Stochastic Simulations of Thermodynamic and Kinetic Properties of Strongly Coupled Quark-Gluon Plasmas in Feynman and Wigner Representation of Quantum Statistical Mechnics.

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OUTLINE

- Phase diagram of strongly coupled quantum Coulomb systems
- Basic assumptions of semi-classical theory for non-Abelian plasma and limits of applicability
- Simulation of thermodynamics of quantum many-particle systems by Feynman path integral Monte Carlo method
- Wigner approach to simulations of quantum dynamics
- Applications to the semi-classical models of quark-gluon plasma
- Applications to the strongly coupled electromagnetic plasma

Interaction and quantum effects in strongly coupled Coulomb systems with different masses of particles.

Classical one-component plasma - COCP

Quantum one-component plasma - QOCP

Classical two-component plasma - CTCP

Quantum two-component plasma model - QTCP

Coulomb interaction: $U_{ab}(r) = e_a e_b / r$

— Nonideality boundary:

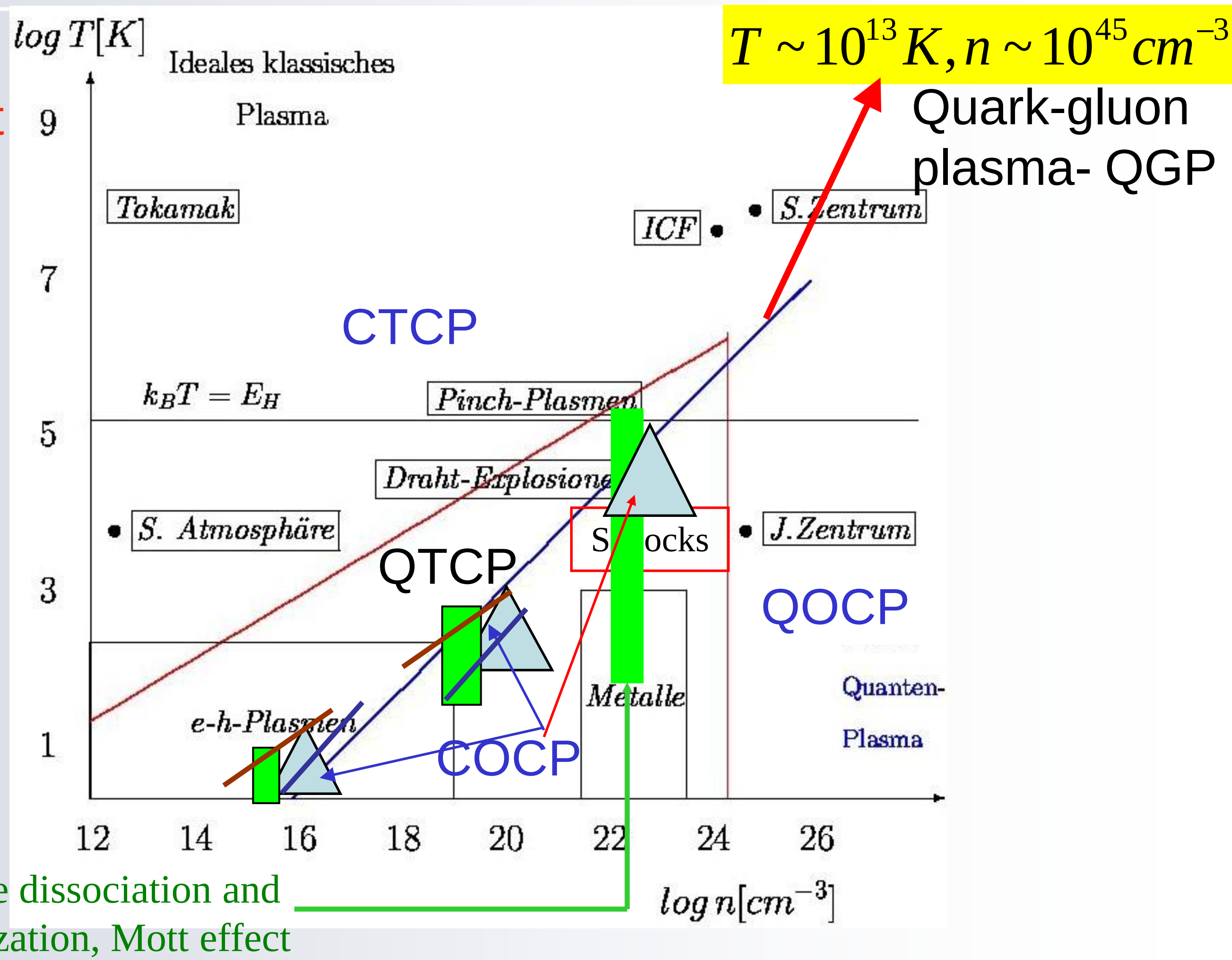
$$< U_{Coul} > = < E_{Kin} >$$

Inside: Strong Coulomb interaction,
Many-body effects
atomes, molecules, clusters

Degeneracy boundary

$$\lambda_e = \bar{r}$$

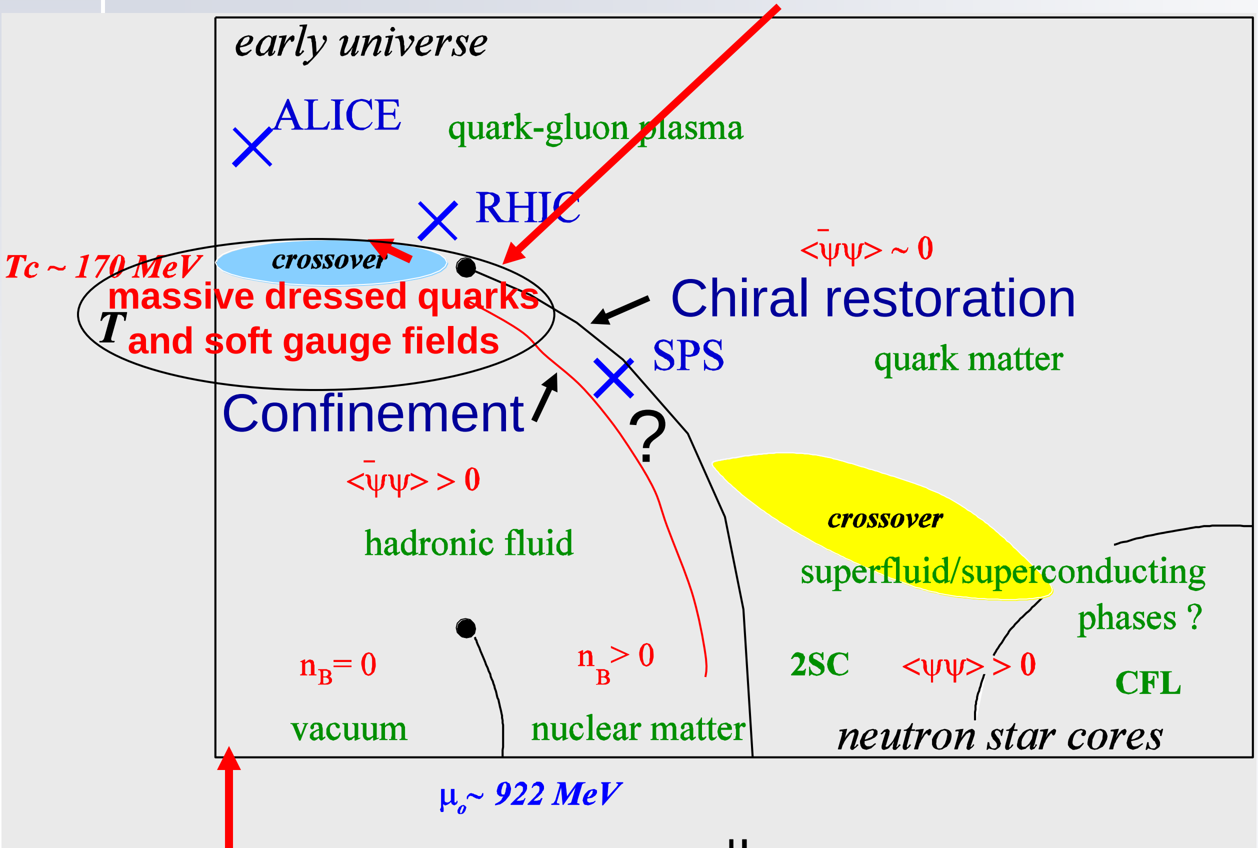
Below: overlapping electron
Wave functions,
Quantum and spin effects



Semi-classical theory for non-Abelian system of color Coulomb quasi-particles

is based on resummation technique and lattice simulations allowing for consideration of quark-gluon plasma as system of dressed quark, antiquark and gluon presented by color Coulomb quasiparticles with T-dependent dispersion curves and width

Litim, Manuel, Stoecker,Bleicher,Feinberg, Richardson,
Bonasera,Maruyama, Hatsuda,Shuryak, Fukushima,….



Phase diagram
(F.Karsch)

Basic asumptions of the semi-classical quasiparticle model of quark – gluon plasma

is based on resummation technique and lattice simulations allowing for consideration of quark-gluon plasma as system of dressed quark, antiquark and gluon presented by color Coulomb quasiparticles with T-dependent dispersion curves and width.

(Phys.Lett.B478,161(2000), Phys. Rev. C, **74**, 044909, (2006))

- All color quasiparticles are massive (m > T) and move non-relastivistically
- Interparticle interaction is domonated by a color Coulomb potential with distance dependent coupling constant.
- The color operators are substituted by their average values
  - classical color vectors in SU(3) (8D vectors with 2 Casimirs condit.).

The model input requires :

- The temperature dependence of the quasiparticle mass.
- The temperature dependence of the coupling constant.

All the input quantiries should be deduced from lattice QCD calculations and substitued in quantum Hamiltonian.

Thermodynamics of quark - gluon plasma in grand canonical ensemble within Feynman formulation of quantum mechanics

$$H_\beta = K_\beta + U_C = \sum_a \sqrt{p_a^2 + m_a^2(\beta)} + U_C \approx$$

$$\approx \sum_a (N_a m_a(\beta) + \frac{p_a^2}{2m_a(\beta)}) + \sum_{a,b} \frac{g^2(|r_a - r_b|, \beta) < \vec{Q}_a | \vec{Q}_b >}{4\pi |r_a - r_b|}, m_a >> T$$

Grand canonical partition function

$$\Omega\left(\mu, \mu_g = 0, V, \beta\right) =$$

$$= \sum_{N_q, N_{\underline{q}}, N_g} \exp(\beta\mu(N_q - N_{\underline{q}}))Q\left(N_q, N_{\underline{q}} N_g, \beta\right) / N_q! N_{\underline{q}}! N_g!$$

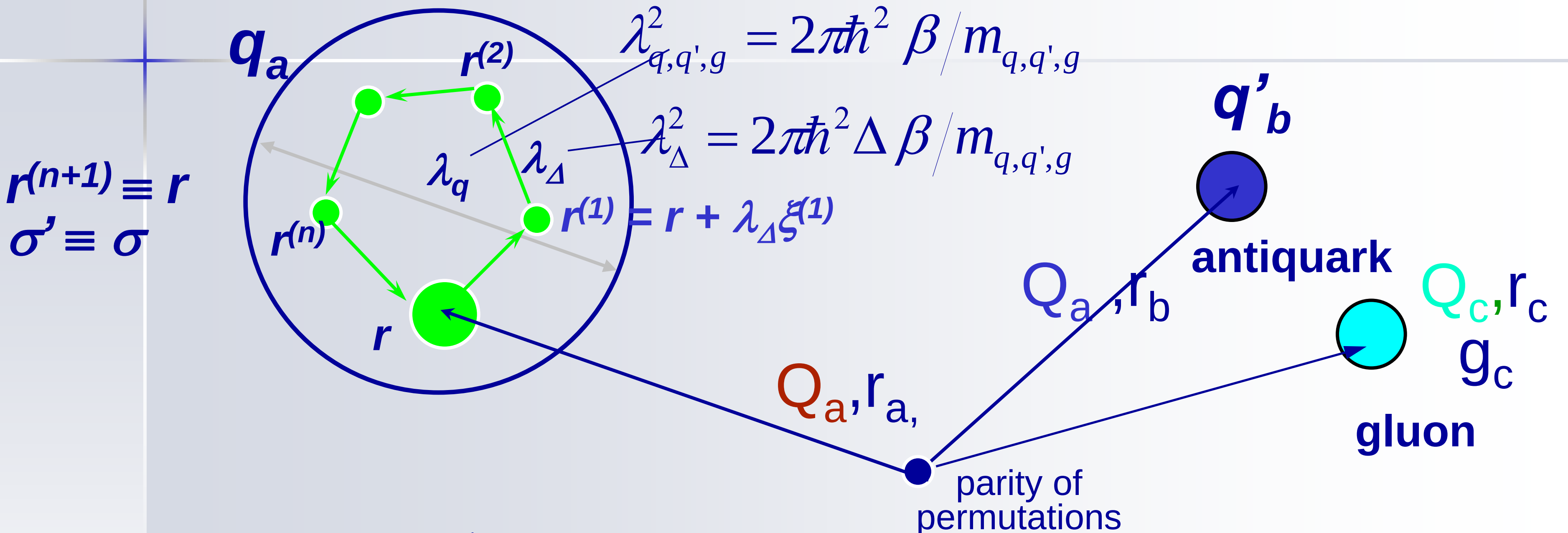
$$Q\left(N_q, N_{\underline{q}}, N_g, \beta\right) = \sum_\sigma \int_V dr d\mathbf{Q} \rho\left(r, \mathbf{Q}, \sigma; \beta\right)$$

$$\rho = \exp\left(-\beta H(\beta)\right) = \underbrace{\exp\left(-\Delta\beta H(\beta)\right) \times \ldots \times \exp\left(-\Delta\beta H(\beta)\right)}_{n+1}$$

$$\beta = 1/kT \qquad \Delta\beta = \beta/(n+1)$$

PATH INTEGRALS
MONTE-CARLO METHOD

quark, antiquark, gluon



$$\rho(r,Q,\sigma;\beta) = \frac{1}{\lambda_{\Delta q}^{3N_q}\lambda_{\Delta q'}^{3N_{q'}}\lambda_{\Delta g}^{3N_g}} \sum_{P=P_q,P_{q'},P_g} (\pm 1)^{\kappa_P} \int_V dr^{(1)}\ldots dr^{(n)} dQ^{(1)}\ldots dQ^{(n)} \times$$

$$\rho\left(r,Q;r^{(1)},Q^{(1)};\Delta\beta\right)\ldots\rho\left(r^{(n)},Q^{(n)};\hat{P}r^{(n+1)},\hat{P}Q^{(n+1)};\Delta\beta\right)S\left(\sigma,\hat{P}\sigma'\right)$$

spin matrix

$$\rho(r^{(l)},Q^{(l)};r^{(l+1)},Q^{(l+1)}) \approx \delta(Q^{(l)}-Q^{(l+1)})\rho(r^{(l)},Q^{(l)};r^{(l+1)},Q^{(l)})$$

Density matrix

$$\sum_{\sigma}\rho(r,Q,\sigma;\beta)=\frac{1}{\lambda_{\Delta}^{3N_q}\lambda_{\Delta}^{3N_{q'}}\lambda_{\Delta}^{3N_g}}\sum_{s=0}^{N_q}\sum_{s'=0}^{N_{q'}}\sum_{s''=0}^{N_g}\rho_{ss's''}([rQ],\beta)$$

$$\rho_{ss's''}([rQ],\beta)=\frac{C_{N_q}^{s}}{2^{N_q}}\frac{C_{N_{q'}}^{s'}}{2^{N_{q'}}}\exp\{-\beta U([rQ],\beta)\}\times$$

$$\times\prod_{l=1}^{n}\prod_{p=1}^{N_e}\phi_{pp}^{l}\det\left|\psi_{ab}^{n,1}\right|_{s}\prod_{p=1}^{N_i}\tilde{\phi}_{pp}^{l}\det\left|\tilde{\psi}_{ab}^{n,1}\right|_{s'}\prod_{p=1}^{N_i}\tilde{\tilde{\phi}}_{pp}^{l}\,per\left|\tilde{\tilde{\psi}}_{ab}^{n,1}\right|_{s''}$$

$$U([rQ],\beta)=\sum_{l=0}^{n}\frac{U_{l}^{qq'g}([r^{(l)}Q],\beta)}{n+1}$$

Pairwise sum of Kelbg potentials for each l=0,…,n

Exchange matrix

$$\left\|\psi_{ab}^{n,1}\right\|_{s}\equiv\left\|\exp\left\{-\frac{\pi}{\lambda_{\Delta}^{2}}\left|(r_a-r_b)+y_a^n\right|^2\right\}\right\|_{s}$$

First studies and testing method within simplified quasiparticle model of quark – gluon plasma.

- All color quasiparticles are massive (m > T) and move non-relastivistically
- All quasiparticle masses are the same.
- We do not distiguish between quark flavors.
- Interparticle interaction is domonated by a color Coulomb potential with interparticle distance dependent coupling constant.
- The color operators are substituted by their average values
  – classical color vectors in SU(2) (3D vec.with 1Cas.) instead of SU(3).

Canonical ensemble instead of grand canonical ensemble.

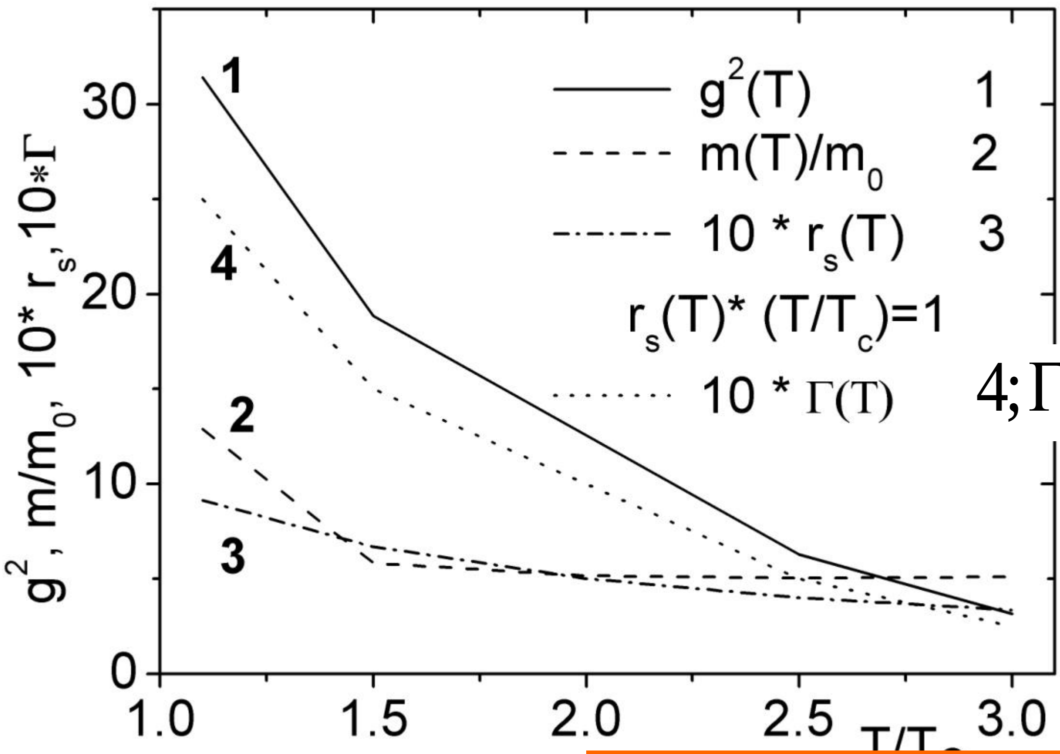
- Numbers of quarks, antiquarks and gluons are equal.

The model input requires :

- The temperature dependence of the quasiparticle mass.
- The temperature dependence of the coupling constant.
- The temperature dependence of the quasiparticle density.

All the input quantiries should be deduced from lattice QCD calculations.

Input quantities from lattice calculations in simplified version



Coupling constant

$$\alpha(T) = g^2(T)/4\pi \sim 1$$

Ratio of potential to kinetic energy per quasiparticle

$$4; \Gamma(T) \sim U/K \sim 1$$

$$< Q_a | Q_b > g^2$$

SU(3) SU(2)

Quasiparticle masses: $m(T)/T_c \approx \frac{0.9}{(T/T_c - 1)} + 3.45 + 0.4T/T_c$

Density: $n\sigma^3 \approx 0.24(T/T_c)^3$ $4\pi r_s^3 n\sigma^3 / 3 = 1$ $\sigma \approx 1.1\,fm$ $r_s(T) = < r > / \sigma \approx 1/(T/T_c)$

[Phys. Rev. C, **74**, 044909, (2006), Phys. Rev. D, 73, 014509, (2006)]

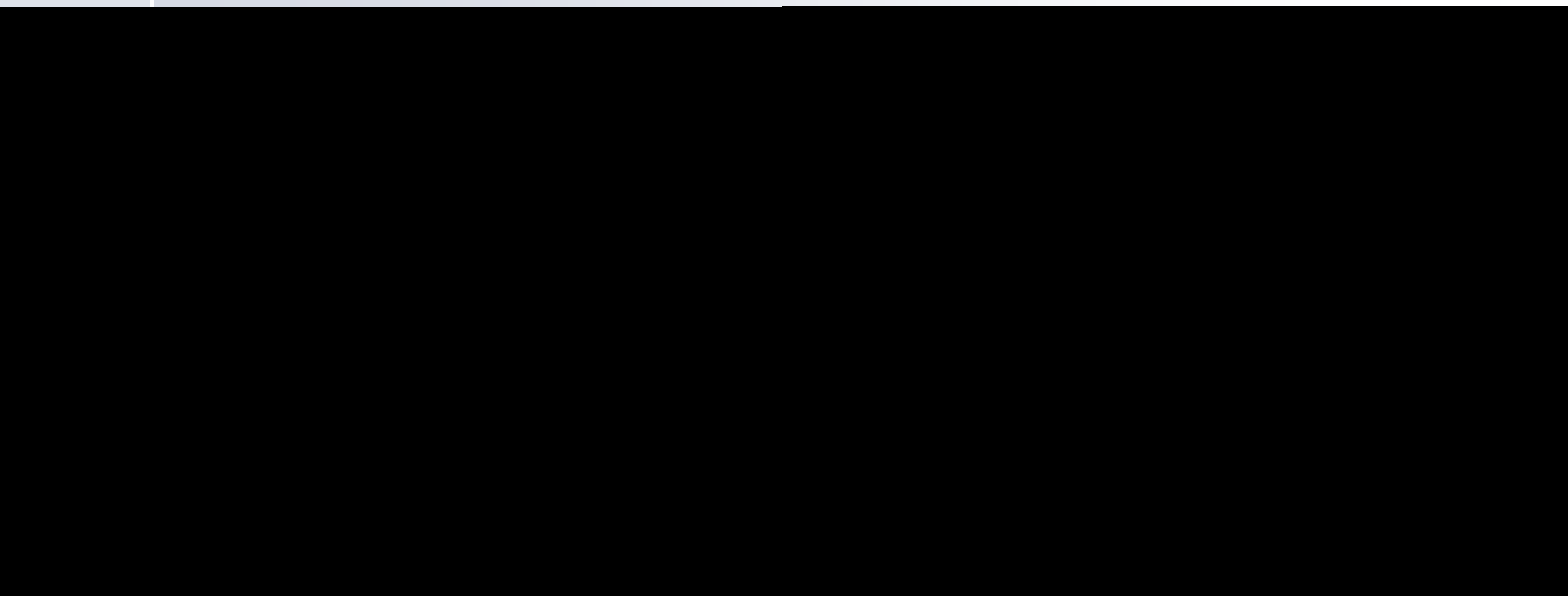
Snapshots of typical configurations

$T=1.1T_0$

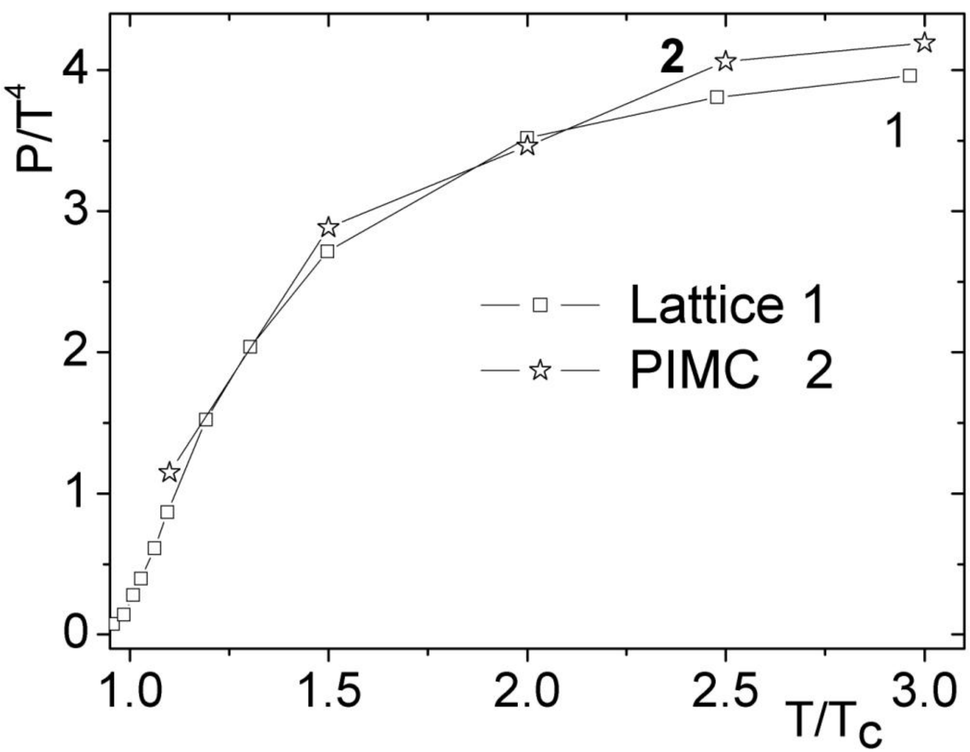
Gas-like rarefied system of 3-4 quasiparticle clusters

$T=3T_0$

Liquid-like dense system of individual quasiparticles



Equation of State. Comparison path integral results with lattice (2+1) QCD



Pair distribution functions in canonical emsemble
Color correlation functions

$$H_\beta \approx \sum_a (N_a m_a(\beta) + \frac{p_a^2}{2m_a(\beta)}) + \sum_{a,b} \frac{g^2(|r_a - r_b|, \beta) C_{ab} < \vec{Q}_a | \vec{Q}_b >}{4\pi |r_a - r_b|}$$

$$Z\left(N_q, N_{q'} N_g, V, \beta\right) = Q\left(N_q, N_{q'} N_g, \beta\right) / N_q! N_{q'}! N_g!$$

$$Q\left(N_q, N_{q'}, N_g, \beta\right) = \sum_\sigma \int_V drd\mathbf{Q} \rho\left(r, \mathbf{Q}, \sigma; \beta\right)$$

$$g_{ab}(|R_1 - R_2|) = g_{ab}(R_1, R_2) = \frac{1}{Q(N_q, N_{q'}, N_g)} \times$$

$$\sum_\sigma \int_V drd\mathbf{Q} \delta(R_1 - r^a{}_1) \delta(R_2 - r^b{}_2) \rho(r, \mathbf{Q}, \sigma; \beta),$$

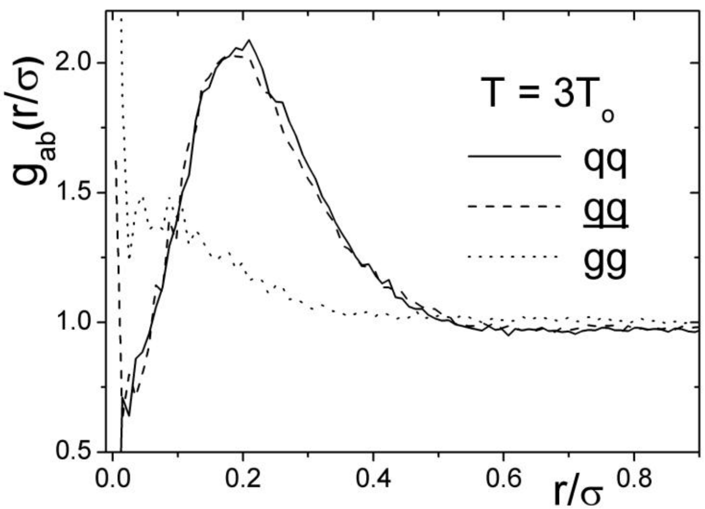
$$c_{ab}(R_1 - R_2)_{Def} = \frac{1}{Q(N_q, N_{q'}, N_g)} \sum_\sigma \int_V drd\mathbf{Q} \times$$

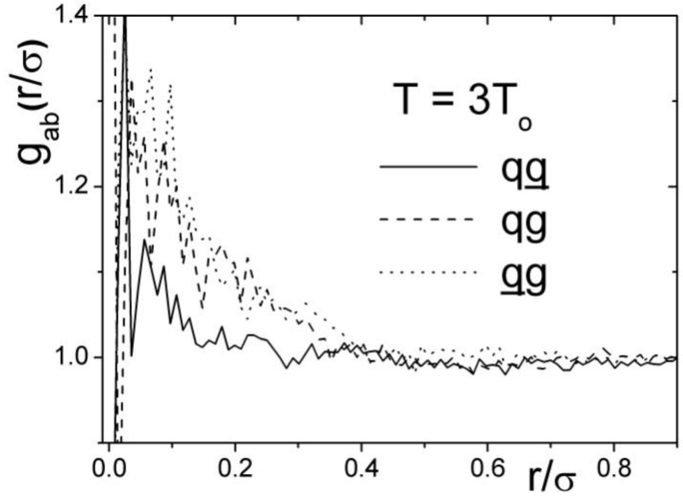
$$\delta(R_1 - r^a{}_1) \delta(R_2 - r^b{}_2) < \mathbf{Q}^1{}_a | \mathbf{Q}^2{}_b > \rho(r, \mathbf{Q}, \sigma; \beta)$$

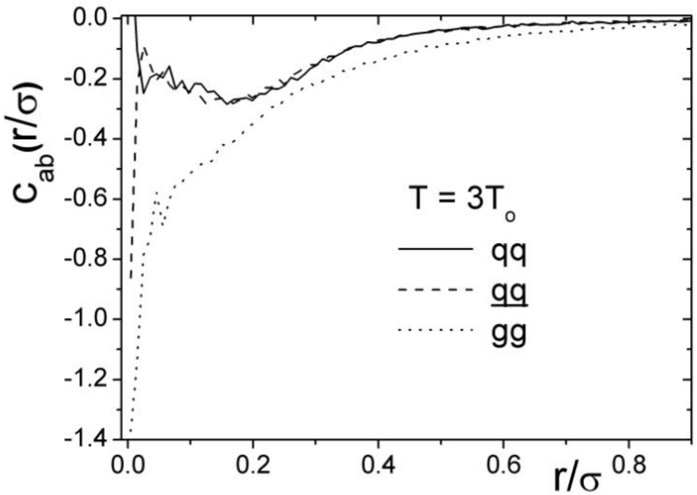
PAIR DISTRIBUTION AND COLOR CORRELATION FUNCTIONS

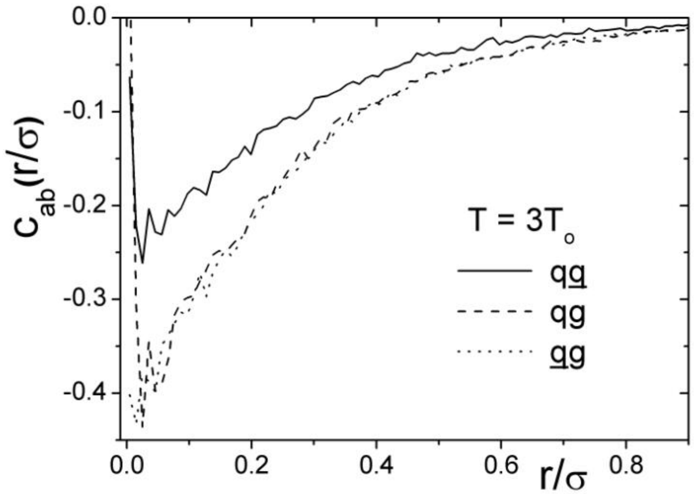
Similar quasiparticles

Different quasiparticles





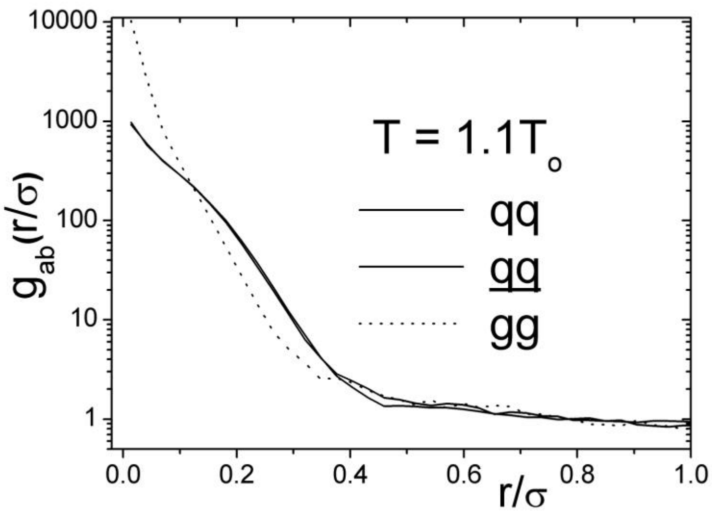


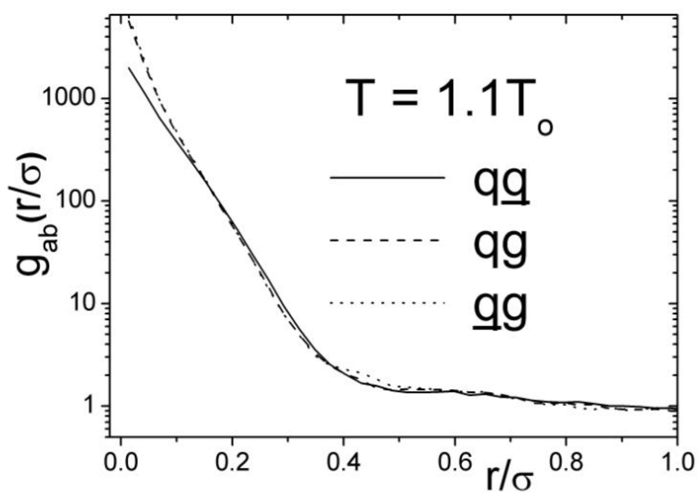


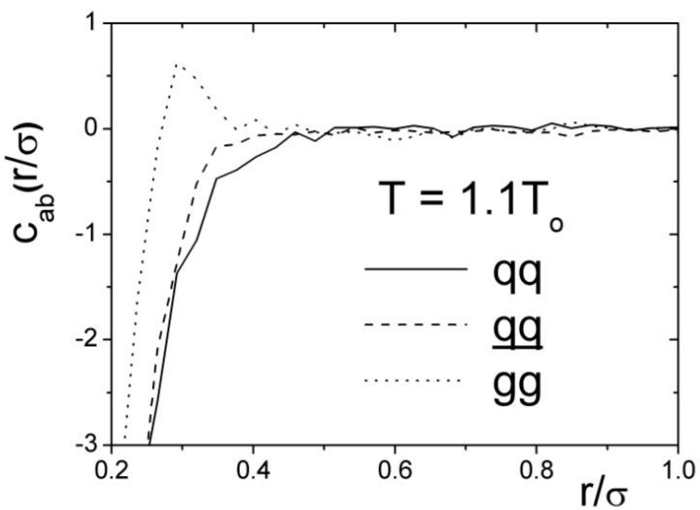
PAIR DISTRIBUTION AND COLOR CORRELATION FUNCTIONS

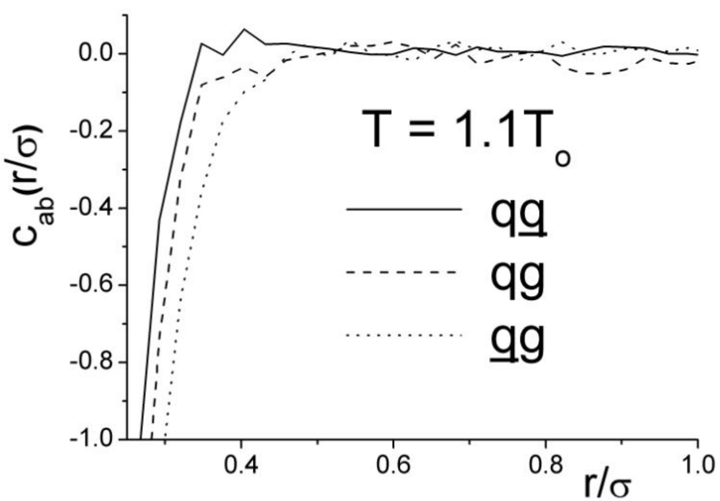
Similar quasiparticles

Different quasiparticles









Estimation of the quasiparticle bound states

The product $r^2 g_{ab}(r)$ has the physical meaning of a probability to find an two quasipartices at a distance |r| from each other. On the other hand, the corresponding quantum mechanical probability is the product of r² and the two-particle Slater sum $\sum_{ab}$

$$\sum_{ab} = 8\pi^{3/2}\lambda_{ab}^3 \sum_{E_\alpha}^{\infty} |\Psi_\alpha(r)|^2 \exp(-\beta E_\alpha) = \sum_{ab}^{d} + \sum_{ab}^{c}$$

$$\sum_{ab}^{d} = 8\pi^{3/2}\lambda_{ab}^3 \sum_{E_\alpha}^{E'} |\Psi_\alpha(r)|^2 \exp(-\beta E_\alpha)$$

$$r^2 g(r) \sim r^2 \left(\sum_{ab}^{d} + \sum_{ab}^{c}\right)$$

$$r^2 g(r) \sim r^2 \sum_{ab}^{d} > r^2, r < a_b$$

$$r^2 g(r) \sim r^2 \sum_{ab}^{c} \sim r^2, r > a_b$$

$$\sum_{ab}^{c} >> \sum_{ab}^{d} \Rightarrow r^2 * \sum_{ab}^{c} \sim r^2$$

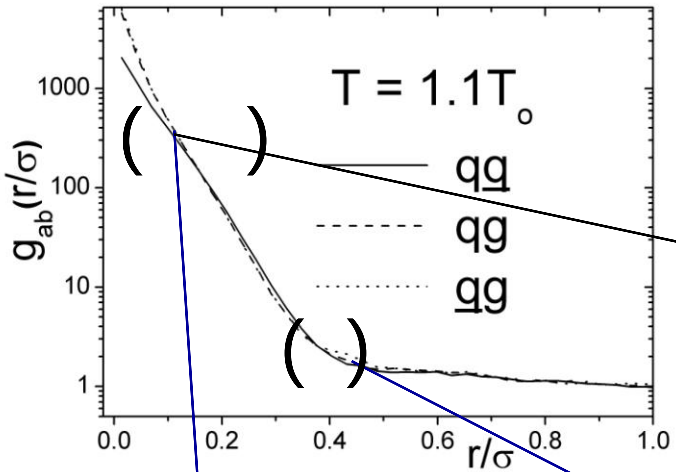
$$\sum_{ab}^{c} << \sum_{ab}^{d} \Rightarrow r^2 * \sum_{ab}^{d} >> r^2$$

Peak related to bound states at interparticle distances of order one Bohr radius exists if discrete bound states in electron-hole or hydrogen plasma are well populated (low temperatures and small densities)

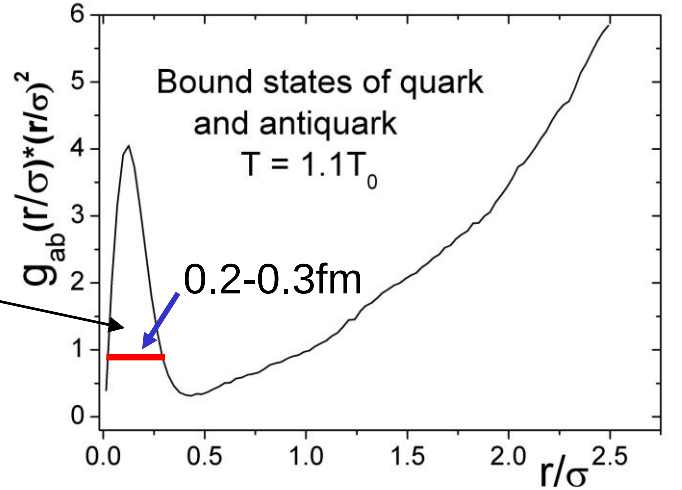
For low densities it is reasonable to choose $E' > -1/\beta$ while for high densities is appropriate $E' = -Ry/r_s$ since the quasiparticle in states with energy $E_\alpha > E'$ can be considered as free particles.

Color bound states and mean force potential ($T=1.1T_c$)

Distribution functions



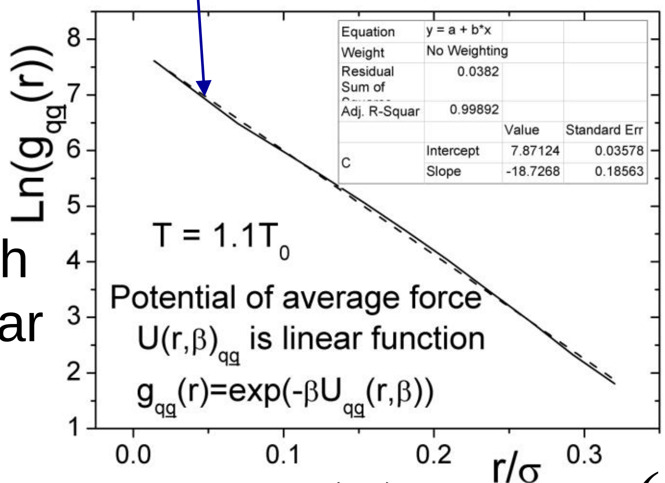
r^2 * Distribution function

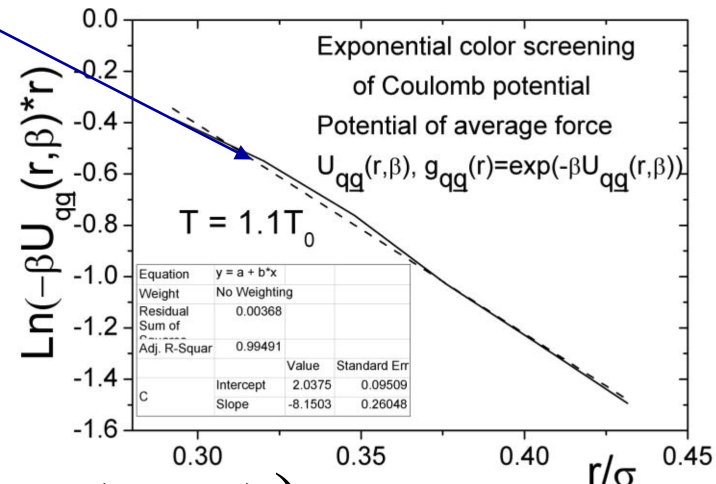


Linear part of the mean force potential

Color screening part of mean force potential

Depth U > 1 GeV agree with lattice near T_c

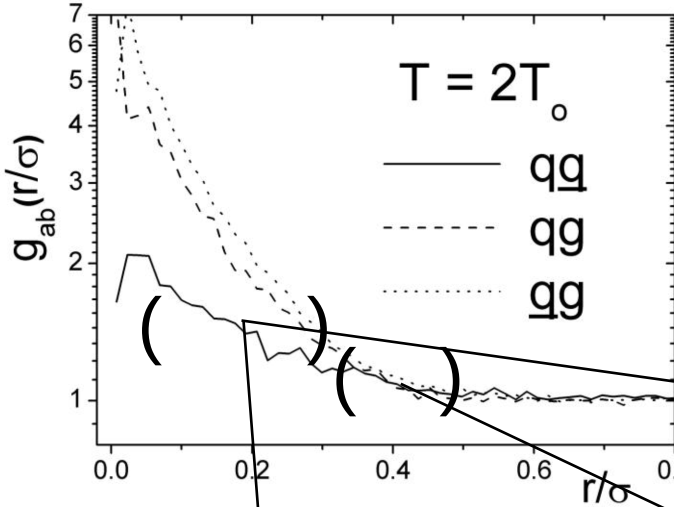




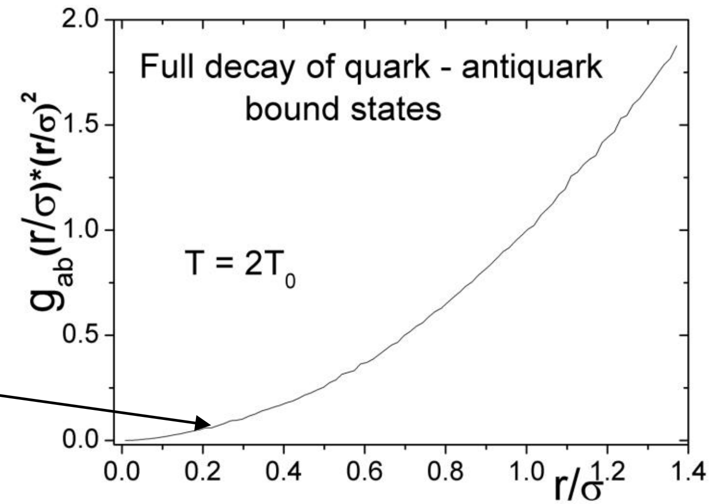
$$g_{q\bar{q}}(r) = \exp(-\beta U_{q\bar{q}}(r,\beta))$$

Decay of color bound states and mean force potential ($T=2T_c$)

Distribution functions



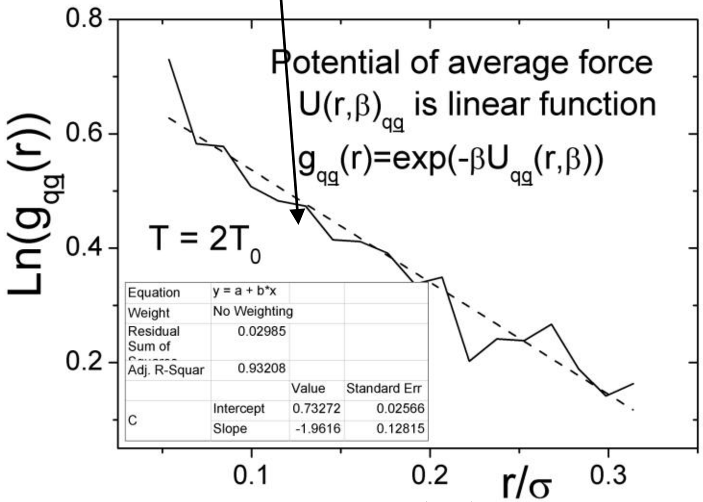
r^2 * Distribution function

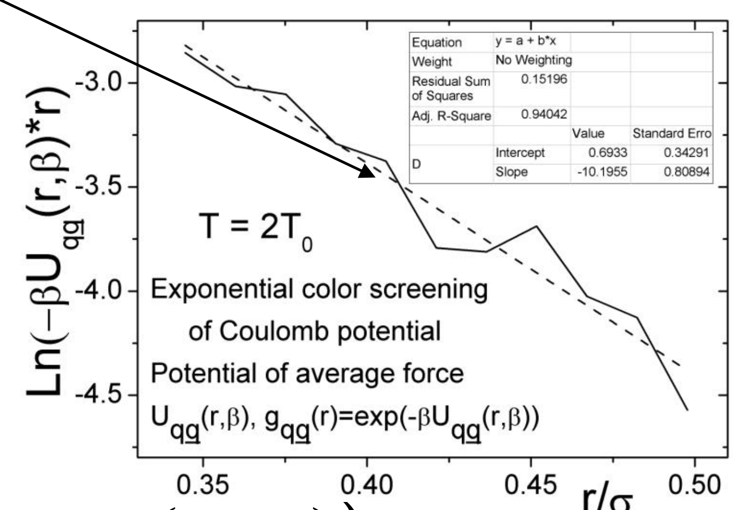


Linear part of the mean force potential

Color screening part of mean force potential

Depth~
175 MeV





$$g_{qq}(r)=\exp(-\beta U_{qq}(r,\beta))$$

Kinetic properties of quark – gluon plasma in canonical ensemble

$$C_{FA}(t) = Z^{-1} Tr\{F \exp(i\frac{Ht_c}{h}) A \exp(-i\frac{Ht_c}{h})\};$$

$$H = K + V(qQ), t_c = t - i\frac{\beta h}{2}, \beta = \frac{1}{kT},$$

$$Z = Tr\{\exp(-\beta H)\}$$

$$C_{FA}(t) = \frac{1}{(2\pi h)^{2\nu}} \iint dQ_1 dp_1 dq_1 dp_2 dq_2 F(p_1, q_1) A(p_2, q_2) \times$$

$$W(p_1, q_1, Q_1; p_2, q_2, Q_1; t; i\beta h),$$

In this model we use approximation

$$\delta(Q_1 - Q_1^{'})\delta(Q_2 - Q_2^{'})\delta(Q_1 - Q_2)$$

$$A(p,q) = \iint d\xi \exp(-i\frac{p\xi}{h}) < q - \frac{\xi}{2} | A | q + \frac{\xi}{2} > \leftarrow$$

Weil symbols of operators

$$W(p_1, q_1, Q_1; p_2, q_2, Q_1; t; i\beta h) = Z^{-1} \iint d\xi_1 d\xi_2 \exp(i\frac{p_1\xi_1}{h}) \exp(i\frac{p_2\xi_2}{h}) \times$$

$$< q_1 + \frac{\xi_1}{2} | \exp(i\frac{Ht_c}{h}) | q_2 - \frac{\xi_2}{2} >< q_2 + \frac{\xi_2}{2} | \exp(-i\frac{Ht_c}{h}) | q_1 - \frac{\xi_1}{2} >$$

Integral equation

$$W(p_1,q_1,Q_1;p_2,q_2,Q_1;t;i\beta h)=\bar{W}(p^0{}_1,q^0{}_1,Q_1^0;p^0{}_2,q^0{}_2,Q_2^0;0;i\beta h)+$$

$$+\int_0^t d\tau\iint ds\iint d\eta W(p_1^\tau-s,q_1^\tau,Q_1^\tau;p_2^\tau-\eta,q_2^\tau,Q_2^\tau;\tau;i\beta h)\gamma(s,q_1^\tau,Q_1^\tau;\eta,q_2^\tau,Q_2^\tau),$$

$$\gamma(s,q_1^\tau,Q_1^\tau;\eta,q_2^\tau,Q_2^\tau)=\frac{1}{2}\{\omega(s,q_1^\tau,Q_1^\tau)\delta(\eta)-\omega(\eta,q_2^\tau,Q_2^\tau)\delta(s)\},F(q,Q)=-\nabla_q V(q,Q)$$

$$\omega(\eta,q,Q)=\frac{4}{(2\pi h)^\nu h}\iint dq'V(q-q',Q)Sin(\frac{2sq'}{h})+F(q,Q)\Box\frac{d\delta(s)}{ds}$$

– –

Positive time direction

$$\frac{dq_1^t}{dt}=\frac{1}{2m}p_1^t,\frac{dp_1^t}{dt}=\frac{1}{2}F(q_1^t,Q_1^t),$$

$$\frac{dQ_{1,i}^{t,a}}{dt}=\frac{1}{2}\sum_{b,c}f^{abc}Q_{1,i}^b\nabla_{Q_{1,i}^c}V(q_1^t,Q_1^t),$$

$$p_1^t(t,p_1,q_1,Q_1)=p_1,q_1^t(t,p_1,q_1,Q_1)=q_1,Q_1^t(t,p_1,q_1,Q_1)=Q_1$$

– –

Color dynamics in SU(2) or SU(3)

Initial conditions

Hamiltonian eqations

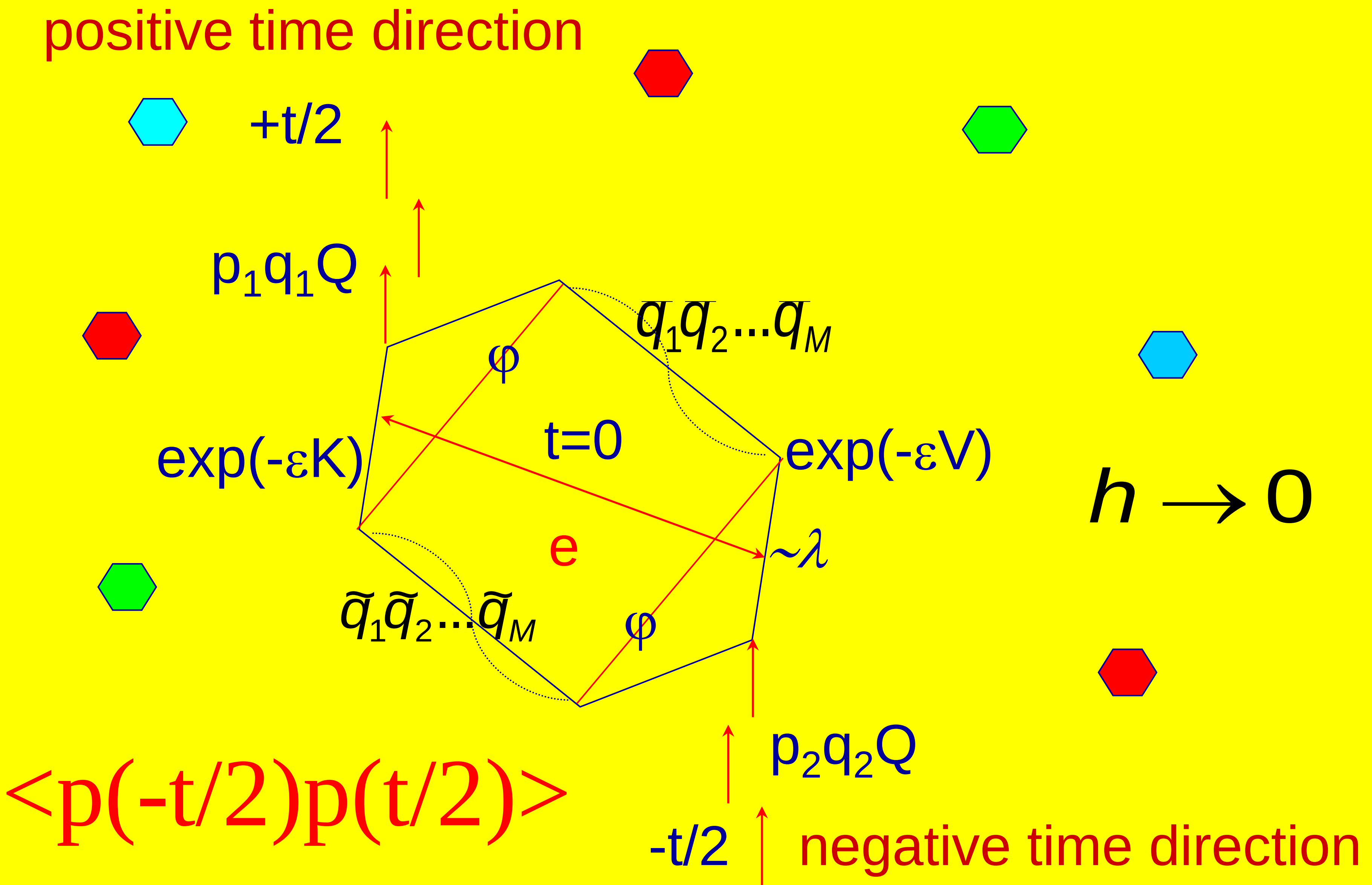
$$\frac{dq_2^t}{dt}=-\frac{1}{2m}p_2^t,\frac{dp_2^t}{dt}=-\frac{1}{2}F(q_2^t,Q_2^t),$$

$$\frac{dQ_{2,i}^{t,a}}{dt}=-\frac{1}{2}\sum_{b,c}f^{abc}Q_{2,i}^b\nabla_{Q_{2,i}^c}V(q_2^t,Q_2^t),$$

Negative time direction

$$p_2^t(t,p_2,q_2,Q_1)=p_2,q_2^t(t,p_2,q_2,Q_1)=q_2,Q_2^t(t,p_2,q_2,Q_1)=Q_1$$

Schematic snapshot
for color phase space dynamics

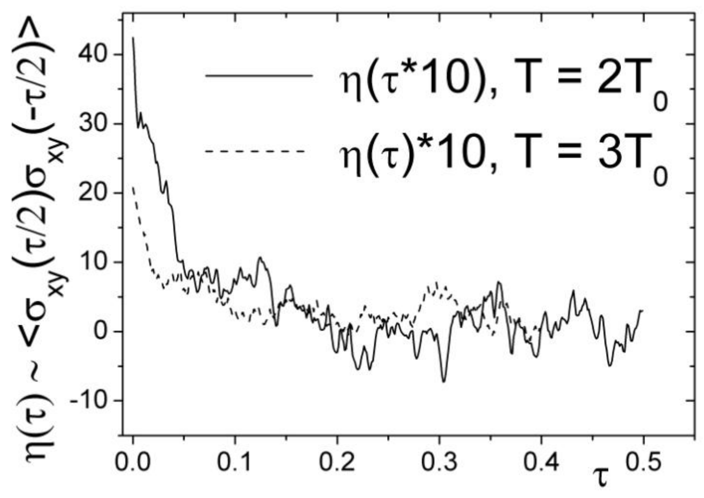


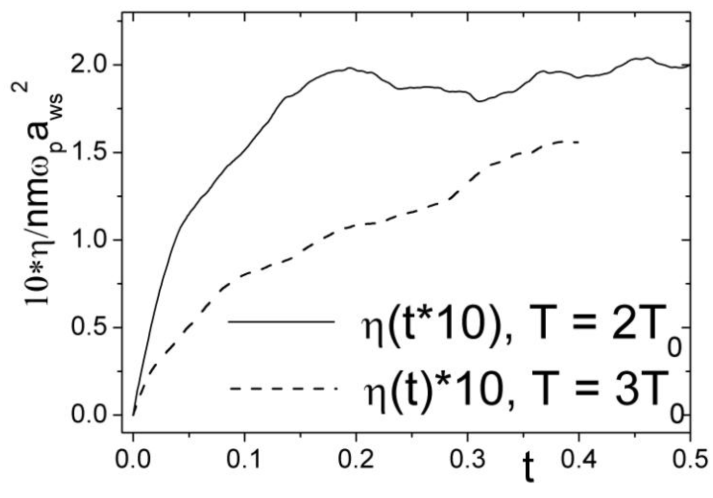
Time autocorrelation function of the stress energy tensor and shear viscosity of quark –gluon plasma

$$\eta(\tau) = \frac{n}{3k_B T}\left\langle \sum_{X<Y} \sigma_{XY}(\tau/2)\sigma_{XY}(-\tau/2) \right\rangle$$

$$\sigma_{XY}(\tau) = \frac{1}{N}\left(\sum_{i=1}^{N} m_i v_{ix} v_{iy} + \frac{1}{2}\sum_{i\neq j} r_{ij,x} F_{ij,y} \right)$$

$$\eta = \lim_{t\to\infty} \eta(t) = \lim_{t\to\infty} \int_0^t d\tau \eta(\tau)$$



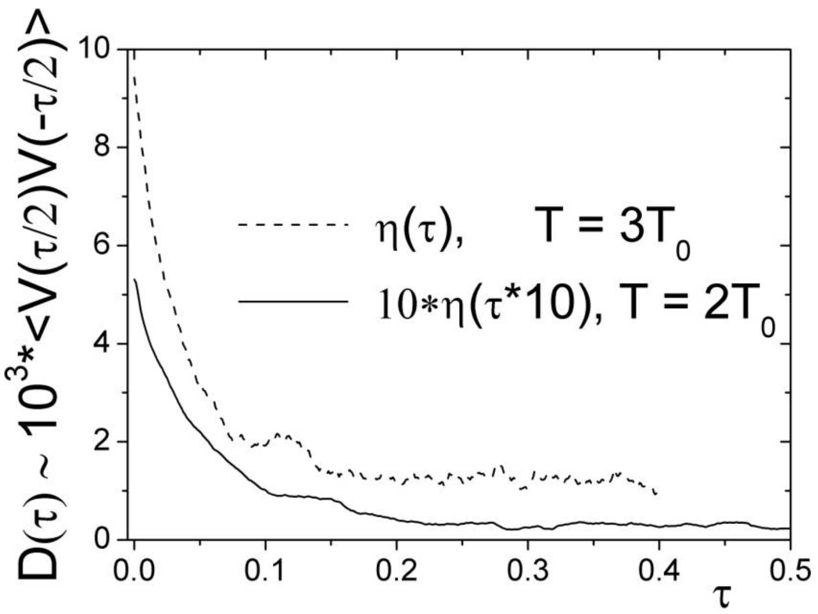


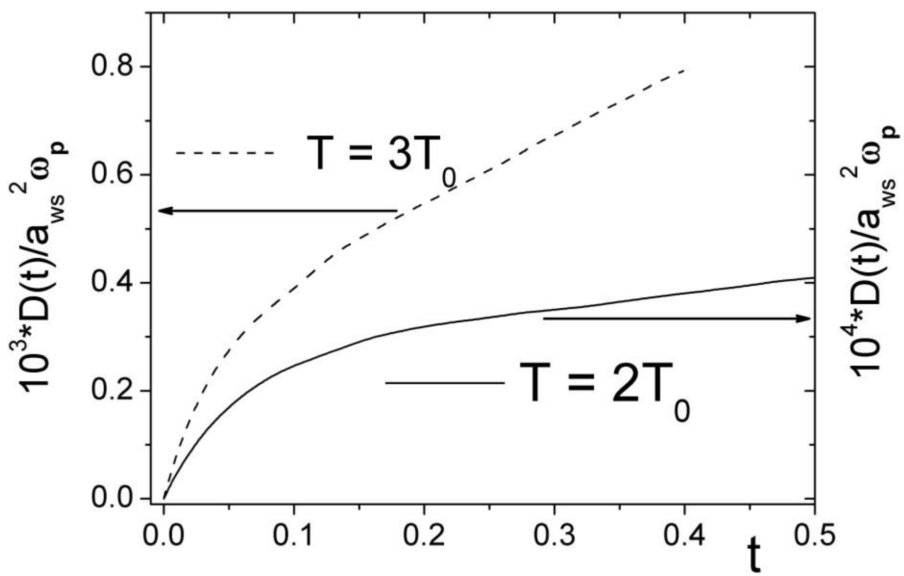
Velocity autocorrelation function and diffusion constant QGP

$$D(\tau) = < v(\tau/2) v(-\tau/2) >=$$

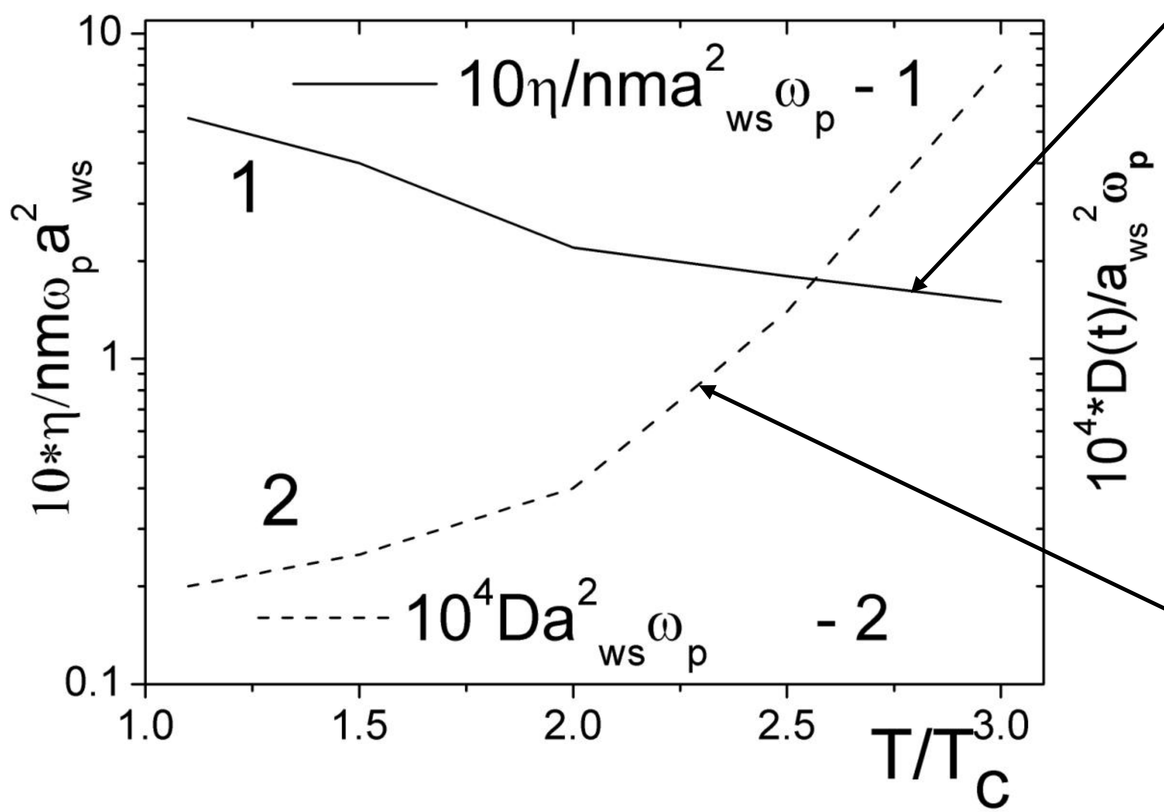
$$= \frac{1}{3N} < \sum_{i=1}^{N} \vec{v}_i(\tau/2) \bullet \vec{v}_i(-\tau/2) >$$

$$D = \lim_{t\to\infty} D(t) = \lim_{t\to\infty} \int_0^t d\tau D(\tau)$$





Diffusion coefficient and shear viscosity



Shear viscosity agrees with Gelman et al., 2006

Diffusion coefficient is ~10^3 lower in comparison with Gelman et al., 2006

Electromagnetic plasma
Crystallization of protons

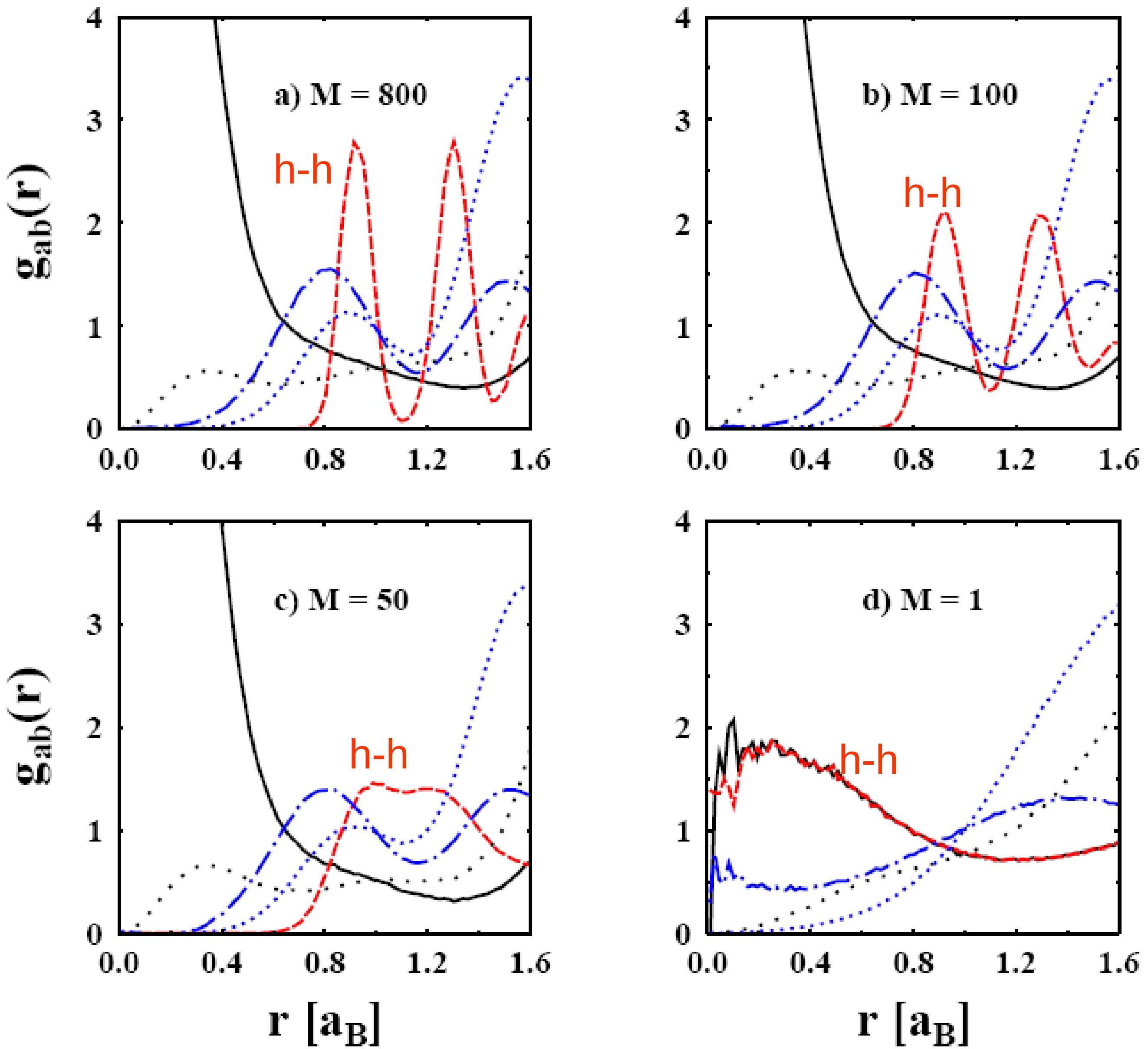
HYDROGEN, PIMC-SIMULATION,
n = 10^{25} cm^{-3}, T=10 000 K^o
Bloch oscillation of electron density in periodic potential

HOLE CRYSTALLIZATION AND QUANTUM MELTING

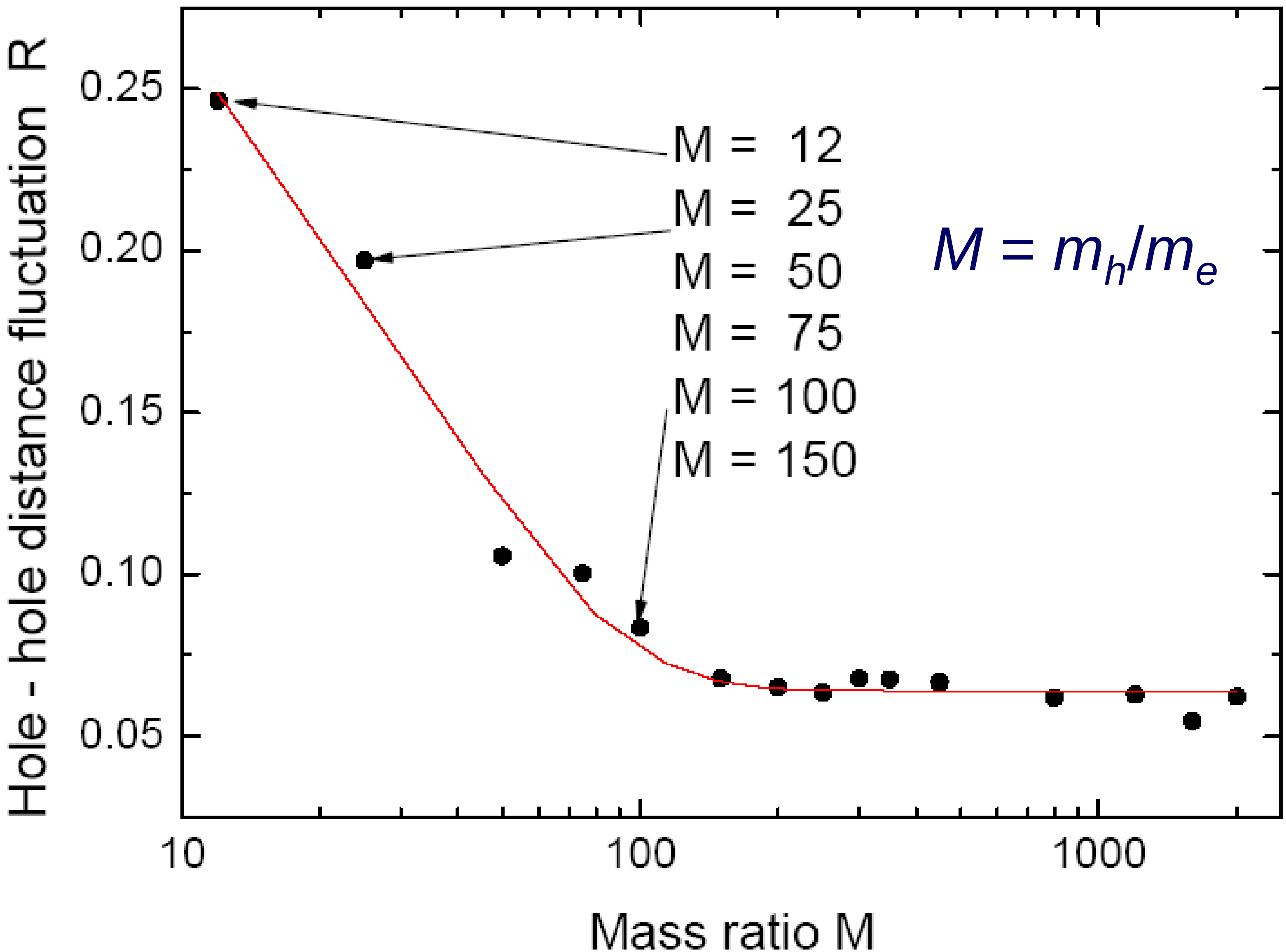
$\langle r \rangle / a_B = 0.63$

$T = 0.064 E_b$

$M = m_h / m_e$

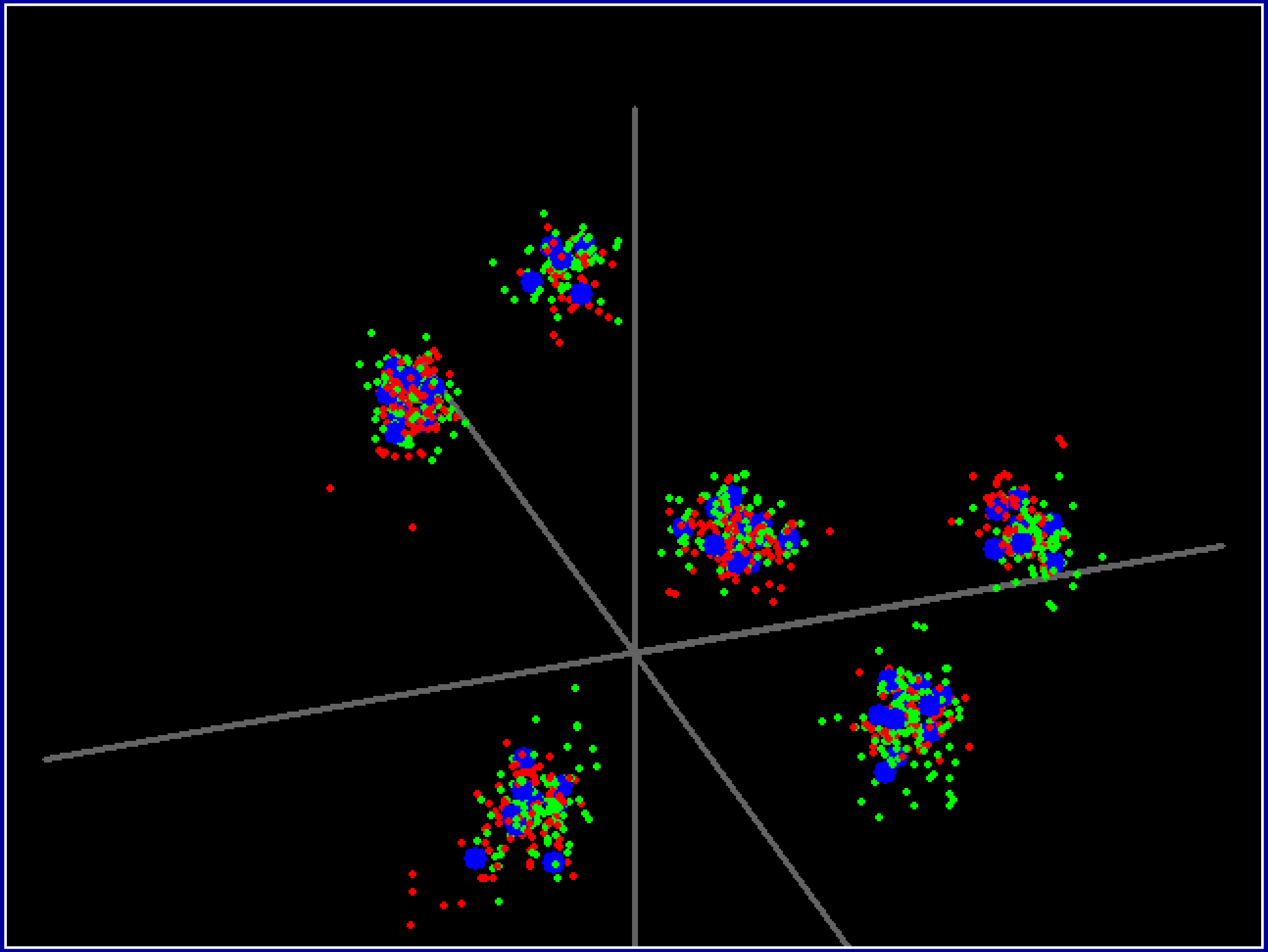


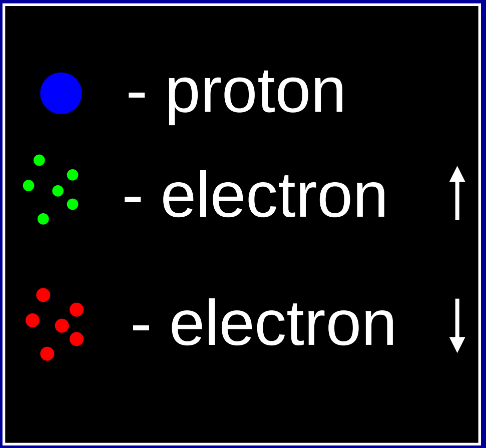
HOLE-HOLE DISTANCE FLUCTUATIONS



Phase transition to metallic state
Metallic drops and many particle clusters in hydrogen plasma

3D quantum two-component plasma.





T = 10000 K, n = 10^{22} cm^{-3}, ρ = 0.0167 g/cm^3

CONCLUSIONS

- Path integral Monte Carlo is a reliable and very fast method of simulation thermodynamic properties in a wide range of plasma parameters
- Quantum dynamics can be constructed on the basis of Feynman and Wigner formulation of quantum mechanics
- The developed numerical approach can be applied to consideration of EM and QG plasmas.
- Results of simulations agree with available theoretical and experimental data.

Thank you for attention.

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Quantum simulations of strongly coupled electromagnetic and quark-gluon plasmas.

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