta_\omega(k,q) - 3\left(\frac{f_\pi}{m_\pi}\right)^2 m_\pi^2\Theta_\pi(p,q)\right],$$

$$\Sigma^0(k_F,k) = \frac{-\gamma}{(2\pi)^3}\frac{g_v^2}{m_v^2}\int_0^{k_F} d^3q - \frac{1}{(4\pi)^2 k}\int_0^{k_F} qdq\left[g_\sigma^2\Theta_\sigma(k,q) + 2g_\omega^2\Theta_\omega(k,q) - 3\left(\frac{f_\pi}{m_\pi}\right)^2 m_\pi^2\Theta_\pi(p,q)\right],$$

where

$$\Theta_i(k,q) = \ln\left|\frac{A_i(k,q) + 2kq}{A_i(k,q) - 2kq}\right|$$

$$\Phi_i(k,q) = \frac{A_i(k,q)\Theta_i(k,q)}{4kq} - 1$$

and $A_i(k,q) = \boldsymbol{k}^2 + \boldsymbol{q}^2 + m_i^2 - \left(E_q - E_k\right)^2;\quad q = |\boldsymbol{q}|,\ k = |\boldsymbol{k}|$

Effective couplings

Goal: Evaluate exchange terms in the DHF selfenergy in closed expressions and relate them to the direct (Hartree) terms through the "effective couplings"

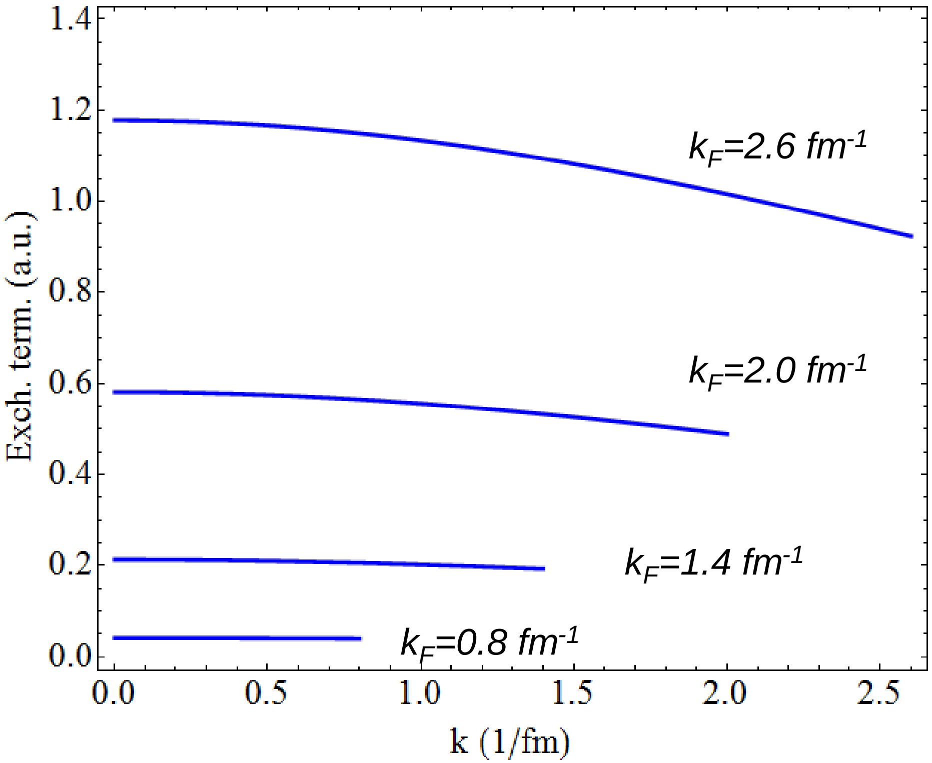
Simplifying assumption: "no retardation" i.e. $(E_q\text{-}E_k)^2$ term is neglected as it is small compared to the masses of exchanged mesons $(E_q\text{-}E_k)^2/m^2_{s,v} \approx 0.01$

Then, each particular contribution to exchange terms depends upon the integral of the type:

$$I_{exc}\left(k_F,k\right)=\frac{1}{k}\int_0^{k_F} qdq\ln\frac{\left(k+q\right)^2+m^2}{\left(k-q\right)^2+m^2}$$

- expanding in k,
- keeping terms up to k^2,
- averaging over Fermi sphere

$$\bar{I}_{exc}\left(k_F\right)\approx 4\left(k_F-m\,\mathrm{Arctg}\left(\frac{k_F}{m}\right)-\frac{k_F^5}{5\left(k_F^2+m^2\right)^2}\right)$$



Density dependence of effective couplings

Vector component of selfenergy:

$$\Sigma^0(k_F) = \frac{g_\omega^{*2}(k_F)}{m_\omega^2}\rho_B(k_F);$$

$$\mathrm{X}(k_F, m) = 4\left(k_F - m\,\mathrm{Arctg}\frac{k_F}{m} - \frac{k_F^5}{5\left(k_F^2 + m^2\right)^2}\right)$$

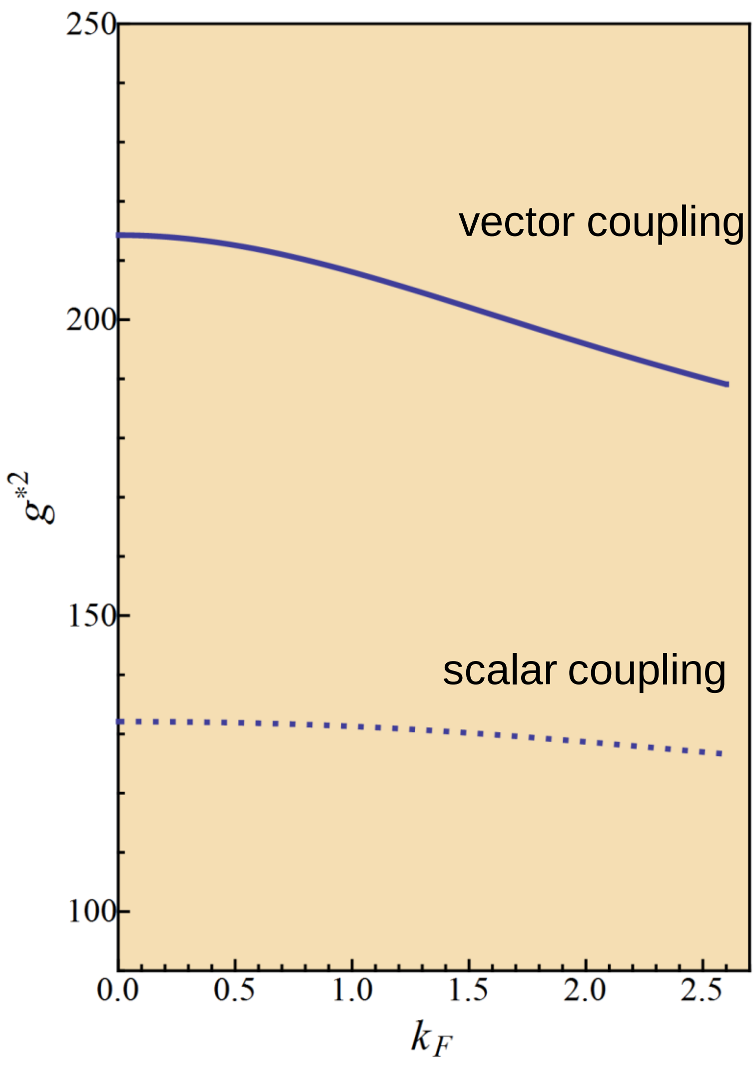
$$g_\omega^{*2}(k_F) = g_\omega^2\left(1 + \frac{3}{32}\frac{g_\sigma^2}{g_\omega^2}\mathrm{X}(k_F, m_\sigma)m_\omega^2 / k_F^3 + \frac{3}{16}\mathrm{X}(k_F, m_\omega)m_\omega^2 / k_F^3 - \frac{9}{32}\frac{f_\pi^2}{g_\omega^2}\mathrm{X}(k_F, m_\pi)m_\omega^2 / k_F^3\right)$$

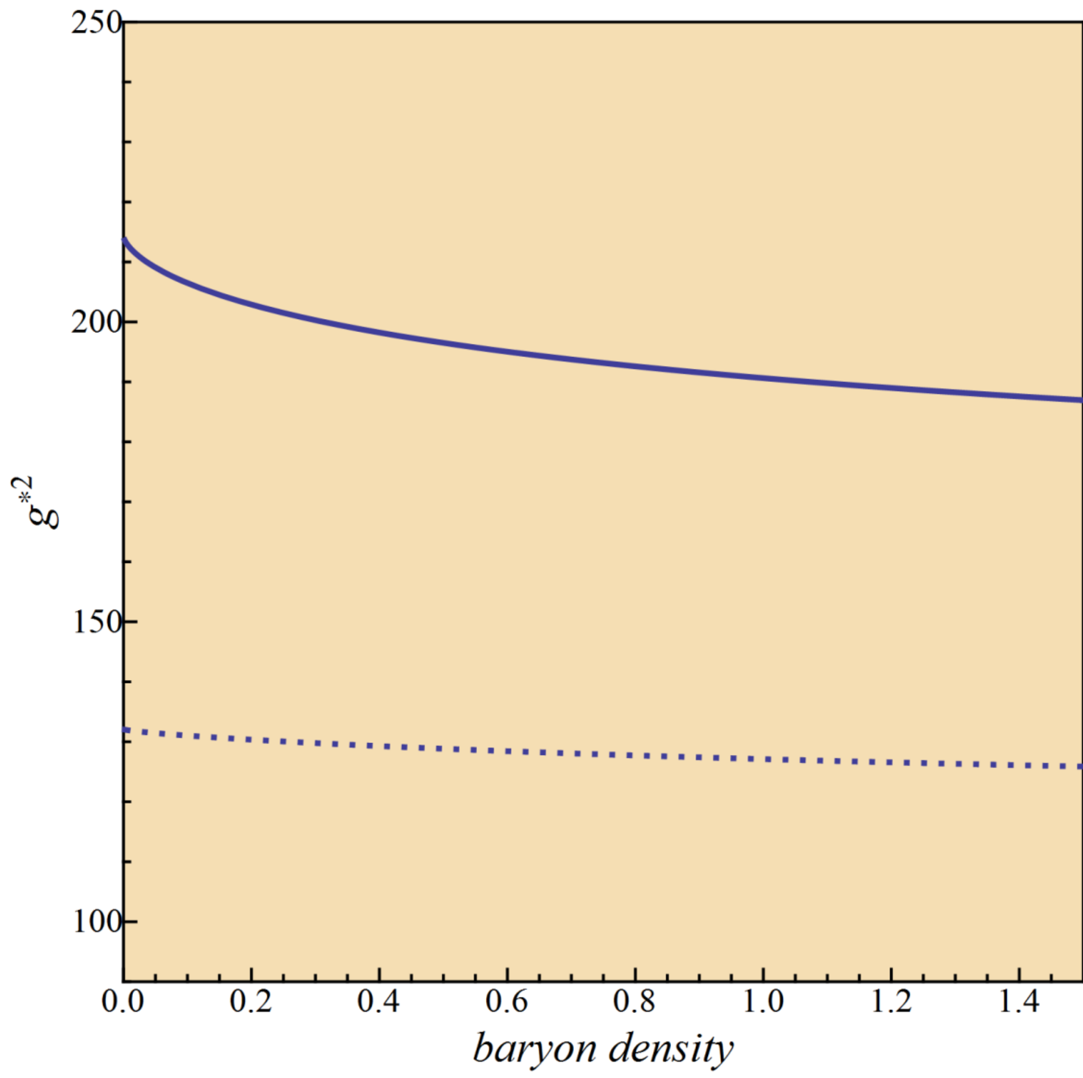
Similarly, for the scalar component of selfenergy:

$$\Sigma^s(k_F) = -\frac{g_\sigma^{*2}(k_F)}{m_\sigma^2}\rho_s(k_F);$$

$$g_\sigma^{*2}(k_F) = g_\sigma^2\left(1 + \frac{3}{8}\frac{g_\omega^2}{g_\sigma^2}\mathrm{X}(k_F, m_\omega)m_\sigma^2 / k_F^3 - \frac{3}{32}\mathrm{X}(k_F, m_\sigma)m_\sigma^2 / k_F^3 - \frac{9}{32}\frac{f_\pi^2}{g_\sigma^2}\mathrm{X}(k_F, m_\pi)m_\sigma^2 / k_F^3\right)$$

Density dependence of effective couplings





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A Bouyssy *et al.*, PRC **36**, 380

DHF for Neutron Matter

Isovector meson contributions to the self-energies become:

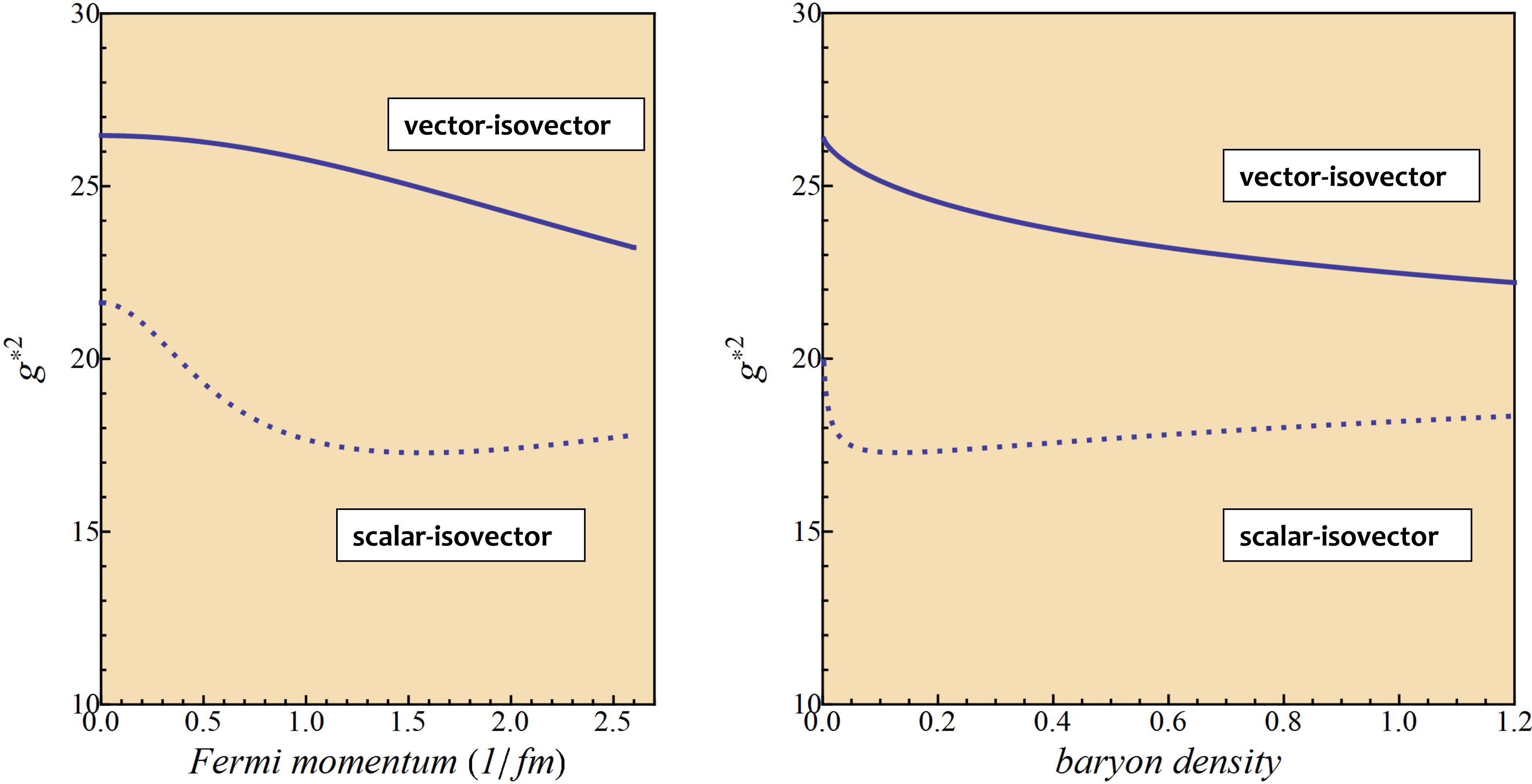
$$\delta\Sigma^s = -\frac{2-\lambda}{\pi^2}\frac{g_\delta^2}{4m_\delta^2}\int_0^{k_F} q^2 \frac{M_q^*}{E_q^*}dq + \frac{2\lambda-1}{4\pi^2 k}\int_0^{k_F} qdq\, \frac{M_q^*}{E_q^*}\left\{\left(\frac{g_\pi}{2M}\right)^2 kq - \frac{1}{4} g_\rho^2\Theta_\rho(k,q) - \frac{1}{4}\left(\frac{g_\pi}{2M}\right)^2 m_\pi^2\Theta_\pi(k,q) + \frac{1}{8} g_\delta^2\Theta_\delta(k,q)\right\},$$

$$\delta\Sigma^0 = -\frac{2-\lambda}{\pi^2}\frac{g_\rho^2}{4m_\rho^2}\int_0^{k_F} q^2 dq - \frac{2\lambda-1}{4\pi^2 k}\int_0^{k_F} qdq\left\{\frac{1}{8} g_\rho^2\Theta_\rho(k,q) - \frac{1}{4}\left(\frac{g_\pi}{2M}\right)^2 m_\pi^2\Theta_\pi(k,q) + \frac{1}{16} g_\delta^2\Theta_\delta(k,q)\right\}$$

$$g_\rho^{*2}(k_F) = g_\rho^2\left(1+\frac{3}{8}\mathrm{X}(k_F,m_\rho)m_\rho^2/k_F^3 + \frac{3}{16}\frac{g_\delta^2}{g_\rho^2}\mathrm{X}(k_F,m_\delta)m_\rho^2/k_F^3 - \frac{3}{4}\left(\frac{g_\pi^2}{2M}\right)^2 m_\pi^2\mathrm{X}(k_F,m_\pi)m_\rho^2/k_F^3\right)$$

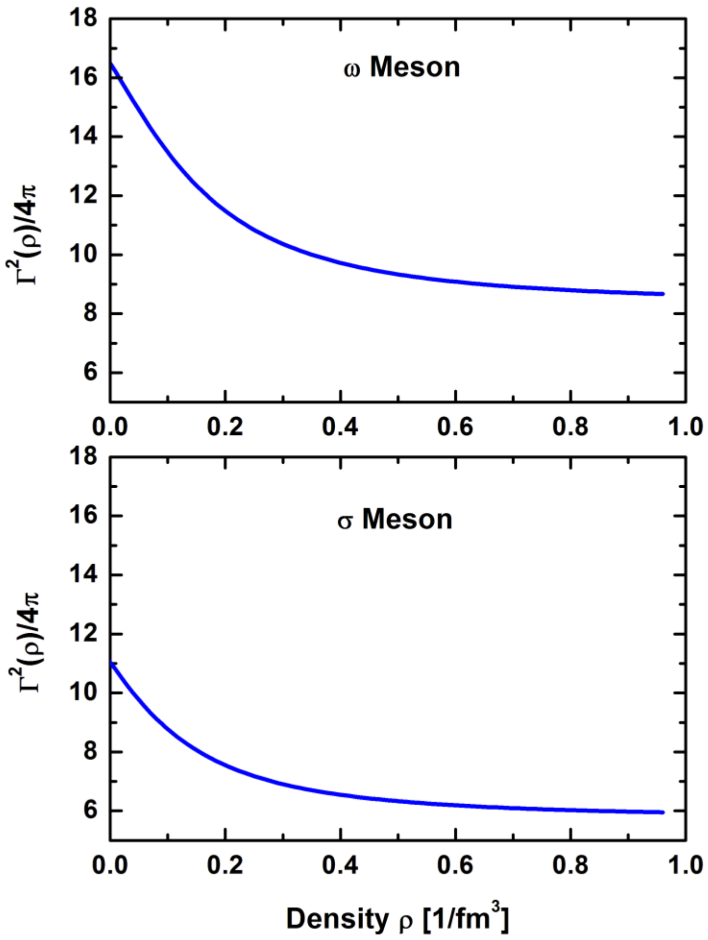
$$g_\delta^{*2}(k_F) = g_\delta^2\left(1-\left(\frac{g_\pi^2}{2M}\right)^2\frac{m_\delta^2}{g_\delta^2} + \frac{3}{16}\frac{g_\rho^2}{g_\delta^2}\mathrm{X}(k_F,m_\rho)m_\delta^2/k_F^3 + \frac{3}{16}\mathrm{X}(k_F,m_\delta)m_\delta^2/k_F^3 - \frac{3}{4}\left(\frac{g_\pi^2}{2M}\right)^2 m_\pi^2\mathrm{X}(k_F,m_\pi)\frac{m_\delta^2}{g_\delta^2}/k_F^3\right)$$

Density dependence of isovector mesons effective couplings

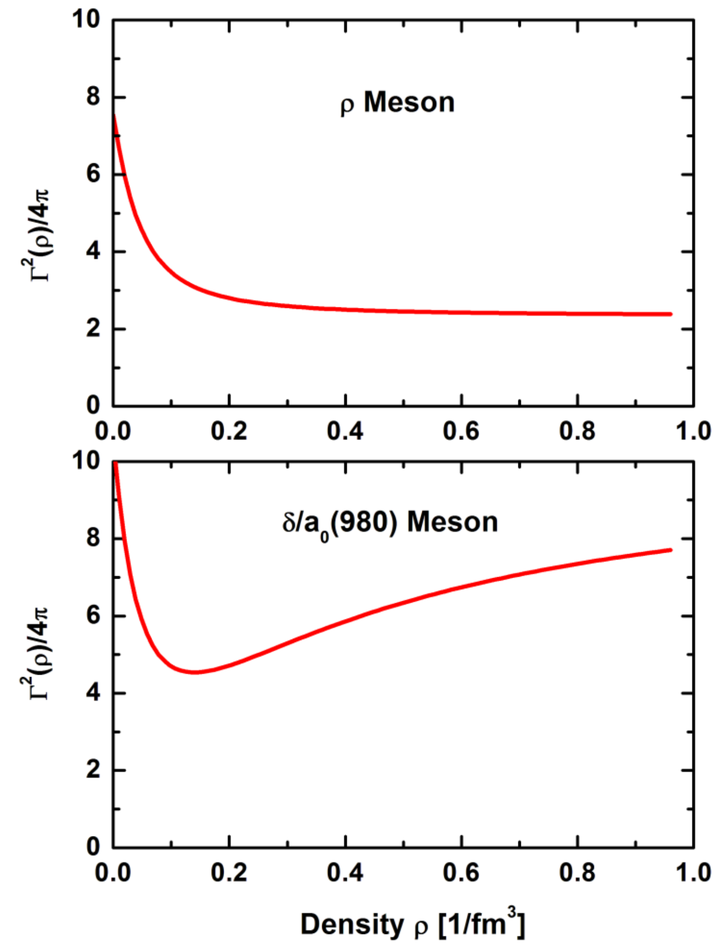


Nuclear Matter DBHF Vertices (Groningen NN-Potential)

H. Lenske, Springer Lecture Notes (2004)



Isoscalar Vertices



Isovector Vertices

Summary

-The exchange contribution of the DHF approach has been projected onto the RMF mean field model at the price of the density dependence of the effective coupling constants,

-The parameter-free closed form expressions for the effective coupling constants have been obtained ,

-The unusual phenomenological density dependence of the coupling constat for the δ/a_o scalar-isovector meson has been qualitatively reproduced.