Infrared Behavior of QCD Running Coupling

Gurjav GANBOLD

Bogoliubov Laboratory of Theoretical Physics, JINR, Dubna

Institute of Physics and Technology, MAS, Ulaanbaatar

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QCD at Large Distances

- ♣ QCD behavior at large distances is an active field of research because many novel behaviors are expected at energies **below 1 GeV (**IR region**).** Understanding of phenomena (quark confinement, QCD running coupling, etc.) requires a correct description of hadron dynamics in the IR region.

- ♣ The PT cannot be used effectively in the IR region and it is required either to supply some phenomenologies, or to use some non-PT methods.

- ♣ One of the fundamental parameters of nature, the QCD effective coupling, can provide a continuous interpolation between the **asymptotical free** state, where PT works well, and the **hadronization regime**, where non-PT techniques must be employed. (e.g. Yu.L.Dokshitzer et al.,1996).

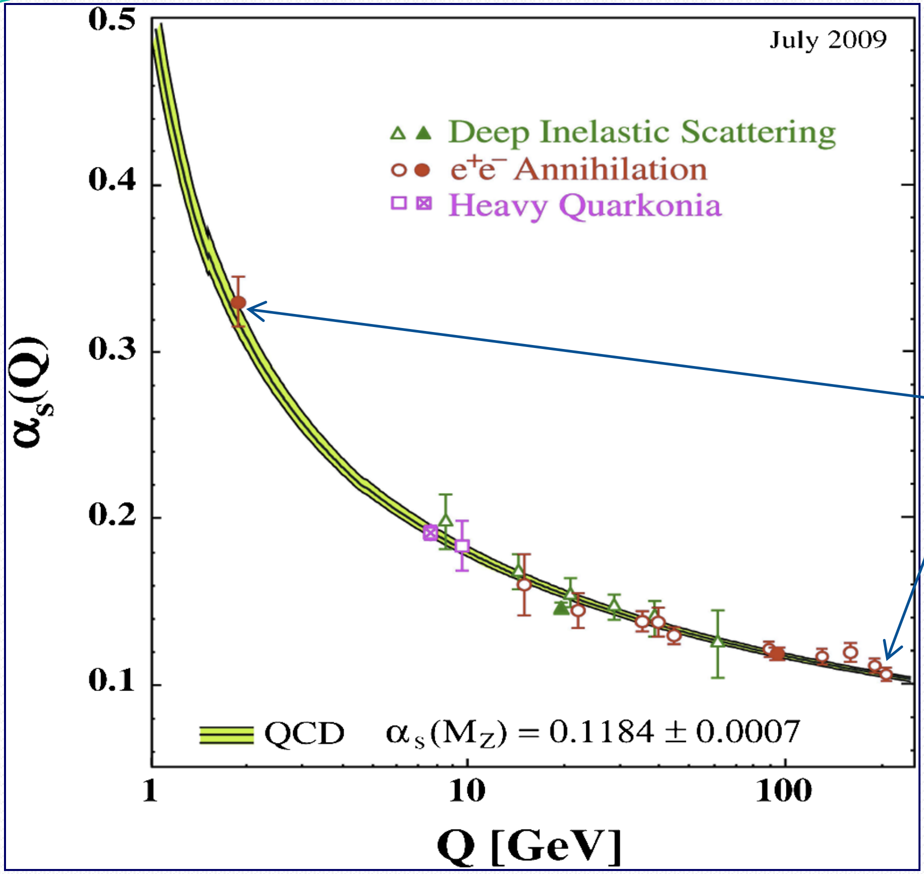
QCD Effective Coupling

The polarization of QCD vacuum causes **two opposite effects**, the color charge ***g*** is **screened** by the virtual quark-antiquark pairs and **antiscreened** by the polarization of virtual gluons.

The competition of these effects results in a variation of the physical coupling $\alpha_s = g^2 / 4\pi$ under changes of distance **1/Q.**

THEORY: QCD predicts a dependence of $\alpha_{s(Q)}$ on energy scale **Q**. This ***dependence*** is described theoretically by the RG equations.

EXPERIMENT: but its ***actual value*** must be obtained from experiment. It is well determined experimentally at relatively high energies Q >2 GeV.



Measurements of α_s as a function of energy scale Q versa QCD predictions.
Ref: PDG-2009

$$\alpha_s(2 < Q < 180) \to measured$$

$$\alpha_s(Q \to \infty) \to 0$$

$$\alpha_s(Q < 1) \to ?$$

D.Zwanziger 1992
Y.Simonov 2001
A.Williams 2001

$$\alpha_s(Q << 1) \to \alpha_s^0$$

Aim:

A self-consistent and physically meaningful prediction of the QCD effective charge in the IR regime remains actual in particle physics.

We determine the QCD effective charge α_s in the low-energy (below ~1 GeV) region by exploiting the hadron spectrum.

- ♣ Consider a relativistic QF model with analytic (IR) confinement
- ♣ Determine the meson masses by solving Bethe-Salpeter eqs.
- ♣ Adjust the model parameters by fitting heavy meson masses.
- ♣ Estimate $\hat{\alpha}_s$ in the low-energy domain below ~1 GeV.
- ♣ Extract a specific IR-fixed point $\hat{\alpha}_S^0 = \hat{\alpha}_s(0)$.
- ♣ Estimate intermediate meson masses (1< M < 9.5 GeV).

The Model

- Consider a relativistic quantum-field model of quark-gluon interaction.
  G.Ganbold, PRD79, 034034(2009); PRD81, 094008 (2010)

$$L = -\frac{1}{4}\left(F^{A}_{\mu\nu} - g\, f^{ABC} A^{B}_{\mu} A^{C}_{\nu}\right)^{2} + \sum_{f}\left(\bar{q}^{a}_{f}\left[\gamma_{\alpha}\partial_{\alpha} - m_{f} + g\,\Gamma^{\alpha}_{C} A^{C}_{\alpha}\right]^{ab} q^{b}_{f}\right)$$

$$F^{B}_{\mu\nu} \equiv \partial_{\mu} A^{B}_{\nu} - \partial_{\nu} A^{B}_{\mu} \qquad \Gamma^{\alpha}_{C} \doteq i\gamma_{\alpha} t^{C}$$

- Partition Functional written in terms of quark and gluon variables

$$Z = \iint \delta\bar{q}\,\delta q \int \delta A \;\exp\left\{-\int dx\, L[\bar{q}, q, A]\right\}$$

Assumptions (in **hadronization region**):

♣ **a)** Quark and gluon propagators are analytic functions.

♣ **b)** the coupling remains weak (<1). Then, ladder BSE is sufficient to estimate the meson (glueball) masses with reasonable accuracy.

Confinement

Confinement and dynamical symmetry breaking are crucial features of QCD.
Color confinement is the result of strong interaction with higher-order terms.

However, in the hadron scales (~ 200 MeV ~ 1 Fermi) QCD becomes non-PT.

Moreover, there is **no analytic proof** that QCD should be color confining.

The reason for confinement may be somewhat complicated.
Some other explanations of confinement:

Analytic Confinement (AC) [H.Leutwyler 1980; G.V.Efimov et al. 1995]
IR Confinement [C.S.Fischer, R.Alkofer, L.von Smekal 2002]
Confinement in lattice MC simulations [C.D.Roberts 1994; F. Lenz 2004]
Confinement in string theory in higher-D [R.Alkofer, J.Greensite, 2007]

IR-finite Propagators

A.G.Williams et. al. [2001]

$$p^2 \tilde{D}(p^2) \xrightarrow[|p|\to 0]{} 0$$

Schwinger-Dyson Eqs. + lattice QCD

$$\tilde{D}(p^2) \xrightarrow[|p|\to 0]{} const \neq 0$$

IR confined:

$$\tilde{D}(p^2) \sim \frac{1}{p^2} = \int_0^\infty ds\, e^{-sp^2} \to \int_0^{1/\Lambda^2} ds\, e^{-sp^2} = \frac{1-\exp(-p^2/\Lambda^2)}{p^2}$$

- **a)** The quark and gluon propagators are entire functions in Euclidean space

G.G., PRD81, 094008 (2010)

$$\tilde{S}^{ab}_{\pm}(\hat{p}) = \delta^{ab}\frac{i\hat{p} + m_f\left[1 \pm \gamma_5 \omega(m_f/\Lambda)\right]}{\Lambda m_f}\cdot \exp\left\{-\frac{p^2 + m_f^2}{2\Lambda^2}\right\}$$

$$\omega(z) \doteq 1/(1 + z^2/4)$$

$$\tilde{D}^{BC}_{\mu\nu}(p) = \delta^{BC}\delta_{\mu\nu}\frac{1 - \exp\left(-p^2/\Lambda^2\right)}{p^2} = \delta^{BC}\delta_{\mu\nu}\int\limits_0^{1/\Lambda^2} ds\, e^{-sp^2}$$

- **b)** Leading-order contributions to quark-antiquark bound state

$$Z_{(\bar{q}q)} = \iint \delta\bar{q}\,\delta q\, \exp\left\{-\left(\bar{q}\,S^{-1}q\right) + \frac{g^2}{2}\left\langle\left(\bar{q}\,\Gamma A q\right)\left(\bar{q}\,\Gamma A q\right)\right\rangle_D\right\}$$

$$\left\langle(\bullet)\right\rangle_D \doteq \int \delta A\; e^{-\frac{1}{2}\left(AD^{-1}A\right)}(\bullet)$$

Quark-Antiquark Bound States

- Allocate one-gluon exchange between colored currents

$$L_{qq} = \frac{g^2}{2} \sum_{f_1 f_2} \iint dx_1 dx_2\, J^{B}_{\mu\, f_1 f_2}(x_1, x_2) D^{BC}_{\mu\nu}(x_1, x_2) J^{C}_{\nu\, f_1 f_2}(x_2, x_1),$$

$$J^{B}_{\mu\, f_1 f_2}(x_1, x_2) \equiv i\, \bar{q}_{f_1}(x_1)\gamma_\mu t^B q_{f_2}(x_2).$$

- Isolate color-singlet combination

$$\left(t^A\right)^{ij} \delta^{AB} \left(t^B\right)^{i'j'} = \frac{4}{9}\delta^{ii'}\delta^{jj'} - \frac{1}{3}\left(t^A\right)^{ii'}\left(t^A\right)^{jj'}$$

- Perform Fierz transformation **(J = S, P, A, V, T)**

- Go to c.m.frame (due to different quark masses)

- Orthonormalized system {U_Q} with quantum numbers Q={n,l, …}:

- Local quark currents and vertices with given quantum numbers

$$J_{QJ\, f_1 f_2}(x) \equiv \bar{q}_{f_1}(x) V_{JQ}(\overleftrightarrow{\partial})\, q_{f_2}(x),$$

$$V_{JQ}(\overleftrightarrow{\partial}) \equiv i^l \int dy\, \sqrt{D(y)}\, \Gamma_J\, U_Q(y)\, e^{\frac{y}{2}\overleftrightarrow{\partial}}$$

- Diagonalization on colorless quark currents

$$L_{qq} = \frac{g^2}{2} \sum_N \int dx\, J_N^+(x)\, J_N(x), \qquad N \equiv \{Q\ J\ f_1\ f_2\}$$

- Gaussian representation: a new path integration over auxiliary fields B:

$$e^{g^2 (J_N^+ J_N)} = \iint \delta B_N^+\, \delta B_N \exp\left\{ -\sum_N (B_N^+ B_N) + g \sum_N [(B_N^+\ J_N) + (J_N^+\ B_N)] \right\}$$

- Explicit path-integration over quark variables and write the effective action

$$S_{eff}[B] = -\frac{1}{2}(B_N B_N) + Tr\left\{\ln[1 + g\,(B_N V_N)\,S]\right\}$$

- Hadronization Ansatz: B_N fields are identified as meson fields with given quantum numbers N

• Generating functional can be rewritten in terms of meson field variables. Isolate all quadratic (kinetic part) field configurations:

$$Z_N = \int \prod_N \delta B_N \exp\left\{ -\frac{1}{2}(B_N [1 + g^2 Tr(V_N S V_N S)] B_N) + W_{resid}[B_N] \right\}$$

• Diagonalization of the quadratic part is equivalent to the solution of the ladder Bethe-Salpeter equation on the orthonormalized system {U_N}

$$g^2 Tr(V_N S\, V_{N'} S) = \left(U_N\, \lambda\, U_{N'} \right) = \lambda_N (-p^2)\, \delta^{JJ'} \delta^{QQ'}$$

• Symmetric Bethe-Salpeter kernel is defined: G.G., PRD79, 034034(2009)

$$\lambda_J(-p^2) = \frac{4 g^2 C_J}{9} \int \frac{d^4 k}{(2\pi)^4} \{V(k)\}^2 \cdot Tr\left\{ \Gamma_J\, \tilde{S}\left(\hat{k} + \mu_1 \hat{p}\right) \Gamma_J\, \tilde{S}\left(\hat{k} - \mu_2 \hat{p}\right) \right\}$$

Meson Mass Equation

- Renormalization: $$U_{REN}(x) \equiv \sqrt{-\dot{\lambda}_N(M_N^2)} \cdot U_N(x)$$

$$\langle U_N | 1 + \lambda_N(-p^2) | U_N \rangle = \langle U_N | 1 + \lambda_N(M_N^2) - \dot{\lambda}_N(M_N^2)(p^2 + M_N^2) | U_N \rangle$$
$$= \langle U_{REN} | (p^2 + M_N^2) | U_{REN} \rangle$$

In relativistic quantum-field theory a stable bound state of "n" massive particles shows up as a pole in the S-matrix with a center of mass energy.

- The meson mass may be derived from the equation:

$$1 + \lambda_N(M_N^2) = 0 \;, \qquad p^2 = -M_N^2 \qquad [Eq.(1)]$$

Spacelike and Timelike Domains

We use the meson mass M as the appropriate characteristic parameter, so our coupling $\hat{\alpha}_s(M)$ is defined in a *timelike* domain.

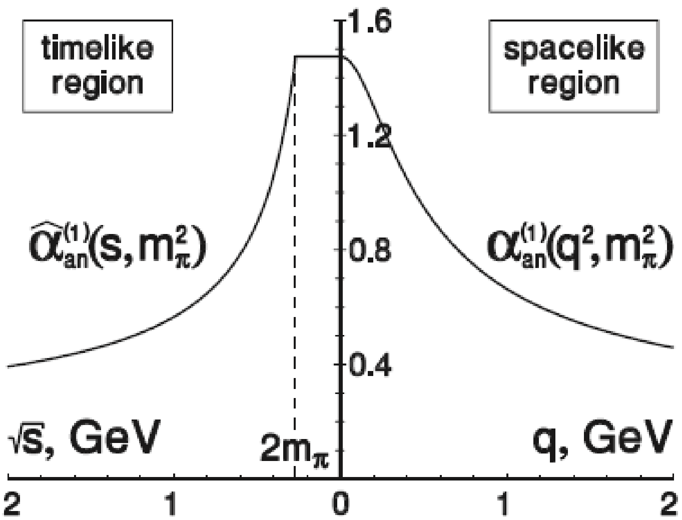
On the other hand, most of known data on $\alpha_s(Q)$ are possible in the *spacelike* region. The continuation of the invariant charge from the spacelike to the timelike region (and vice versa) was elaborated by making use of the integral relationships between the QCD running coupling in Euclidean and Minkowskian domains (e.g. K.A.Milton 1999, A.V.Nesterenko 2003):

$$\alpha_s(Q^2) = \int_0^\infty \frac{dt}{(1+t)^2} \, \hat{\alpha}_s(t \cdot Q^2)$$

It is easy to show that

$$\alpha_s(0) = \hat{\alpha}_s(0) \underbrace{\int_0^\infty \frac{dt}{(1+t)^2}}_{=1} = \hat{\alpha}_s(0)$$

G.G., PRD81, 094008 (2010)



The one-loop massless analytic coupling in the spacelike and timelike domains

Meson Spectrum and Running Coupling

Consider the most established sectors of hadron spectroscopy,
the pseudoscalar (P) and vector (V) mesons.

The polarization BS kernel is real and symmetric that allows us to find a simple variational solution to this problem. Choose a trial Gaussian function

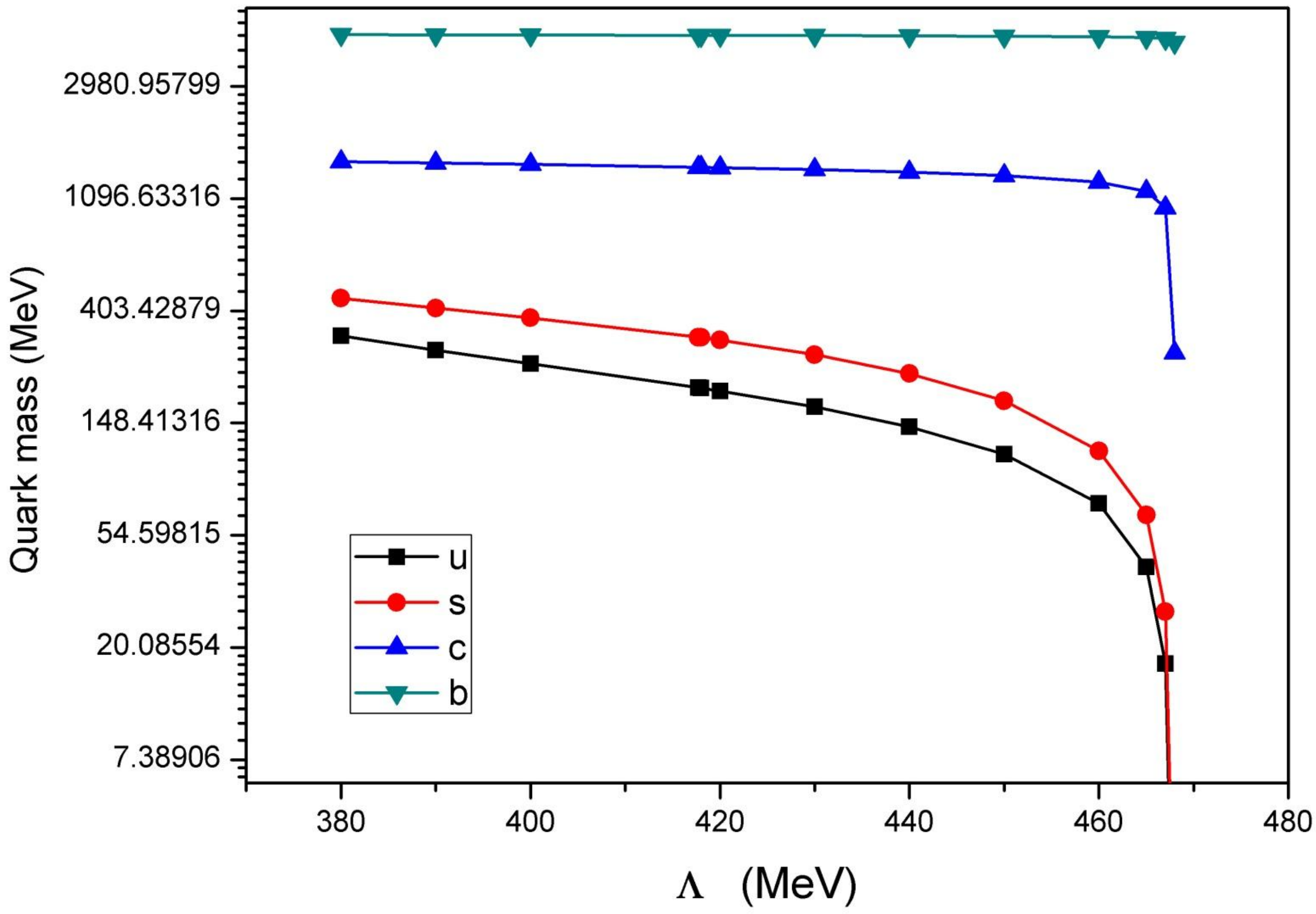
$$U(x) = \frac{2a}{\pi} \exp\left(-a\Lambda^2 x^2\right), \qquad a > 0$$

Variational equation for meson masses:

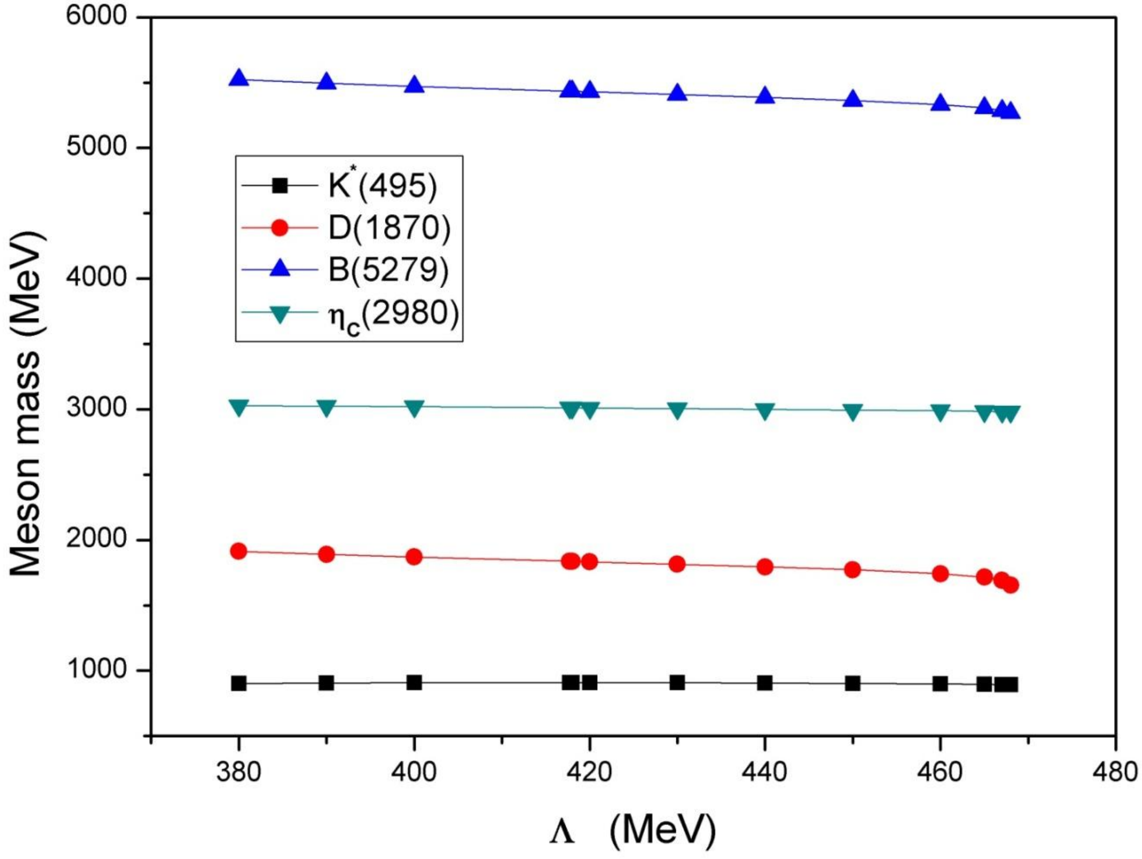
$$1 = \frac{8\,\hat{\alpha}_s C_J}{3\pi^2} \max_{a>0} \Phi\left(a, J, \frac{m_1}{\Lambda}, \frac{m_2}{\Lambda}, \frac{M_J}{\Lambda}\right), \qquad J = \left\{P, V\right\} \qquad [Eq.(2)]$$

This kind of scale dependence is most pronounced in leading-order QCD and often used to test and specify uncertainties of theoretical calculations for physical observables.

G.G., PRD79, 034034 (2009)



G.G., PRD79, 034034 (2009)



R.Feynman:

$$E \approx \langle \Psi | H | \Psi \rangle \quad \rightarrow \quad \Delta\Psi / \Psi \sim 0.1 \quad \rightarrow \quad \Delta E / E \sim 0.01$$

Fixing Model Parameters

Extract intermediate values the effective coupling in interval 2–10 GeV from a smooth interpolation of known data (Table 1). Particularly,

$$\hat{\alpha}_s(9460) = 0.1817, \qquad \hat{\alpha}_s(3097) = 0.2619,$$
$$\hat{\alpha}_s(2112) = 0.3074, \qquad \hat{\alpha}_s(2010) = 0.3138$$

As a particular case, for $\Lambda = 345 MeV$ we solve Eq.2 and fix parameters

$$m_{ud} = 192.56, \qquad m_s = 293.56,$$
$$m_c = 1447.59, \qquad m_b = 4692.51$$

Finding Effective Coupling in Region Below 1 GeV

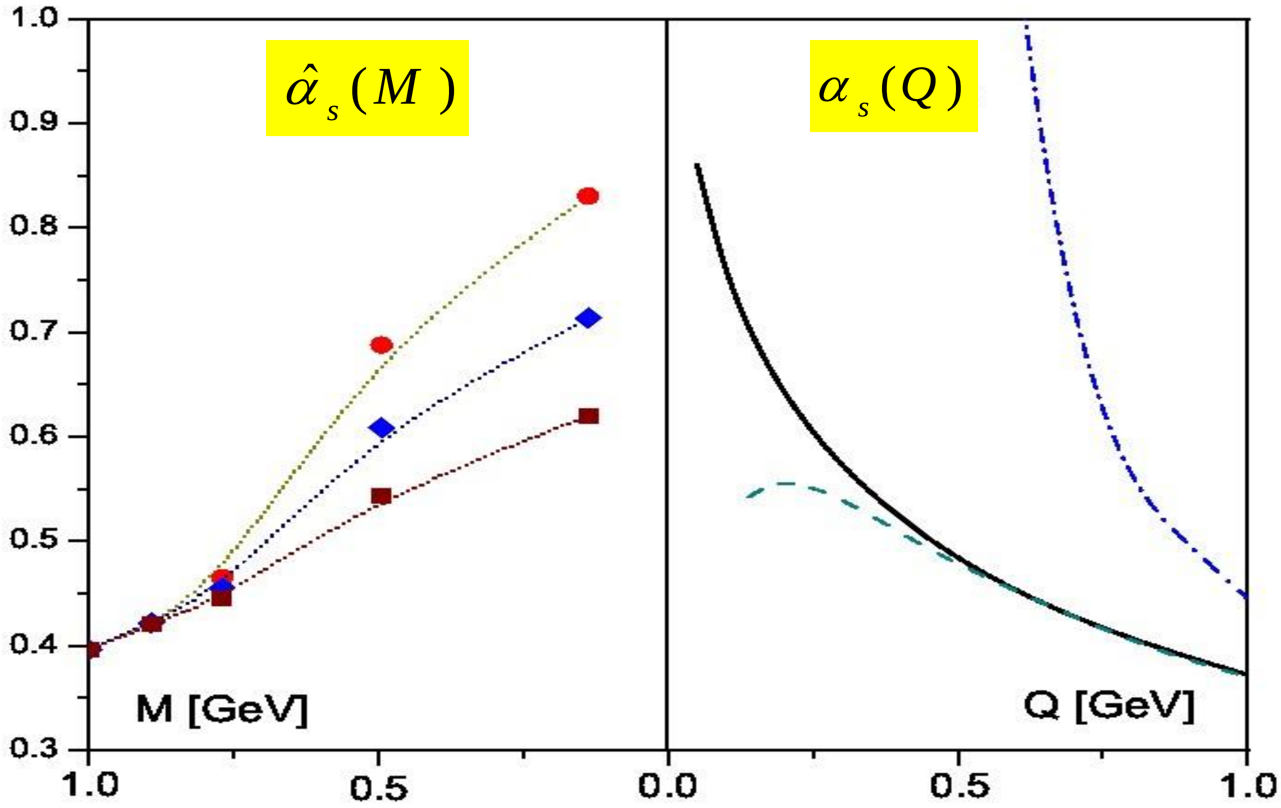
Having fixed the model parameters, we solve an inverse problem and find the effective coupling in the region below 1 GeV as follows:

$$\hat{\alpha}_s(138) = -\lambda_P^{-1}(\Lambda, 138, m_{ud}, m_{ud}) = 0.7131,$$
$$\hat{\alpha}_s(495) = -\lambda_P^{-1}(\Lambda, 495, m_{ud}, m_s) = 0.6086,$$
$$\hat{\alpha}_s(770) = -\lambda_V^{-1}(\Lambda, 770, m_{ud}, m_{ud}) = 0.4390,$$
$$\hat{\alpha}_s(892) = -\lambda_V^{-1}(\Lambda, 892, m_{ud}, m_s) = 0.4214.$$

[G.G., PRD81, 094008 (2010)]

Our estimates of $\hat{\alpha}_s(M)$ at
red dots Λ=330 MeV;
blue diamonds Λ=345 MeV;
brown squares Λ=360 MeV)

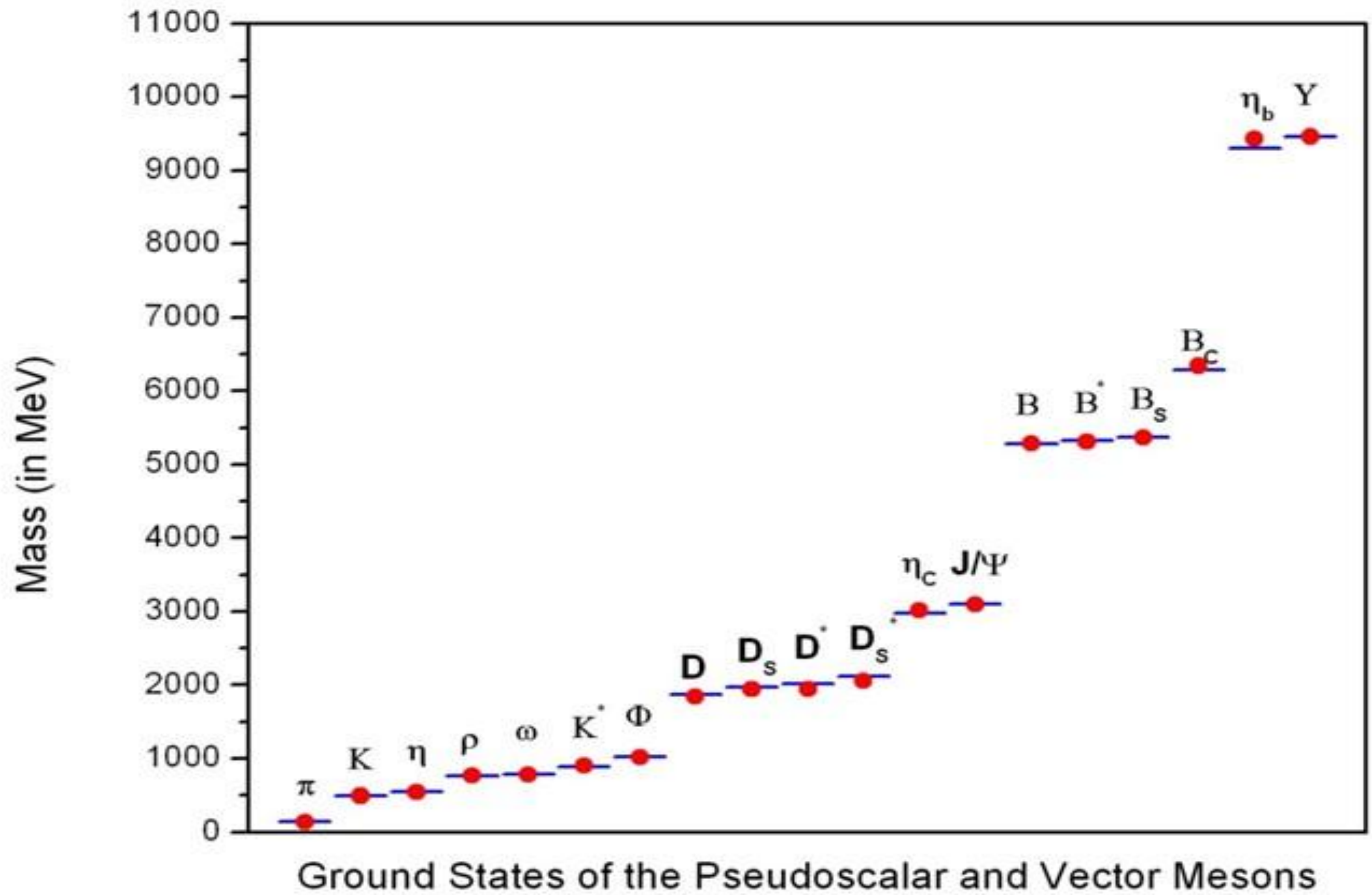
compared with [M.Baldicci et al. 2008]
PT (dot-dashed),
3-loop analytic coupling (solid),
massive 1-loop analytic coupling (dashed)



G.Ganbold, PRD81, 094008 (2010)

Calculating Intermediate Meson Masses

|relative errors| < **3%**



IR-finite Behavior of Effective Coupling

The possibility that the QCD coupling constant features an IR-finite behavior has been extensively studied in recent years [S.Brodsky 2004, A.C.Aguilar 2004, D.V.Shirkov 2003, etc.].

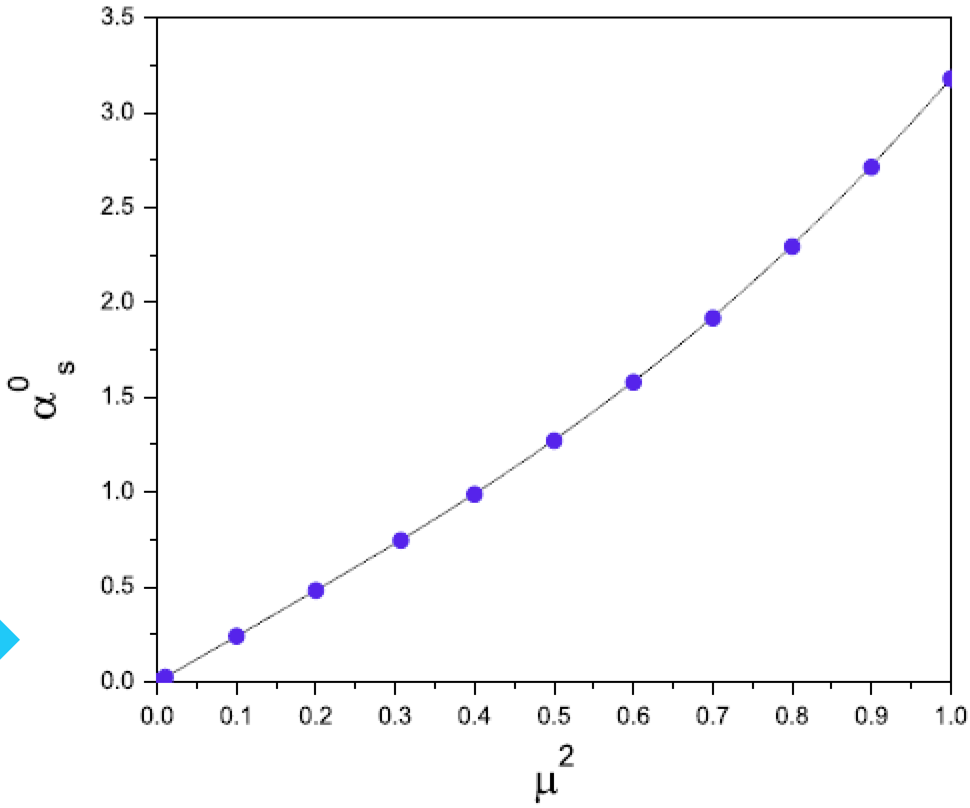
The phenomenological evidence for effective coupling finite in the IR region is much more numerous.

Consider the IR-fixed point

$$\hat{\alpha}_s^0 = \hat{\alpha}_s(0)$$

by evaluating Eq.(2) for M=0 , m1= m2=m and

$$\mu = m/\Lambda$$



G.G., PRD81, 094008 (2010)

Note that a value of order ~2 or larger would be definitely out of line with many other phenomena, such as nonrelativistic potentials for a charmonium [A.M.Badalian 2000] and analytic perturbation theory [D.V.Shirkov 1997]. Obviously, this constrains the value of constituent quark mass:

$$\mu^2 < 0.8 \ \ or \ \ m < 0.8\Lambda$$

Since we are searching the IR-fixed point, it is reasonable to choose the lightest (up/down) quark mass. Particularly, for

m=192.56 MeV and Λ =345 MeV we obtain

$$\hat{\alpha}_s^0 = 0.757, \quad or \quad \hat{\alpha}_s^0/\pi = 0.241$$

G.G., PRD81, 094008 (2010)

Our result is in reasonable agreement with often-quoted estimates:

$$\alpha_s^0/\pi = 0.19 - 0.25 \quad [S.Godfrey\ 1985],$$
$$\alpha_s^0/\pi = 0.265 \quad [T.Zhang\ 1991],$$
$$\alpha_s^0/\pi = 0.26 \quad [F.Halzen\ 1993],$$
$$\left\langle \alpha_s^0/\pi \right\rangle_{1GeV} = 0.2 \quad [M.Baldicchi\ 2008]$$

By interpolating smoothly obtained results into the intermediate-energy region we calculate $\hat{\alpha}_s(M)$ on a wide interval of energy scale 0.14–9.5 GeV.

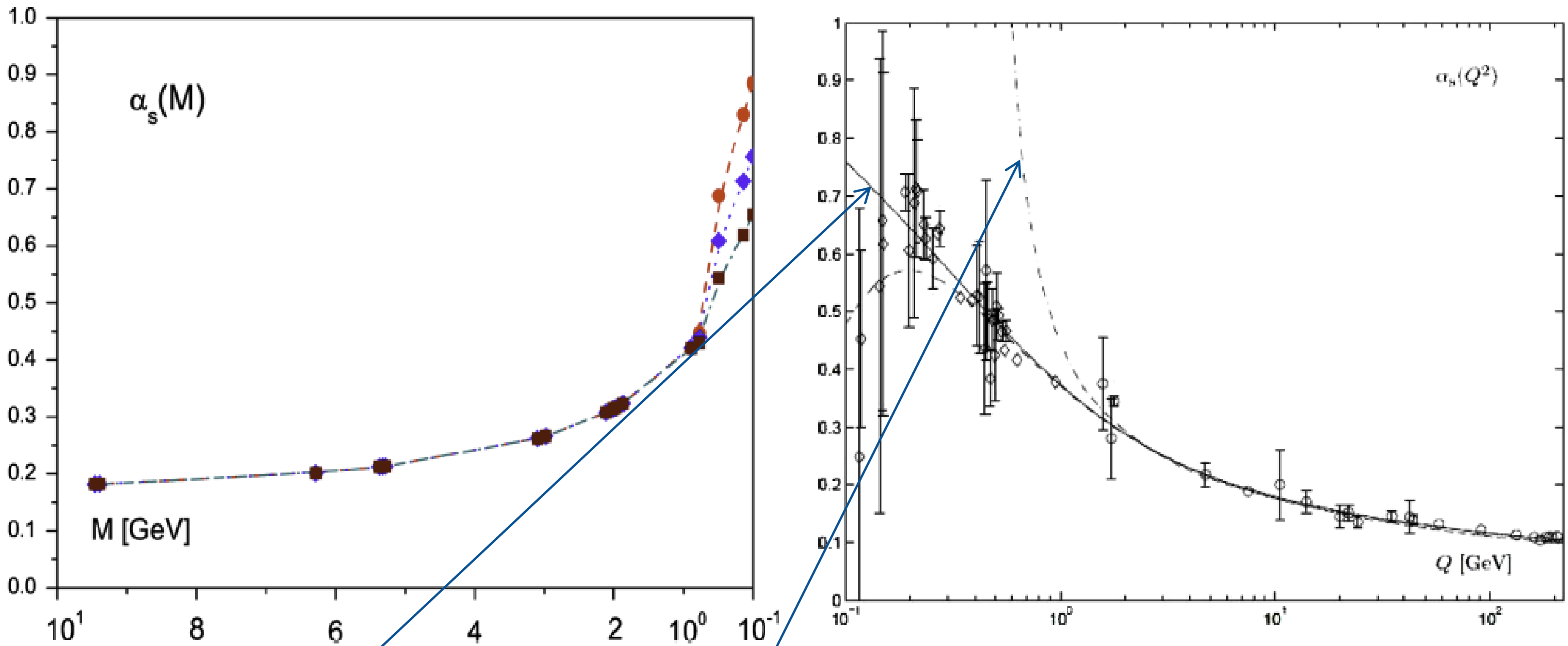


FIG. 5 (color online). Summary of estimates of $\hat{\alpha}_s(M)$ in interval from 0 to 10 GeV at different values of confinement scale. In the left panel, $\Lambda = 330$ MeV (red dots), $\Lambda = 345$ MeV (blue diamonds), and $\Lambda = 360$ MeV (black squares) compared with $\alpha_s(Q)$ (in the right panel) defined in low-energy (open diamonds) and high-energy (open circles) experiments. Also shown are the three-loop analytic coupling (solid curve), its perturbative counterpart (dot-dashed curve), both normalized at the Z-boson mass, and the massive one-loop analytic coupling (dashed curve) (for details see Ref. [31]).

Summary:

♣ We show that taking into account the correct symmetry structure of the quark-gluon interaction **in the confinement region** (reflected in simple forms of propagators) can result in reasonable description of physical characteristics in low-energy particle physics.
Particularly, the QCD running coupling and conventional meson spectrum may be explained reasonably in the framework of a simple relativistic quantum-field model of quark-gluon interaction based on **analytic** (or, IR) **confinement**.

♣ Despite its model origin, the approximations used, and questions about the very definition of the coupling in the IR region, our approach exhibits a new, independent, and specific **IR-finite behavior of QCD coupling**.

♣ The merit of our model is the ability to address simultaneously different sections of the low-energy particle physics. The consideration can be extended to other problems (glueballs, decay constants, exotic mesons, qq-gg mixed states, multiquark BS, baryons, …etc.).