Bremsstrahlung and pair production processes at low energies, multi-differential cross section and polarization phenomena

E. A. Kuraev[1], Yu. M. Bystritskiy[1], M. Shatnev[2], E. Tomasi-Gustafsson[3]

[1] JINR, BLTP, Dubna, Russia
[2] National Science Centre "Kharkov Institute of Physics and Technology", Kharkov, Ukraine
[3] CEA, IRFU, SPhN, Saclay, 91191 Gif-sur-Yvette Cedex, France and
CNRS/IN2P3, Institut de Physique Nucléaire, Orsay, France

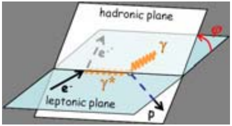
The XX International Baldin Seminar on High Energy Physics Problems
"Relativistic Nuclear Physics and Quantum Chromodynamics"
October 6, 2010

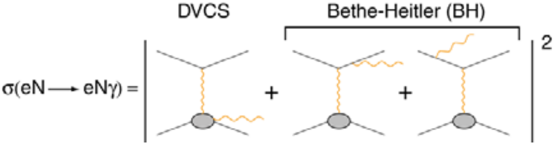
Talk Plan

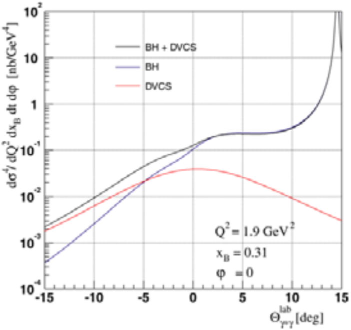
1. Motivation
2. Details of calculation
    1. Bremsstrahlung amplitude
    2. Kinematics
    3. Cross section and polarization observables
    4. Pair production
    5. Screening and Coulomb corrections
3. Numerical results
4. Conclusions

Motivation

Studying the process of polarized photon electroproduction on proton it is possible to find out more detailed information on proton structure (A. V. Belitsky, D. Mueller, and A. Kirchner, Nucl. Phys. **B629**, 323 (2002)).







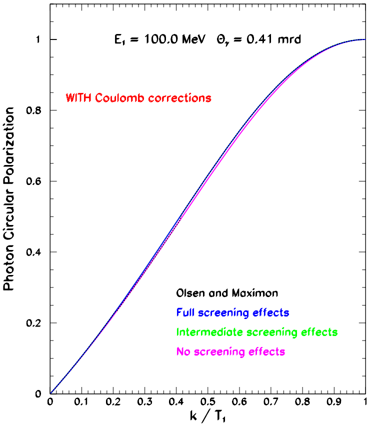
The Bethe-Heitler (BH) process where the real photon is emitted either by the incoming or outgoing electron interferes with Deep-Virtual Compton Scattering (DVCS).

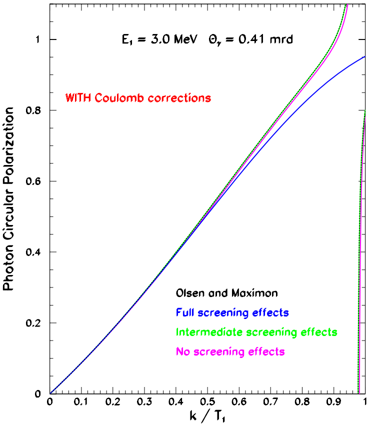
DVCS and BH are indistinguishable but the BH amplitude is exactly calculable and known at low momentum transfer.

See for details talk of E. Voutier.

Motivation: Point of Problem

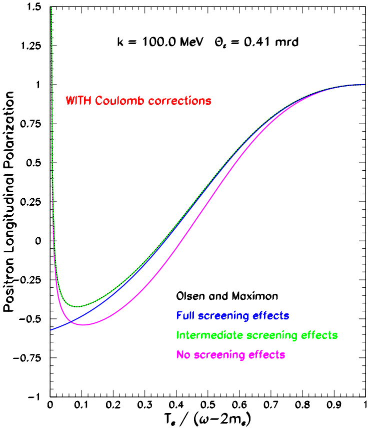
To calculate the bremsstrahlung contributions for polarized electron scattering on nucleus or photoproduction of the lepton pair in the nucleus field to small scattering angle the results of well-known paper of Olsen and Maximon (H. Olsen and L. C. Maximon, Phys. Rev. **114**, 887 (1959)) are used. In particular the code of Monte Carlo modeling in Geant 4 implements this results.

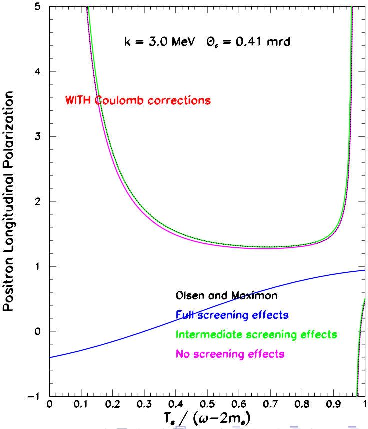




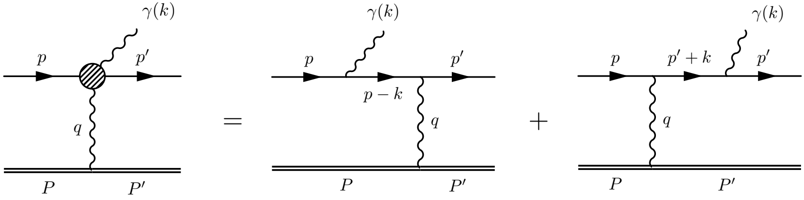
Motivation: Point of Problem

To calculate the bremsstrahlung contributions for polarized electron scattering on nucleus or photoproduction of the lepton pair in the nucleus field to small scattering angle the results of well-known paper of Olsen and Maximon (H. Olsen and L. C. Maximon, Phys. Rev. **114**, 887 (1959)) are used. In particular the code of Monte Carlo modeling in Geant 4 implements this results.





The process $e^- p \to e^- p \gamma$:



is described by the following amplitude:

$$\mathcal{M} = -e^3 Z \left[\bar{u}\left(P'\right)\gamma^\nu u\left(P\right)\right]\frac{g_{\mu\nu}}{q^3} \times$$
$$\times \left[\bar{u}\left(p'\right)\left(\frac{\gamma^\mu\left(\not{p} - \not{k} + m_e\right)\not{\varepsilon}}{\left(p-k\right)^2 - m_e^2} + \frac{\not{\varepsilon}\left(\not{p}' + \not{k} + m_e\right)\gamma^\mu}{\left(p'+k\right)^2 - m_e^2}\right)u\left(p\right)\right],$$

where ε is 4-vector of bremsstrahlung photon polarization and Z is the electric charge of target in units of proton charge $e = |e| = \sqrt{4\pi\alpha}$.

Light-Cone (Sudakov) Kinematics

Since we're going to consider the small-angle scattering it is convenient to use light-cone parametrization for 4-momenta (V. V. Sudakov, Sov. Phys. JETP **3**, 65 (1956), V. N. Baier, E. A. Kuraev, V. S. Fadin, and V. A. Khoze, Phys. Rept. **78**, 293 (1981)). We use two light-cone vectors $\tilde{p}$ and $\tilde{P}$ as basis:

$$\begin{aligned} \tilde{p} &= p - P\frac{m_e^2}{s} = E\,(1,0,0,1)\,, & p^2 = m_e^2, \quad P^2 = M_p^2, \\ \tilde{P} &= P - p\frac{M_p^2}{s} = \frac{M_p}{2}\,(1,0,0,-1)\,, & \tilde{p}^2 = \tilde{P}^2 = 0, \end{aligned}$$

where $s = 2\,(pP)$ is the total energy. Then any vector is decomposed in terms of these basis:

$$\begin{aligned} q &= \alpha\tilde{P} + \beta\tilde{p} + q_\perp, \\ p' &= \alpha'\tilde{P} + x\tilde{p} + p'_\perp, \\ k &= \alpha_k\tilde{P} + \bar{x}\tilde{p} + k_\perp, \\ \varepsilon &= \alpha_e\tilde{P} + e_\perp, \end{aligned}$$

where $\bar{x} = 1 - x$ and
$(q_\perp\tilde{p}) = \left(q_\perp\tilde{P}\right) = (p'_\perp\tilde{p}) = \left(p'_\perp\tilde{P}\right) = (k_\perp\tilde{p}) = \left(k_\perp\tilde{P}\right) = (e_\perp\tilde{p}) = \left(e_\perp\tilde{P}\right) = 0$
and $q_\perp^2 = -\mathbf{q}^2$, ${p'_\perp}^2 = -\mathbf{p}^2$, $k_\perp^2 = -\mathbf{k}^2$, $e_\perp^2 = -\mathbf{e}^2$.

Phase Volume and Particles Polarization

In these parametrization phase volume $d\Phi_3$ can be written in standard form:

$$d\Phi_3 = \delta^4\left(p + P - p' - P' - k\right) \frac{d^3\mathbf{k}}{2E_k} \frac{d^3\mathbf{p}'}{2E_{p'}} \frac{d^3\mathbf{P}'}{2E_{P'}} = = \frac{1}{4s} \frac{dx}{x\bar{x}}\, d\mathbf{p}\, d\mathbf{q}. \quad (1)$$

Final photon polarization we parameterize in terms of Stokes parameters $\xi_{1,2,3}$:

$$\sum_\lambda \varepsilon_i^\lambda \varepsilon_j^{*\lambda} = \frac{1}{2} \begin{pmatrix} 1+\xi_3 & \xi_1 - i\xi_2 \\ \xi_1 + i\xi_2 & 1-\xi_3 \end{pmatrix}_{ij}, \qquad i,j = x,y. \quad (2)$$

Polarization state of initial electron is parameterized using density matrix:

$$\sum_s u^s(p)\,\bar{u}^s(p) = \left(\not{p} + m_e\right)\left(1 - \gamma_5 \not{a}\right), \quad (3)$$

where a is the polarization vector which is equal to $a = \lambda_e \left[\left(m_e/s\right)\tilde{P} - \left(1/m_e\right)\tilde{p}\right]$ in case of longitudinal polarization with helicity λ_e, and $a = a_\perp$ in case of transversal polarization.

Unpolarized Cross Section

Thus the expression for unpolarized cross section is:

$$d\sigma_0^{ep\to ep\gamma} = \frac{2\alpha^3 Z^2}{\pi^2 (DD')^2 \left(q^2\right)^2} R_p (1-x) d\mathbf{q} d\mathbf{p} dx, \quad (4)$$

$$R_p = \mathbf{q}^2 (1+x^2) DD' - 2x m_e^2 (D-D')^2, \quad (5)$$

where

$$2pk = \frac{D}{\bar{x}}, \qquad D = \mathbf{k}^2 + m_e^2 \bar{x}^2, \qquad \mathbf{k} = \mathbf{q} - \mathbf{p};$$

$$2p'k = \frac{1}{x\bar{x}} D', \qquad D' = \mathbf{r}^2 + m_e^2 \bar{x}^2, \qquad \mathbf{r} = x\mathbf{q} - \mathbf{p}.$$

Polarized Cross Section

Thus the general expression for cross section with possible polarization of initial and final particles has the form:

$$\begin{aligned} d\sigma^{ep\to e\gamma p} &= d\sigma_0^{ep\to ep\gamma}\; P_e, \\ P_e &= 1+\lambda_e\xi_2 P_T+\lambda_e\xi_2 P_a+\tau_{pp}+\tau_{pq}+\tau_{qq}, \end{aligned} \tag{6}$$

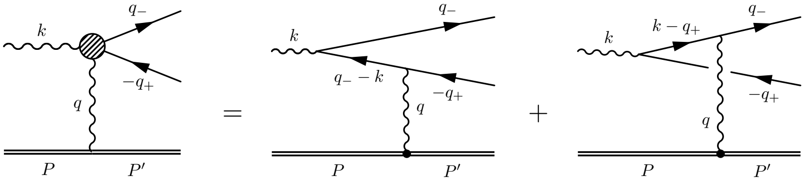
where the effective degrees of polarization are

$$\begin{aligned} P_T &= \frac{1}{R_p}\left[-\mathbf{q}^2 DD'\left(1-x^2\right)+2m_e^2 x\bar{x}\left(D-D'\right)^2\right], \\ P_a &= \frac{2xm_e}{R_p}\left[\left(\mathbf{pa}\right)\left(D-D'\right)^2+\left(\mathbf{qa}\right)\left(D-D'\right)\left(D'-xD\right)\right], \\ \tau_{pp} &= \frac{\mathbf{p}^2}{R_p}\frac{2x\left(D-D'\right)^2}{\bar{x}^2}\left[\xi_3\cos\left(2\phi_p\right)+\xi_1\sin\left(2\phi_p\right)\right], \\ \tau_{qq} &= \frac{\mathbf{q}^2}{R_p}\frac{2x\left(xD-D'\right)^2}{\bar{x}^2}\left[\xi_3\cos\left(2\phi_q\right)+\xi_1\sin\left(2\phi_q\right)\right], \\ \tau_{pq} &= \frac{1}{R_p}\frac{4x\left(D-D'\right)\left(xD-D'\right)}{\bar{x}^2}\left|\mathbf{q}\right|\left|\mathbf{p}\right|\left[\xi_3\cos\left(\phi_p+\phi_q\right)+\xi_1\sin\left(\phi_p+\phi_q\right)\right], \end{aligned} \tag{7}$$

where ϕ_q and ϕ_p are the azimuthal angles of vectors $\mathbf{q}$ and $\mathbf{p}$.

Pair Creation in $\gamma - p$ collisions

The process of pair creation:



is described by the following amplitude:

$$\mathcal{M} = -e^3 Z \left[\bar{u}\left(P'\right)\gamma^{\nu} u\left(P\right)\right] \frac{g_{\mu\nu}}{q^3} \times$$

$$\times \left[\bar{u}\left(q_-\right)\left(\frac{\not{\varepsilon}\left(\not{q}_- - \not{k} + m_e\right)\gamma^{\mu}}{\left(q_- - k\right)^2 - m_e^2} + \frac{\gamma^{\mu}\left(\not{k} - \not{q}_+ + m_e\right)\not{\varepsilon}}{\left(k - q_+\right)^2 - m_e^2}\right) v\left(q_+\right)\right],$$

where ε is 4-vector of initial photon polarization and Z is the electric charge of target in units of proton charge $e = |e| = \sqrt{4\pi\alpha}$.

Screening

The effect of complete screening of nucleus charge by electron shields can be taken into account in frames of Molière model (G. Molière, Z. Naturforsch. **A2**, 133 (1947)) by replacing

$$\frac{4\pi\alpha}{-q^2} \to \frac{4\pi\alpha\left[1 - F\left(-q^2\right)\right]}{-q^2}, \tag{8}$$

where

$$\frac{1 - F\left(-q^2\right)}{-q^2} = \sum_{i=1}^{3} \frac{\alpha_i}{\mathbf{q}^2 + m^2\beta_i},$$

and α_i and β_i are some parameters:

$$\alpha_1 = 0.1, \qquad \alpha_2 = 0.55, \qquad \alpha_3 = 0.35,$$

$$\beta_i = \left(\frac{Z^{1/3}}{121}\right) b_i, \qquad b_1 = 6.0, \qquad b_2 = 1.2, \qquad b_3 = 0.3.$$

Coulomb Corrections

For large values of Z, Coulomb corrections due to an arbitrary number of photons interacting with the charged leptons and with the nuclei has to be taken into account. For instance, the total cross sections of pair photoproduction in case of absence of screening is then has a form:

$$\sigma^{\gamma} = \frac{28}{9}\frac{Z^2\alpha^3}{m^2}\left[\ln\left(\frac{2\omega}{m}\right) - \frac{109}{42} - f(Z)\right], \tag{9}$$

and in case of complete screening

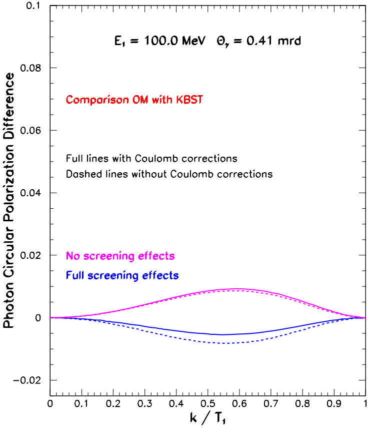
$$\sigma^{\gamma} = \frac{28}{9}\frac{Z^2\alpha^3}{m^2}\left[\ln\left(183Z^{-1/3}\right) - \frac{1}{42} - f(Z)\right], \tag{10}$$

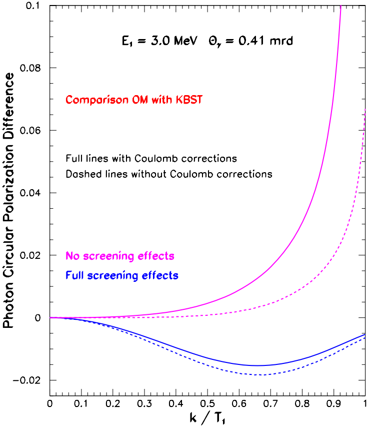
where $f(Z)$ is the Bethe-Maximon-Olsen function

$$f(Z) = (Z\alpha)^2 \sum_{n=1}^{\infty} \frac{1}{n\left[n^2 + (Z\alpha)^2\right]}. \tag{11}$$

Numerical results

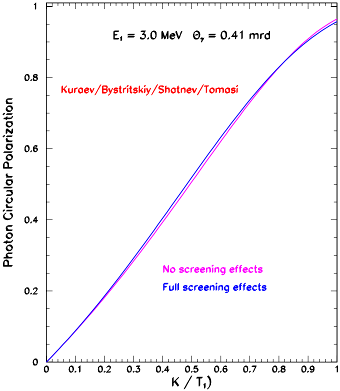
Comparison with the results of Olsen and Maximon gives good agreement for relatively high energies and sufficiently disagrees in case of small energies:

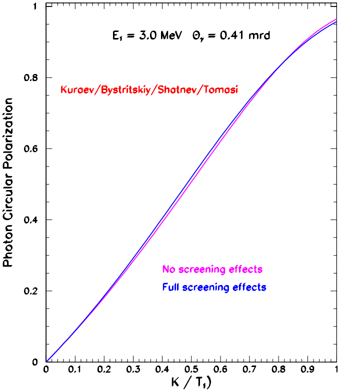




Numerical results

For bremsstrahlung with the production of circularly polarized photon we have symmetric and stable results:





Conclusions

- The revision of bremsstrahlung and lepton pair photoproduction in nucleus field is done in case of small scattering angles and relatively small energies.
- The resultant formulae gives numerically stable quantities and requires to be implemented in Monte Carlo modeling code.

All details of present calculations can be found in paper
E. A. Kuraev, Y. M. Bystritskiy, M. Shatnev, and E. Tomasi-Gustafsson, Phys. Rev. **C81**, 055208 (2010), arXiv:0907.5271.