P-adic Coverage Method in Fractal Analysis of Shower

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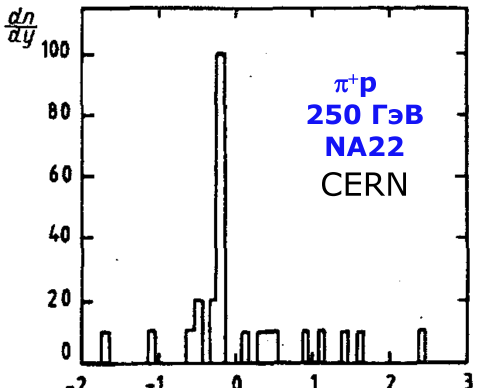
Outline

- **Self-similarity and fractality in multiple production at high energies**
- **Analysis results of the Box Counting Method for determination of the fractal dimension**
- **The method of analysis of regular showers**
- **Summary**

Self-similarity & Fluctuation & Intermittency

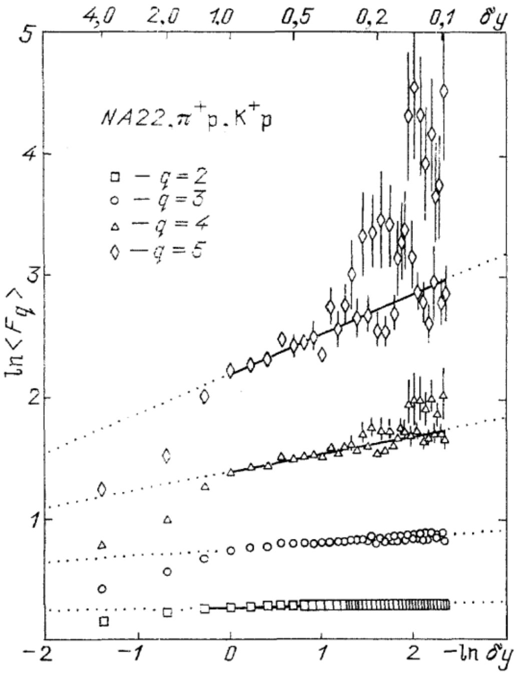


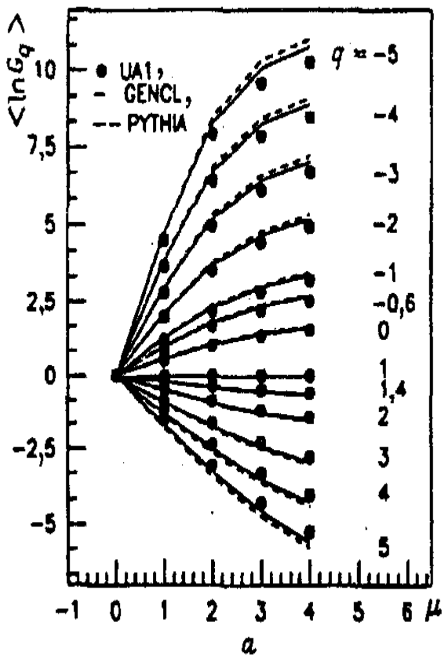
Intermittency: abnormal events with large multiplicity fluctuation were observed in h-h interaction.

NA22: $\delta y=0.1$, $dN/dy=100$, $dN/dy / \langle dN/dy\rangle \approx 60$

Observable fluctuations are dynamical and reflects self-similarity of interaction

Bialas A. // Nucl. Phys. 1986 B273, p.703
Hwa R. // Phys.Rev. 1990. D41, p.1456





Power Law dependence of factorial moments $F_q(\delta y)$, $G_q(\delta y)$ on bin widths δy

$$F_q(\delta y) \sim (\delta y)^{-\phi(q)}$$

$$G_q(\delta y) \sim (\delta y)^{-\tau(q)}$$

$$F_q(\delta y) = M^{q-1} \frac{\left\langle \sum_{k=1}^{M} n_k (n_k-1)\ldots(n_k-q+1) \right\rangle}{}$$

$$G_q = \sum_{m=1}^{M}{}' p_m^q \; , \; p_m = n_m/n, \; n = \sum_{m=1}^{M} n_m$$

$M=2^{\mu}$-number of bins with width δy,
n_k- number of particles in k-bin

Self-similarity & z-Scaling

High-p_T inclusive particle spectra is described by the dimensionless function Ψ depending on a single variable z

$$\Psi(z) = \frac{\pi \cdot s}{(dN/d\eta)\cdot \sigma_{inel}} \cdot J^{-1} \cdot E\frac{d^3\sigma}{dp^3}$$

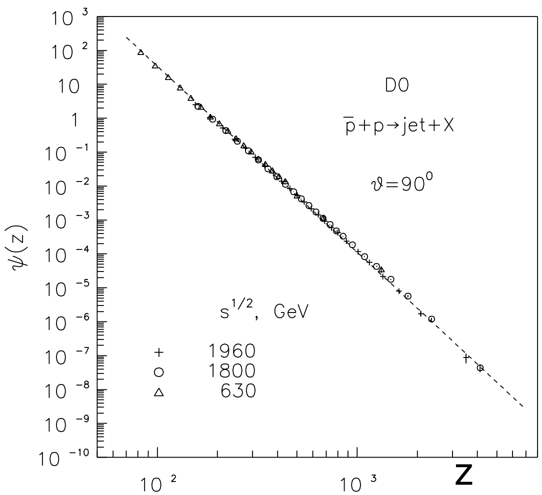
$$z = z_0\, \Omega^{-1}$$

$$\Omega = (1 - x_1)^{\delta_1}(1 - x_2)^{\delta_2}$$

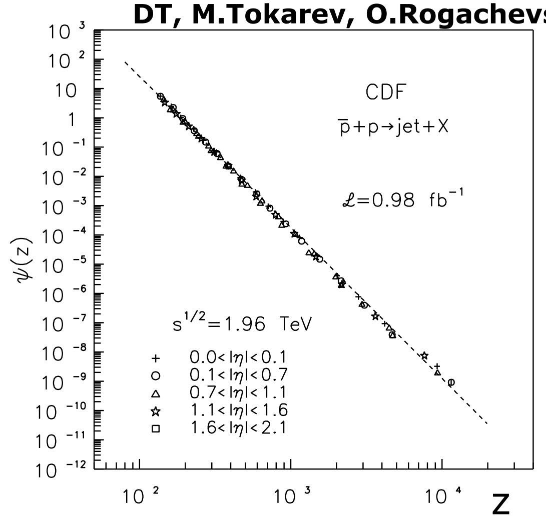
s- the energy of colliding, dN/dη- the multiplicity density, σ_{inl}- inelastic cross section, $Ed^3\sigma/dp^3$- the inclusive cross section, J- coefficient depend on the kinematical variable. x_1,x_2: Max Ω(x1,x2) and conservation laws 4-momentum of the exclusive subprocess

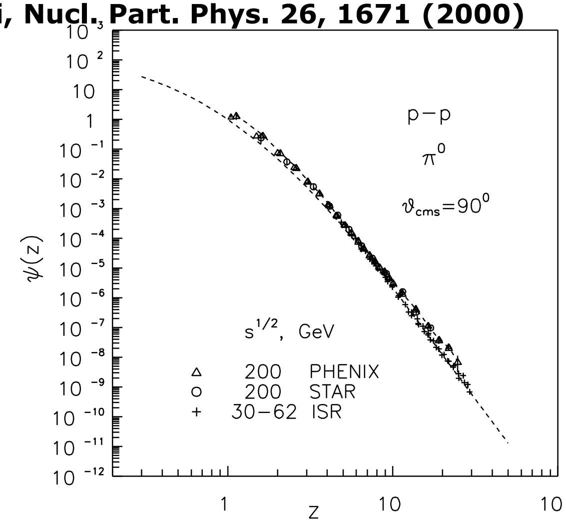
$$(x_1P_1 + x_2P_2 - p)^2 = (x_1M_1 + x_2M_2 + m_2)^2$$

Zborovsky , Panebratsev , Tokarev , Skoro Phys. Rev. D54, 5548 (1996)



DT, M.Tokarev, Mod. Phys. A 15, 3495 (2000)
DT, M.Tokarev, O.Rogachevski, Nucl. Part. Phys. 26, 1671 (2000)





Energy, angular independence of Ψ(z)
Power low $\Psi(z) \sim z^{-\beta}$ over a wide z-range
It indicates on self-similarity of hadron production at various scale

Self-similarity & Fractality & Multiple production

$F_q(\delta y) \sim (\delta y)^{-\phi(q)}$, $G_q(\delta y) \sim (\delta y)^{-\tau(q)}$, $\Psi(z) \sim z^{-\beta}$

Power Laws established experimentally, and characterizing self-similarity of particles production on different scales **are typical for fractals**

Fractal is the self-similar object with **nonintegral (fractal) dimension**

Fractal dimension is the value D_F which provides the finite limit

N(δ) - is number of probes size δ, covering an object

$$\lim_{\delta \to 0} N(\delta) \cdot \delta^{D_F} = const$$

Relationship of fractal and multiple production

Power Law exponent τ(q)

(Intermittency: $G_q(\delta y) \sim (\delta y)^{-\tau(q)}$)

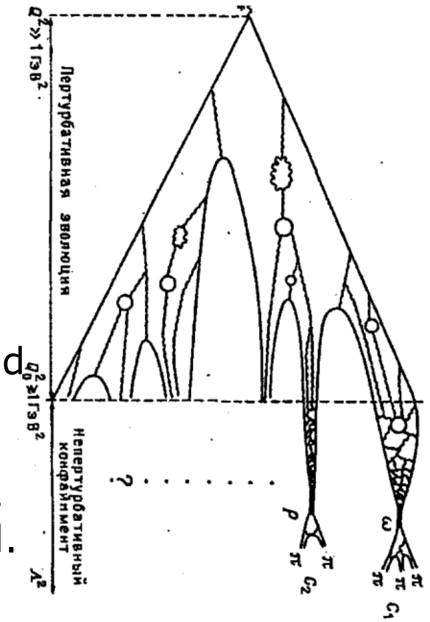
defines

spectrum of fractal dimension

(generalized fractal dimension)

$$D(q) = \tau(q)/q-1$$

- Set of hadrons produced in inelastic interaction are set of points of the three -dimensional phase-space (p_T, η, φ)
- The distribution of points in phase-space is non-uniformly and is determined by the interaction dynamics.
- Set of these points in the phase-space are considered as a fractal.



Models of multiple production & Intermittency

- **Cascade process (branching self-similar jets)**
  Dremin. et. all. // Usp. Fiz. Nauk. 163, 1-60, 1993
  - Partons cascade: Q_0^2 = 0.4 ГэВ^2, hardronization keeps the fractality
  - JETSET: hardronization
  - ARIADNE: Q_0^2, hard and soft process relation
- **Ginzburg-Landau second order phase transition model:**
  (Formation of QGP, thermodynamic equilibrium, phase transition)
  R.C. Hwa, M.T. Nazirov. Phys.Rev.Lett. 1992, v.69, p.741

Are most promising in describing the Intermittency
$G_q(\delta y) \sim (\delta y)^{-\tau(q)}$

Spectrum of fractal dimension D(q) = t(q)/q-1

Cascade process: D(q) – is a linear
Theory of phase transition: D(q) - is const

Determination of fractal dimensions is important for reconstruction of interaction dynamics

Box Counting method

- Choice of some set of different bin widths (probes) δ_i.
  It is defined by the δ_{min}, δ_{max}, law of δ-change
- Construction the distributions of analyzed value y for every bin width δ_i
- Counting the number of non-zero bins $N(\delta_i)$
- Plotting the graph in double-log scale $\ln N(\delta_i)$ vs. $\ln \delta_i$
- If analyzed space is fractal the **graph is linear** and fractal dimension D_b is equal to the slope parameter $b(\delta)$

The Box Counting method has a single parameter - set of bin widths

Test Box Counting Method

Parton final-state shower is used as a test fractal

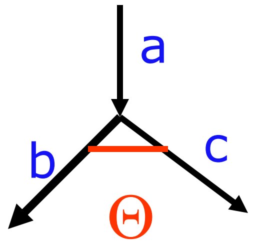
Laws of final-state parton shower (PYTHIA)

- At each step of shower parton branch into two daughter partons **$a \to bc$**
- Final state shower is time-like (**m_{eff}** >0, depends on **Q_0**) **$m_{eff,g}$**$=1/2\cdot$**Q_0**, **$m_{eff,q}$**$=(m_q^2+1/4\cdot$**Q_0**$^2)^{1/}$
- If **$m_{eff}^2 \geq Q_0^2$** parton can branch
- The kinematic of process is described by the energy fraction **z:** $E_b=zE_a$, $E_c=(1-z)E_a$
- The range **of admissible values** $z_-(m_{eff}) < z < z_+(m_{eff})$ **is defined by the effective mass**

$$Z_\pm = 1/2 + \{1+(m_{effb}^2-m_{effc}^2)/m_{effa}^2 \pm |\mathbf{p_a}|/E_a \cdot \sqrt{(m_{effa}^2-m_{effb}^2-m_{effc}^2)^2-4m_{effc}^2 m_{effb}^2}/m_{effa}^2\}$$

- **The range of a opening angle: $\Theta_-(z_-) < \Theta < \Theta_+(z_+)$ is defined by the values Z_+, Z_-**

$$\theta_\pm \approx 1/(\sqrt{z_{a\pm}(1-z_{a\pm})}) \cdot m_{aeff}/E_a$$



branching

- **The opening angles are ordered:** $\Theta_b, \Theta_c < \Theta_a$
  (opening angle of a daughter parton can't be more parent)

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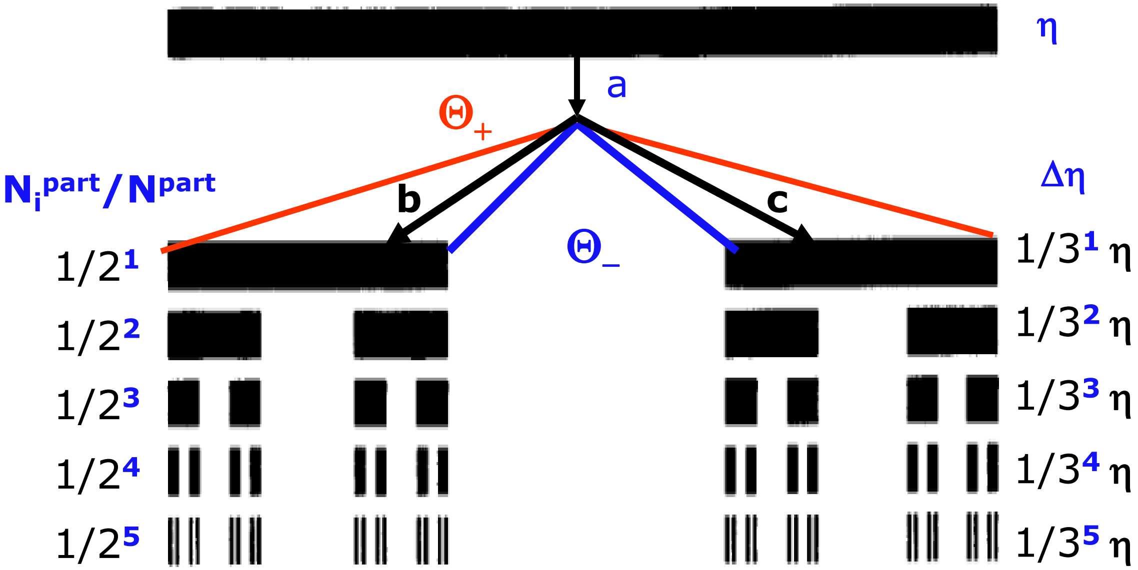
Scenario of parton shower – Triad Cantor Set



- Outgoing from hard process parton branch a→bc

  $\theta_{\pm}$- admissible opening angle
  Black rectangles –permissible parts η

- Branching process is repeated

Power Law:

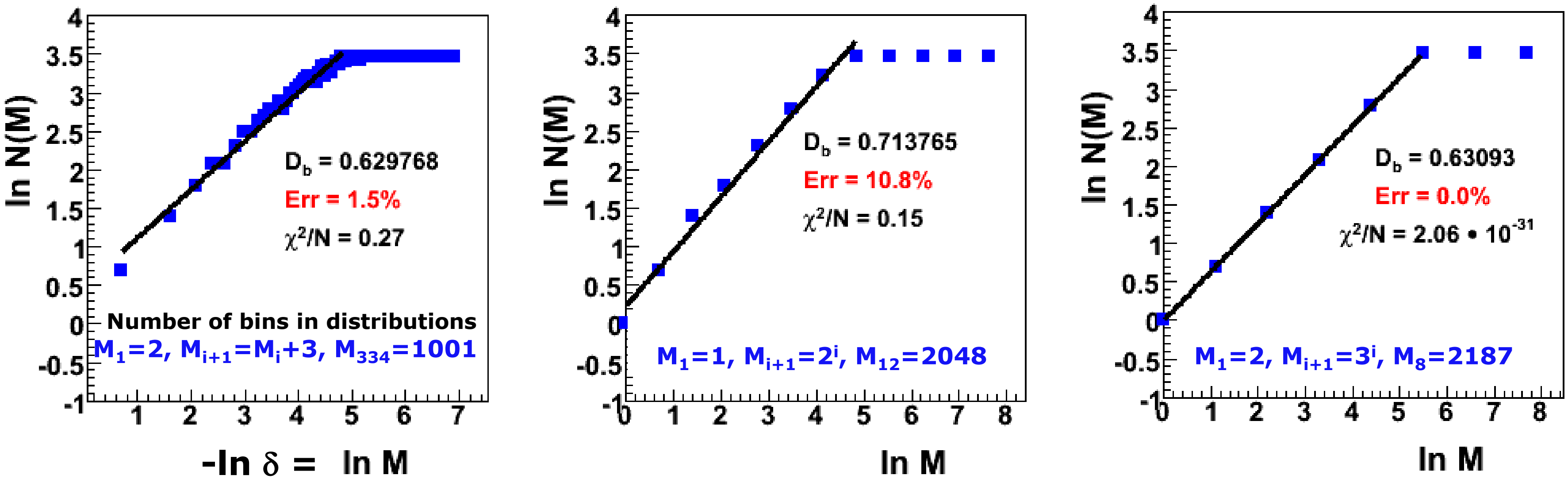
$$N_i^{part}/N^{part}(\Delta\eta) \sim (\Delta\eta)^d$$

$$d = D_F = \ln 2/\ln 3 \approx 0.63093$$

The shower is regular, binary cascade with permissible 1/3 η parts

- 5 level (32 partons) of this cascade are used to test of the Box Counting method to define the fractal dimension
- Pseudorapidity distribution $1/N \cdot dN/d\eta$ of final partons is analyzed
- Pseudorapidity is chosen randomly from permissible range for each final parton

Restoration accuracy of D_F vs. Set of bin widths



- All presented graphics have a plateau. The plateau corresponds to distributions in which the maximum bin content is 1 parton.
- Set of bins ($M_1=2$, $M_{i+1}= M_i+3$, $M_{334}=1001$) is redundant. In it specifies numerous intermediate plateau
- Set of bins $M_{i+1} = 2^i$ defines the value of D_F with Err=10.8%

- Set of bins $M_{i+1} = 3^i$ is an optimum one. It provides exact restoration of D_F and infinitely small $\chi 2/N$
- The optimum set is connected with the law of cascade formation
  $M_i = (1/3)^i$ $\Delta\eta = (1/3)^i \eta$ (Triad Cantor Set)

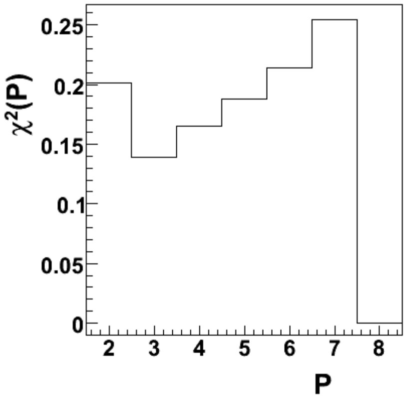
P-adic Coverage Method

- Read out data – $\{\eta_i\}$ of partons
- Construction of pseudorapidity distributions for different set of bins.

  In each set the number of bins are changed as degree of **basis P** **(** $M_i=P^{i-1}$).

  These distributions are various P-adic Coverages of the pseudorapidity space.
- Counting the number of binary cascade level $N_{lev} = LOG_2 N^{part}$, N^{part} is number of partons
- Plotting the graph ln N(M) vs. ln M (number of points N_{lev} +1 in graph)
- Determination of the values of χ^2 (P) and the slope parameter b(P)
- Determination of the optimum set of bins. It corresponds to $\chi^2(P) < \chi_{lim}^2$
- The optimum set of bins M(P) defines:

1. The fractal dimension D_F = b(P) with maximum accuracy.
2. Permissible parts in η-space, 1/k=1/P).



Choice $\chi_{lim}^2 = 10^{-5}$

- The dependence of χ2 (P) for regular binary cascade with permissible 1/8 η -part has a local and global minimum
- The global minimum $\chi 2 < 10^{-21}$ corresponds to the optimum set
- The value of $\chi_{lim}^2 = 10^{-5}$ uniquely defines the optimum set of bins for binary cascade with different permissible 1/k (k = 3÷20) parts η,

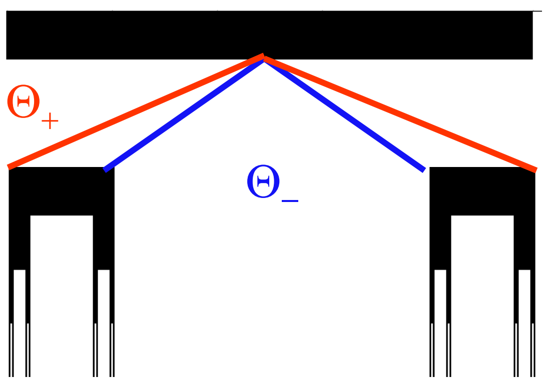
Binary cascades with permissible 1/k parts of η

The Cascades with permissible 1/5 parts of η

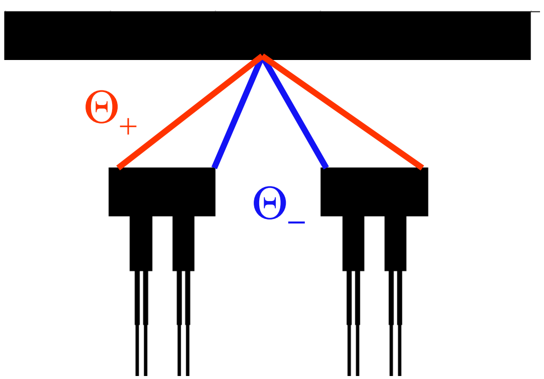
The Fractal dimensions of both cascades are the same $D_F = \ln 2/\ln 5$

Partons fill

1 and **5** parts of η



2 and **4** parts of η



Different scenarios of parton cascade correspond
to different admissible range of the opening angle

Values of D_F and permissible parts 1/k of η are not enough
and
the structure of filled space (η-space) is necessary
for
restoration of law of the cascade formation

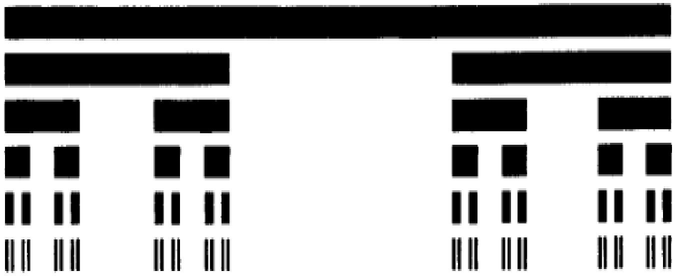
Structure of the filled space

- Record of level structures (Structure arrays for each bin widths in the set)
- Determination of regularity for each level (structures within level must be self-similar)
- Determination of cascade regularity (each level must be regular and level structures must be self-similar)
- For regular cascade the filled parts of space are defined by the arrays Structure
  They are self-similar i.e. they are same structure at different scales

Level structure of regular binary cascade with filled 1 and 3 part of η

The 1-level structure is defined by the one array. It consists of 3 elements (101). This array reflects the structure of the filled space.

The 2-level structure consists of two arrays. Each of arrays corresponds to structure of the filled parts of the first level.



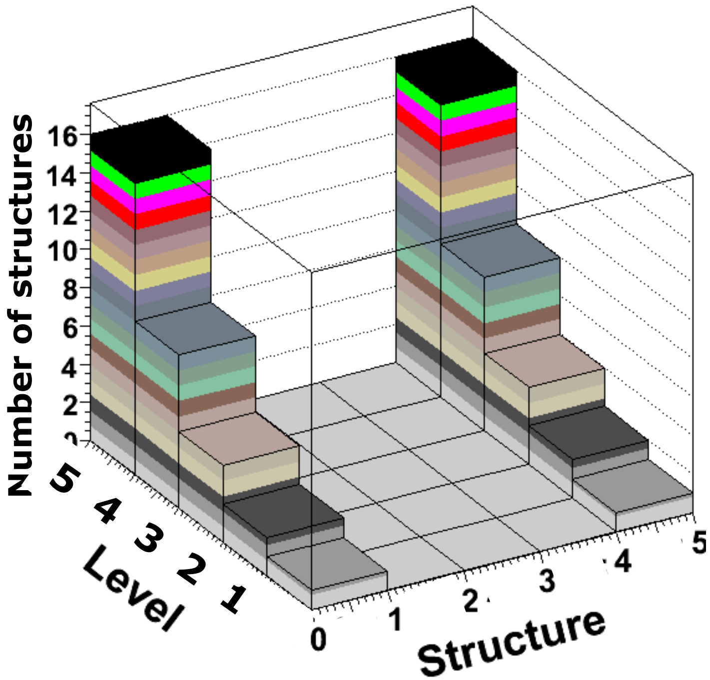
Structure Arrays

1 : (101)
2 : (101) (101)
3 : (101) (101) (101) (101)
4 : (101) (101) (101) (101) (101) (101) (101) (101)

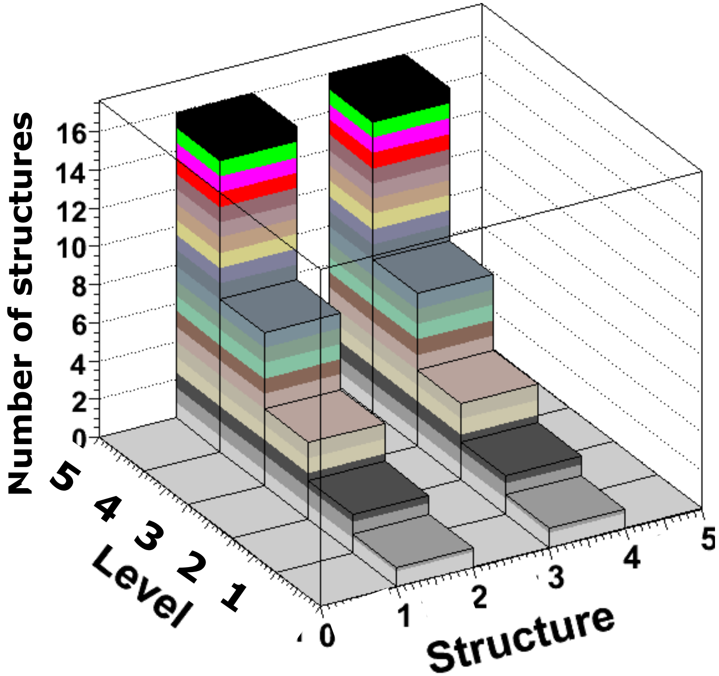
Analysis of the arrays Structure

Input data - {η_i } of partons

two cascades with permissible 1/5 part of η:
1, 5 parts η filled and 2, 4 parts η filled

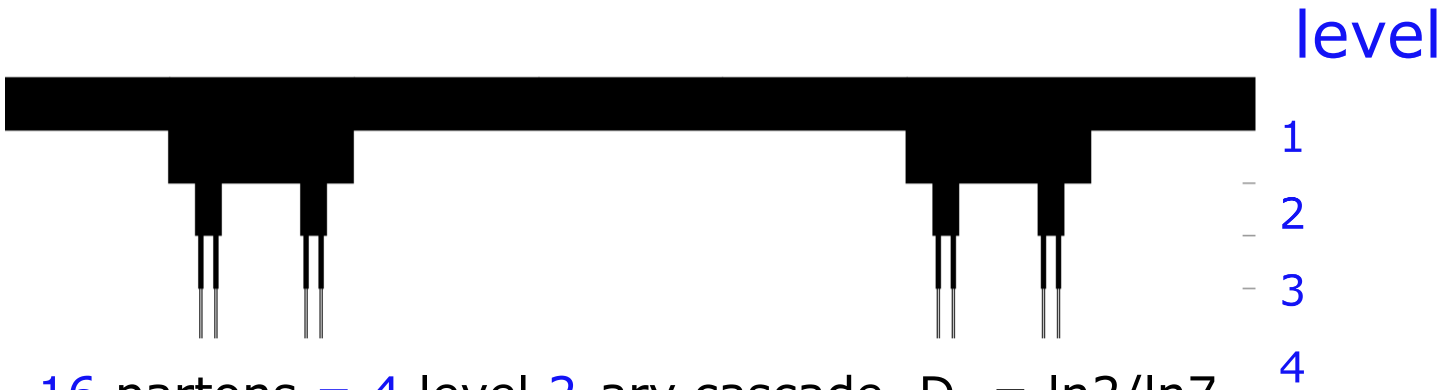


Analysis of the Arrays Structure



Restored cascade structure
Define filled parts η space at each cascade level

Complex cascades (1 → N partons) - N-ary cascades

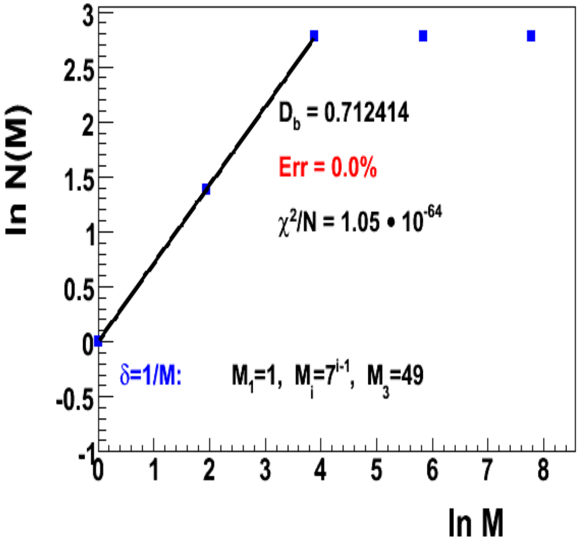


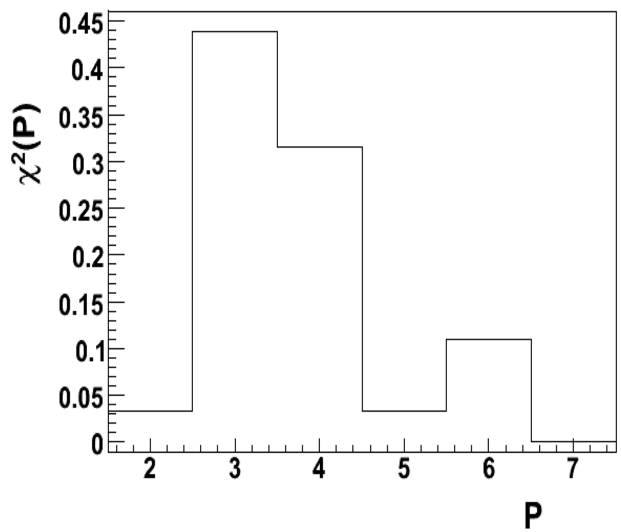
16 partons = 4 level 2-ary cascade D_F = ln2/ln7

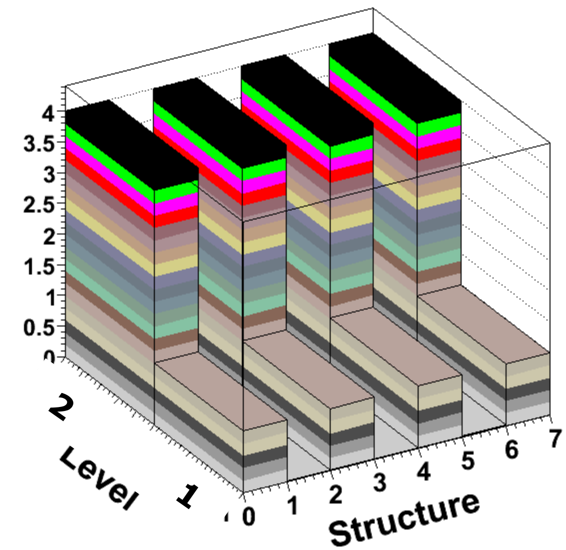
16 partons = 2 level 4-ary cascade D_F = ln4/ln7

Number of partons in the final state doesn't allow to determine number of level and number of partons at branching

Reconstruction of complex cascade (4-ary)







- Dependences ln N on ln M for ten sets of bins ($M_i=P^{i-1}$, p=1…10) have a plateau. Existence of plateau is connected to redundancy of considered levels.
- Region without plateau is fitted (N_{fit}), points $\Delta N_i=(N_i-N_{i-1})=0$, i=N…0 is excluded
- Value χ^2 (P)<10^{-5} correspond the set of bins M: $M_i=7^{i-1}$. It set defines

1. Fractal dimension D_F with maximum accuracy
2. Level structure (filled parts of space)
3. Number of cascade level ($N_{lev} = N_{fit}-1$)
4. Number of partons at branching N_{part} (number of non-zero element in the array Structure for 1-level)

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P-adic Coverage Method - I

- Read out data - $\{\eta_i\}$ of partons
- Construction of pseudorapidity distributions for different set of bins.

  M: $M_i = P^{i-1}$ (P-adic Coverage)
- Counting the number of binary cascade level $N_{lev} = LOG_2 N^{part}$, N^{part} - is number of partons
- Plotting the graph ln N(M) vs. ln M (number of points N_{lev} +1 in graph)
- Determination of the values of χ^2 (P) and the slope parameter b(P) (region without plateau)
- Determination of the optimum set of number of bins. It corresponds to $\chi^2(P) < 10^{-5}$
- The optimum set of bins M(P) defines:
  1. The fractal dimension $D_F = b(P)$ with maximum accuracy.
  2. Level structure (filled parts of space)
  3. Number of cascade level ($N_{lev} = N_{fit} - 1$)
  4. Number of partons at branching $N_{part} = N(M_2)$

Summary

- Box counting method was analyzed

- It was established existence the optimum set of bins.
  It provides exact restoration of D_F and is connected with the law of cascade formation.

- The P-adic Coverage Method of fractal analysis was proposed.

  It was used for regular cascade with permissible 1/N parts η

  This method allows to define:

  - Fractal dimension $D_F = b(P)$ with maximum accuracy
  - Cascade structure (filled parts of space)
  - Number of cascade level
  - Number of partons at branching

Thank You for attention