Self-similarity of Particle Production in Soft and Hard p_T Region

I. Zborovský[*] and M.V. Tokarev[**]

[*]Nuclear Physics Institute
Řež, Czech Republic

[**]Joint Institute for Nuclear Research
Dubna, Russia

Part.Nucl.Lett. 3 (2006) 312
PRD 75 (2007) 094008
arXiv:0809.1033

Contents

- Inclusive reactions at high energies
  (motivation, principles, symmetries)
- Self-similarity in hadron production
  (variable z, ideas, definitions)
- z-Scaling in pp/$p\bar{p}$ collisions
  - properties, soft & hard p_T region
  - kinematics of constituent sub-processes
- Scaling features in AA collisions at RHIC
  - constituent sub-processes (kinematics, energy losses)
- Structural relativity - symmetry at small scales
- Conclusions

Principles & Symmetries

- Motivation: Search for phenomenological description of inclusive cross sections aiming to grasp main principles which influence the particle production at high energies.
- *Self-similarity.*
- *Locality.*
- *Fractality.*

There exists special symmetry inherent to them.
Symmetry with respect to structural degrees of freedom
(space-time structural relativity).

Self-similarity of inclusive reactions

$$M_1+M_2\rightarrow m+X$$

The self-similarity of inclusive reactions concerns similarity of hadron interactions at constituent level. It is connected with dropping of certain parameters out of physical description of inclusive distributions.
The reduction is achieved by grouping of suitable parameters into an adequate, physically meaningful, but still simple self-similarity variable z.

Parameters of inclusive reactions entering into the variable z are:

1. Reaction characteristics (A_1, A_2, P_1, P_2)
2. Particle characteristics (m, p, θ)
3. Structural and dynamical characteristics of the interaction (δ, ε,… $dN/d\eta$,...)

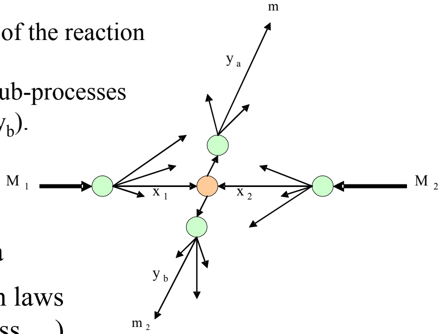
We search for a universal function $\psi(z)\approx Ed^3\sigma/dp^3$ which reflects the self-similarity, locality, and fractality of hadron interactions as revealed by data on inclusive distributions at high energies.

Locality of hadron interactions

Gross features of single particle distributions of the reaction

$$M_1+M_2\rightarrow m+X$$

can be expressed in terms of the constituent sub-processes
$(x_1M_1) + (x_2M_2)\rightarrow(m/y_a) + (x_1M_1+x_2M_2+m_2/y_b)$.



x_1,x_2,y_a,y_b are momentum fractions
of the corresponding momenta

m_2 is introduced for internal conservation laws
(isospin, baryon number, strangeness,…)

We consider the sub-process as subject to 4-momentun conservation law

$$(x_1P_1+x_2P_2-p/y_a)^2 = M_X^2$$

Recoil mass: $M_X= x_1M_1+x_2M_2+m_2/y_b$

Fractality of hadron interactions at small scales

- Fractality at small scales is related to constituent substructure of the interacting objects (hadrons, nuclei).
- The constituents consist of subtle nets of partons (QCD q, $\bar{q}$, and g) which evolve with increasing resolution.
  The resolution is connected with p_T.
- Fractal properties are revealed by increasing the resolution with respect to all constituent sub-processes which underlay the inclusive reactions at high energies.

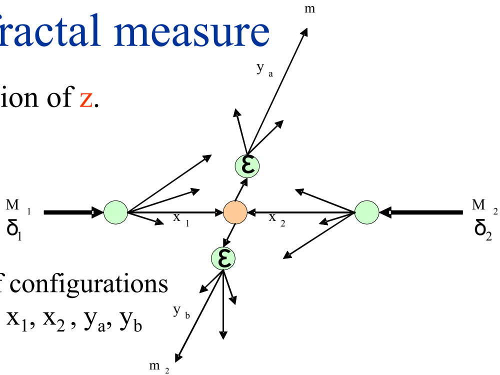
Hypothesis of fractality at small scales:
Hadron constituent sub-structure does not exhaust
with increasing resolution.

Scaling variable z as fractal measure

The fractality is reflected in definition of z.

$$z = z_0 \Omega^{-1}$$

$$\Omega = (1-x_1)^{\delta_1} (1-x_2)^{\delta_2} (1-y_a)^{\varepsilon} (1-y_b)^{\varepsilon}$$



Ω is proportional to relative number of configurations containing a sub-process with fractions x_1, x_2 , y_a, y_b of the corresponding 4-momenta.

$\delta_1, \delta_2, \varepsilon$ - parameters characterizing structure of the colliding objects and fragmentation process

Ω^{-1} characterizes resolution at which a constituent sub-process can be singled out of the inclusive reaction.

$$z(\Omega) \to \infty \quad \text{if} \quad \Omega^{-1} \to \infty$$

Resolution Ω^{-1} (x_1,x_2,y_a,y_b)

Principle of minimal resolution :

The fractions x_1, x_2, y_a, y_b are determined to minimize the resolution Ω^{-1} of the fractal measure $z=z_0\Omega^{-1}$ with respect to all constituent sub-processes in which the inclusive particle with the momentum p can be created.

This corresponds to maximum of

$$\Omega=(1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon}(1-y_b)^{\varepsilon}$$

with the condition

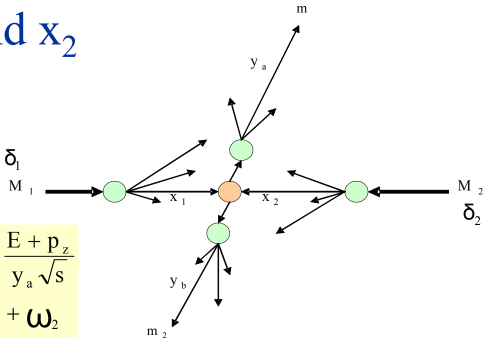
$$(x_1P_1+x_2P_2-p/y_a)^2 = (x_1M_1+x_2M_2+m_2/y_b)^2.$$

Decomposition of x_1 and x_2

Principle of minimal resolution

$\Downarrow$

$$x_i = \lambda_i + \chi_i$$



$$\lambda_1 \approx \frac{1}{y_a}\frac{(P_2 p)}{(P_1 P_2)} \approx \frac{E - p_z}{y_a\sqrt{s}} \qquad \chi_1 = [\mu_1^2 + \omega_1^2]^{1/2} - \omega_1$$

$$\lambda_2 \approx \frac{1}{y_a}\frac{(P_1 p)}{(P_1 P_2)} \approx \frac{E + p_z}{y_a\sqrt{s}} \qquad \chi_2 = [\mu_2^2 + \omega_2^2]^{1/2} + \omega_2$$

$$\omega_i = \mu_i U \qquad U = \frac{\alpha - 1}{2\sqrt{\alpha}}\xi \qquad \alpha = \frac{\delta_2}{\delta_1}$$

μ_i and ξ are simple analytic functions of λ_1, λ_2 and y_b

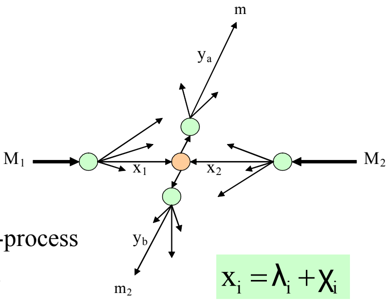
$$(x_1 M_1) + (x_2 M_2) \rightarrow m/y_a + (x_1 M_1 + x_2 M_2 + m_2/y_b)$$

$$(\lambda_1 + \chi_1) + (\lambda_2 + \chi_2) \rightarrow (\lambda_1 + \lambda_2) + (\chi_1 + \chi_2)$$

Scaling variable z

$z = z_0 \Omega^{-1}$

$$z_0 = \frac{s_\perp^{1/2}}{(dN_{ch}/d\eta|_0)^c m_N}$$



$s_\perp^{1/2} = T_a + T_b$ - transverse kinetic energy of the sub-process consumed on production of m & m_2

$$x_i = \lambda_i + \chi_i$$

$$T_a = y_a(\sqrt{s_\lambda} - M_1\lambda_1 - M_2\lambda_2) - m \qquad T_b = y_b(\sqrt{s_\chi} - M_1\chi_1 - M_2\chi_2) - m_2$$

$$p_T/y_a = \bar{p}_T/y_b$$

$$s_\lambda = (\lambda_1 P_1 + \lambda_2 P_2)^2 \qquad s_\chi = (\chi_1 P_1 + \chi_2 P_2)^2$$

$$0 \le z \le \infty$$

$dN_{ch}/d\eta|_0$ - multiplicity density of the charged particles at $\eta=0$

c - parameter interpreted as a “specific heat” of created medium

m_N - arbitrary constant (fixed at the value of nucleon mass)

In the central region: $T_a \cong \sqrt{p_T^2 + m^2} - m \qquad T_b \cong \sqrt{\bar{p}_T^2 + m_2^2} - m_2$

Properties of the scaling function $\psi(z)$ in $pp/p\bar{p}$ collisions

- Energy independence ($s^{1/2} > 20$ GeV)
- Angular independence ($\theta_{cms}=3^0$-90^0)
- Multiplicity independence ($dN_{ch}/d\eta=1.5$-26)
- Power law, $\psi(z) \sim z^{-\beta}$, at high z ($z>4$)
- Flavor independence ($\pi, K, \varphi, \Lambda, .., D, J/\psi, B, \Upsilon, \ldots$)
- Saturation at low z ($z<0.1$)

NB: Different shapes of $\psi(z)$ for pp & $p\bar{p}$ at large z
for $h^{\pm}$, π^0, direct γ, and jets.

Scaling function $\psi(z)$

$$\psi(z) = \frac{1}{N\sigma_{in}} \frac{d\sigma}{dz} \quad \Rightarrow \quad \psi(z) = \frac{\pi}{(dN/d\eta)\sigma_{in}} J^{-1} E \frac{d^3\sigma}{dp^3}$$

- σ_{in} - inelastic cross section
- N – averaged multiplicity of the corresponding hadron species
- $dN/d\eta$ - pseudorapidity multiplicity density at angle θ (η)
- $J(z,\eta;p_T^2,y)$ – Jacobian
- $Ed^3\sigma/dp^3$ - inclusive cross section

Normalization of $\psi(z)$:

$$\int_0^\infty \Psi(z)dz = 1$$

Scale transformation of z

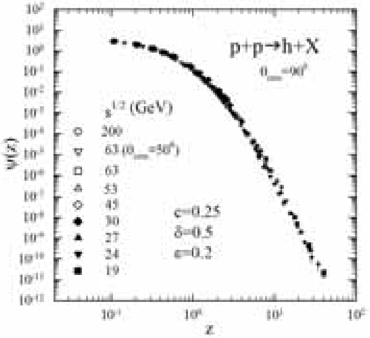
$$z \to \alpha_F z \quad \Psi \to \alpha_F^{-1} \Psi$$

preserves the normalization condition.

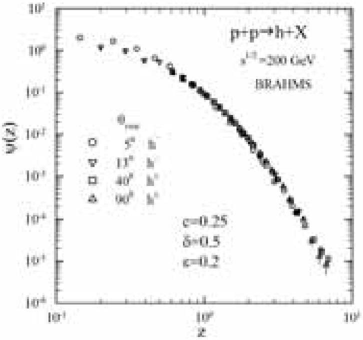
Energy & angular independence of ψ(z)

Charged hadrons in pp collisions

FNAL: PRD 19 (1979) 764
ISR: NPB100 (1975) 237; NPB 208 (1982) 1



STAR: PRL 91 (2003) 172302
BRAHMS: PRL 93 (2004) 242303



- The spectra at different energies & angles are described by a single ψ(z)
- A power law, $\psi(z) \sim z^{-\beta}$, at large z

$$\Omega = (1-x_1)^{\delta}(1-x_2)^{\delta}(1-y_a)^{\varepsilon}(1-y_b)^{\varepsilon}$$

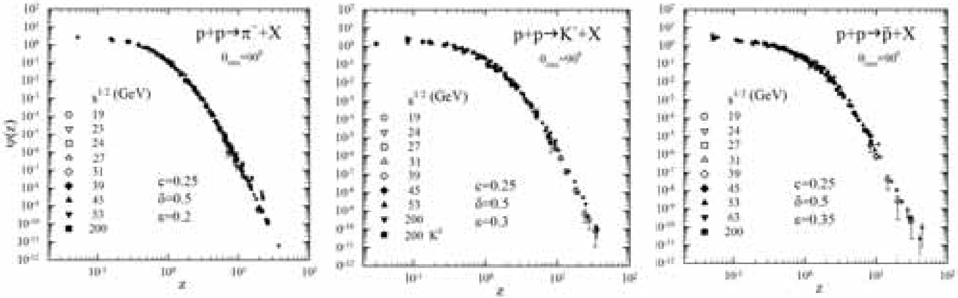
$$z = \frac{s_{\perp}^{1/2}}{(dN_{ch}/d\eta|_0)^c m_N} \cdot \Omega^{-1}$$

Energy independence of $\psi(z)$

π^-, K^-, $\bar{p}$ in pp collisions

FNAL: PRD 19 (1979) 764 ; PRD 40 (1989) 2777
ISR: NPB 100 (1975) 237

STAR: PLB 637 (2006) 161; PLB 616 (2005) 8.
J. Adams, M. Heinz, nucl-ex/0403020



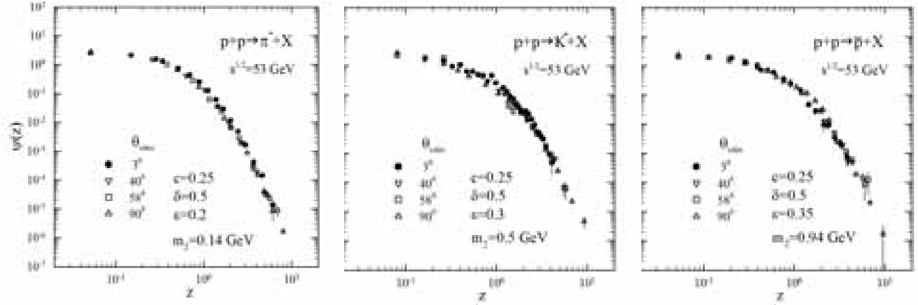
- The shape of $\psi(z)$ is the same for different hadrons
- The power law, $\psi(z) \sim z^{-\beta}$, at large z
- $\psi(z)$ is sensitive to δ and ε at large z
- ε increases with the particle mass

$$\Omega = (1-x_1)^{\delta}(1-x_2)^{\delta}(1-y_a)^{\varepsilon}(1-y_b)^{\varepsilon}$$

Angular independence of ψ(z)

π^-, K^-, $\bar{p}$ in pp collisions

ISR: NPB 56 (1973) 333; NPB 100 (1975) 237



$m_2=m(\pi)$ | $m_2=m(K)$ | $m_2=m(p)$

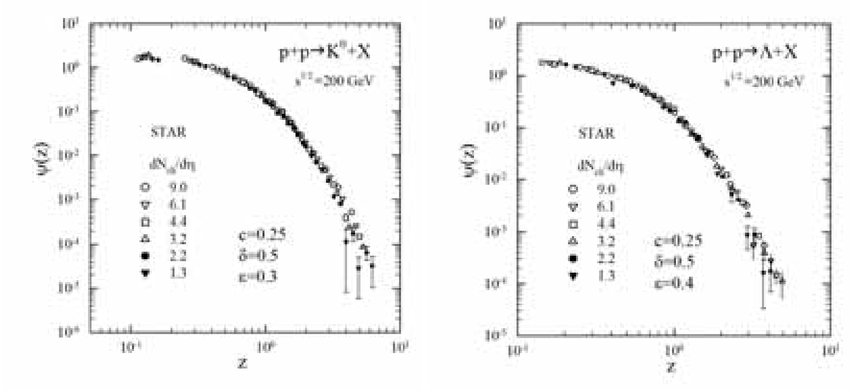
- Sensitivity of ψ(z) to m_2 in the fragmentation region ($\theta_{cms}=3^0$)
- ε increases with the particle mass

$$(x_1P_1+x_2P_2-p/y_a)^2 = (x_1M_1+x_2M_2+m_2/y_b)^2$$

Multiplicity independence of ψ(z)

K_S^0 & Λ in pp collisions

R. Witt, J.Phys.G: 31 (2005) S863



- Multiplicity independence of ψ(z) over a wide rage of $dN_{ch}/d\eta$.
- The multiplicity selection criteria give strong restriction on the parameter c.

$$z_0 = \frac{s_\perp^{1/2}}{(dN_{ch}/d\eta|_0)^c m_N}$$

Variable z & Entropy

$$z = z_0 \Omega^{-1} \qquad z_0 = \frac{s_\perp^{1/2}}{(dN_{ch}/d\eta|_0)^c m_N} \qquad \Omega = (1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon}(1-y_b)^{\varepsilon}$$

$$z \cong \frac{s_\perp^{1/2}}{W} \qquad W = (dN_{ch}/d\eta|_0)^c \cdot \Omega$$

- relative number of such constituent configurations which contain the configuration $\{x_1, x_2, y_a, y_b\}$

Statistical entropy:

$$S = \ln W$$

Thermodynamical entropy for ideal gas :

$$S = c_V \ln T + R \ln V + S_0$$

$$S = c \cdot \ln(dN_{ch}/d\eta|_0) + \ln[(1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon}(1-y_b)^{\varepsilon}] + \ln W_0$$

- $dN_{ch}/d\eta|_0$ characterizes "temperature" of the colliding system.
- Provided local equilibrium, $dN_{ch}/d\eta|_0 \sim T^3$ for high temperatures and small μ.
- c has meaning of a "specific heat" of the produced medium.
- Fractional exponents $\delta_1, \delta_2, \varepsilon$ are fractal dimensions in the space of $\{x_1, x_2, y_a, y_b\}$
- Entropy increases with $dN_{ch}/d\eta|_0$ and decreases with increasing resolution Ω^{-1}

Maximal entropy S $\Leftrightarrow$ minimal resolution Ω^{-1} of the fractal measure z

Scale transformation of z

$$z \approx \frac{s_\perp^{1/2}}{W}$$

$$z' = z/W_0 \quad \psi'(z') = W_0 \psi(z)$$

The scale transformation of the variable z
is connected with arbitrariness
in the choice of the absolute values of entropy.

$$S = \ln W + \ln W_0$$

W_0 is connected with absolute number of the constituent configurations which can be realized in a specific inclusive reaction.
W_0 is very large (infinite) and drops out of the z-scaling by a renormalization of z (dimensional normalization with m_N)
W_0 depends on type of the inclusive particle.

Scaling functions for different hadrons are reduced
to a single curve by the transformation

$$z \rightarrow \alpha_F z \quad \Psi \rightarrow \alpha_F^{-1} \Psi$$

$\alpha_F = W_0(F)/W_0(\pi)$ for the corresponding hadron (F)

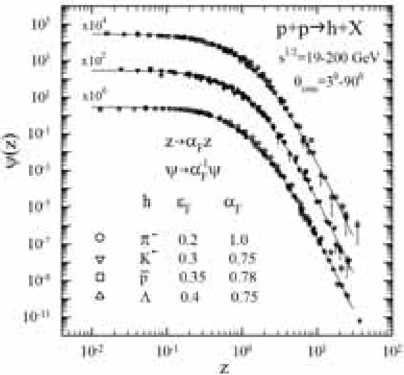
F-independence of ψ(z) & saturation at low z

π ,K^-, $\bar{p}$, Λ in pp collisions

FNAL:
PRD 75 (1979) 764

ISR:
NPB 100 (1975) 237
PLB 64 (1976) 111
NPB 116 (1976) 77
(low p_T)
NPB 56 (1973) 333
(small angles)

STAR:
PLB 616 (2005) 8
PLB 637 (2006) 161
PRC 75 (2007) 064901



- $\psi(z) \sim z^{-\beta}$ at large z
- ε_F, α_F independent of p_T, $s^{1/2}$, θ_{cms}

- Energy & angular independence
- Flavor independence (π, K, p, Λ)
- Saturation for z<0.1

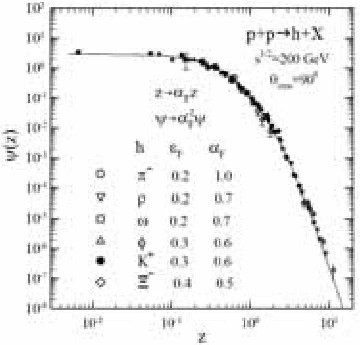
F-independence of $\psi(z)$ and saturation at low z

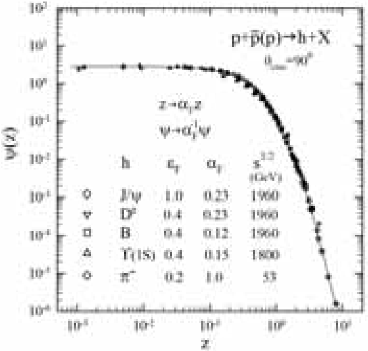
π , ρ, ω, ϕ, K*, Ξ, J/ψ, D, B, Υ in pp/p$\bar{p}$ collisions

STAR:
PRL 92 (2004) 092301
PLB 612 (2005) 181
PRC 71 (2005) 064902
PRC 75 (2007) 064901

PHENIX:
PRC 75 (2007) 051902

CDF:
PRL 88 (2002) 161802
PRL 91 (2003) 241804
PRD 71 (2005) 032001





- Energy independence
- Flavor independence (ρ, ω, ϕ, Ξ, J/ψ, D, B, Υ)
- Saturation for z = 0.001-0.1

- Power law $\psi(z) \sim z^{-\beta}$ at large z
- ε_F, α_F independent of p_T, $s^{1/2}$

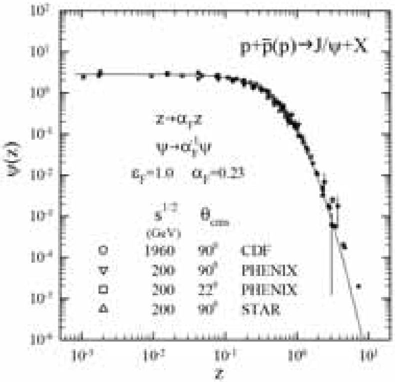
F-independence of ψ(z) and saturation at low z

J/ψ in pp/$p\bar{p}$ collisions

CDF:
PRD 71 (2005) 032001

PHENIX:
PRL 98 (2007) 232002

STAR:
QM2008,Jaipur,India
arXiv:0804.4846

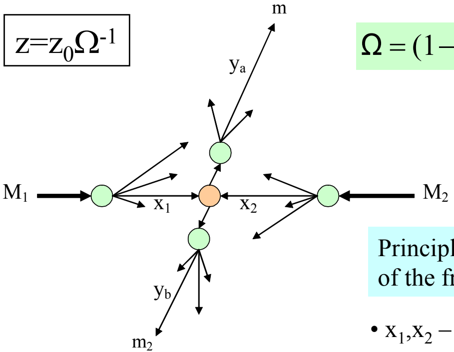


- $\psi(z)\sim z^{-\beta}$ at large z
- ε_F, α_F independent of p_T, $s^{1/2}$, θ_{cms}

- Energy & angular independence
- Saturation for z=0.001-0.1
- Extra large ε=1

Kinematics of constituent sub-process in pp

$z = z_0 \Omega^{-1}$



$$\Omega = (1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon}(1-y_b)^{\varepsilon}$$

$$(x_1P_1 + x_2P_2 - p/y_a)^2 = M_X^2$$

$$M_X = x_1M_1 + x_2M_2 + m_2/y_b$$

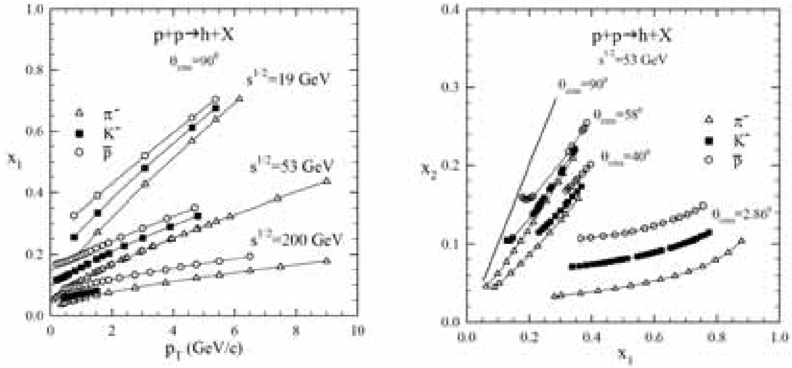
Principle of minimal resolution Ω^{1} of the fractal measure z gives:

- x_1, x_2 – energy of the sub-process
- y_a – energy losses (dissipation) by the production of the inclusive particle
- $M_X = x_1M_1 + x_2M_2 + m_2/y_b$ – recoil mass
- y_b – multiplicity of the recoil system

Larger ε = smaller y = larger energy losses in the final state & larger recoil mass

Momentum fractions x_1 & x_2

FNAL, ISR, RHIC

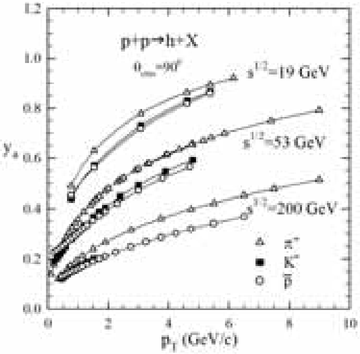


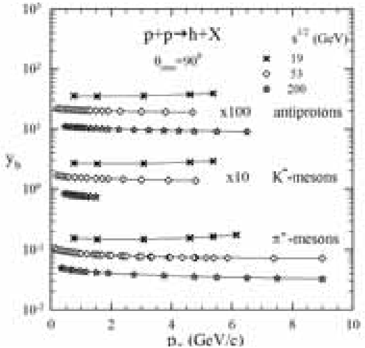
- x_1, x_2 increase with p_T and decrease with $s^{1/2}$
- x_1, x_2 increase slightly with the particle mass
- $x_1=x_2$ at $\theta_{cms}=90^0$; $x_1>>x_2$ at $\theta_{cms}=2.86^0$
- Considerable increase of the small fraction x_2 with the particle mass at $\theta_{cms}=2.86^0$

$(x_1P_1+x_2P_2-p/y_a)^2 = (x_1M_1+x_2M_2+m_2/y_b)^2$

Momentum fractions y_a & y_b

FNAL, ISR, RHIC



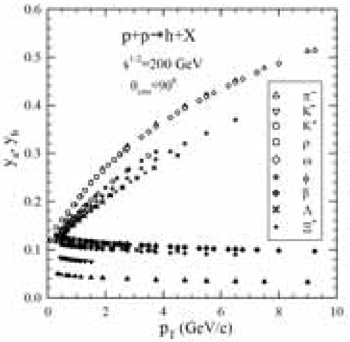


- y_a increases with $p_T \Rightarrow$ energy losses decrease with p_T
- y_a decreases with $s^{1/2} \Rightarrow$ energy losses increase with $s^{1/2}$
- y_a slightly decreases with m $\Rightarrow$ energy losses increase with particle mass
- y_b is flat with $p_T \Rightarrow$ week dependence of M_X on p_T
- $y_b \ll y_a$ for $p_T > 1$ GeV/c $\Rightarrow$ production of (m) is balanced by soft (high multiplicity) recoil M_X
- y_b slightly increases with m $\Rightarrow$ heavier particles are balanced by harder recoil M_X

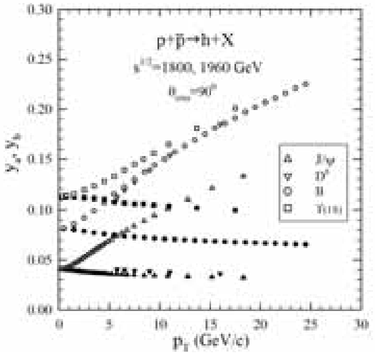
$$(x_1P_1+x_2P_2-p/y_a)^2 = (x_1M_1+x_2M_2+m_2/y_b)^2$$

Momentum fractions y_a & y_b

RHIC



Tevatron

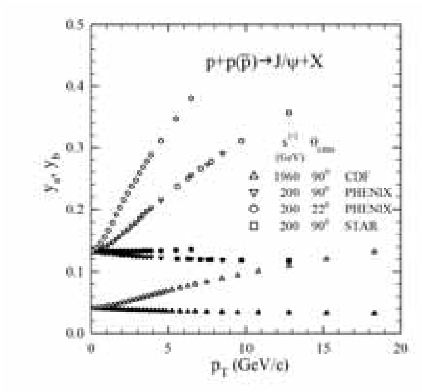


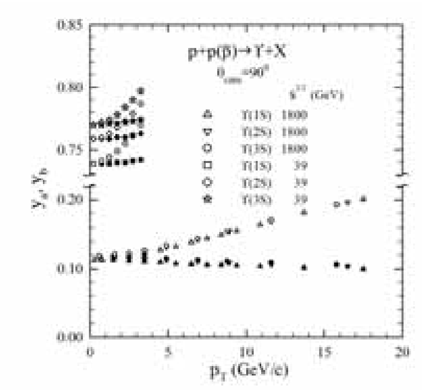
- y_a increases with $p_T \Rightarrow$ energy losses decrease with p_T for various flavors
- y_b is flat with $p_T \Rightarrow$ weak dependence of M_X on p_T for various flavors
- $y_b \cong y_a$ at low $p_T \Rightarrow$ $M_X \cong m/y_a$ (for heavy particles)
- Anomaly small y_a for J/ψ ⇒ extra large energy losses for J/ψ production
- Anomaly small y_b for J/ψ ⇒ extra soft (high multiplicity) recoil M_X for J/ψ production

$(x_1P_1+x_2P_2-p/y_a)^2 = (x_1M_1+x_2M_2+m_2/y_b)^2$

Momentum fractions y_a & y_b

FNAL, RHIC, Tevatron





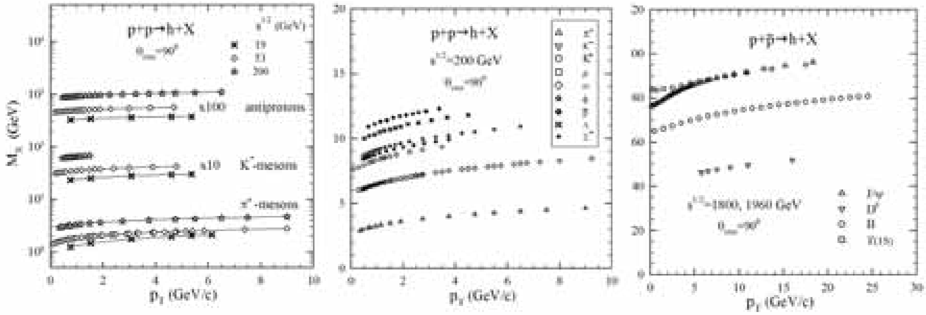
- y_a significantly decreases with $s^{1/2}$ ⇒ energy losses considerably increase with $s^{1/2}$
- y_b significantly decreases with $s^{1/2}$ ⇒ large increase of the recoil multiplicity with $s^{1/2}$
- y_a is larger for small θ_{cms} ⇒ smaller energy losses in the fragmentation than in the central region
- dependence of y_a, y_b on the respective Υ state vanish at higher energy
- $y_a \cong y_b$ at small p_T ⇒ $M_X \cong m/y_a$ is independent on $s^{1/2}$ for heavy quarkonia

$$(x_1P_1+x_2P_2-p/y_a)^2 = (x_1M_1+x_2M_2+m_2/y_b)^2$$

Recoil mass M_X

FNAL, ISR, RHIC, Tevatron

$$M_X = x_1 M_1 + x_2 M_2 + m/y_b$$



- M_X slightly increases with p_T
- M_X increases with $s^{1/2}$ for all particle species
- M_X increases with the particle mass
- Extra large M_X for J/ψ

$(x_1P_1+x_2P_2-p/y_a)^2 = (x_1M_1+x_2M_2+m_2/y_b)^2$

Summary of z-scaling in $pp/p\bar{p}$ collisions

- The energy, angular, and multiplicity independence of $\Psi(z)$ for $h^{\pm}$, π^{-}, K^{-}, $\bar{p}$,K_S^0, Λ
- Consistence of z-scaling with ρ, ω, Ξ, B, D, J/ψ, and Υ production
- The flavor independence over large region of z
- The saturation at low z
- The power law at high z
- "Specific heat" for the $pp/\bar{p}p$ system: c=0.25
- Proton fractal dimension: δ=0.5
- Fragmentation dimension ε increases with the particle mass:
  $\varepsilon(\pi)=0.2$, $\varepsilon(K)=0.3$, $\varepsilon(p)=0.35$, $\varepsilon(\Lambda)=0.4$
- c, δ, ε are independent of p_T, $s^{1/2}$, θ, and multiplicity.

Scaling features in AA collisions

Additivity of fractal dimensions in pA collisions: $\delta_A = A\delta$

The property is connected with factorization of the resolution Ω^{-1} of the fractal measure $z = z_0 \Omega^{-1}$ for small values of $x_2 \equiv x_A$.

$\bar{x}_2 \equiv A x_2$ -momentum fraction of the interacting nucleus expressed in units of the nucleon mass

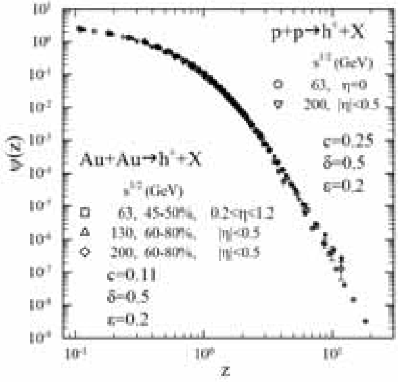
$$\Omega \simeq (1-x_1)^{\delta}(1-\bar{x}_2/A)^{A\delta} \approx (1-x_1)^{\delta}(1-\bar{x}_2)^{\delta}$$

is relative number of initial configurations in a single nucleon interaction regime ($x_2 < A^{-1}$).

$\delta_A = A\delta$ is consistent with z-scaling in pD, pBe, pTi, pW collisions PRC 59 (1999) 2227

$\delta_1 = A_1\delta$ & $\delta_2 = A_2\delta$ in AA collisions

Charged hadrons in peripheral AuAu collisions



ISR: NPB 208 (1982)1

STAR: PRL 89 (2002) 202301;
PRL 91 (2003) 172302

PHOBOS: PRL 94 (2005) 082304

pp collisions:
$dN_{ch}/d\eta|_0$ for non-single-diffractive events
AA collisions:
$dN_{ch}/d\eta|_0$ for corresponding AA centrality

- The energy independence of $\Psi(z)$ in peripheral AuAu
- The same shape of $\Psi(z)$ for pp & peripheral AuAu
- "Specific heat" $c_{AuAu}=0.11 < c_{pp}=0.25$
- The same ε in pp & peripheral AuAu

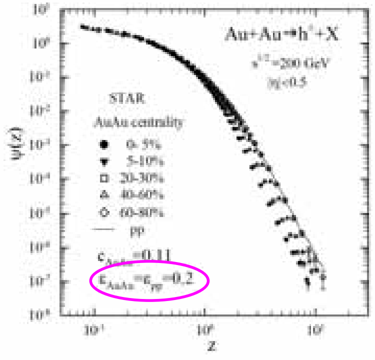
$$\Omega=(1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon}(1-y_b)^{\varepsilon}$$

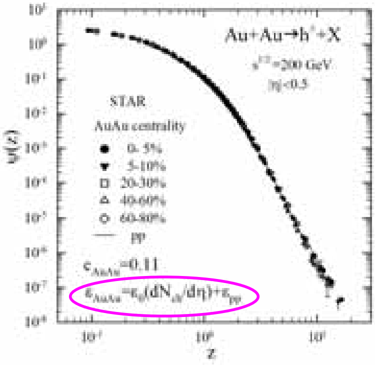
$$z=\frac{s_\perp^{1/2}}{(dN_{ch}/d\eta|_0)^c\, m_N}\cdot\Omega^{-1}$$

Charged hadrons in 200 GeV AuAu collisions

different centralities

STAR: PRL 91 (2003) 172302





- Suppression of $\Psi(z)$ in the central AuAu for $\varepsilon_{AuAu}=\varepsilon_{pp}$

- The same $\Psi(z)$ in pp & AuAu for all centralities when ε_{AuAu} depends on AuAu multiplicity
- "Specific heat" c_{AuAu}=0.11 for all centralities

$$\Omega=(1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon}(1-y_b)^{\varepsilon}$$

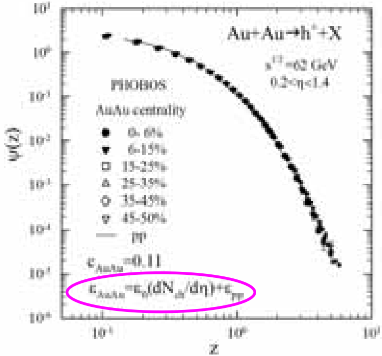
$$z=\frac{s_{\perp}^{1/2}}{(dN_{ch}/d\eta|_0)^c m_N}\cdot\Omega^{-1}$$

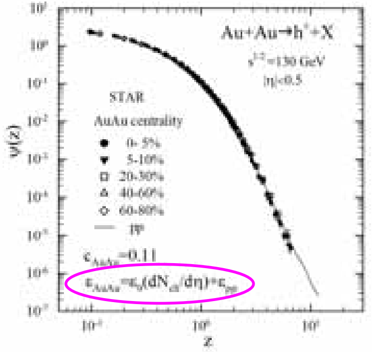
Charged hadrons in 62 & 130 GeV AuAu collisions

different centralities

PHOBOS: PRL 94 (2005) 082304
ISR: Z.Phys.C69 (1995) 55; NPB 208 (1982)1

STAR: PRL 89 (2002) 202301; PRL 91 (2003) 172302





- The same Ψ(z) in AuAu & pp for ε_{AuAu} dependent on AuAu multiplicity
- "Specific heat" c_{AuAu}=0.11 (constant with $s^{1/2}$)
- ε_0 increases with $s^{1/2}$: $\varepsilon_0(62\text{GeV})=0.0018 < \varepsilon_0(130\text{GeV})=0.0022 < \varepsilon_0(200\text{GeV})=0.0028$ (AuAu)

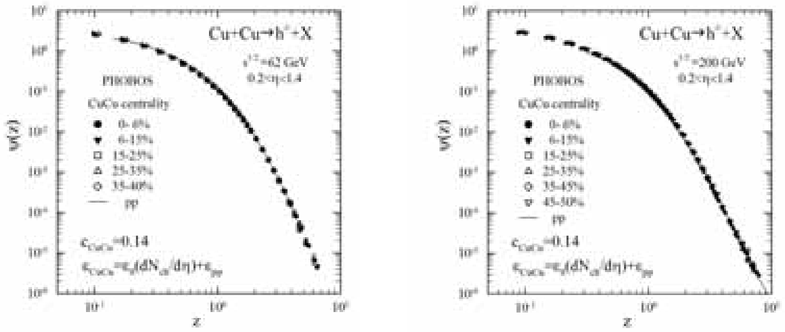
$$\Omega=(1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon}(1-y_b)^{\varepsilon}$$

$$z=\frac{s_\perp^{1/2}}{(dN_{ch}/d\eta|_0)^c m_N}\cdot\Omega^{-1}$$

Charged hadrons in 62 & 200 GeV CuCu collisions

different centralities

ISR: Z.Phys.C69 (1995) 55; NPB 208 (1982) 1 PHOBOS: PRL 96 (2006) 212301 STAR: PRL 91 (2003) 172302



- The same Ψ(z) in CuCu & pp for ε_{CuCu} dependent on CuCu multiplicity
- "Specific heat" c_{CuCu}=0.14 is constant with $s^{1/2}$
- ε_0 increases with $s^{1/2}$: ε_0(62GeV) =0.005< ε_0(200GeV) =0.008 (CuCu)

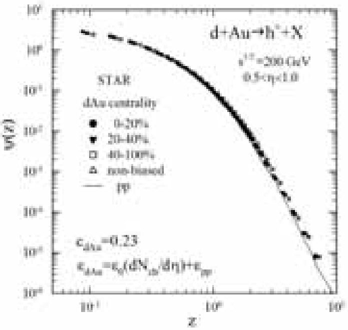
$$\Omega = (1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon}(1-y_b)^{\varepsilon}$$

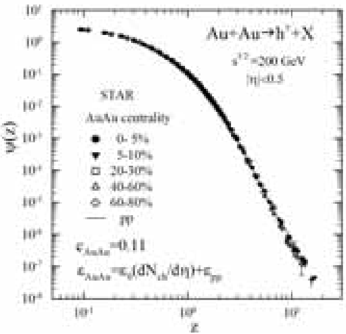
$$z = \frac{s_\perp^{1/2}}{(dN_{ch}/d\eta|_0)^c m_N} \cdot \Omega^{-1}$$

Charged hadrons in 200 GeV dAu & AuAu collisions

different centralities

STAR: PRC 70 (2004) 064907; PRL 91 (2003) 172302





- The same shape of Ψ(z) in dAu, AuAu, and pp for ε_{AA} dependent on AA multiplicity
- "Specific heat" c decreases with the system size:

  c_{pp}=0.25> c_{dAu}=0.23 > c_{CuCu}=0.14> c_{AuAu}=0.11
- ε_0 decreases with the system size:

  ε_0(dAu) =0.04> ε_0(CuCu) =0.008> ε_0(AuAu) =0.0028 ($s^{1/2}$=200GeV)

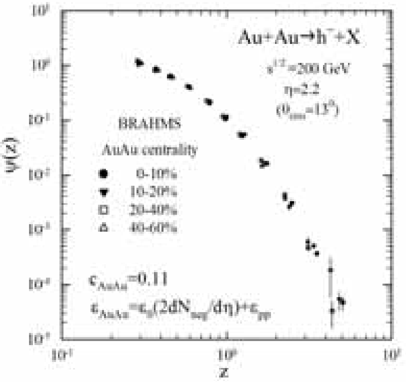
$$\Omega=(1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon}(1-y_b)^{\varepsilon}$$

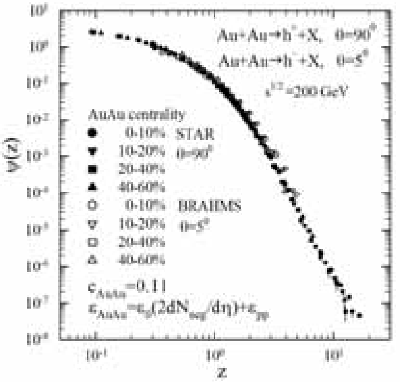
$$z=\frac{s_\perp^{1/2}}{(dN_{ch}/d\eta|_0)^c m_N}\cdot\Omega^{-1}$$

Negative hadrons in 200 GeV AuAu collisions

different angles and centralities

BRAHMS: PRL 91 (2003) 072305   STAR: PRL 91 (2003) 172302





- The multiplicity independence of $\Psi(z)$ in the central and fragmentation region
- The same dependence of ε on $(dN_{neg}/d\eta)$ in the central and fragmentation region
- The sensitivity of $\Psi(z)$ to m_2 in the fragmentation region

$$\Omega = (1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon}(1-y_b)^{\varepsilon}$$

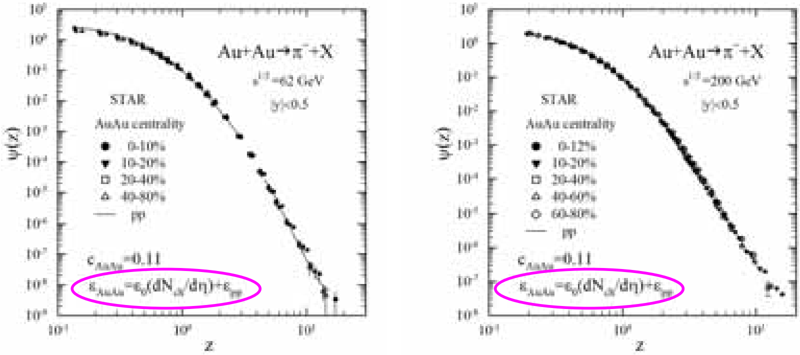
$$z = \frac{s_\perp^{1/2}}{(dN_{ch}/d\eta|_0)^c m_N} \cdot \Omega^{-1}$$

Negative pions in 62 & 200 GeV AuAu collisions

different centralities

ISR: NPB 100 (1975) 237; NPB 208 (1982) 1

STAR: PLB 637 (2006) 161; PRL 97 (2006) 152301; PLB 655 (2007) 104

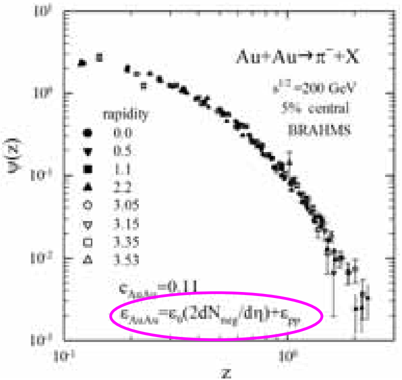


- The same Ψ(z) in AuAu & pp for ε_{AuAu} dependent on AuAu multiplicity
- "Specific heat" c_{AuAu}=0.11 is the same as for negative hadrons
- ε_0 is the same as for negative hadrons

Negative pions in 200 GeV AuAu collisions

different rapidities in central collisions

BRAHMS: PRL 94 (2005) 162301



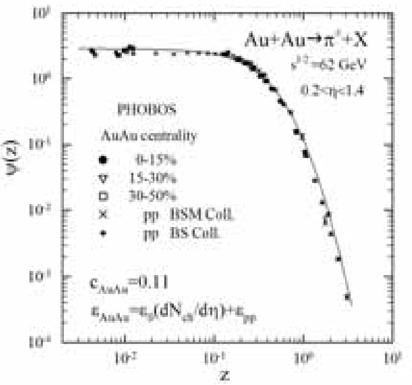
- The same Ψ(z) for ε_{AuAu} dependent on AuAu multiplicity at different rapidities
- "Specific heat" c_{AuAu}=0.11 does not depend on rapidity
- ε_0 does not depend on rapidity

Saturation of ψ(z) at low z in AuAu collisions

π in pp & AuAu collisions

PHOBOS:
PRC 75 (2007) 024910

ISR:
NPB 100 (1975) 237
PLB 64 (1976) 111 (low p_T)



$$z \cong \frac{s_{\perp}^{1/2}}{(dN_{ch}/d\eta|_0)^c}$$

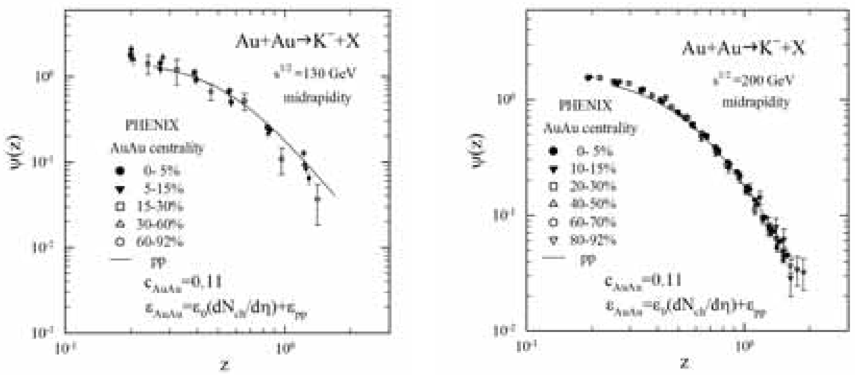
at low z
(low p_T)

- The saturation of ψ(z) in AuAu for z<0.1
- The saturation in AuAu extrapolates the saturation in pp up to z=0.004
- The centrality (multiplicity) independence of ψ(z) in AuAu

Negative kaons in 130 & 200 GeV AuAu collisions

different centralities

PHENIX: PRC 69 (2004) 034909; PRC 74 (2006) 024904

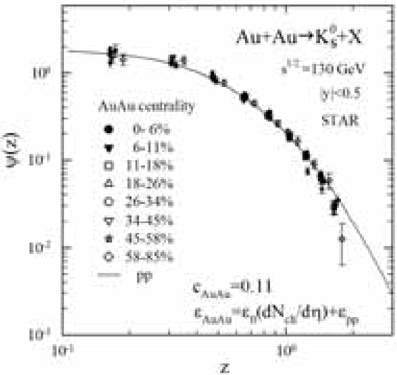


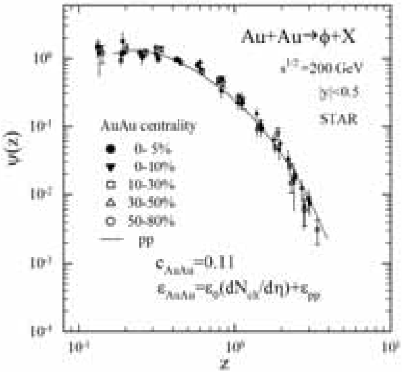
- The same $\Psi(z)$ in AuAu & pp for ε_{AuAu} dependent on AuAu multiplicity
- "Specific heat" c_{AuAu}=0.11 is the same as for negative hadrons
- ε_0 is the same as for negative hadrons

K_s^0 & ϕ mesons in 130 & 200 GeV AuAu collisions

different centralities

STAR: PLB 595 (2004) 143; PRC 75 (2007) 064901; PLB 612 (2005) 181

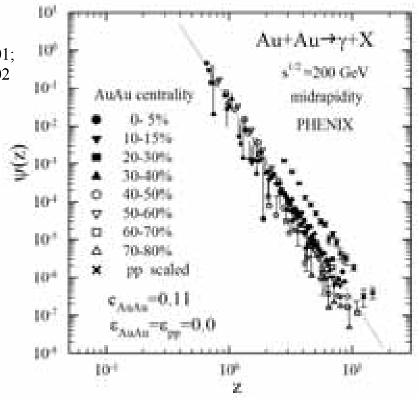


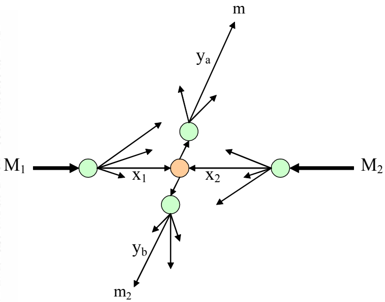


- The same Ψ(z) in AuAu & pp for ε_{AuAu} dependent on AuAu multiplicity
- "Specific heat" c_{AuAu}=0.11 is the same as for negative hadrons
- ε_0 is the same as for negative hadrons

Direct photons in AuAu collisions

PHENIX:
PRL 94 (2005) 232301;
PRL 98 (2007) 012002



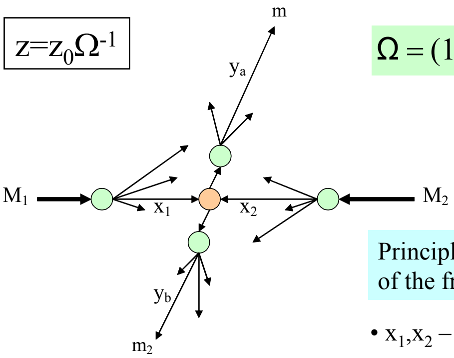


- Data prefer $\varepsilon \approx 0$ (y=1) - direct production of γ in the sub-process with no (or small) energy losses.
- But error bars are too large to make strong conclusion on ε

$$\Omega = (1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon}(1-y_b)^{\varepsilon}$$

Kinematics of constituent sub-process in AA

$z = z_0 \Omega^{-1}$



$$\Omega = (1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon}(1-y_b)^{\varepsilon}$$

$$(x_1P_1 + x_2P_2 - p/y_a)^2 = M_X^2$$

$$M_X = x_1M_1 + x_2M_2 + m_2/y_b$$

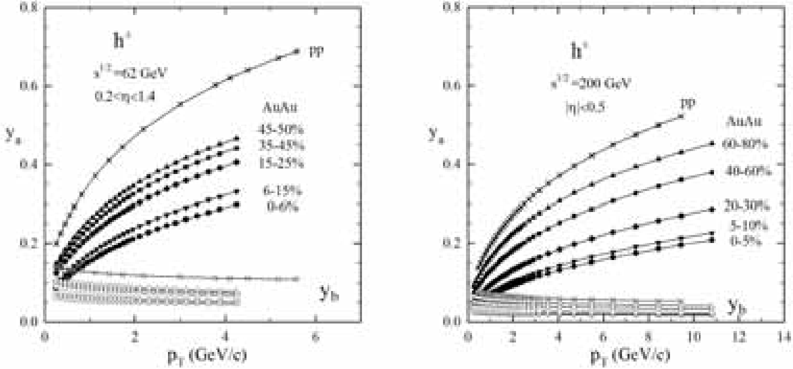
Principle of minimal resolution $\Omega^{\;1}$
of the fractal measure z gives:

- x_1, x_2 – energy of the sub-process
- y_a – energy losses (dissipation) by production of the inclusive particle
- $M_X = x_1M_1 + x_2M_2 + m_2/y_b$ – recoil mass
- y_b – multiplicity of the recoil system

Larger ε = smaller y = larger energy losses in the final state & larger recoil mass

Momentum fractions y_a, y_b in pp & AuAu

PHOBOS charged hadrons STAR



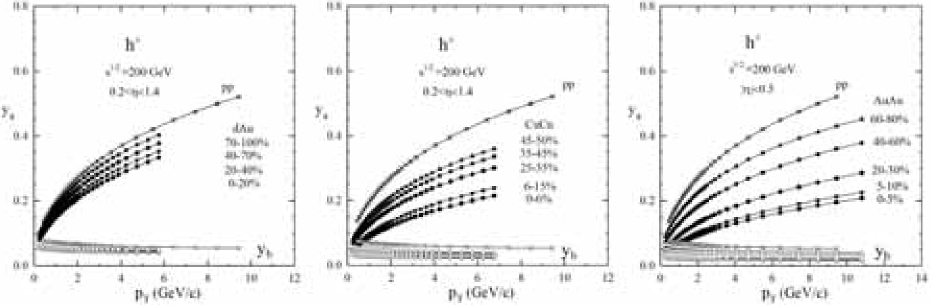
- y_a decreases with centrality $\Rightarrow$ energy losses increase with centrality
- The decrease of y_a in AuAu relative to pp is larger at higher $p_T \Rightarrow$ relative energy losses AuAu/pp increase with p_T
- The decrease of y_a in AuAu relative to pp (at same p_T) is larger at higher $s^{1/2} \Rightarrow$ relative energy losses AuAu/pp increase with $s^{1/2}$

$(x_1P_1+x_2P_2-p/y_a)^2 = (x_1M_1+x_2M_2+m_2/y_b)^2$

Momentum fractions y_a, y_b in dAu, CuCu, AuAu

charged hadrons

PHOBOS PHOBOS STAR

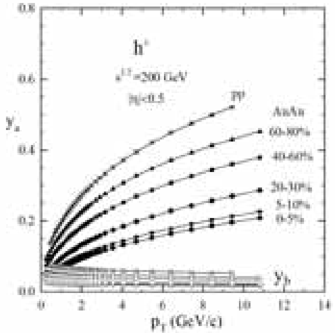


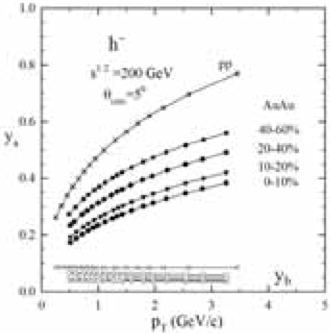
- y_a decreases with centrality in all AA ⇒ energy losses increase with centrality for all AA systems
- The decrease of y_a in AA relative to pp is larger for heavier AA systems ⇒ relative energy losses AA/pp increase with the system size
- The relative energy losses dAu/pp are considerably small.

$(x_1P_1+x_2P_2-p/y_a)^2 = (x_1M_1+x_2M_2+m_2/y_b)^2$

y_a, y_b in the central and fragmentation regions

STAR charged/negative hadrons BRAHMS





- Considerable decrease of y_a in AuAu relative to pp at $\theta_{cms}=5^0$ $\Rightarrow$ considerable energy losses in AuAu relative to pp in the fragmentation region
- y_a is larger at $\theta_{cms}=5^0$ than for $\theta_{cms}=90^0$ $\Rightarrow$ smaller energy losses in the fragmentation region than in the central interaction region
- The decrease of y_a in AuAu relative to pp (at same p_T) is larger at $\theta_{cms}=5^0$ than for $\theta_{cms}=90^0$ $\Rightarrow$ relative energy losses AuAu/pp are larger in the fragmentation than in the central region (at same p_T).

$(x_1P_1+x_2P_2-p/y_a)^2 = (x_1M_1+x_2M_2+m_2/y_b)^2$

Summary of scaling features in AA collisions

- Assuming multiplicity dependence of the fragmentation dimension

$$\varepsilon_{AA} = \varepsilon_0 (2dN_{neg}/d\eta)+\varepsilon_{pp}$$

  an approximate energy, angular, and centrality independence of $\psi(z)$ in AA collisions for $h^{\pm}$, π^-, K^-, K^0_S, ϕ can be obtained.
- ε_0 depends on $s^{1/2}$ and system size
- ε_0 is independent of the particle type, angle, and centrality
- "Specific heat" c is independent of $s^{1/2}$, particle type, angle, and centrality.
- "Specific heat" c decreases with the system size:

$$c_{pp}=0.25 > c_{dAu}=0.23 > c_{CuCu}=0.14 > c_{AuAu}=0.11.$$

Internal symmetry of hadron production at small scales

(space-time structural relativity at high p_T)

The symmetry connects ***Self-similarity, Locality,*** and ***Fractality.***
It is the symmetry with respect to structural degrees of freedom.
The structural relativity is realized by structural transformations.
The transformations connect kinematical variables expressed relative
to different fractal structures A and B.
The fractal structures are characterized by different fractal dimensions δ_1 and δ_2.

The structural relativity is a symmetry of hardon production at small scales expressed by special Lorenz-like transformations at constant spatial resolution ξ:

$$E^A = \gamma(u)\left(E^B - u p_z^B\right)$$
$$p_z^A = \gamma(u)\left(p_z^B - u E^B\right)$$
$$p_T^A = p_T^B$$

$$\gamma(u) = (1-u^2)^{-1/2}$$ u is the structural velocity

$$\frac{u}{\sqrt{1-u^2}} = \frac{\alpha - 1}{2\sqrt{\alpha}}\xi$$ $б = д_2/д_1$ is ratio of the fractal dimensions

$$\xi = \sqrt{\frac{\lambda_1\lambda_2}{(1-\lambda_1)(1-\lambda_1)}}$$ ξ is the spatial resolution

$$\lambda_{1,2} \cong (P_{2,1}p)/(P_A P_B)$$

Structural relativity in collisions of structural objects A & B

The requirement of minimum resolution of the fractal measure z gives specific decomposition of the momentum fractions:

$$x_1 = \lambda_1 + \chi_1$$
$$x_2 = \lambda_2 + \chi_2$$

$$\lambda_i \cong (P_j p)/(P_A P_B)$$

$$\chi_i = \sqrt{\mu_i^2 + \omega_i^2} \mp \omega_i$$

$$\omega_i = \mu_i U$$

$$U = \frac{u}{\sqrt{1-u^2}} = \frac{\alpha - 1}{2\sqrt{\alpha}}\xi$$

$$\chi_1 - \chi_2 = \gamma(u)\left[(\mu_1 - \mu_2) - u(\mu_1 + \mu_2)\right]$$
$$\chi_1 + \chi_2 = \gamma(u)\left[(\mu_1 + \mu_2) - u(\mu_1 - \mu_2)\right]$$

$$\gamma(u) = (1 - u^2)^{-1/2}$$

The fractions χ_i, μ_i are expressed via momenta of the colliding and produced particles:

$$\chi_i = \frac{(P_j \bar{p}^B)}{(P_1 P_2)} \qquad \mu_i = \frac{(P_j \bar{p}^A)}{(P_1 P_2)}$$

$$\chi_1 - \chi_2 = \frac{2}{\sqrt{s}}\bar{p}_z^A, \quad \chi_1 + \chi_2 = \frac{2}{\sqrt{s}}\bar{E}^A$$
$$\mu_1 - \mu_2 = \frac{2}{\sqrt{s}}\bar{p}_z^B, \quad \mu_1 + \mu_2 = \frac{2}{\sqrt{s}}\bar{E}^B$$

$$\chi_1 \chi_2 = \frac{1}{\sqrt{s}}\sqrt{\left(\bar{p}_T^A\right)^2 + m_2^2}$$
$$\mu_1 \mu_2 = \frac{1}{\sqrt{s}}\sqrt{\left(\bar{p}_T^B\right)^2 + m_2^2}$$

$\chi_1 \chi_2 = \mu_1 \mu_2$ is α independent function of ξ

Structural transformations:

$$\bar{p}_z^A = \gamma(u)\left(\bar{p}_z^B - u\bar{E}^B\right)$$
$$\bar{E}^A = \gamma(u)\left(\bar{E}^B - u\bar{p}_z^B\right)$$

Invariant:

$$\bar{p}_\perp^A = \bar{p}_\perp^B$$

The structural transformations connect (E,p) expressed relative to different structures A and B at the same scale ξ

Structural velocity transformations

Lorenz-like transformations:

$$E^A = \gamma(u)\left(E^B - u p_z^B\right)$$
$$p_z^A = \gamma(u)\left(p_z^B - u E^B\right)$$
$$p_T^A = p_T^B$$

Velocity transformations:

$$u = \frac{u_1 + u_2}{1 + u_1 u_2}$$

$$\frac{u}{\sqrt{1-u^2}} = \frac{\alpha - 1}{2\sqrt{\alpha}}\xi$$

$$\alpha = \delta_2/\delta_1$$

Kinematical limit ⇔ fractal limit:

$$\xi = 1 \Leftrightarrow z = \infty$$

$$u = \frac{\alpha - 1}{\alpha + 1} \quad \Leftrightarrow \quad \alpha = \alpha_1 \alpha_2$$

Space-time structural relativity preserves motion relativity at any spatial resolution ξ.

Conclusions

- The main features of z-presentation of inclusive spectra at high energies were summarized.
- New properties of the z-scaling in $pp/p\bar{p}$ collisions – flavor independence and saturation at low z, were established.
- z-Scaling reflects the self-similarity, locality, and fractality of hadron interactions at constituent level.
- The scaling features of charged hadron, negative pion and kaon production in AA collisions were demonstrated.
- Kinematic properties of the constituent sub-processes were discussed.
- Estimates of the energy losses in pp &AA collisions in terms of the momentum fractions were obtained.
- Structural relativity connected with the self-similarity, locality, and fractality of hadron production at high p_T was discussed.
- The results may be of interest in searching for new physics in soft and hard p_T region of particle production at RHIC, Tevatron, and LHC.

Thank You for Attention