Weyl group, CP and the kink-like field configurations in the effective SU(3) gauge theory

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Starting points

- nonzero vacuum gluon condensate

$$\lim_{V\to\infty} \langle V^{-1} \int_V d^4x g^2 F^2 \rangle \neq 0$$

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- importance of extracting the Abelian $\hat{B}_\mu(x)$ part of the gauge fields

$$\hat{A}_\mu(x) = \hat{B}_\mu(x) + \hat{X}_\mu(x), \quad [\hat{B}_\mu(x), \hat{B}_\nu(x)] = 0$$

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Effective Lagrangian

Consider the following Ginsburg-Landau effective Lagrangian for the soft gauge fields satisfying the requirements of invariance under the gauge group $SU(3)$ and space-time transformations,

$$L_{\text{eff}} = -\frac{1}{4}\left(D^{ab}_{\nu}F^{b}_{\rho\mu}D^{ac}_{\nu}F^{c}_{\rho\mu} + D^{ab}_{\mu}F^{b}_{\mu\nu}D^{ac}_{\rho}F^{c}_{\rho\nu}\right) - U_{\text{eff}}$$

$$U_{\text{eff}} = \frac{1}{12}\text{Tr}\left(C_1\hat{F}^2 + \frac{4}{3}C_2\hat{F}^4 - \frac{16}{9}C_3\hat{F}^6\right),$$

where

$D^{ab}_{\mu} = \delta^{ab}\partial_{\mu} - i\hat{A}^{ab}_{\mu} = \partial_{\mu} - iA^{c}_{\mu}(T^{c})^{ab}, \quad F^{a}_{\mu\nu} = \partial_{\mu}A^{a}_{\nu} - \partial_{\nu}A^{a}_{\mu} - if^{abc}A^{b}_{\mu}A^{c}_{\nu},$

$\hat{F}_{\mu\nu} = F^{a}_{\mu\nu}T^{a}, \quad T^{a}_{bc} = -if^{abc} \quad \text{Tr}\left(\hat{F}^2\right) = \hat{F}^{ab}_{\mu\nu}\hat{F}^{ba}_{\nu\mu} = -3F^{a}_{\mu\nu}F^{a}_{\mu\nu} \le 0,$

and the gauge coupling constant is absorbed into the gauge field: $gA_{\mu} \to A_{\mu}$.

$$\lim_{V\to\infty}\langle V^{-1}\int_V d^4xF^2\rangle \neq 0 \longrightarrow C_1 > 0,\ C_2 > 0,\ C_3 > 0.$$

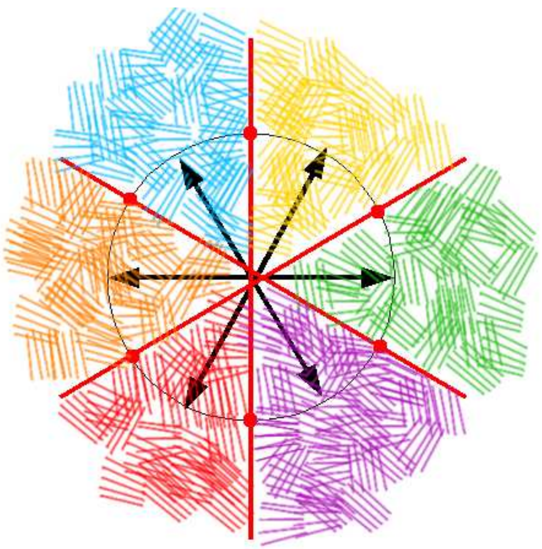
$$F^{a}_{\mu\nu}F^{a}_{\mu\nu} = 4b^2_{\text{vac}}\Lambda^4 > 0,\quad b^2_{\text{vac}} = \left(\sqrt{C_2^2 + 3C_1C_3} - C_2\right)/3C_3.$$

Consider A_μ fields with the **Abelian field strength**

$$\hat{F}_{\mu\nu} = \hat{n} B_{\mu\nu},$$

where matrix $\hat{n}$ can be put into Cartan subalgebra

$$\hat{n} = T^3 \cos\xi + T^8 \sin\xi, \; 0 \le \xi < 2\pi.$$



It is convenient to introduce the following notation:

$$\hat{b}_{\mu\nu} = \hat{n} B_{\mu\nu}/\Lambda^2 = \hat{n} b_{\mu\nu}, \; b_{\mu\nu} b_{\mu\nu} = 4 b_{\text{vac}}^2,$$

$$e_i = b_{4i}, \; h_i = \frac{1}{2}\varepsilon_{ijk} b_{jk}, \; \mathbf{e}^2 + \mathbf{h}^2 = 2 b_{\text{vac}}^2.$$

$$(\mathbf{eh}) = |\mathbf{e}|\,|\mathbf{h}| \cos\omega, \;\; (\mathbf{eh})^2 = \mathbf{h}^2 \left(2b^2 - \mathbf{h}^2\right) \cos^2\omega.$$

Hence the effective potential takes the form

$$U_{\text{eff}} \; = \; \Lambda^4 \left[-C_1 b_{\text{vac}}^2 + C_2 \left(2 b_{\text{vac}}^4 - (\mathbf{eh})^2 \right) + \frac{1}{9} C_3 b^2 \left(10 + \cos 6\xi\right) \left(4 b_{\text{vac}}^4 - 3\,(\mathbf{eh})^2 \right) \right].$$

There are twelve discrete global degenerated minima at the following values of the variables h, ω and ξ

$$\mathbf{h}^2 = b_{\text{vac}}^2 > 0, \;\; \omega = \pi k \;\; (k = 0, 1), \;\; \xi = \frac{\pi}{6}(2n+1)\,(n = 0, \dots, 5).$$

Kink-like configurations

Discrete minima mean there exist kink-like field configurations interpolating between these minima. For instance, for the angle ω

$$L_{\rm eff} = -\frac{1}{2}\Lambda^2 b_{\rm vac}^2 \partial_\mu\omega\partial_\mu\omega - b_{\rm vac}^4\Lambda^4\left(C_2 + 3C_3 b_{\rm vac}^2\right)\sin^2\omega,$$

with the sine-Gordon equation of motion

$$\partial^2\omega = m_\omega^2 \sin 2\omega, \quad m_\omega^2 = b_{\rm vac}^2\Lambda^2\left(C_2 + 3C_3 b_{\rm vac}^2\right),$$

with kink solution

$$\omega = 2\arctan\left(\exp\left(\sqrt{2}m_\omega x_1\right)\right),$$

which can be treated as domain wall.

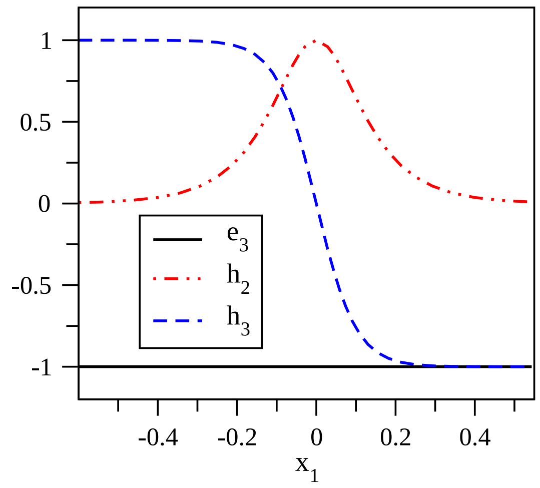


Figure 1: The gauge field flips from the anti-self-dual at $t \ll -1/\mu$ to self-dual at $t \gg 1/\mu$ configuration: $h_3 = b_{\rm vac}\cos\omega$, $h_2 = b_{\rm vac}\sin\omega$, $e_i = \delta_{i3}b_{\rm vac}$. Here $b_{\rm vac} = 1$, $\mu = 10$.

Spectrum of the charged field in the kink-like background

Consider the eigenvalue equation of the charged field in the kink-like background

$$-\left(\partial_\mu - iB_\mu(x)\right)^2 \phi = \lambda\phi.$$

Bulk: We have homogeneous (anti-)self-dual fields

$$B_\mu(x) = B_{\mu\nu}x_\nu,\ \ \tilde{B}_{\mu\nu} = \pm B_{\mu\nu},\ B_{\mu\alpha}B_{\nu\alpha} = B^2\delta_{\mu\nu},\ B = \Lambda^2 b_{\rm vac},$$

The eigenvalue equation can be rewritten as follows

$$\left[\beta^+_\pm\beta_\pm + \gamma^+_+\gamma_+ + 1\right]\phi = \frac{\lambda}{4B}\phi$$

where creation and annihilation operators $\beta_\pm$, $\beta^+_\pm$, $\gamma_\pm$, $\gamma^+_\pm$ are expressed in terms of the operators α^+, α,

$$\beta_\pm = \frac{1}{2}\left(\alpha_1 \mp i\alpha_2\right),\ \gamma_\pm = \frac{1}{2}\left(\alpha_3 \mp i\alpha_4\right),\ \alpha_\mu = \frac{1}{\sqrt{B}}(Bx_\mu + \partial_\mu),$$

$$\beta^+_\pm = \frac{1}{2}\left(\alpha^+_1 \pm i\alpha^+_2\right),\ \gamma^+_\pm = \frac{1}{2}\left(\alpha^+_3 \pm i\alpha^+_4\right),\ \alpha^+_\mu = \frac{1}{\sqrt{B}}(Bx_\mu - \partial_\mu).$$

The eigenvalues and the square integrable eigenfunctions are

$$\lambda_r = 4B\left(r+1\right),\ \ r = k+n\ (\text{for self}-\text{dual field}),\ \ r = l+n\ (\text{for anti}-\text{self}-\text{dual field}) \quad (1)$$

$$\phi_{nmkl}(x) = \frac{1}{\sqrt{n!m!k!l!\pi^2}}\left(\beta^+_+\right)^k\left(\beta^+_-\right)^l\left(\gamma^+_+\right)^n\left(\gamma^+_-\right)^m\phi_{0000}(x),\ \phi_{0000}(x) = e^{-\frac{1}{2}Bx^2}, \quad (2)$$

Discrete spectrum. Absence of periodic solutions is treated as confinement of the charged field.

Domain Wall:

$$B_2 = 0,\ B_1 = 2Bx_3,\ B_3 = 0,\ B_4 = 2Bx_3 \quad (H_i = 2B\delta_{i2},\ E_i = -2B\delta_{i3})$$

$$\phi(x) = \exp(-ip_4x_4 - ip_2x_2)\chi(x_3)$$
$$\left[p_2^2 - \partial_3^2 + (p_4 + 2Bx_3)^2 + 4B^2x_3^2\right]\chi = \lambda\chi$$

Square integrable over x_3 solution

$$\chi_n(p_4|x_3) = \exp\left\{-2\sqrt{2}B\left(x_3 + \frac{p_4}{4B}\right)^2\right\} H_n\left(2^{3/4}\sqrt{B}\left(x_3 + \frac{p_4}{4B}\right)\right)$$

$$\lambda_n(p_2^2, p_4^2) = 2\sqrt{2}B\left(2n + 1 + \frac{p_2^2}{2\sqrt{2}B} + \frac{p_4^2}{4\sqrt{2}B})\right)$$

Continuous spectrum similar to Landau levels. The presence of periodic eigenfunction can be treated as localization of charged field on domain wall.

Conclusions

- Effective action for pure Yang-Mills gauge fields invariant under the standard space-time and local gauge $su(3)$ transformations is considered.

- It is demonstrated that a set of twelve degenerated minima of the potential exists as soon as a nonzero gluon condensate is postulated. The minima are connected to each other by the parity transformations and Weyl group transformations associated with the color $su(3)$ algebra.

- The presence of degenerated discrete minima in the effective potential leads to kink-like gauge field configurations interpolating between different minima. In Euclidean space-time these configurations represent domain walls. In the real space-time they are solitons.

- In the bulk (outside the domain wall) charged fields are confined. The charged field is localized on the domain wall.

Thank you for your attention!